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Longstaff and Schwartz model for American option pricing

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1. Introduction

Deriving closed form solution for plain European style option, which can be exercised only at time of maturity, had been fairly discussed in literature. Black and Scholes (1973) and Merton (1973) had derived an interesting method for finding closed form solution for European option. However, despite the recent important advances, Pricing of an American option and especially options whose price depends on multiple factor changes still require necessary implementation of various numerical methods.

Boyle (1977), which discussed pricing of European option using simulation based on stochastic process followed by underlying asset price, namely Monte Carlo method, had been popular since. Underlying simulation techniques are rather simple and flexible in one dimensional cases. Monte Carlo method has importance also in valuation of derivatives with underlying path-dependent stochastic processes such as jump diffusions and non-Markovian process, as discussed in Merton (1976), Cox and Ross (1976), Heath, Jarrow and Morton (1992) and had been a significant numerical approach for higher dimensional cases (Broadie & Glasserman, 1997a).

Main focus of our examination is to look into an important use of Monte carlo approach for valuation of American option, which primarily depends on the optimal stopping time and corresponding payoff as given in equation 1. In any American style option, option holder determines the worth of their option by comparing between immediate exercise payoff and continuation value at each time point. In short, value is given by optimal stopping time choice, such that

$$V_0 = \max_{\tau \in \mathcal{T}} \mathbb{E}[\hat{g}(S\tau)] \tag{1}$$

Where $\hat{g}$ is the measurable (discounted payoff) function that is dependent on stochastic process $(S_t)_{0 \le t \le T}$. For finding our optimal time $\tau \in \mathcal{T}([0,T])$ to exercise the option, we require some method to determine the conditional expected payoff from holding the option for future. In this report, we look into this problem in perspective of Longstaff and Schwartz (2001) model, this technique is called Least Square Monte Carlo (LSM). The idea of the approach is to determine the continuation value by applying least square regression on discounted cash flows and some special functions of simulated asset price. Such as for l-th path and the continuation value is

$$E\left[\hat{g}\left(S_{t_{i+1}}^{l}\right)\middle|\mathcal{F}_t\right] = \alpha_o + \alpha_1 f_1\left(S_{t_i}^{l}\right) + \cdots + \alpha_M f_M\left(S_{t_i}^{l}\right) \tag{2}$$

Using regression on simulated asset's prices with orthogonal basis functions $f_1(X)$ ... $f_M(X)$ which can span on any point in $\mathcal{L}^2$ space, we find our conditional expectation function for each time point t_i (in the discretized time interval $\mathcal{T} = \{t_1, t_2, \dots, T\}$). In this paper we keep our focus on the practical application of this algorithm, its convergence results for option pricing and suitable choices for basis functions to minimize error.

In our section 2 we look into another similar Monte Carlo method and compare the techniques. We describe the framework of Longstaff and Schwartz (2001) and underlying algorithm in section 3. Further in the report's section 4 we look into the application of LSM in comparison to the Finite difference approach for American Put option. Convergence property of LSM is discussed in section 5 and finally in section 6, we discuss the suitable form for our regression function.

2. Monte Carlo approaches

Monte Carlo simulation algorithm is used for sampling numerous paths for stochastic processes, such as asset prices and portfolio values. Value of the given option is average of discounted payoff from option from all the paths. For American option, finding payoff on each time point and path is very complex and can be computationally infeasible.

To mitigate first issue with continuous time frame, we rely on estimations on only a finite set, $\mathcal{T} = \{\, 0 = t_1, t_2, \dots, t_K = T \,\}$ of time points with $t_1 < t_2 < \cdots < t_K$ and each point is $0 \leq t \leq T$ i.e. option is priced as Bermundan style, which can only be exercised on some discrete time points but whenever time steps $K \rightarrow \infty$ then valuation process mimics pricing of continuously-exercisable option.

To estimate the conditional expectation we use value given by equation 2, we call it Least square Monte Carlo, this method may not always produce low error but has fair advantages in computational handling and simplicity when comparing to methods developed before its arrival. Longstaff and Schwartz (2001) and Tsitsiklis and Van Roy (2001) both introduced LSM for American option. While latter proved the idea of estimating continuation value from combination of linear basis function on all cross-sectional simulated stochastic prices and prior used only in-the-money paths.

Prior approach proved to be more efficient since it produces low bias (Tang, 2015), however it may require higher number of basis functions for partial convergence (Clement, Protter, & Lamberton, 2002) to find an efficient non arbitrage price estimate.

3. Least square Monte Carlo algorithm

As defined in our earlier section, this idea corresponds to the projecting selected simulated stock prices to produce continuation value. The framework for underlying price processes and mathematical space is rather commonly used in option pricing[1]. We assume a complete probability space $(\Omega, \mathcal{F}_T, \mathcal{P})$ in a finite time interval $[0, T]$, within the option expires. Set Ω consists all possible values of random process ω, which can be realized on different paths. $\mathcal{F}_t$ is the sigma field containing all the relevant price information until the period t, $\mathcal{P}$ is the probability measure containing the probabilistic characteristics of sample paths.

[1] Details of general framework is also common to Black and Scholes (1973), Merton (1973), Harrison and Kreps (1979). Harrison and Pliska (1981), Cox. Ingersoll, and Ross (1985), Heath, Jarrow, and Morton (1992).

Under no arbitrage or risk neutral world assumption, we derive the price of derivative under a measure which is called equivalent martingale measure Q[2].

Pricing American-style derivative at time $t_k \in \mathcal{T}$ as described in introduction is based on idea of Snell envelope, so at each time and in our simulated path, one required to make exercises decision based on current exercise premium against the present value of future cash generated, $C(\omega, t_j; t_k, T)$, which is dependent on realized sample path and path-wise optimal stopping time $t_j \in \mathcal{T}$ for $t_j \geq t_k$. Importantly LSM only suited to derivatives whose payoff has finite-variance distribution or more importantly square-integrable so its payoff is part of $\mathcal{L}^2$-space and we can generate the projection of continuation value which is

$$F(\omega; t_k) = E_Q\left[\sum_{t=k+1}^{t_j} \exp\left(-\int_{t_k}^{t_j} r(w,s)ds\right) C(\omega, t_j; t_k, T) | \mathcal{F}_{t_k}\right] \qquad (3)$$

by using backward dynamic programming, introduced by Carriere (1996) for pricing American put option, where payoff from each path at any time step can be defined by the preceding value (backward induction) by use of some reliable functions.

These functions in our method are well defined orthogonal basis of $\mathcal{L}^2 - space$

$$B_1, \dots, B_M : \mathbb{R}^d \to \mathbb{R}$$

We use such basis and find coefficients $\alpha_0, \alpha_1, \dots \alpha_M \in \mathbb{R}$ to define equation 2

$$\hat{F}(\omega, t_j; t_k, t) = \sum_{m=0}^{M} \alpha_m B_m \qquad (4)$$

so that for all in-the-money paths, we have

$$\min_{\alpha_1, \alpha_2, \dots \alpha_M \in \mathbb{R}} \sum_{l=1}^{l \leq N} ((F(\omega_l, t_j; t_k, t) - \hat{F}(\omega_l, t_j; t_k, t))^2 \qquad (5)$$

[2] The probability measure in which price of underlying derivative is equal to discounted cash flows generated under risk neutral world.

We can use $\hat{F}(\omega, t_j; t_k, t)$ in place of $F(\omega, t_j; t_k, t)$ and compare it with immediate exercise value $\hat{g}(t_k)$. Main difference for LSM from many other methods is that the only paths being cosidered are in-the-money. The classic Longstaff and Schwartz (2001) uses weighted laguerre polynomials, where in litterature there are examples for use of other different types of basis such as Hermite, Legendre, Chebzshev, Jacobi polynomial as well as simple higher degree polynomials. To understand the theory behind the use of basis functions which suitably span over the space of possible continuation values based on Markovian price process, we refer to Abramowitz and Stegun (1992), where in our last section we will look into some choices for equation 4 available to us.

4. Application of Least square Monte Carlo

LSM algorithm has specific purpose of pricing American style derivatives, which can be exercised before maturity. In this section we provide you with practical application of this algorithm by pricing an American put option. Notice that LSM however is not most efficient or most simple available approach but it certainly has more importance in higher dimensions such in 20-factor string model and for complex derivatives such as Asian options, amortizing swaps or with jump diffusion process (see Longstaff and Schwartz, 2001).

In standard case, we assume the price process S_t follows geometric Brownian motion (GBM)

$$dS = rSdt + \sigma SdZ \tag{6}$$

with constant drift (risk neutral) rate r, volatility σ and Z is standard Brownian motion. In our example we solve this problem as Bermudan option with only 50 exercising points but similar to continuous case, we are looking for price $P(S, t)$ which satisfies partial differential equation namely Black Scholes PDE

$$\left(\frac{\sigma^2 S^2}{2}\right) P_{SS} + rSP_S - rP + P_T = 0 \tag{7}$$

and boundary payoff condition

$$P(S,T) = max(0, K - S_T) \tag{8}$$

We now price an American put option with different parameters and different methods and results are given in Table 1. The stock is non dividend paying and underlying price follows equation 6. Main focus of our work is to estimate the results from different methods and compare early exercise premium. We apply Longstaff and Schwartz's LSM algorithm with antithetic-Monte Carlo process. We take 50 time steps, 100000 simulations (50000 plus 50000 antithetic) and only use first 3 weighted Laguerre polynomials[3] as

$$L_n(X) = \exp\left(-\frac{X}{2}\right)\frac{e^X}{n!}\frac{d^n}{dX^n}(X^n\, e^{-X}) \tag{9}$$

Due to early exercise property, American Put options are always priced above the European put options. We derive the early exercise value in column 6 and 9 by subtracting value of European option, which is priced using Black-Scholes model. As a benchmark price, we use Finite difference method with 40000 time steps and 2000 mesh points.

The results from Table 1 show that even with different parameters our results are still close to benchmark, where positive value in column 10 imply that LSM price estimates are consistently lower. Original work suggests that LSM generally provides a lower bound for the price and Finite difference is a high estimate method but in fact due to standard error (section 5.1) LSM price may deviate. These results in Table 1 are also identical to results provided in Longstaff and Schwartz (2001) for up to 2 cents. This suggests that the algorithm is robust to simulations.

5. Properties of Least square Monte Carlo

Least square Monte Carlo like the other simulation based algorithms do not provide any closed form solutions since the paths are randomly created however we should have a clear idea that under correct assumptions the convergence is

[3] See provided matlab code.

possible; choice and complexity of our functional is model defining i.e. here large number of basis minimizes the error in the linear regression and solution is unique if sample paths tend to grow to infinity.

5.1 **Convergence**

For LSM method due computational limitation one cannot either induce large degree of complexity, M, nor can simulate, N, infinite paths. However, convergence of the results from the LSM have been discussed in details in Clement et. al. (2002). Given the least square approach the rate of convergence is dependent on number of basis functions in regression and simulated paths[4], it is also not only available remedy to assess convergence but has more importance in high dimension cases in terms of reliability (Curse of dimensionality).

Given below in equation 10 and 11 are propositions from Longstaff and Schwartz. For proof of proposition see appendix of Longstaff and Schwartz (2001), where now we elaborate two important points given by the writers.

1. For a Markovian underlying process $X \in (0, \infty)$, LSM algorithm provides us with low price the option value $V(X)$ however its approaches closer to actual value for large M, such as

$$V(X) \geq \lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} LSM(\omega_i; M, \mathcal{T}). \qquad (10)$$

2. For American Put option, LSM estimated $\widehat{F_M}(\omega; t_k)$ and also continuous conditional expectation $F(\omega; t_k)$, if both satisfy

$$\int_0^\infty e^{-X} F^2(\omega; t_k) dX < \infty$$

$$\int_0^\infty e^{-X} \hat{F}_M^2(\omega; t_k) dX < \infty$$

[4] Cf 1196-97 Least Squares Monte Carlo approach to American Option Valuation 93-1203, @2004 INFORMS

Table 1 Comparison Finite Difference and Least square Monte Carlo

S_0	σ	T	Finite Difference Method	Black Scholes European option	Early exercise value	Least Square Monte Carlo	Standard Error	Early exercise value	Difference in early exercise
36	**0.2**	**1**	4.492	3.844	0.648	4.464	0.009	0.620	0.028
36	**0.2**	**2**	4.833	3.763	1.070	4.822	0.011	1.059	0.012
36	**0.4**	**1**	7.113	6.711	0.401	7.066	0.019	0.354	0.047
36	**0.4**	**2**	8.498	7.700	0.798	8.465	0.023	0.765	0.033
38	**0.2**	**1**	3.265	2.852	0.413	3.239	0.009	0.387	0.026
38	**0.2**	**2**	3.755	2.991	0.764	3.725	0.011	0.734	0.030
38	**0.4**	**1**	6.161	5.834	0.327	6.103	0.019	0.269	0.058
38	**0.4**	**2**	7.670	6.979	0.691	7.619	0.022	0.640	0.051
40	**0.2**	**1**	2.329	2.066	0.262	2.292	0.009	0.226	0.037
40	**0.2**	**2**	2.904	2.356	0.548	2.862	0.011	0.506	0.043
40	**0.4**	**1**	5.327	5.060	0.267	5.299	0.018	0.239	0.028
40	**0.4**	**2**	6.926	6.326	0.600	6.856	0.022	0.530	0.070
42	**0.2**	**1**	1.628	1.465	0.164	1.602	0.008	0.138	0.026
42	**0.2**	**2**	2.229	1.841	0.387	2.188	0.010	0.346	0.041
42	**0.4**	**1**	4.596	4.379	0.217	4.538	0.017	0.159	0.058
42	**0.4**	**2**	6.255	5.736	0.519	6.191	0.021	0.455	0.064
44	**0.2**	**1**	1.118	1.017	0.101	1.102	0.006	0.085	0.016
44	**0.2**	**2**	1.703	1.429	0.274	1.669	0.009	0.240	0.034
44	**0.4**	**1**	3.960	3.783	0.177	3.895	0.016	0.112	0.065
44	**0.4**	**2**	5.653	5.202	0.451	5.590	0.021	0.388	0.062

Comparison of the Finite difference method to LSM with first 3 weighted Laguerre polynomials. Column 4 and 7 show the price of American Put option. Column 1-3 show the underlying parameters. Column 5 show the BS model prices for similar European style option. Bench Mark prices are given using Explicit finite difference method, with 40000 time steps and 2000 grid points for stock prices, and LSM with 100000 (50000 and 50000 antithetic) simulations and time steps are 50. Column 8 gives standard error of LSM prices. Early exercise in column 6 and 9 is difference of both methods from European option in column 5. Column 10 shows underpricing of LSM model from finite difference prices.

then for any arbitrary small value ϵ there $\exists M$ such that

$$\lim_{N\to\infty} Pr\left[|\, V(X) - \frac{1}{N}\sum_{i=1}^{N} LSM(\omega_i; M, \mathcal{T}) \,| > \epsilon \right] = 0 \qquad (11)$$

which implies that for almost sure convergence we require $M \to \infty$ & $N \to \infty$.

5.2 Numerical Results

For analysis of put options with underlying simple GBM process, we can find the convergence rather easily however in it not very clear for multi-asset or Jump diffusion process. For analysis, we have taken into consideration weighted Laguerre, Hermite and Higher order polynomials performance with increasing number of simulations and basis functions.

Basis Functions and Order of Polynomials	**1**	**2**	**3**	**4**	**5**
Weighted Laguerre	3.958	4.078	4.278	4.465	4.467
Probabilistic Hermite	4.412	4.458d	4.468	4.474	4.469
High Order Polynomials	3.952	4.468	4.461	4.461	4.472

Table 2 Comparison of LSM method with increasing number of basis functions for American Put option with S_o=36, σ=0.20, r=6%, T=1, K=40, N=10^4 and time steps are 500.

In line with the Proposition 1, we see in Table 2 all the above functions provides only low estimators in comparison to Finite Difference method ($V(X) = 4.492$). However, surprisingly weighted Laguerre functions do not perform as good as Hermite or simple polynomials functions. This is the case due to low bias factor of Laguerre functions where other basis are quite flexible. We discuss the further properties of our basis function in next section.

For Proposition 2, almost sure convergence of LSM is still justifiable from Figure 1. Here we also see that the results do not grow instantaneously after some level of simulations but it does not have much theoretical base yet there is clear impact of number of basis functions improving the output.

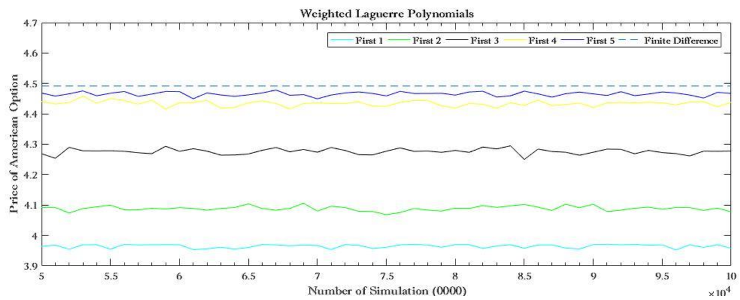

Figure 1 Comparison of LSM method with increasing number of basis functions for American Put option with S_o=36, σ=0.20, r=6%, T=1, K=40, N=50000 to 10^4 and time steps are 50.

As proved by Clement et. al. (2002)., large order of basis functions indeed help LSM price to converges partially to actual option value such $\lim_{N \to \infty} V_N^M(X) \xrightarrow{p} V(X)$ but one need to be careful since for optimal and efficient results, one require only suitable number of basis for any given simulations[5].

5.3 Accuracy

One important point, which have not been fairly discussed in original paper is that the method is prone to different error types. Previously, we discussed main two type of error in results, one of which is basically modelling error (irreducible) and other is approximation error, remedy is increasing number of basis M and N. Second source is also effected due to variance in coefficients estimations of our linear model however one can work with various other parametric and non-parametric models to reduce coefficient error. Longstaff and Schwartz (2001) suggests linear model is very effective because it provides simplicity and since

[5] Glasserman and Yu (2004) has that for Brownian and Geometric Brownian underlying process the number of simulations grows faster with increasing basis functions for same level of accuracy in our regression.

only in-the-money paths are being considered this method is enough to span over possible continuation value with low standard error.

Another important error type comes from the dynamic programming procedure, which generates pricing for a discretized version of American option. With only finite exercise points the dynamics of pricing models are effected sometime severely and produces high error. There are many other models, which suffers from the same problem. One can see in Figure 2 that how by increasing time steps in the model improves the accuracy of our solution.

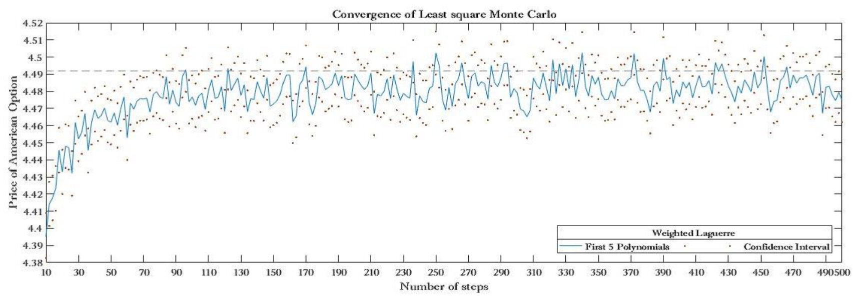

Figure 2 Price of American Put option with So=36, σ=0.20, r=6%, T=1, K=40, N=50000, M=5 and time steps are 10 to 500.

Estimates suggest for accurate pricing of continuously exercisable options, it is required that time steps should go to infinity. As discussed in Stentoft (2004) this is neither computationally feasible nor guarantee possible execution of LSM since the span required on each steps may not be possibly generated by any defined basis, which hinders the solution outcome of our system of equations.

6. Model selection

Problem in hand now is to understand suitable functional form for equation 2, which is very important for accuracy since realistically neither $N \to \infty$ or $M \to \infty$ is possible. However, one can improve the results by using efficient functional form without compromising heavily on computational capacity. In this section, we divide this problem into three main issues. Type of model, choice of basis and order of polynomial to be used in the model.

There is no single procedure to determine the suitable functional form i.e. linear or non-linear model which can minimizes the error, however, we see that non-parameteric models have better estimation and more techniques are available to compress the error. One of such example is given in Lee (2008), who uses local avaraging method with Gaussian Kernels and different basis functions to find the continuation value as similar to LSM. This method rather an improvement over because it provides lower bias (Singyun, 2015. pp 95-110).

The well-known choice for basis functions to define our regression are Laguerre, Hermite, Legendre, Chebyshev, Jacobi Polynomial as well as the simple higher order polynomial such as $S, S^2, \ldots, S^M$, Fourier or trigonometric series. The detailed work of Navas and Mareno (2003) shows that numerical results of our least square method are robust from the order of basis functions, however, not each set of functions is as efficient and powerful. For the sake of accuracy one need to understand that if basis are not powerful and underlying price movement is complicated, such as with 2-degree polynomial function

$$\hat{F}(\omega, t) = \alpha_0 + \alpha_1 S + \alpha_2 S^2 \quad (12)$$

then it may cause high bias but if either complexity is too extreme, M is large, or basis is too wiggly then there will be too much flexibility in the model. Table 3 explains this tradeoff between bias and variance. Where we show that how different choices of basis effects our numerical results as well as standard error of LSM.

Each column of Table 3 shows the number of basis in our regression from equation 4 with simulations $N = 5000, 75000, 100000$ and time steps $K = 50$. As we discussed in section 5.2, simulation indeed improves the approximation by reducing the bias and standard error (given in red values under the price of option).

On one side, we can see the improvement in $\hat{V}(X)$ from using Hermite and High order polynomails compared to Weighted Laguerre for each simulation N, with

Simulations and Order of Polynomials		**1**	**2**	**3**	**4**	**5**
Weighted Laguerre	50000	3.9952	4.0580	4.2634	4.4448	4.4750
		0.0015	0.0059	0.0088	0.0112	0.0117
	75000	3.9952	4.0577	4.2233	4.4201	4.4897
		0.0012	0.0049	0.0070	0.0093	0.0101
	100000	3.9956	4.0561	4.2456	4.4351	4.4765
		0.0010	0.0040	0.0063	0.0080	0.0087
Probabilistic Hermite	50000	4.4212	4.4633	4.4863	4.4853	4.4996
		0.0129	0.0124	0.0124	0.0127	0.0127
	75000	4.4188	4.4670	4.4802	4.4884	4.4848
		0.0105	0.0101	0.0101	0.0104	0.0101
	100000	4.4118	4.4832	4.4938	4.4840	4.4935
		0.0091	0.0092	0.0090	0.0088	0.0086
High Order Polynomials	50000	4.4038	4.4642	4.4878	4.4822	4.5015
		0.0129	0.0125	0.0128	0.0123	0.0123
	75000	4.4196	4.4821	4.4904	4.4797	4.4922
		0.0105	0.0102	0.0104	0.0105	0.0103
	100000	4.4020	4.4814	4.4820	4.4785	4.4770
		0.0092	0.0088	0.0090	0.0087	0.0091

Table 3 Bias-Variance tradeoff of LSM

only first two Hermite and higher order polynomials the regression provides slighlty better resutls (by around 0.5 cent) then the function of 5 weighted laguerre basis with N =100000.

Although, last two methods save the computational power but comes at a price of higher variance. Standard error is important component in option pricing and with increasing number of basis, the error is increasing at faster rate except for weighted laguerre, which is more contrained among all three methods. In many cases, one can rely on the model selection techniques, for any basis type and size, and choosing functional form that provides lowest MSE hence optimal bias-variance tradeoff.

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Frankfurt, 05/26/2019.

Narendar Kumar