(1) rate $= \lambda$

(1) $x_1, x_2 \ldots x_n$

$f(x, \lambda) = \lambda \exp(-\lambda x)$

mean $E(x) = \mu = 1/\lambda$

$Var[x] = \sigma^2 = 1/\lambda^2$

median $= x_{0.5} = \dfrac{\log(2)}{\lambda}$

$\Rightarrow$ two statistics

mean (1) $T_1 = \bar{X} = \dfrac{\sum_{i=1}^{n} x_i}{n}$

med (2) $T_2 = x/\log 2$

(3) $Var(T_1) = \dfrac{1}{n^2} \sum_{i=1}^{n} var(x_i)$

$\Rightarrow \dfrac{n \cdot var(x_1)}{n^2} = \dfrac{\sigma^2}{n} = \dfrac{1}{n\lambda^2}$

(a) Central limit, what is asymptotic distrib of

$\sqrt{n}\left(\bar{x} - \dfrac{1}{\lambda}\right)$

The variance will be less than infinity for an Central limit theorem.

$\Rightarrow x_1, x_2, \ldots x_n$ is Considered IID.

$\Rightarrow \sqrt{n} * \left(\bar{x} - \dfrac{1}{\lambda}\right) \Rightarrow \sqrt{n}.\ (\bar{x} - \mu)$ Converges to

$\boxed{N \sim (0, \sigma^2)}$

and $\mu = \dfrac{1}{\lambda}$, which is same as above.

$\sqrt{n} * \left(\bar{x} - 1/\lambda\right)$ also Converges to $N \sim (0,$

(2) Asymptotic distribution of $\sqrt{n}\left(\bar{x} - \frac{\log(2)}{\lambda}\right)$

↳

$$\sqrt{n}\left(\bar{x} - x_{0.5}\right) \Rightarrow N\left(0, 1/4(f(x_{0.5}))^2\right)$$

and we are already given that,

$$x_{0.5} = \log(2)/\lambda$$

$$f(0.5) = \lambda \cdot e^{-\lambda \cdot \log 2/\lambda}$$

$$\Rightarrow \lambda/2$$

(a) so, $N\left(0, 1/4[f(x_{0.5})]^2 \Rightarrow N\sim(0, \sigma^2)\right) \quad \because \sigma^2 = 1/\lambda^2$

(3)

$$\text{Median} = N\sim(0, \sigma^2)$$

$$\text{variance}(T_2) = \left(1/\log 2\right)^2 \cdot \left(\sigma^2/n\right)$$

$\because var(cT) = c^2 var(T)$

$$Var(T_2) = 2.081\left(\sigma^2/n\right)$$

$$var(T_2) = 2.08\,\frac{\sigma^2}{n}$$

(4)

$$ARE(T_1, T_2) = var(T_1) / var(T_2)$$

so, from the above information 4

Proofs did,

$$\Rightarrow (\sigma^2/n) / \left(2.081\left(\frac{\sigma^2}{n}\right)\right)$$

$$\Rightarrow \frac{1}{2.08} \Rightarrow 0.48$$

So, ARE = 48% efficient.

(5) If we compare the two statistic T_1 & T_2, Their variance is to be considered. [The dataset with less variance is better.]

if, $V_1 < V_2$ then V_1 is better

here $v(T_1) < v(T_2)$ $\quad v(T) \Rightarrow var(T)$

$$var(T_1) < var(T_2)$$

so, T_1 is better

(6) Just draw the level plot.
its drawn at the first of the assignment.