Naren Suri – Bayesian Stats – Assignment 1 – nsuri@iu.edu

1. (2.1)

2.1 Marginal and conditional probability: The social mobility data from Section2.5 gives a joint probability distribution on (Y1, Y2)= (father's occupation, son's occupation). Using this joint distribution, calculate the following distributions:

a) The marginal probability distribution of a father's occupation;

b) The marginal probability distribution of a son's occupation;

c) The conditional distribution of a son's occupation, given that the father is a farmer;

d) The conditional distribution of a father's occupation, given that the son is a farmer.

Table 1: Probability of joint distribution based on father and son occupations

Father \ Son	Farm	Operatives	Craftsmen	Sales	Professional
Farm	0.018	0.035	0.031	0.008	0.018
Operatives	0.002	0.112	0.064	0.032	0.069
Craftsmen	0.001	0.066	0.094	0.032	0.084
Sales	0.001	0.018	0.019	0.010	0.051
Professional	0.001	0.029	0.032	0.043	0.130

Rule of marginal probability

$$P_r(P1) = \sum_{k=1}^{K} P_r(P1 \cap P2_k)$$

a) The marginal probability distribution of father's occupation.

Based on the rule of marginal probability

Pr(P1 = f arm) = 0.018 + 0.035 + 0.031 + 0.008 + 0.018 = 0.11

Pr(P1 = operatives) = 0.002 + 0.112 + 0.064 + 0.032 + 0.069 = 0.279

Pr(P1 = craftsmen) = 0.001 + 0.066 + 0.094 + 0.032 + 0.084 = 0.277

Pr(P1 = sales) = 0.001 + 0.018 + 0.019 + 0.010 + 0.051 = 0.099

Pr(P1 = professional) = 0.001 + 0.029 + 0.032 + 0.043 + 0.130 = 0.235

b) The marginal of son's occupation

Pr(P2 = farm) = 0.018 + 0.002 + 0.001 + 0.001 + 0.001 = 0.023

Pr(P2 = operatives) = 0.035 + 0.112 + 0.066 + 0.018 + 0.029 = 0.26

Pr(P2 = craftsmen) = 0.031 + 0.064 + 0.094 + 0.019 + 0.032 = 0.24

Pr(P2 = sales) = 0.008 + 0.032 + 0.032 + 0.010 + 0.043 = 0.125

Pr(P2 = professional) = 0.018 + 0.069 + 0.084 + 0.051 + 0.130 = 0.352

c) The conditional distribution of sons occupation given that father is a farmer

Pr(P2 = f armer|P1 = farmer) = Pr(P2 = farmer ∩ P1 = farmer) / (P1 = farmer)

= 0.018/0.11 = 0.163

Pr(P2 = operatives|P1 = farmer) = Pr(P2 = operatives ∩ P1 = farmer) /(P1 = farmer)

=0.035/0.11

= 0.318

Pr(P2 = craftsmen|P1 = farmer) = Pr(P2 = craftsmen ∩ P1 = farmer) /(P1 = farmer)

=0.031/0.11

= 0.281

Pr(P2 = sales|P1 = farmer) = Pr(P2 = sales ∩ P1 = farmer) /(P1 = farmer)

=0.008/0.11

= 0.072

Pr(P2 = professional|P1 = farmer) = Pr(P2 = professional ∩ P1 = farmer) / (P1 = farmer)

=0.018/0.11

= 0.163

d) The conditional distribution of fathers occupation given that son is a farmer

Pr(P1 = f armer|P2 = farmer) = Pr(P1 = farmer ∩ P2 = farmer) / (P2 = farmer)

=0.018/0.023

= 0.78

Pr(P1 = operatives|P2 = farmer) = Pr(P1 = operatives ∩ P2 = farmer)/(P2 = farmer)

=0.002/0.023

= 0.086

Pr(P1 = craftsmen|P2 = farmer) = Pr(P1 = craftsmen ∩ P2 = farmer)/(P2 = farmer)

=0.001/0.023

= 0.0435

Pr(P1 = sales|P2 = farmer) = Pr(P1 = sales ∩ P2 = farmer)/(P2 = farmer)

=0.001/0.023

= 0.0435

Pr(P1 = professional|P2 = farmer) = Pr(P1 = professional ∩ P2 = farmer)/(P2 = farmer)

=0.001/0.023

= 0.0435

2.3 Full conditionals: Let X, Y,Z be random variables with joint density (discrete

or continuous) p(x, y, z) / f(x, z)g(y, z)h(z). Show that

a) p(x|y, z) / f(x, z), i.e. p(x|y, z) is a function of x and z;

b) p(y|x, z) / g(y, z), i.e. p(y|x, z) is a function of y and z;

c) X and Y are conditionally independent given Z.

a.

Given: $p(x, y, z) \propto f(x, z)g(y, z)h(z)$

We shall assume this to be a continuous joint density.

To Prove $p(x|y, z) \propto f(x, z)$

we have p(x|y, z) = p(x, y, z)/p(y, z)

we can say that, p(x, y, z) = kf(x, z)g(y, z)h(z), where k is a constant.

We can represent p(y, z) = integral (p(x, y, z)dx) ,This can be further be simplified as follows

p(y, z) = Integral [f(x, z)g(y, z)h(z)dx]

= p(y, z) = g(y, z)h(z) Integral (f(x, z)dx)

Now g(y,z) and h(z) are constant. Substituting this in the first equation we get

p(x|y, z) = k f(x, z)g(y, z)h(z) / g(y, z)h(z) integral f(x, z)dx

From this we can easily say that $p(x|y, z) \propto f(x, z)$ / R f(x, z)dx

This implies that p(x|y, z) is a function of x and z alone.

b

To prove that $p(y|x, z) \propto g(y, z)$

we have $p(y|x, z) = p(x, y, z) / p(x, z)$

But from the given, $p(x, y, z) = kf(x, z)g(y, z)h(z)$

Where k is a constant.

We can represent p(x, z) = INTEGRAL (p(x, y, z))dy

This can be simplified as follows p(x, z) = INTEGRAL f(x, z)g(y, z)h(z)dy

= p(x, z) = f(x, z)h(z) INTEGRAL g(y, z)dy

f(x,z) and h(z) acts like a constant.

Substituting this in the first equation we get
p(y|x, z) = kf(x, z)g(y, z)h(z) / f(x, z)h(z) INTEGRAL g(y, z)dy

From this we can say that $p(y|x, z) \propto g(y, z)$ / INTEGRAL g(y, z)dy

This implies that p(y|x, z) is a function of y and z alone.

C.

To show that X and Y are conditionally independent given Z.

We know $p(x, y|z) = p(x, y, z) / p(z)$ But from the given we can say that,

$p(x, y, z) = k\, f(x, z)g(y, z)h(z)$ Where k is a constant.

We can represent p(z) = Double INTEGRAL p(x, y, z)dxdy

= h(z) Double INTEGRAL f(x, z)g(y, z)dxdy

From these equations we can say that $p(x, y|z) \propto f(x, z)g(y, z)$ / Double INTEGRAL f(x, z)g(y, z)dxdy

From this we can conclude that X and Y are conditionally independent given Z