The formalization of an investment strategy

The uptrend process

(from "Technical Analysis of Stock Trends")

1. ….You, along with many others, bought XYZ at 12, carried it up to 24, decided that was plenty high for it, and cashed in.
2. Thereupon XYZ reacted to 20, and you congratulate yourself on your astuteness.
3. But then, unexpectedly, it turns around and rushes up to 30. Now you don't feel so smart; that was a better stock than you gave it credit for being. You wish you had it back. You will not pay more for it, but if it comes back down to 24, the price at which you sold, you'll "reinstate your position."

Perhaps you have never been in either of these situations. Perhaps your own reactions wouldn't, in such cases, have been the same as those we have indicated. But, if you have had a fair amount of experience in the market — have some knowledge **of the psychology of the "average investor"** — you know that the pictures we have described are typical. …"

Elements of an investment strategy
(operating)

Technical analysis converts the information contained in past price dynamics in technical signals. These signals should to be converted in an INVESTMENT STRATEGY to be formalized **before cash is invested**

- Essential elements of an investment strategy are:
  - **current price**, compared to what we build scenarios.
  - **scenario,** or actions resulting from the possible evolutions of the current price.
  - **critical price**, an influential level of support/resistance; an asset value that will allow to choose among pre-defined scenarios.
  - **stop loss**, a price value at which**, in any case**, closing the open position before the predetermined target is reached, generating a loss.
  - **stop gain**, a price value at which**, in any case**, closing the open position at the predetermined target, realizing the gains of the position (lock in profits).
  - **performance**, the gap between critical price and stop gain/loss.
  - **Probability assessment** (subjective; see confluence's principle)
  - **cash management: position sizing and duration,** the determination of the best cash allocation among present and future (i.e. possible) investment determined by the achievement of the critical price. Sometimes the management of unrealized profits/losses.

Possible scenarios (buy)

(actions resulting from current prices' future dynamics)

Buy up (buy break): Identifies a scenario in which traders will open long positions only if **the current price will surpass the critical price**. A move above resistance is used as a buy signal because an increase in upward momentum often follows the breakout.

Buy area identifies a scenario in which traders will open long positions only if **the current price will decrease approaching the critical price**. Traders expect that - in a consolidation phase - price will move within a well-defined pattern or barrier so that any move toward the support will be followed by an increase of asset's price.

Possible scenarios (sell)

(actions resulting from current prices' future dynamics)

Sell down identifies a scenario where traders will open short positions when **the current price will drop below the critical price**.
A move below support is used as a sell signal because an increase in downward momentum often follows the breakout.

Sell area identifies a scenario in which traders will open short positions only if **the current price will increase approaching the critical price**. Traders expect that - in a consolidation phase - price will move within a well-defined channel so that any move toward the resistance will be followed by a decrease of asset's price.

Stop gain (profit) and Stop Loss

The identification of a **profit target** – a price level where is mandatory to close existing positions (achieving a gain) - imply a subjective forecast about the direction and strength of prices trends associated with different scenarios. Technicians infer profit target using **measure rules**, that is through empirical and statistical analysis of previous graphic patterns.

The identification of a **stop loss** "target" – a price level where is mandatory closing existing positions (loosing cash) - **is more difficult and subject to personal judgment** than the identification of a stop gain. Anyway the stop loss levels are not only based on “how much” traders are willing to lose.

Technicians suggest to consider:

1. the traded asset volatility combined with the risk aversion and the personal ability to stay on the market, as a stop loss makes it clear that the market has "denied" the scenario assumed by the trader.
2. the relationship with the overall scenario and - in particular - with the expected stop gain. **The ratio between expected profits and losses should in fact be between 3 and 5**; in that way a winning trading can 'balance' from 3 to 5 losses. It is 'strongly' advised not to go below this minimum.

Stop loss and stop gain

Some traders prefer to place **trailing stop orders** to protect profits. As the stock price goes up they only sell if it falls of a pre-determinated amout from its highest price ever.

The decision to use dynamic stop levels, must be evaluated (and taken) BEFORE any investment. Change the scenario's subjective likelihood, - that is: exit a trading strategy without reaching pre-determinated loss or gain - is one of the most common situations leading traders to negative returns.

Operationally consider:

- **Confluence principle**: never use just one technical analysis tool to indicate levels of exit from an investment, but the combination of multiple instruments and ideas "converging" into one comprehensive strategy.
- **Stop-loss levels are much more difficult to identify - and respect – than stop gains**

Trading strategies (scenarios)

An example

Asset	Actual price	Scenarios	Critical price	Stop Loss	Stop Gain	Perf. ratio profit/loss	Subj. prob.
XZTA	9.6	Buy up	10	9.5	12	4	++
XZTA	9.6	Buy area	8.5	8	9.9	2.8	+
XZTA	9.6	Sell down	8	8.5	7	2	-
XZTA	9.6	Sell area	9.9	10	8.5	14	--
SCFA	1.9	Buy up	2	1.85	2.45	3	++
GUYT	4.4	Buy area	4	3.7	4.15	1.5	-
ABDE	5.2	Sell area	6	6.25	5	4	+

Buy up (10): a long position on XZTA will be opened only when (if) the actual price surpass 10 euro. The position will be closed only if the price reaches 12 euro (profit of 2) or 9.5 euro (losing 0.5).
Buy area (8.5): a trader will open long positions on XZTA only if the current price will decrease approaching 8.5 euro. It will exit the position when (if) the price hits 9.9 or 8 euro (losing) .
Sell down (8): identifies a scenario where traders will open short positions on XZTA when the current price will drop below 8. They short covering if price move downward reaching 7 (gaining 1), or pull-back to 8.5 (losing 0.5 euro).
Sell area (9.9): identifies a scenario in which traders will open short positions only if the current price will increase approaching 9.9. they will exit the open position when (if) the price hits 8.5 (gaining 1.4 euro) or 10 (losing 10 cents) .

Money management (hints)

Position sizing

Position Sizing is a technique that consists of adjusting the size or the number of shares/contracts of a position before (or after) initiating a buy or a short trading order.
It can be done with very complex algorithms, not covered by this course. However, with the sole purpose of introducing this important (applied) aspect, some (simple) examples follow:

Fixed cash amount: this basic money management technique consists of entering a fixed currency amount for each new trade. Applying this technique the number of positions in the portfolio will increase as portfolio equity increases and it will decrease when the portfolio equity decreases.
e.g If the portfolio equity is equal to $10,000 and you want to invest only $1000 per trade then you will take approximately 10 positions. When your portfolio equity increases to $50,000, your portfolio will have about 50 positions.

Fixed cash amount position sizing allow you to allocate exactly the same amount of money for each trade you enter. Regardless of the stock, its price or its volatility, the number of shares for each new trade is calculated based on the amount per trade you would like to allocate

Money management (hints)

Position sizing

Fixed position number: once the maximum number of position in the portfolio is defined it remains defined a threshold of cash investment.
e.g. Your portfolio equity is euro 10,000 and the maximum number of positions allowed in the portfolio is five. This means that each position will get around 20% of the portfolio capital.

It is worth to adjust the threshold depending on both:

- **the asset volatility** (risk) and
- **the subjective probability** that the trader assigns to the achievement of the target price in each scenario

→

Fixed position number with volatility adjustment: The historical volatility of each new asset to buy/short is analyzed and then the amount of cash to enter is updated according to the asset's volatility. The higher the volatility the higher the risk and therefore, traders –normally - will reduce the investment value. Many volatility measures can be considered. Traders can infer the best measure considering different scenarios that occur during the simulation/backtest of a trading strategy.

Money management (hints)

Position sizing

Fixed position number with subjective probability adjustment: the probability that the trader assigns to the achievement of the target price in each scenario will lead the amount of cash to enter each new position

The “average” share of the cash per-position, will be strongly increased if there are many technical signals suggesting to enter an investment (see confluence principle).
Tipically: a resistance breakout with high volume, some oscillators nearby the overbought area and a diffusion index close to 100% (see market breadth)

The management of unrealized profits/losses (hints)

Lock in profits

Often, unexpert traders tend to lock in profits, cashing – too soon - the gains of a position by exiting the market at a profit level far away from the scenario's "stop gain" level. Conversely they do not close loosing positions to avoid to realize a "real" loss.
The net result of this behaviour is:

maximize losses and minimize profits!

For this reason it is generally recommended to always respect faithfully the loss and gain levels pre-determinated in the trading strategy

avoiding the management of unrealized profits / losses

However

the most experienced traders can close the position for any number of reasons: reduce exposure, generated cash, etc..
All profits are unrealized until the position is closed. By locking in, that portion of the investment is no longer exposed to risks and it become possible to open new positions

Money management (hints)

Duration (holding period)

The time period between the opening and closing of a position in a security indicates the holding period for the security.
This holding period may vary widely, depending on the investor's preference and the type of security.
For example, day traders generally close out trading positions on the same day that they were opened, while a long-term investor may close out a long position in a blue-chip stock many years after the position was first opened.

The expected duration has a direct effects on liquidity management as to generate new cash is necessary to lock in positions. Lack of liquidity are not uncommon expecially when the price actions display a strong momentum. As a result traders are often unable to open all of the recommended positions

Consider that - at a given volatility level - **the duration of the investment is expected to be a direct function of the width of the scenario's price targets**.

And finally the report!

As a broker you should provide written acknowledgment indicating that a trade has been completed. This includes details such as:

the date, price, commission, fees and settlement terms of the trade.

- the date of purchase / sale;
- actual purchase price (specifying whether gross or net of fees);
- number of shares held;
- actual performance of the portfolio;
- equivalent in euro of the position;
- the remaining cash.

It is also appropriate to introduce some of the comments that make it possible to synthesize the most recent actions.

A simple example with «basic» elements

Stock	date of purchase	price (gross)	# shares	Load value	Stop loss	Stop gain	Market price	Market value	Performance	Performance %
Enel	20-giu	2.76	5000	13800	2.67	3.2	2.96	14,800	1,000	7.2%
Unicredit	01-lug	4.6	2700	12420	4.5	5.26	5.18	13,986	1,566	12.6%
Telecom	05-mar	0.66	15000	9900	0.62	0.76	0.63	9,450	- 450	-4.5%
Remaining cash								**1900**		
Total								**40,136**	**2,116**	5.6%

Comment:
we exit for stop gain achievement Banca Mediolanum (+8.5%), Turin-Milan motorway in a draw and Luxottica for stop-loss hit (-3.1%). Given the lack of liquidity any new investment decision could be financed selling Unicredit

(simple) Trading rules

MAs in trading

(Instrument for trend detection)

- Moving averages dampen out most of the fluctuations shorter than the length of the moving average. Thus they are used to determining when the trend is changing.
- A moving average, in fact, is graphically represented by a curve that follows the line of prices remaining below/above in the case of uptrend/downtrend.

- When a moving average changes direction, technicians said that the trend represented by that moving average has changed direction (with a lag depending on the MA's amplitude and formula) A rising moving average indicates a rising trend over the period of the moving average. A declining moving average indicates a declining trend.

- Markets have many different trends going at one time. Generally, shorter-period moving averages represent shorter price trends, and longer moving averages represent longer price trends.

MAs in trading
(Crossover rule)

- Moving averages are seen as dynamic resistances or supports (like the trendlines). As the idea behind computing moving averages is to smooth an otherwise volatile series, when the price penetrates the moving average (or a short term moving average penetrates a long term one), a trend is identified

- Thus **buy and sell signals** are generated by an average (two averages) of the prices level and the price itself (a long period average and a short period average)
- The strategy is expressed as buying (or selling) when the price (the short-period moving average) rieses above (falls below) the moving average (the long period moving average)

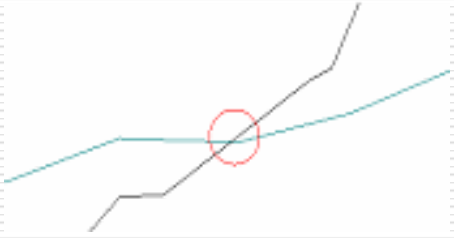
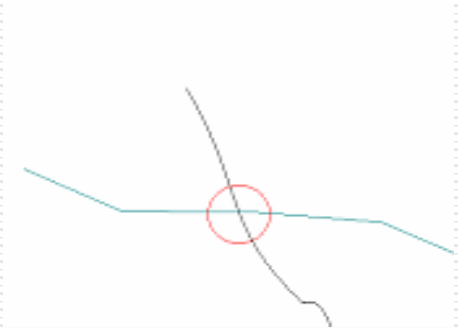
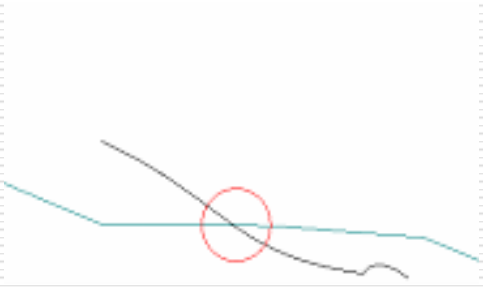
- The intersection between price and MA is called (in the jargon of technical analysts) **"crossover".**

- The following diagram illustrated the use of moving averages in trading.

* Brock, William, Josef Lakonishok, and Blake LeBaron. 1992. "Simple Technical Trading Rules and the Stochastic Properties of Stock Returns." *Journal of Finance* 47:5, 1731–64.

MAs in trading

The quality of the trading signals

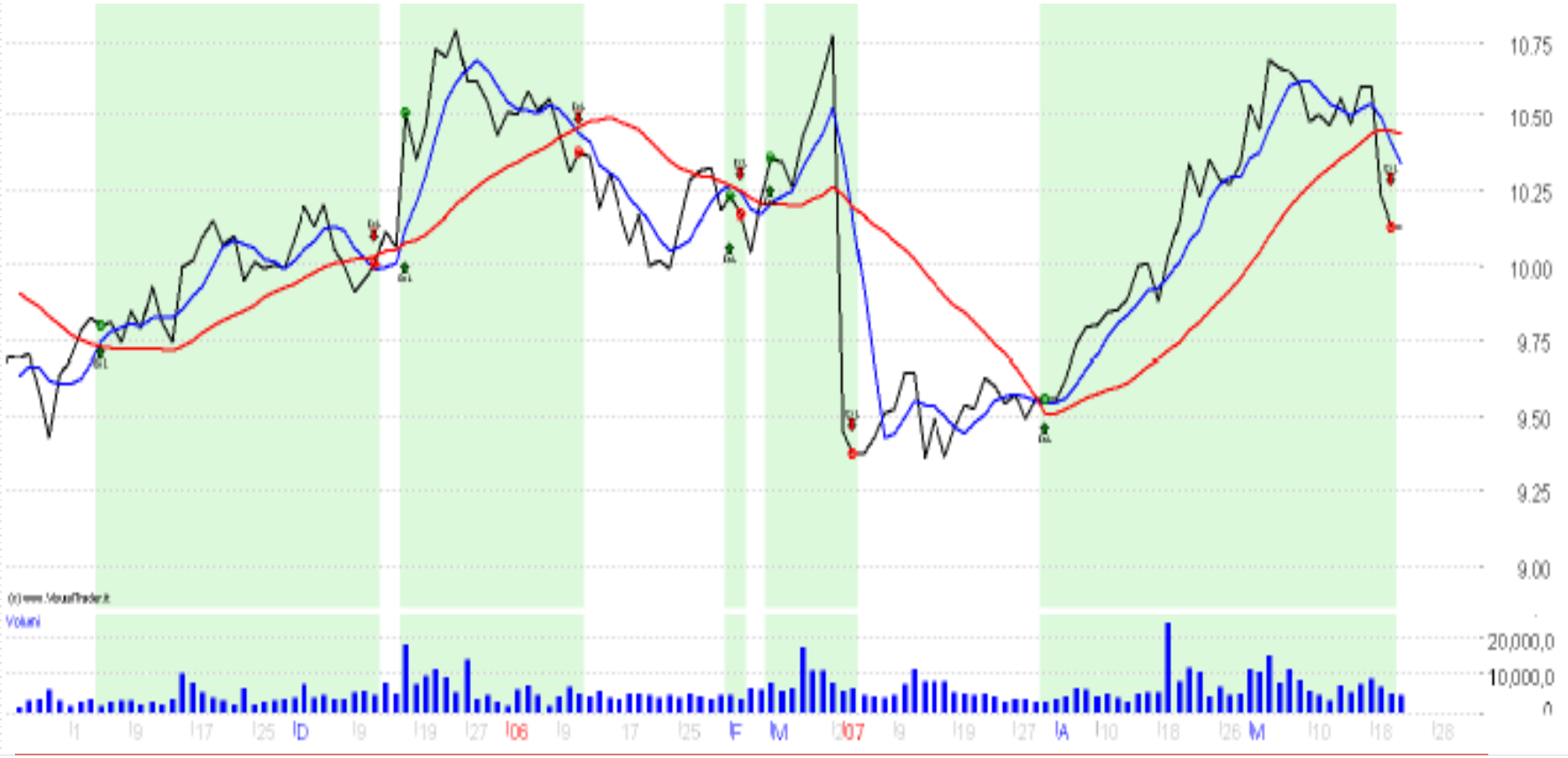
	Weak Signals	Strong Signals
Buy signals		
Sell signals		

The best signals are when the prices line (in black) cross the moving averages (in blue) with a reduced gradient (in both the upside and the downside). Intersections with strong slope are often the result of a "jump" in prices driven by the erratic component.

MAs in trading

(An Example: Alleanza Assicurazioni)

Dayly graph ; MA(21), MA(5).



MAs in trading

The quality of the trading signals

- It should be stressed that all those types of filters generate "signals" delayed with respect to the real behavior of the price. The lag in turning, however, has an advantage as it allows to surety of the signal.
- A longer-period length includes more data and more information. Each specific data point becomes less important. A large change in specific data thus has less influence on the longer moving average. However, if this large change in data is the beginning of a significant change in trend, it takes longer for the trend change to be recognized. The longer moving average is slower to pick up trend changes but less likely to indicate a trend change incorrectly from a short-term blip in the data.
- In this conflict between **"reliability" and "accuracy"** of a signaling system (the trade off between the ability to filter out the error component and the ability to react quickly to changes in trend direction), **reliability is a preferable characteristic as it reduces losses** although it limits the gains by delaying the time the trader enters/exits the market.

MAs in trading

The quality of the trading signals

- Experience suggests that the crossover rule generates a huge number of false signals when applied in accumulation/distribution phases or in short term trend detection. For this reason, moving average crossover systems are more commonly used favoring their reliability against their late signals.
- A flat trend results in moving averages oscillating horizontally and crossovers not followed by directional change in prices. This causes "whipsaws" in signals whereby a buy signal is followed by a sell signal at or below the buy signal price, and vice versa. Here trader loses money following "fluctuating signals".
- It is thus important that the moving average period lengths be long enough to bypass any flat trends in the price. Because this is not always possible, moving average crossover systems have a high rate of false signals.

- **Give up part of the strategy's "potential" return, is the only way to limit the number of false signals.**

MAs in trading

How to reduce false signals (in the signaling system)

- One possibility to reduce these whipsaws is given by the use of a moving average crossover system based on more than one moving average over different periods.
- Substituting the price with a short moving average, is possible to filter out part of the erratic component which often manifests itself in a large change in a specific price data.
- Trading signals are given when the shorter moving average will cross over and under the longer moving average.
- A crossover of a fast moving average and a slow moving average will tend to occur near the turning point of the slow moving average, and thus, while occurring long after the actual turn in prices, **it is more reliable as a signal.**
- The unknown variables in such a system are the lengths of the two moving averages. We can calibrate what those lengths should be, or we can optimize the data to see what lengths give the most reliable signals.

MAs in trading

How to reduce false signals (in the signaling system)

- Give up part of the strategy return is the only way to limit the number of false signals. You can choose:
  - to wait for a while to find confirmation of the trend reversal (to open or close market positions). This solution is feasible, however, only in the case of trading on the medium - long period when the strategies margins are such as to expect positive returns.
  - to consider a "tolerance"*(g)* around the average. The *g* value can be a fixed percentage or a function of prices volatility; shifting upwards and downwards the MA, we get a channel whose upper extreme provides buying signals when it is penetrated (upwards) from the price line while the lower extreme provides selling signals when it is penetrated (downwards) from the line of the price. Analytically:

$$\text{Buying sig.} \quad P_{t-1} \le MA(N)_t \cdot (1+g) \quad P_t > MA(N)_t \cdot (1+g)$$

$$\text{Selling sig.} \quad P_{t-1} \ge MA(N)_t \cdot (1-g) \quad P_t < MA(N)_t \cdot (1-g)$$

MAs in trading

How to reduce false signals (in the signaling system)

- The percentage change (g) should be a function of:
  - the trading approach (i.e. long term investor/speculator and therefore of the frequency of analysis (time, day, month ...)
  - the volatility of the stock.

- In time of high volatility in fact there is a greater probability to observe "false" intersections of the average (or channel) with the price (i.e. intersections due to the erratic component and not to a change in the trend). Thus is possible to tie the width of the channel to the volatility as follows:

$$\text{Buying sig.} \quad \begin{cases} P_{t-1} \le MA(N)_t \cdot \left(1 + g \cdot SD_{t(N)}\right) \\ P_t > MA(N)_t \cdot \left(1 + g \cdot SD_{t(N)}\right) \end{cases}$$

$$\text{Selling sig.} \quad \begin{cases} P_{t-1} \ge MA(N)_t \cdot \left(1 - g \cdot SD_{t(N)}\right) \\ P_t < MA(N)_t \cdot \left(1 - g \cdot SD_{t(N)}\right) \end{cases}$$

Where SD is a price volatility measure – function of t – calculated over intervals of width N.

MAs in trading
(the objectivity….)

- The trading rule is "objective" that is replicable even by those who have less experience in graphical analysis or academics (after they found that moving average crossover signals generate statistically significant stock market directional signals) ….
  - …"This paper tests two of the simplest and most popular trading rules—moving average and trading range break…" "…Overall, our results provide strong support for the technical strategies. The returns obtained from these strategies are not consistent with four popular null models: the random walk, the AR(1), the GARCH-M, and the Exponential GARCH." (the patterns uncovered by technical rules cannot be explained by correlation or volatility models) "… Buy signals consistently generate higher returns than sell signals, and further, the returns following buy signals are less volatile than returns following sell signals."… *Brock, William, Josef Lakonishok, and Blake LeBaron (1992). "Simple Technical Trading Rules and the Stochastic Properties of Stock Returns." **Journal of Finance** 47:5, 1731–64.*

- This objectivity makes the trading rules based on moving averages suitable to be implemented in automated trading systems (a computer trading program that automatically submits trades to an exchange)

Trend extraction with graphical approach

Support and Resistance

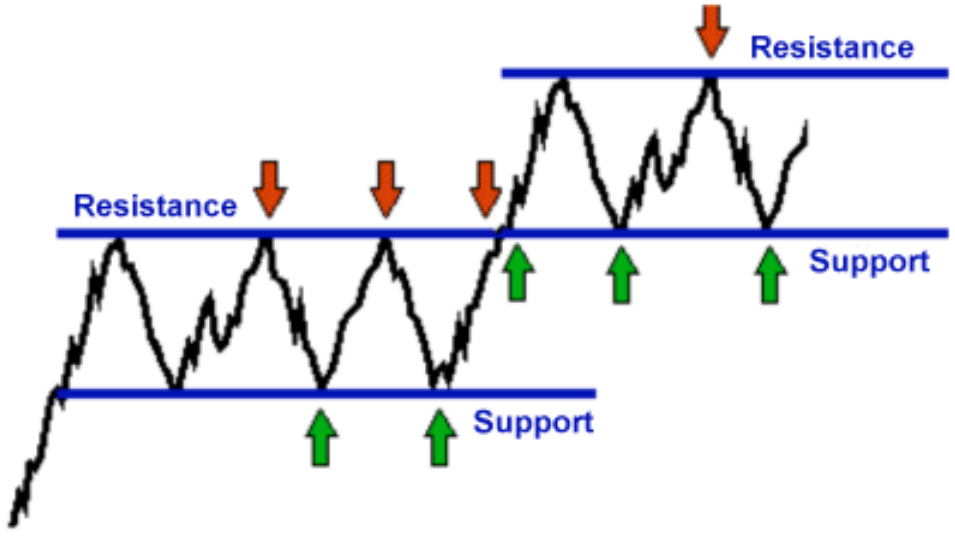
- A **support level** is a price level (or a range) where the price tends to find support as it is going down. This means the price is more likely to "bounce" off this level rather than break through it.
- A **resistance level** is the opposite of a support level. It is where the price tends to find resistance as it is going up. This means the price is more likely to "bounce" off this level rather than break through it.

- Practitioners drown support and resistence on historical price's graphs in order to build probalized scenarios on future trends and build investment strategies.
- Once the price has passed a resistence (or a support), by an amount exceeding some noise, **it is likely that it will continue** rising (dropping) until it finds another resistance (support) level.
- Traders will place orders when the price moves beyond identified support/resistance levels.

Support and resistance

(graphical approach)

SUPPORT: The support level (or the area of support) is a price level which, historically, a stock has had difficulty falling below. It is thought of as the level at which a lot of buyers tend to enter the stock.



RESISTANCE: An area of resistance or resistance level indicates that the stock or index is finding it difficult to break through it, and may head lower in the near term.

Trading range break-out.

- Breakout trading is used by active investors to take a position within a trend's early stages. Generally speaking, this strategy can be the starting point for major price moves with expansions in volatility.

- **A breakout is** a stock price that moves outside defined **support** or **resistance** level with (often) increased volume. A breakout trader enters a long position after the stock price breaks above resistance or enters a short position after the stock breaks below support.
- Once the stock trades beyond the price barrier, volatility tends to increase and prices usually trend in the breakout's direction.

- In many circumstances, breakouts are the starting point for major price trends (the starting point for future volatility increases and large price swings).
- Regardless of the time frame, ranges are easy to spot, making the range breakout strategy very popular. Whether you use intraday, daily or weekly charts, the concepts are universal. You can apply this strategy to day trading, swing trading or any style of trading.

Trading range break-out.

- **Breakout trading welcomes volatility**. The volatility experienced after a breakout is likely to generate emotion because prices are moving quickly and in a volatile fashion. Unfortunately volatility means «risk» and many traders lose money on this strategy, mainly because of false breakouts, corrections to the breakout point and unrealistic expectations.
- Range breakout examples are often used to show a stock or commodity breaking out and making a large percentage sprint, this is not always the case. With potentially hundreds of ranges being traded in different instruments in markets around the world, the probability of picking the few that will eventually explode is not high, but yet it is the dream of breakout traders to have that trade and ride it out for a fabulous gain.
- Unfortunately, large moves (and large gains) are rare, and given the difficulty to be patient and waiting for the breakout to happen, the trader is often not even in the trade when that move finally does occur.

… not only quantitative finance instruments (techniques)

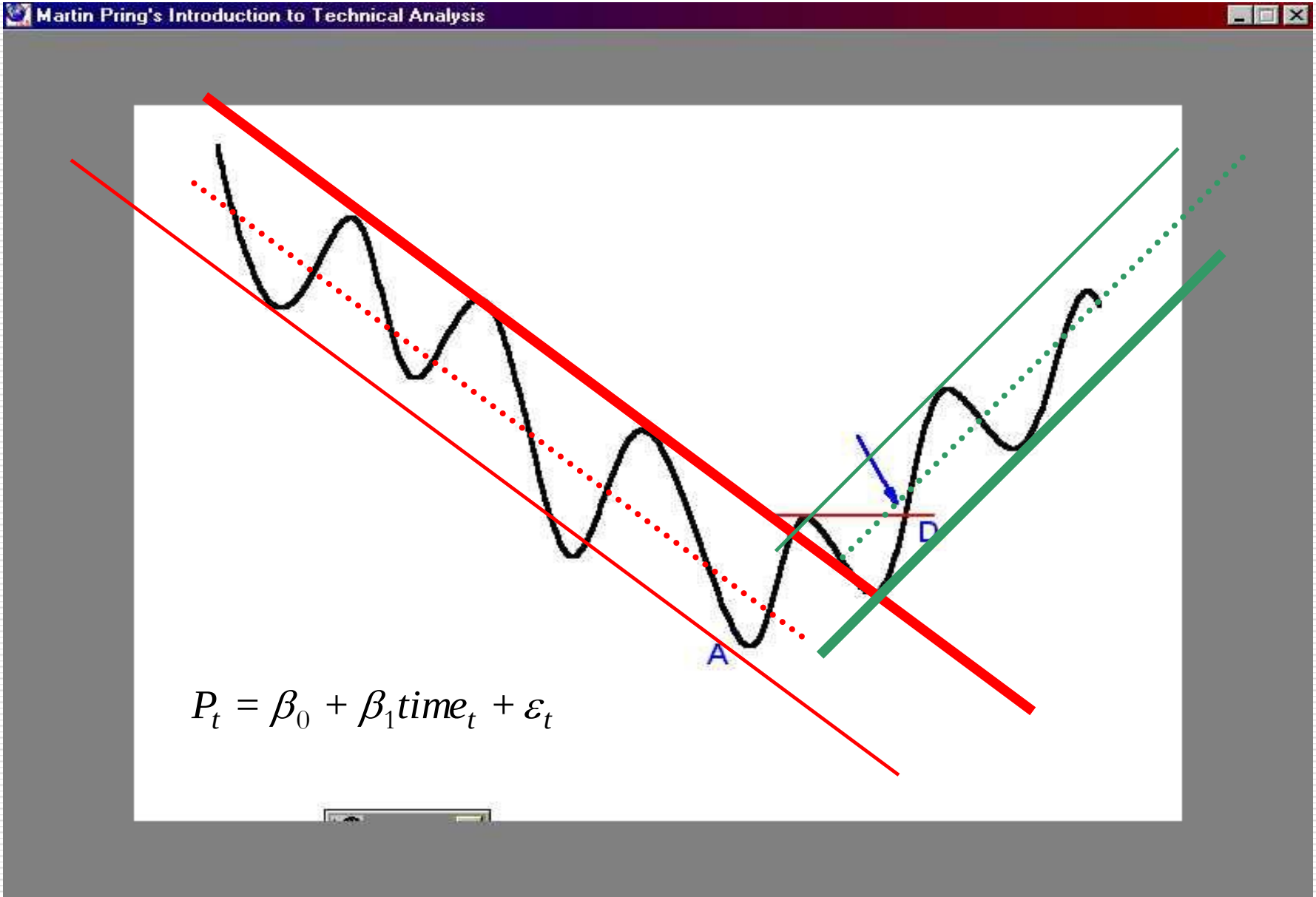
… but psycology and strategy

A statistical view of practitioners trading rules based on trends (support and resistance)

- If is possible to assume that the observed time series of financial asset is generated by an ergodic DGP **variance stationary,** whose expected value could be approximated locally by a **linear trend**

.....

A statistical view of practitioners trading rules based on trends (support and resistance)



A statistical view of practitioners trading rules based on trends (support and resistance)

(in case of bull market)

Trend line = $\hat{P}_t = b_0 + b_1 time_t - z_\alpha \sigma_\varepsilon$

Trend = $\hat{P}_t = b_0 + b_1 time_t$

Return line = $\hat{P}_t = b_0 + b_1 time_t + z_\alpha \sigma_\varepsilon$

z_α ?

(Asymmetrical time invariant) interval with bootstrap

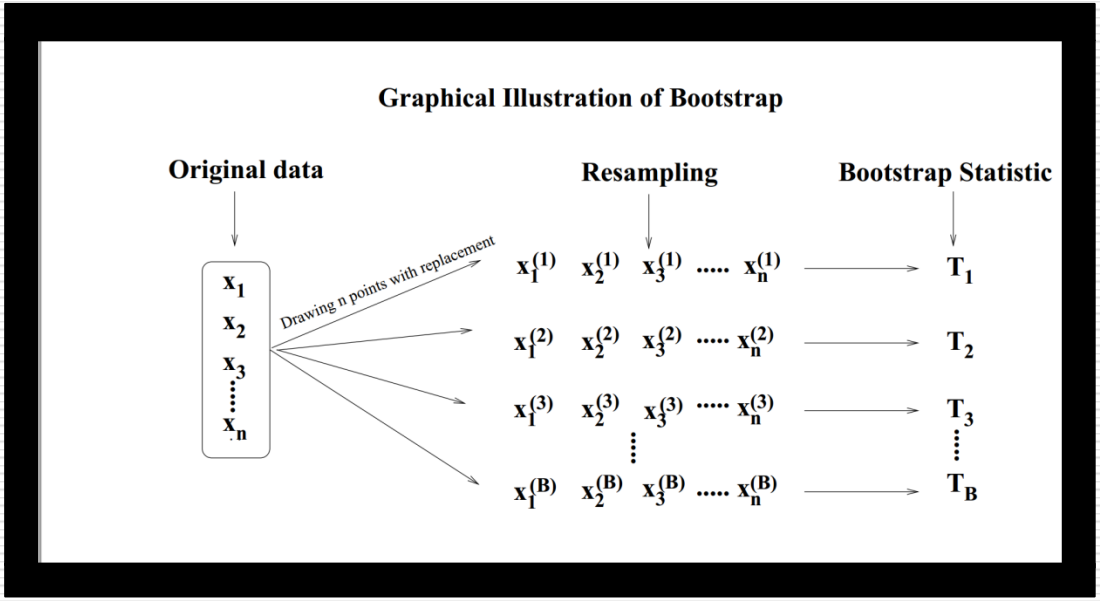
When normal distributions may not good enough to approximate the sampling errors distribution (e.g. errors display a asymmetric distribution around a «trend») Z_α values could be derived from the empirical distribution

An alternative way for constructing Confidence Intervals is bootstrap*

Bootstrap draws samples from the Empirical Distribution of data {x1, x2, ⋯ , xn} – e.g. the error distribution - to replicate statistic T to obtain its sampling distribution. The Empirical Distribution is just a Uniform distribution over {x1, x2, ⋯ , xn}.
Therefore Bootstrap is just drawing i.i.d samples from {x1, x2, ⋯ , xn}.
The procedure is illustrated by the following figure.

***** *Trend coefficients' estimate are influenced by the fact that, using prices as dependent variable, the errors' autocorrelation tends to 1 showing non-linear dynamics.*
The coefficients are also conditioned by the chosen optimization criterion (that could be different from the least squares).
A more general solution for determin probabilized scenarios is given in Phillips (2001), « Descriptive Econometric for financial time series with empirical illustration», *J. Appl. Econ.* **16**: 389–413.

(Asymmetrical) interval with bootstrap



Bootstrap Confidence Intervals

• Simple Method

To obtain the 95% Confidence Interval, the simple method is by taking 2.5% and 97.5% quantiles of the B replication T1, T1, ⋯ , TB as the lower and upper bound respectively.

• More Sophisticated Method

When the distributions are skewed we need do some adjustment. One method which is proved to be reliable is BCa method (BCa stands for Bias-corrected and accelerated).

For the details please refer to DiCiccio,T.J. and Efron B. (1996) [2]

A statistical view of practitioners trading rules based on ... (support and resistance)

More in general

- trend could take any functional form

Lower confidence band = $\hat{P}_t = T_t - z_\alpha \sigma_\varepsilon$

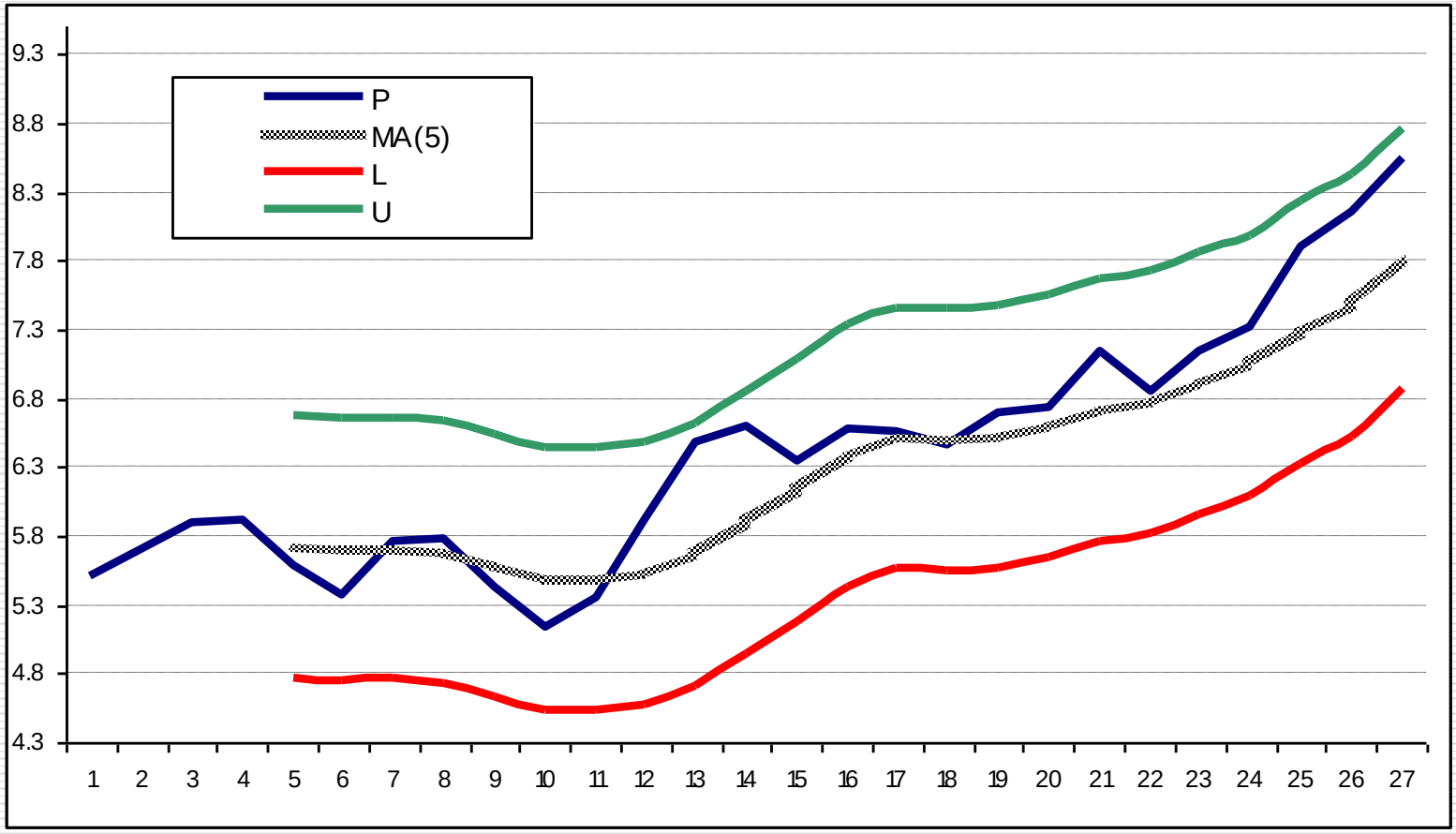
Trend = $\hat{P}_t = T_t$

Upper confidence limit = $\hat{P}_t = T_t + z_\alpha \sigma_\varepsilon$

If trend is extracted with an average (moving)

$$P_t = \mu_t + \varepsilon_t \longrightarrow \hat{P}_t = E(P_t|_{(t-m)} P_t) = \mu_t$$

A statistical view of practitioners trading rules based on ... (moving average channel)



A general approach to smoothing for trading
(The choiche of λ parameter under a quadratic loss function?)

- In the **exponential smoothing** approach the basic relationship is given by the weighted arithmetic mean where the weights follow an exponential distribution:

$$ES_{t+1} = \sum_{i=0}^{t-1} \lambda^i P_{t-i} \Big/ \sum_{i=0}^{t-1} \lambda^i ; \quad \lambda \in]0,1]$$

- The λ parameter may be estimated by many statistical software with (OLS), but that there are practical reasons suggesting that is also possible to calibrate the smoothing parameter. Filtering roules applies:
  - λ has to be close to zero when the error component is "relevant" compared to trend.
  - λ has to be close to 1 when the trend component is predominant.

A general approach to smoothing for trading
(The choiche of λ parameter under a quadratic loss function?)

- Anyway practitioners' goal is not to fit a curve minimizing the sum of squared residuals; they wants a smoother that yields maximum return.
- With the LS principle:
  - A 1 euro forecasting error has the same "relevance" indipendently from the absolute level reached by price. (which would suggest to apply the OLS priciple to data in logarithms).
  - an error of "2" does not weigh twice as an error of "1" (but weighs 4 times).
  - the sign of the percentage error is ininfluent.

$$\text{p.e.} \equiv \left[\left(\hat{T}_{t+1} - P_{t+1}\right) / P_{t+1}\right]$$

  while, if the realized yield is 0.3%, an error of 0.8% is not the same when calculated with respect to forecasts which were, respectively, 1.1% or -0.5%. *…in the first case I earn less than what expected, in the second case I lose money…*
  - Investors generally are more interested in maximize the trend accuracy (the number of correctly predicted signs) rather than minimizing a function of the forecast errors

…so it is not written in the "tables of the law" that the least squares principle is the best way to draw a linear trend

A general approach to smoothing for trading
(what about volatility clusters?)

- Time series of financial asset returns often exhibit the *volatility clustering* property: large changes in prices tend to cluster together, resulting in persistence of the amplitudes of price changes.

  In such circumstances, the assumption of constant variance (homoskedasticity) for the DGP is inappropriate.
- If we assume that the price time series of a financial asset is generated by a DGP build on a sequence of normally distributed random variables,

$$N\left(\pi_t;\sigma_t^2\right) \quad t=1,2,\ldots,T$$

- whose time-varing expected values and variances could be approximated locally respectly by a moving average and a moving variance (af the same amplitude $N<T$), we get intervals of time-varying width
- Practitioner call this "procedure" **Bollinger bands.**

A statistical view of practitioners trading rules
(based on smoothing trends, allowing for time varing variance)

Dynamic lower trend line = $\hat{P}_t = ES_t(N) - z_\alpha \sigma_t$

Trend = (expected value) = $\hat{P}_t = ES_t(N)$

Dinamic upper trend line = $\hat{P}_t = ES_t(N) + z_\alpha \sigma_t$

Bollinger bands $\lambda = 1 \quad and \quad z_\alpha = 2$ (usually)

A statistical view of practitioners trading rules
(based on smoothing trends, allowing for time varing variance)

Bollinger Bands MA(21) of dayly closing prices:

$Z\alpha=2.4$; $Z\alpha=1.4$

Choosing the optimal smoothing parameter

(the size in case of a moving average)

- The period of the analysis, the series volatility and the market phase (lateral or directional) influence the choice of the optimal smoothing parameter (i.e the trend's "shape" or the best trading rule)
- Practitioners run simulations over the past; the optimal smoothing parameter (λ) or the optimal length for a moving average (or combination of sizes, in the case of trading with 2 moving averages) is that displaying the best performance (in terms of subjective loss function (e.g. yielded return after fees).
- This criterion is called **"historic simulation"**. Is frequently used even if it conceals traps that are well known by statisticians: overfitting and the difficulty to generalize the result.

Choosing the optimal smoothing parameter

(the size in case of a moving average)

- **Overfitting (the relation between the sample size and the trading complexity):** When **FEW** data are considered it is easy to "optimize" the parameter of the trend smoother in order to have positive historical returns. However focusing on a "small" sample increases the risk that the attained good performance depends on both: a "simple" trend (increasing, decreasing or lateral) and a "simple" realized erratic component. Considering wider optimization periods partially shelters from this problem, unless a very complex model (with many optimized parameters) is used. In fact, complex models can "replicate" highly complex erratic dynamics that will never reproduced in the future.
- **Scarce possibility of generalization:** in sample (knowing the series realizations to be predicted) good results are often attained, but this does not ensure that the same performance can be obtained in the future. In other words, the historical performance usually overestimate the future results of the trading technique. This bias is not constant among different amplitudes, preventing to use past performances as acriterion to choose an amplitude

Choosing the optimal smoothing parameter

(the size in case of a moving average)

- First, as a general premise, the investor's objectives should be considered.
- If a long term investment is wanted, then 'slow' averages are to be preferred (on a sample size covering at least 6 to 12 months). This yields a few reversal signals of the trend component, that are delayed but 'certain'.
- Conversely, if the objective is a speculative investment, the sample size must be narrowed. Even a few days of observation if investors look at intraday prices (e.g. hourly/15 minutes -or even less- closing prices)
- In general it must be taken into account that in the lateral phases it is advisable to widen the MA amplitude, to filter out the erratic effects. In fact, without a trend, price's dynamic are essentially determined by the unpredictable component. A too fast MA (i.e. that follows the erratic oscillations too close) necessarily produces a lot of false signals.

Choosing the optimal smoothing parameter

(the size in case of a moving average) - Ex-post simulation -

- A better solution consists in detaching the optimization phase (the search for the moving average's window size yielding the highest profit) from the simulation phase (the scenario about profits that can be attained in the future).
- In pratice, the price time series can be divided into an:

  - optimization interval, where to search for the best performing average;

  - simulation interval, after the former, where to gauge (on ' unknown' observations, i.e. *ex-post*) the returns yielded by the found average.
- The analyst must choose the proportion of the available observations to allocate between the 2 intervals. Both should be as wide as possible: the optimization one to avoid overfitting; the simulation one to ensure that the measured *ex-post* performance is less dependent by a 'favorable' realization of the erratic component. In general the simulation interval includes from 5% to 20% of the available observations).
- **The optimal moving average window size is that yielding the best performances on the simulation sample.**

Ex-post simulation for trading system

An example

1. Define:

- how to allocate observations between optimization and simulation interval;
- the trading pattern: long-term (wide size) or speculative (narrow size);
- a loss function (e.g. return, risk, risk-return, max drawdown).

2. Evaluate the performances of *H* different strategies, on the optimization interval (e.g. building a matrix: H x *K)*. If MM(3) and MM(56) crossover is employed, operating only with RSI>75%, the corresponding row of the matrix is the experiment vector: Vh=(3, 56, 75%)); Vh1=(3, 56, 70%) if operating only with RSI>70%.

3. Select the strategies yielding the best *h*<<*H* performances.

4. Apply the *h* selected strategies to the simulation sample and detect the *h**<<*h* ones with the best performance.

5. Repeat steps 1.-4. *R* times, each time modifying:

- time frame width (daily, weekly…),
- allocation between optimization and simulation interval,
- asset (possibly better to try first with different assets of the same sector).

6. Assess similarities between the *Rh** selected models, based on their respective experiment vectors *Vh* , so that the most numerous cluster provides information about the parameters range (here the window sizes and RSI) that is optimal with reference to the adopted loss function and robust to overfitting.

Working with two assets
relative approach: ratio or spread

- **Divergence Spread:** is a trend following technique. Is based on opening a long stock position and simultaneous a stock short position for an equivalent value. What matters it is that the "right" stock perform better than the other.

- In practice…
  - First choose two stocks in the same market group (their dynamic should be correlated). Lets say A and B
    - *Note: pay attention to the hypothesis of normality when testing correlation.*
  - Second draw a normal stock market graph plotting the result of two stocks ratio (A/B) applying two moving averages to the chart
    - Usually a fast and a slow MA are chosen by optimization, that is two means that in the past have provided "good" results.
  - Signals are given by the simple crossing of moving average lines, forming a stop and reverse trading, (always on the market).
    - When the fast moving average crosses downward (from top to bottom) the slow moving average, the spread is to be sold, ie, short first stock (A) and long second stock (B). The opposite when the fast moving average crosses upward from the bottom up the slow moving average, (long on "A" and short on "B").

Working with two assets
relative approach: ratio or spread

- **Put/call ratio** …the ratio is considered to be a contrarian indicator, so that an extreme reading **above 1.0** (~1.02) is actually a bullish signal, and a lower reading **(~0.6)** of the ratio indicates bearish signal (is a bullish sentiment) ….. **from Wikipedia**

- **Convergence Spread** Is a contrarian approach that exploits the property of a couple of **RELATED, NON STATIONARY** stocks to return to their common average price ("mean reverting"). In practice, it is suggested to sell the stock that - in relation to the other - is excessively overvalued as it had an upward "unusual" (respect to the average dynamics), buying the weaker one. *(… Exactly the opposite of what suggested with divergence spreads …)*

Working with UNRELATED non-stationary variables

Spurious regression

Source: Non-Stationary Time Series, Cointegration and Spurious Regression; (by H. B. Nielsen)

- Assume that X_{1t} and X_{2t} are two totally unrelated I(1) variables.
  Then we would like the static regression

$$X_{1t} = \mu + \beta_2 X_{2t} + u_t,$$

  to reveal that $\beta_2 = 0$ and $R^2 = 0$.

- This turns out not to be the case!
  The standard regression output will indicate a relation between X_{1t} and X_{2t}.
  This is called a spurious regression or nonsense regression result.

Spurious regression

Source: Non-Stationary Time Series, Cointegration and Spurious Regression; (by H. B. Nielsen)

- Consider two presumably unrelated variables:

  CONS Danish private consumption in 1995 prices.
  BIRD Number of breeding cormorants (skarv) in Denmark.

  And consider a static regression model

$$\log(\text{CONS}_t) = \beta_0 + \beta_1 \cdot \log(\text{BIRD}_t) + u_t.$$

  We would expect (or hope) to get $\widehat{\beta}_1 \approx 0$ and $R^2 \approx 0$.

- Applying OLS to yearly data $1982 - 2001$ gives the result:

$$\log(\text{CONS}_t) = \underset{(80.90)}{12.145} + \underset{(6.30)}{0.095} \cdot \log(\text{BIRD}_t) + u_t,$$

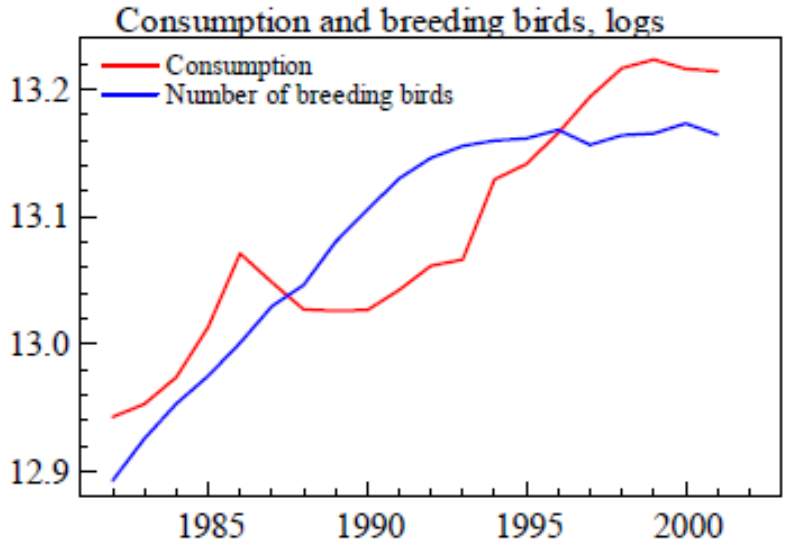
  with $R^2 = 0.688$.

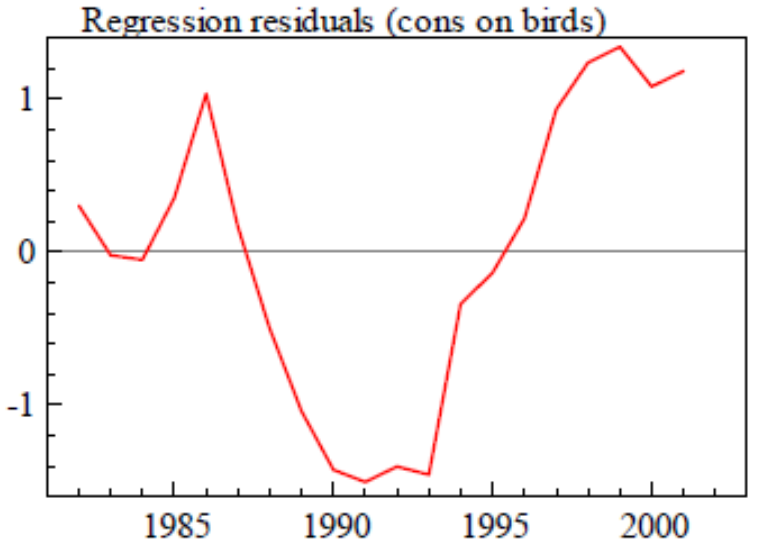
- It looks like a reasonable model. But it is complete nonsense: spurious regression.

Spurious regression

Source: Non-Stationary Time Series, Cointegration and Spurious Regression; (by H. B. Nielsen)

- The variables are non-stationary.
  The residual, u_t, is non-stationary and standard results for OLS do not hold.
- In general, regression models for non-stationary variables give spurious results.
  Only exception is if the model eliminates the stochastic trends to produce stationary residuals: Cointegration.





Spurious regression (syntesis)

- The phenomenon of spurious regression is present under different forms of nonstationarity in the DGP. In particular, when the variables Y_t and X_t are nonstationary,independent of each other, ordinary least squares applied to the regression model:

$$Y_t = \beta_0 + \beta_1 X_t + \varepsilon_t$$

have the following implications:

- 1) the OLS estimator of $\beta 1$ does not converge to its true value of zero
- 2) the t-statistic for testing the null hypothesis H0 : $\beta 1 = 0$ [t(β)] diverges, thus indicating the presence of an asymptotic (spurious) relationship between Y_t and X_t .

  The rate at which t(β) diverges depends on the type of nonstationarity present in the process generating Y_t and X_t .
- 3) R square tend to be overestimated
- Granger and Newbold (1974) suggest that the problem of spurious regression can be eliminated by differencing the data, but this implies the loss of long-run information content in the data.

Working with RELATED non-stationary variables

Cointegration

A second threat arises with truly related nonstationary variables that are integrated of order 1.

Taking Granger and Newbold (1974) counsel, in this case if one specifies the regression model in term of changes in variables only one misses the error-correction mechanism that connects cointegrated variables.

Regressions involving the change of cointegrated variables should also involve the lagged levels of those variables, but with the constraint of the cointegrating relationship imposed. This is a central point of the of Engle and Granger's (1987) work.

Working with RELATED non-stationary variables

Estimate the cointegration parameter

- Often, economic theory suggests that the dynamic of some variables can be linked by a long-term equilibrium relationship. Although the variables may deviate from equilibrium, it is expected that economic forces or government actions lead again variables in balance.

- To introduce the formal concept of co-integration between two time series, you must first define the concept of integration of order 1 (I (1)). A time series {Xt} is said integrated of order 1, if it is not stationary in the levels, but is stationary in the first differences $X_t = X_t - X_{t-1}$.

- Two time series $\{X_t\}$ e $\{Y_t\}$ - integrated of order 1 - are cointegrated if there is a coefficient β (called cointegration parameter) such that the difference $Y_t - \beta X_t$ is a stationary time series.

Working with RELATED non-stationary variables

The cointegration parameter

- Consider the following regression $\boldsymbol{y_t = c + \beta x_t + u_t}$

  where x_t is I(1) such that $x_t = x_{t-1} + e_t$ but also y_t is I(1). β is the cointegration parameter.

- The OLS estimator of β under the assumptions:
  - $u_t \sim N(0, \sigma_u)$
  - u_t ed e_t are independent (x_t is exogenous respect to u_t)
  - It is consistent (if T→∞, converges to the true value of β)
  - $T * (\beta_{ols} - \beta)$ has a normal limit distribution.
  - The t-statistic used to test if $\beta_{ols} = \beta$ converges to a standard normal distribution.

Working with RELATED non-stationary variables

Cointegration test

It is possible to test the existence of a cointegration relationship (Engle and Granger) considering whether the linear combination:

$$u_t = y_t - \beta x_t \qquad \text{is I(0).}$$

The basic idea is to use EGDF (Engle-Granger-Dickey-Fuller) approach to for the presence of unit root in u_t.

- Reference is made to three alternative specifications
  - $y_t = \beta x_t + u_t$
  - $y_t = c + \beta x_t + u_t$
  - $y_t = c + \mu t + \beta x_t + u_t$
- It is advisable to take in account the residual autocorrelation using the augmented version ot the test:

$$\Delta u_t = c + \mu t + \varphi x_t + \sum_{i=1}^{k} \gamma_i \Delta u_{t-i} + \varepsilon_t$$

The two series are cointegrated, if is possible to reject the hypothesis $H_0: |\varphi| = 1$

Working with RELATED non-stationary variables

Error correction model (ECM)

To avoid misspecification …. regressions involving the change of cointegrated variables should also involve the lagged levels of those variables, but with the constraint of the cointegrating relationship imposed….

The ECM takes the following form:

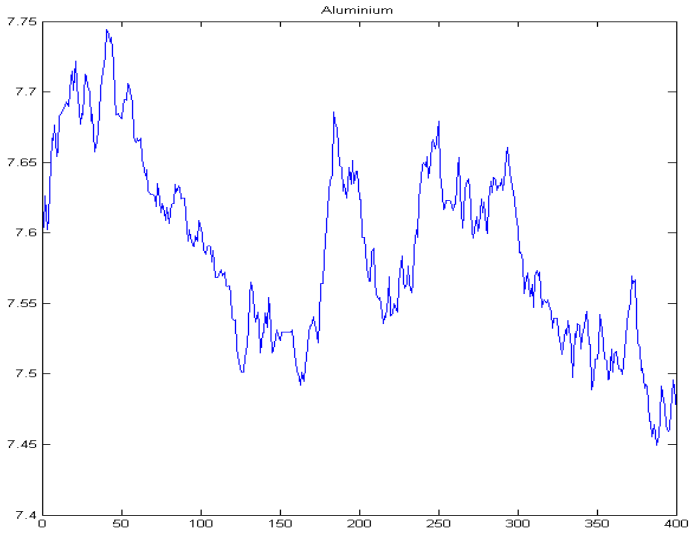
$$\Delta y_t = c + \alpha(y_{t-1} - \beta x_{t-1}) + \sum_{i=1}^{k} \gamma_i \Delta y_{t-i} + \sum_{i=1}^{k} \gamma_i \Delta x_{t-i} + \varepsilon_t$$

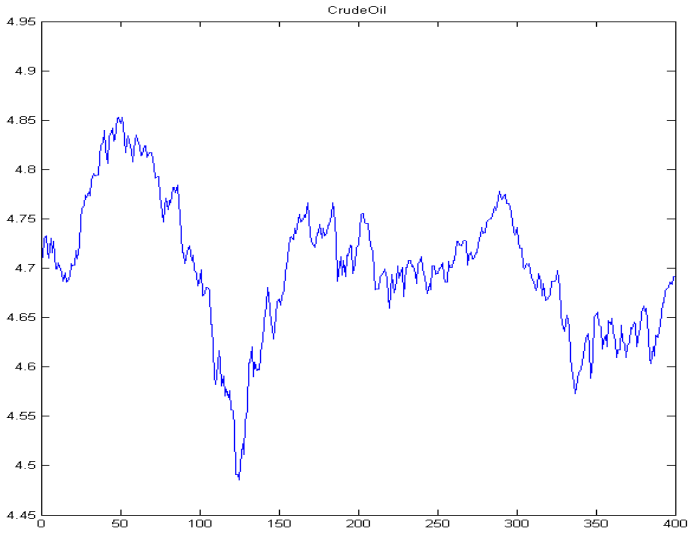
In particular $\alpha(y_{t-1} - \beta x_{t-1})$ can be described as a "equilibrium error" or a disequilibrium term relative to the previous period. If it is different from zero the model is out of equilibrium, and vice versa. Thus is expected it take values around zero **until the estimated cointegration relationship holds.**

This feature is the basis of pairs trading (or convergence spread)

Convergence spread
an example

- Y_t is a vector of the values assumed - at the instant t - by two series, such as Aluminum and Oil. In the example below a two-year prices up to May 2014 is reported.
- It can be observed they have a common (negative) trend

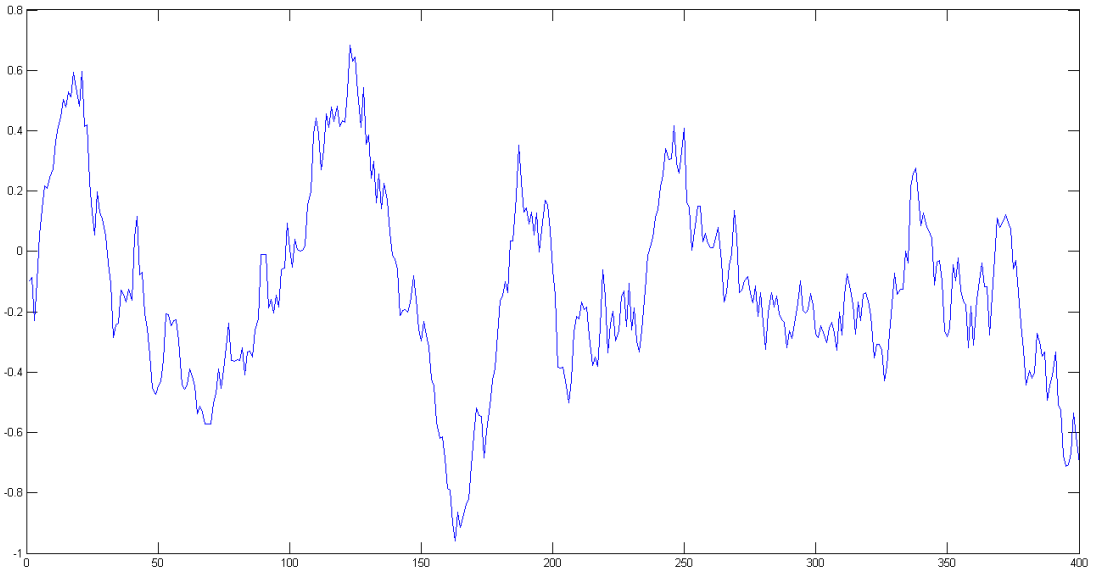




Convergence spread

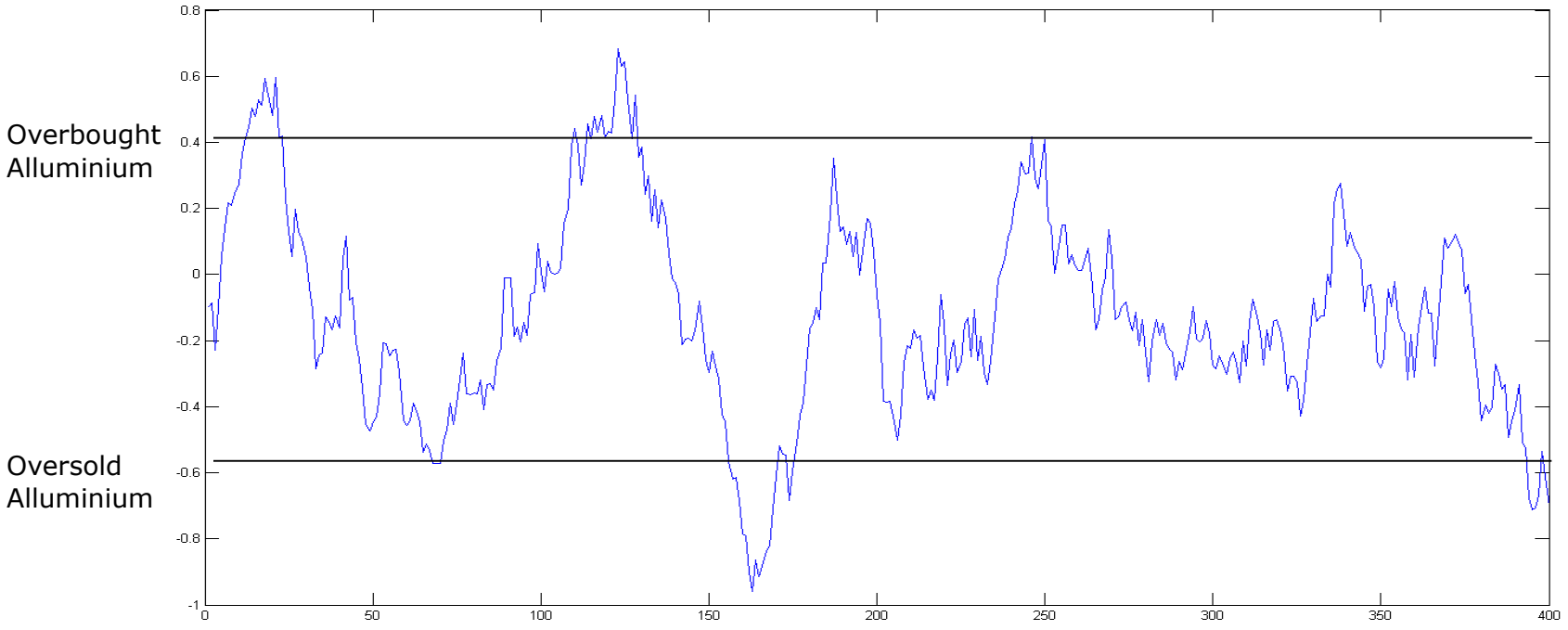
$$\beta' Y_t = Alluminium_t - \beta_2 \, Oil_t$$

is the co-integration relationship representing the long term balance shared by the two series. *It is a stationary zero-mean time series.*



Trading strategy
(convergence spread or pairs trading)

- As in the case of the oscillators, one can draw two areas corresponding to an excessive departure from equilibrium relationship

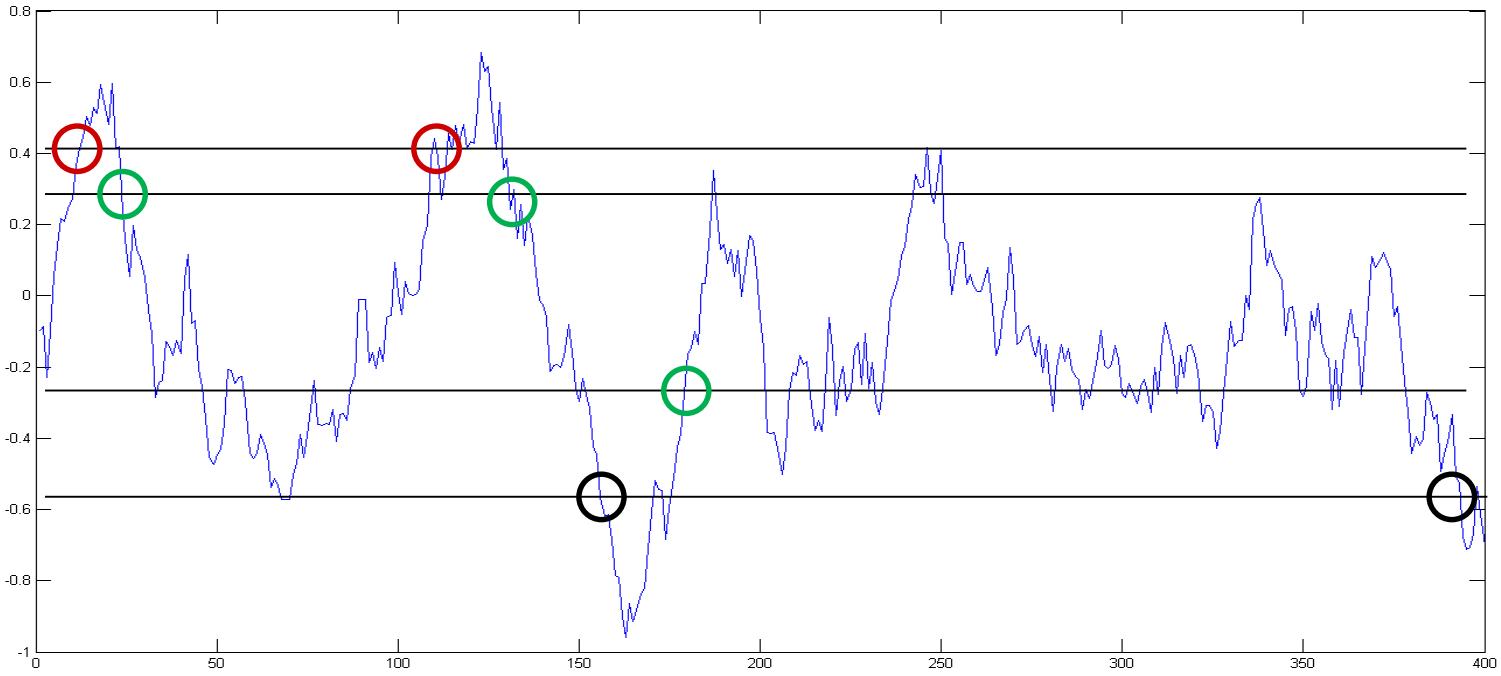


Trading optimization
(convergence spread or pairs trading)

Long Alluminium; short Oil

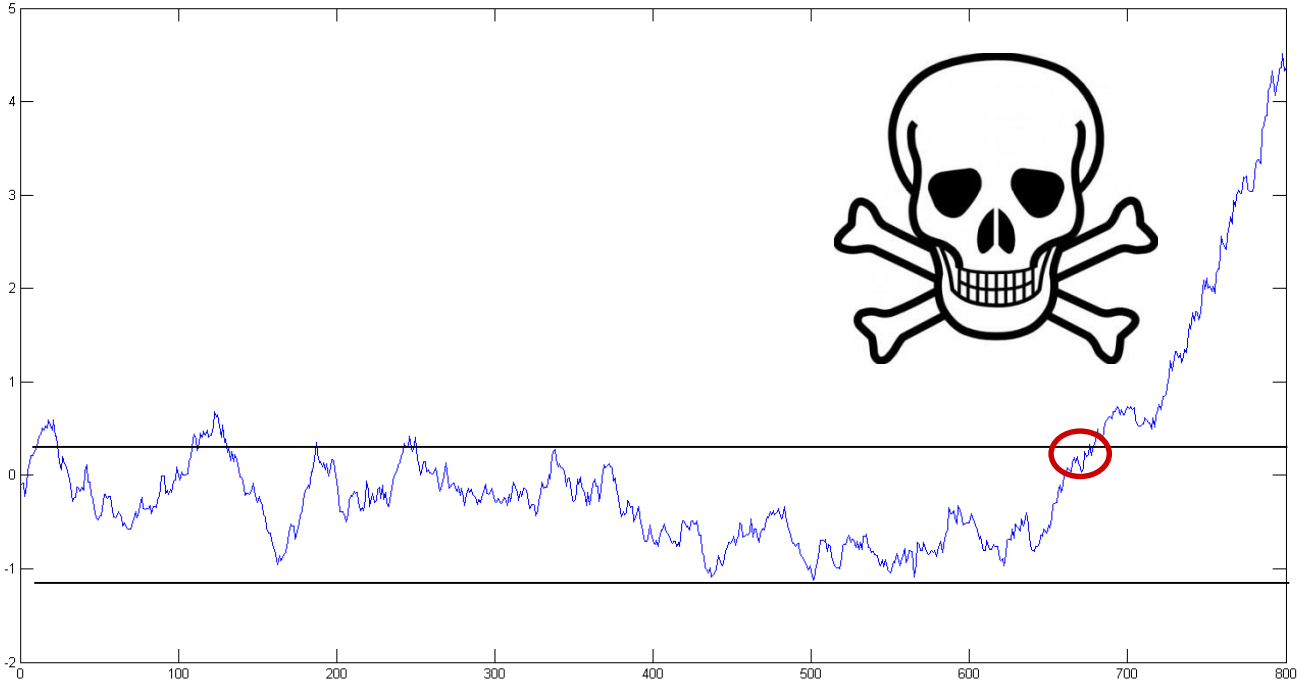
Close Positions

Long Oil; short Aluminium



False signals
(convergence spread or pairs trading)

- The risk breaking of co-integration relationship (cointegration risk)
  - (That really occurred as showed in the following graph considering data until April 2015)



Be rational…

- Diversify your portfolio (analyze and invest on several cointegrated pairs)

- Risk management: Place stop losses:
  - Subjective (or "technical"): based on the graphical approach
  - Objective (model driven based on forecasted dynamics of the co-integration relationship: you close the position when the observed relationship value is external to his confidence interval (given an alpha value).