

- Theorem : **Intermediate value property**

- statement: Let  $f$  be a continuous on  $[a, b]$  and let  $f(a) < f(b)$  (  $s$  is a value which is intermediate between two values taken by  $f$  ) then there exists  $x$  such that  $a < x < b$  and  $f(x) = s$ .

- Proof :

- Let  $S = \{x \in [a, b] : f(x) \leq s\}$ . Since  $a \in S$ , we have  $S \neq \emptyset$  and  $S$  is bounded above by  $b$ . Let  $c$  be the least upper bound of  $S$ .

- We claim that  $f(c) = s$ . Since  $c$  is the least upper bound of  $S$ , there exist a sequence  $\{x_n\}$  from  $S$  such that  $x_n \rightarrow c$ . By the continuity of  $f$ ,  $f(x_n) \rightarrow f(c)$ . Since for all  $n$ , we have  $f(x_n) \leq s$ . Note that  $b > c$ . Consider a sequence  $y_n = c + (b - c)/n$ . As  $y_n$  It follows that  $f(c) = s$ .