UNIT - III

Flexure and Shear

Analysis of sections for Flexure :

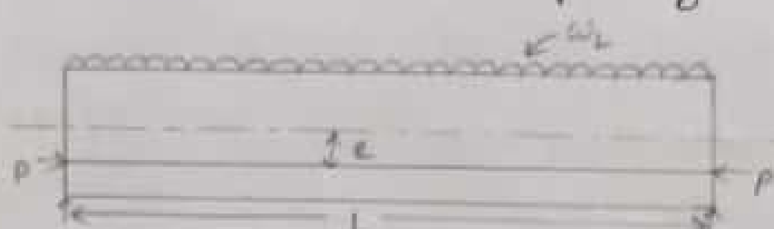
Q) What are the basic assumptions made for the analysis of prestressed sections for flexure (2m)

Basic Assumptions :

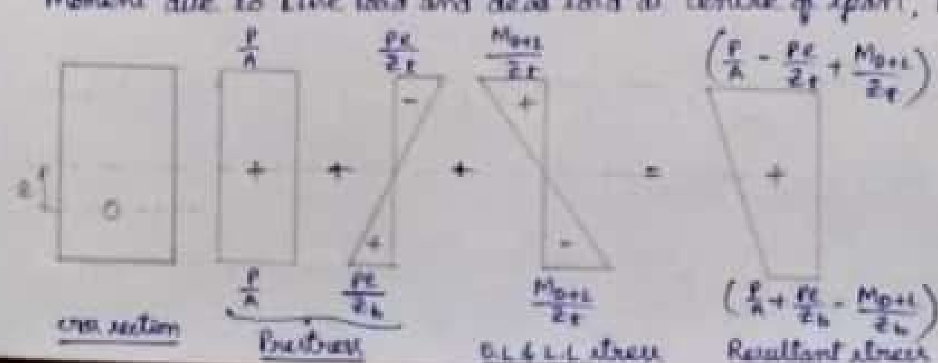
1) Concrete is homogenous, elastic and isotropic material.
2) Within the range of working stresses, both concrete and steel behave elastically not withstanding the small amount of creep which occurs in both the materials under sustained loading.
3) A plane section before bending is assumed to remain plane even after bending, which implies linear strain distribution across the depth of the member.

Q) Draw the stress distribution and resultant stress diagrams for a PSC rectangular beam with a prestressing force 'P' at a constant eccentricity 'e'. The beam is having a dead load of intensity ω_d and supports an uniformly distributed live load of intensity ω_L. Take cross sectional area of beam as A, section modulus for top and bottom as Z_t and Z_b respectively. (3m)

Sol)



Moment due to live load and dead load at centre of span, $M_{D+L} = \frac{(\omega_d+\omega_L)L^2}{8}$



a) A rectangular concrete beam 100 mm wide by 250 mm deep spanning over 8 m is prestressed by a straight cable carrying an effective prestressing force of 250 kN located at an eccentricity of 40 mm. The beam supports a live load of 1.2 kN/m.

a) Calculate the resultant stress distribution for the centre of span cross section of the beam assuming the density of concrete as 24 kN/m³.

b) Find the magnitude of the prestressing force with an eccentricity of 40 mm which can balance the stresses due to dead and live loads at the soffit of the centre span section. (10 m)

2d) <u>Given</u>:

Prestressing force, $P = 250$ kN

Cross sectional area of beam, $A = 100 \times 250 = 25 \times 10^3$ mm^2.

Eccentricity, $e = 40$ mm

Live load on the beam, $\omega_L = 1.2$ kN/m

Span, $L = 8$ m.

→ Self weight of the beam, $\omega_d = 24(0.1 \times 0.25) = 0.6$ kN/m

Total load on the beam, $\omega = (\omega_d + \omega_L) = 0.6 + 1.2 = 1.8$ kN/m

→ Section Modulus, $Z_t = Z_b = \frac{bd^2}{6} = \frac{100 \times 250^2}{6} = 1.04 \times 10^6$ mm^3

→ B.M at the centre of span section, $M = \left[\frac{wl^2}{8}\right] = \left[\frac{1.8 \times 8^2}{8}\right] = 14.4$ kN.m

→ Bending stress due to loads at top and bottom fibres,

$$\boxed{M_t \times M_b} \quad \frac{M}{Z_t} = \frac{M}{Z_b} = \left[\frac{14.4 \times 10^6}{1.04 \times 10^6}\right] = 13.8 \text{ N/mm}^2.$$

→ Prestress at top and bottom fibres,

$$\left[\frac{P}{A} \pm \frac{Pe}{Z}\right] = \left[\frac{250 \times 10^3}{25 \times 10^3} \pm \frac{250 \times 10^3 \times 40}{1.04 \times 10^6}\right] = [10 \pm 9.6] \text{ N/mm}^2.$$

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(a) Resultant stresses in concrete

At top fibre = $f_{top} = \left[\frac{P}{A} - \frac{Pe}{Z_t}\right] + \left[\frac{M}{Z_t}\right]$

$= (10 - 9.6 + 13.8) = \boxed{14.2}$ N/mm² (compression)

At bottom fibre = $f_{c\,bottom} = \left[\frac{P}{A} + \frac{Pe}{Z_b}\right] - \left[\frac{M}{Z_b}\right]$

$= (10 + 9.6 - 13.8) = \boxed{5.8}$ N/mm² (compression)

b) If P is the prestressing force required to balance the stresses at soffit

then, $\left[\frac{P}{A} + \frac{Pe}{Z_b}\right] = \left[\frac{M}{Z}\right]$

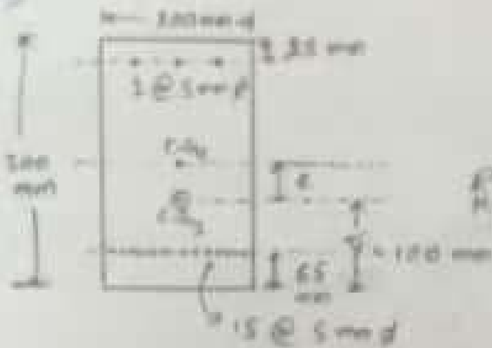
$\rightarrow P\left[\frac{1}{25 \times 10^3} + \frac{40}{1.04 \times 10^6}\right] = \left[\frac{14.4 \times 10^6}{1.04 \times 10^6}\right]$

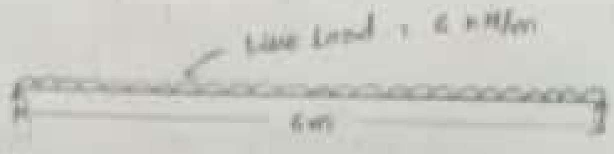
Solving, $\boxed{P = 176 \times 10^3 \text{ N}}$

$\therefore \boxed{P_{required} < P_{provided}}$

Q) A rectangular concrete beam of cross section 30 cm deep and 20 cm wide is prestressed by means of 15 wires of 5 mm diameter located 6.5 cm from the bottom of the beam and 3 wires of 5 mm diameter, 2.5 cm from the top. Assuming the prestress in the steel as 840 N/mm², calculate the stresses at the extreme fibres of the mid span section when the beam is supporting its own weight over a span of 6m. If a uniformly distributed live load of 6 kN/m is imposed, evaluate the maximum working stress in concrete. The density of concrete is 24 kN/m³. (10m)

2d)





→ Distance of the centroid of the prestressing force from the base,

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2}$$

$$= \frac{\left[\frac{\pi}{4}(5^2)(15)\right](65) + \left[\frac{\pi}{4}(5^2)(3)\right](275)}{\frac{\pi}{4}(5^2)\left[15+3\right]}$$

$$= \frac{(15 \times 65) + (3 \times 275)}{18} = 100 \text{ mm}$$

∴ Eccentricity, $e = 150 - 100 = \boxed{50 \text{ mm}}$

→ Prestressing force, $P = \text{Prestress} \times A_{st}$

$$= 840 \left(18 \times \frac{\pi}{4}(5^2)\right)$$

$$= 296.8 \times 10^3 \text{ N}$$

Area of cross section of beam, $A = (300 \times 200) = 6 \times 10^4 \text{ mm}^2$.

Moment of Inertia, $I = \frac{bd^3}{12} = \left[\frac{200 \times 300^3}{12}\right] = 45 \times 10^7 \text{ mm}^4$

(Dist. of c.G$_c$ from the pt. of calculation of stress) $y_t = 150$ mm, $y_b = 150$ mm

→ Self weight of the beam, $w_d = 0.3 \times 0.2 \times 24 = 1.44$ kN/m

Self wt. ~~of load~~ moment, $M_d = \frac{1.44 \times 6^2}{8} = 6.48$ kN-m

Live load moment, $M_L = \frac{6 \times 6^2}{8} = 27$ kN-m

Direct stress due to prestress, $\left[\frac{P}{A}\right] = \frac{296.8 \times 10^3}{6 \times 10^4}$

$= \boxed{4.95}$ N/mm² (at top & bottom)

Bending stress due to prestress, $\left[\frac{Pe}{I} \cdot y\right] = \left[\frac{(296.8 \times 10^3)(50)}{45 \times 10^7} \times 150\right]$

$= \boxed{4.95}$ N/mm² (at top & bottom)

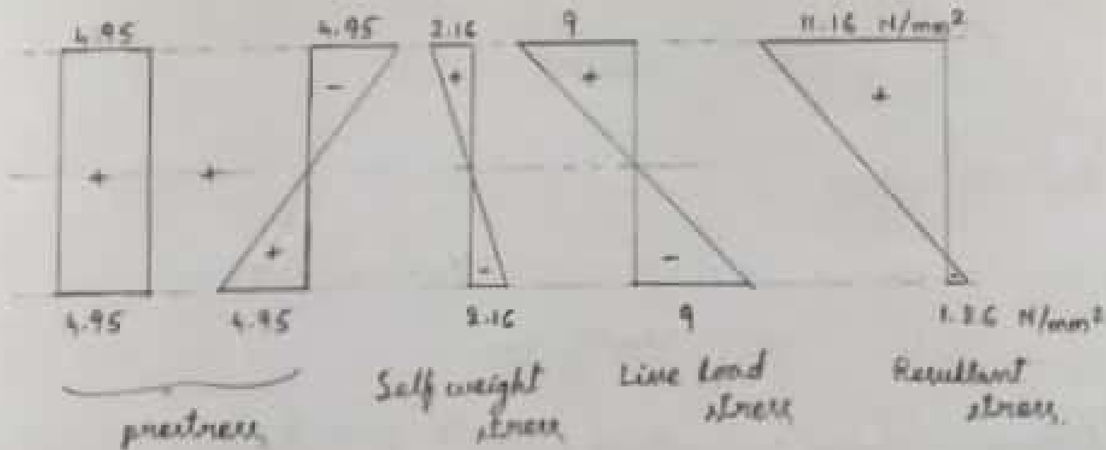
Self weight stress, $\frac{M_d}{I} \cdot y = \frac{6.48 \times 10^6}{45 \times 10^7} \times 150$

$= \boxed{2.16}$ N/mm² (at top & bottom)

Live load stress, $\frac{M_L}{I} \cdot y = \frac{27 \times 10^6}{45 \times 10^7} \times 150$

$= \boxed{9}$ N/mm² (at top & bottom)

The resultant stresses at the extreme fibers of mid span section due to (self wt. + prestress + live load) are as shown below



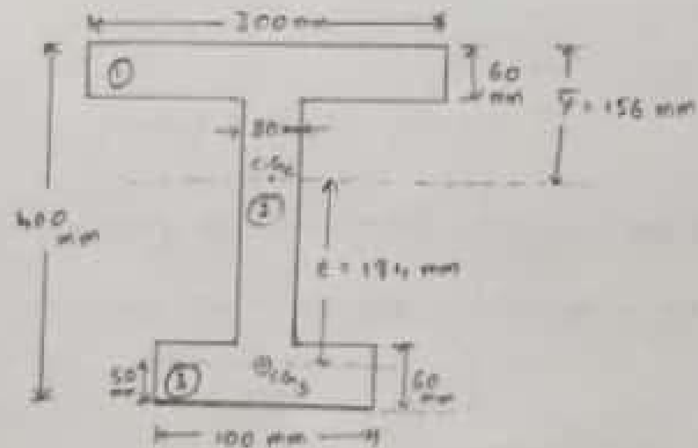
∴ Maximum working stress in concrete = 11.16 N/mm² (compression)

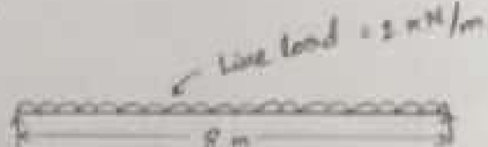
Q) An unsymmetrical I-section beam is used to support an imposed load of 2 kN/m over a span of 8 m. The sectional details are top flange 300 mm wide and 60 mm thick; bottom flange, 100 mm wide and 60 mm thick; thickness of the web = 80 mm; overall depth of the beam = 400 mm. At the centre of the span, the effective prestressing force of 100 kN is located at 50 mm from the soffit of the beam. Estimate the stresses at the centre of span section of the beam for the following load conditions.

(a) Prestress + Self weight

(b) Prestress + Self weight + Live load. (10 m)

Ad)





Prestressing force, P = 100 kN

Area of concrete, $A = (300 \times 60) + (80 \times 280) + (100 \times 60)$

$= 46400 \text{ mm}^2$

Distance of centre of gravity of concrete from top.

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3} = \frac{(300 \times 60)(30) + (80 \times 280)(200) + (100 \times 60) \times (370)}{(300 \times 60) + (80 \times 280) + (100 \times 60)}$$

$\boxed{\bar{y} = 156 \text{ mm}}$ (from top)

$\therefore e = 400 - 156 - 50 = 194 \text{ mm}$

Moment of Inertia, $I = [I_{G_1} + A_1 h_1^2] + [I_{G_2} + A_2 h_2^2] + [I_{G_3} + A_3 h_3^2]$ ④

where $I_{G_1} = \frac{bd^3}{12} = \frac{300 \times 60^3}{12} = 54 \times 10^5$

$I_{G_2} = \frac{80 \times 280^3}{12} = 146.34 \times 10^6$

$I_{G_3} = \frac{100 \times 60^3}{12} = 18 \times 10^5$

$\therefore I = \left[(54 \times 10^5) + (300 \times 60)(126)^2\right] + \left[(146.34 \times 10^6) + (280 \times 80)(44)^2\right]$

$+ \left[(18 \times 10^5) + (100 \times 60)(214)^2\right]$

$= \boxed{756.43 \times 10^6}\ mm^4$

→ $y_t = 156$ mm , $y_b = 244$ mm

$Z_t = \frac{I}{y_t} = \frac{756.43 \times 10^6}{156} = 484.9 \times 10^6\ mm^3$

$Z_b = \frac{I}{y_b} = \frac{756.43 \times 10^6}{244} = 310 \times 10^4\ mm^3$

→ Dead Load, $w_d = 24 \times 0.0464 = 1.12$ kN/m

Live Load, $w_L = 2$ kN/m

→ Moment due to dead load, $M_d = \frac{1.12 \times 8^2}{8} = 8.96$ kN-m

Moment due to live load, $M_L = \frac{2 \times 8^2}{8} = 16$ kN-m

→ Prestress at top, $\frac{P}{A} - \frac{Pe}{Z_t} = \frac{100 \times 10^3}{46400} - \frac{100 \times 10^3 \times 194}{4.84 \times 10^6}$

$= 2.15 - 4 = \boxed{-1.85}\ N/mm^2$

at bottom, $\frac{P}{A} + \frac{Pe}{Z_b} = \frac{100 \times 10^3}{46400} + \frac{100 \times 10^3 \times 194}{3.1 \times 10^6}$

$= 2.15 + 6.25 = \boxed{8.4}\ N/mm^2$

→ Self weight stress, at top, $\frac{M_d}{Z_t} = \frac{5.96 \times 10^6}{4.8 \times 10^6} = \boxed{1.86}$ N/mm²

at bottom, $-\frac{M_d}{Z_b} = \frac{-8.96 \times 10^6}{3.1 \times 10^6} = \boxed{-2.89}$ N/mm²

→ Live Load stress, at top, $\frac{M_L}{Z_t} = \frac{16 \times 10^6}{4.8 \times 10^6} = \boxed{3.33}$ N/mm²

at bottom, $-\frac{M_L}{Z_b} = \frac{-16 \times 10^6}{3.1 \times 10^6} = \boxed{-5.16}$ N/mm²

Resultant Stresses

(a) Due to (Prestress + Self weight stress)

At top $= -1.85 + 1.86 = \boxed{0}$ N/mm²

At bottom $= 8.4 - 2.89 = \boxed{5.51}$ N/mm²

(b) Due to (Prestress + Self weight stress + Live Load stress)

At top $= -1.85 + 1.85 + 3.33 = \boxed{3.33}$ N/mm²

At bottom $= 8.4 - 2.89 - 5.16 = \boxed{0.35}$ N/mm²

Q) A PSC beam of section 200 mm wide by 300 mm deep is used over an effective span of 6m to support an imposed load of 4 kN/m. The density of concrete is 24 kN/m³. At the centre of span section of the beam, find the magnitude of

(a) The concentric prestressing force necessary for zero fibre stress at the soffit when the beam is fully loaded and

(b) the eccentric prestressing force located 100 mm from the bottom of the beam which would nullify the bottom fibre stresses due to loading.

Sol) Area of cross section, $A = 200 \times 300$

$= 6 \times 10^4 \text{ mm}^2$

Section modulus, $Z_b = Z_t = \left[\frac{200 \times 300^2}{6}\right]$

$= 3 \times 10^6 \text{ mm}^3$

Dead load, $w_d = 24 \times 0.2 \times 0.3 = 1.44 \text{ KN/m}$.

Moment due to dead load, $M_d = \frac{1.44 \times 6^2}{8}$

$= 6.48 \text{ KN-m}$

Moment due to live load, $M_L = \frac{4 \times 6^2}{8} = 18 \text{ KN-m}$.

Tensile stress at the bottom fibre due to dead and live loads

$= \left[\frac{(6.48 + 18) \times 10^6}{3 \times 10^6}\right] = 8.16 \text{ N/mm}^2$.

(a) If P = concentric prestressing force, for zero stress at the soffit of the beam under loads

$$\frac{P}{A} = 8.16$$

$$\therefore P = (8.16)(6 \times 10^4)$$

$$= \boxed{489.6} \text{ KN}$$

(b) If P = eccentric prestressing force (e = 50 mm), for zero stress at the soffit of the beam under loads.

$$\left(\frac{P}{A}\right) + \left(\frac{Pe}{Z_b}\right) = 8.16$$

$$P\left[\frac{1}{6 \times 10^4} + \frac{50}{3 \times 10^6}\right] = 8.16$$

$$P = \boxed{244.8} \text{ KN}$$

∴ The magnitudes of P clearly indicate the advantage of eccentric prestressing.

Cable Profile and Cable Layout

Q) Explain briefly about Load Balancing Concept (5 m)

Load Balancing Concept :

→ In this method, the moment, upward thrust and upward deflection due to prestress in tendons are calculated. The upward thrust will be provided such that it balances part (or) full of superimposed load.

→ Cable profile is selected such that the transverse component of cable force balances the given type of external loads.

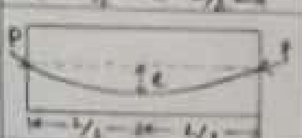
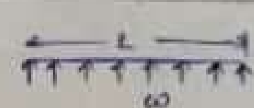
→ The straight portions of the cable do not induce any reactions vertically except moments at ends.

→ Curved cables result in uniformly distributed loads.

→ Sharp angles in a cable induce concentrated loads.

→ In general, this requirement will be satisfied if the cable profile in a prestressed member corresponds to the shape of BMD resulting from the external loads.

→ Thus concept of load balancing is useful in selecting the tendon profile, which can supply most desirable system of forces in concrete.

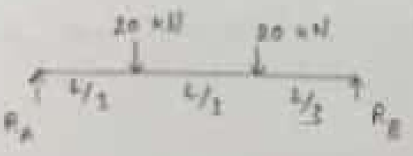
Tendon Profile	Equivalent Moment/Load	Equivalent Loading	BMD
	$M = P.e$ (hog)		
	$M = P.e = \frac{WL}{4}$ $W = \frac{4Pe}{L}$		
	$M = \frac{\omega L^2}{8}$ $\omega = \frac{8Pe}{L^2}$		
	$M = W\,aL$ $W = \frac{Pe}{aL}$		

Q) A rectangular concrete beam 200 mm × 800 mm supports two concentrated loads 20 KN each at the third point of a span of 9 m.

a) Suggest a suitable cable profile, if the eccentricity is 100 mm for the middle 3rd portion of beam. Calculate the prestressing force reqd. to balance the bending effect of the concentrated loads (neglect self weight of the beam).

b) For the same cable profile, find the effective force in the cable if the resultant stress due to self wt., imposed loads and prestressing force is zero at the bottom fibre of the mid span section (Assume unit weight of concrete as 24 KN/m³) (5 m)

sd)



Support Reactions:

$\Sigma F_y = 0 \longrightarrow R_A + R_B = 40$ KN.

$\Sigma M_A = 0 \longrightarrow R_B(L) - 20\left(\frac{2L}{3}\right) - 20\left(\frac{L}{3}\right) = 0$

$R_B(L) = 20\frac{(3L)}{3}$

$R_B = \boxed{20}$ KN.

$\therefore R_A = \boxed{20}$ KN.

Moment (max) at the centre of span,

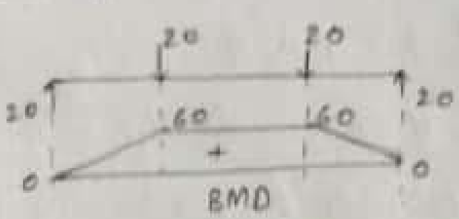
$M = 20\left(\frac{L}{2}\right) - 20\left(\frac{L}{6}\right)$

$= \frac{20L}{3}$

span of the beam, $L = 9$ m.

$\therefore M = \frac{20(\cancel{9}^{3})}{\cancel{3}} = \boxed{60}$ KN-m

(a) Bending Moment Diagram :



A trapezoidal cable profile is selected, since the BMD due to the two concentrated loads is trapezoidal in shape.

Section modulus, $Z = \frac{300 \times 800^2}{6} = 32 \times 10^6 \ mm^3$

Let 'P' be the prestressing force required to balance the B.M

$\therefore \quad P \cdot e = 60 \ KN\text{-}m$

$P = \frac{60 \times 10^6}{100} = \boxed{6 \times 10^5} N.$

(b) Self weight of the beam, $w_d = 24 \times 0.3 \times 0.8$

$= 5.76 \ KN/m$

Self weight moment, $M_d = \frac{5.76 \times 9^2}{8} = 58.32 \ KN\text{-}m$

For the resultant stress, to become '0' at the bottom

$$P\left[\frac{1}{A} + \frac{e}{Z_b}\right] - \frac{M_d + M_L}{Z_b} = 0$$

$$P\left[\frac{1}{300 \times 800} + \frac{100}{32 \times 10^6}\right] = \frac{(58.32 + 60) \times 10^6}{32 \times 10^6}$$

$$P = 507 \times 10^3 \ N$$

$$= \boxed{507} \ KN$$

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Kern Points and Pressure Line

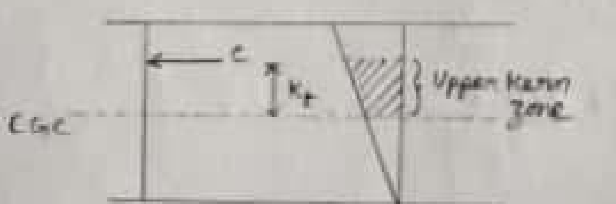
Q) Explain briefly about a) Kern Points
b) Cracking Moment
c) Pressure Line (10 m)

a) Kern Points :

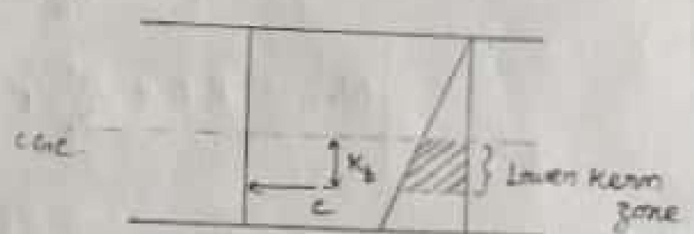
→ When the resultant compressive force (c) is located within a specific zone of a section, tensile stresses are not generated. This zone is called kern zone of a section.

→ When the resultant compression (c) under service loads is located at the upper kern point, the stress at bottom edge is zero. Similarly when resultant compression is located at bottom kern point at transfer of prestress the stress at upper edge is zero.

→ The levels of upper and lower kern points from cgc are denoted as K_t and K_b respectively.



Resultant stress under service loads.



Resultant stress at transfer of prestress

$$\frac{C}{A} - \frac{C K_t \cdot y_b}{I} = 0$$

$$\frac{C}{A} = \frac{C K_t \cdot y_b}{I}$$

$$\boxed{K_t = \frac{I}{A \cdot y_b} = \frac{r^2}{y_b}}$$ where, radius of gyration, $r = \sqrt{\frac{I}{A}}$

Similarly $$\boxed{K_b = \frac{I}{A \cdot y_t} = \frac{r^2}{y_t}}$$

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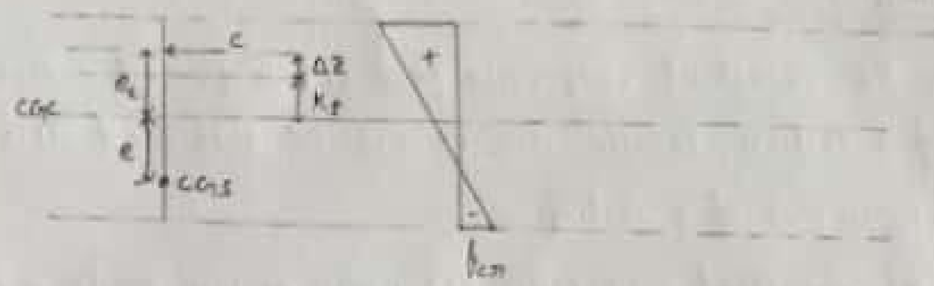
b) Cracking Moment using Kern Points :

→ The Kern points can be used to determine the cracking moment (M_{cr}). The cracking moment will be slightly greater than moment causing zero stress at bottom. Thus resultant compression will be slightly above k_t to cause cracking tensile stress (f_{cr}) at bottom.



cracking moment, $M_{cr} = c$ (lever arm)

$$= c(e + e_c)$$

$$= c(e + k_t + \Delta z)$$

$$M_{cr} = c(e + k_t) + \frac{f_{cr} \cdot I}{y_b}$$

$$= ce + \frac{r^2}{y_b} \cdot c + \frac{f_{cr} \cdot I}{y_b}$$

$$\boxed{M_{cr} = P.e + \frac{r^2}{y_b} \cdot P + \frac{f_{cr} \cdot I}{y_b}}$$

c) Pressure Line :

→ The pressure line in a beam is the locus of the resultant compression along the length. It is also called as the 'Thrust Line' (or) 'C-Line'.

→ It is used to check whether 'c' at transfer and at service loads is falling within the Kern zone of the section.

→ The eccentricity of pressure line (e_c) from cgc should be less than k_b (or) k_t to ensure that resultant compression falls within the kern zone.

$$\boxed{e_c = z - e}$$

If $e_c > k_t$ → tension is generated at the bottom of the beam.

If $e_c < k_b$ → tension is generated at the top of the beam.

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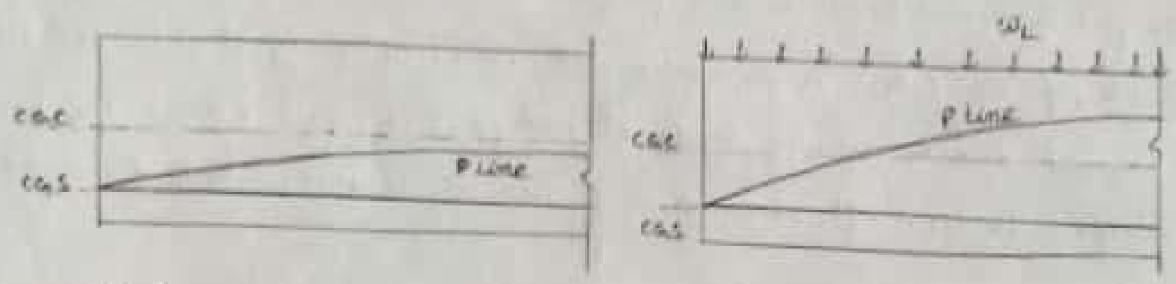


(a) Pressure line at transfer under self weight

(b) Pressure line under service loads

→ Pressure line gets further shifted from CGC with the increase of moment under service loads.

Q) What are the different types of prestressed members based on limiting zone. (3 m)

Based on limiting zone, prestressed members are divided into 3 types

1) Type 1 - Fully prestressed member :

→ For this type limiting zone is defined as the zone for placing the c.G of tendons such that resultant compressive force always lies within kern zone.

→ Tension is not allowed in the member at transfer as well as under service loads

2) Type 2 - Partially prestressed member :

→ For this type limiting zone is defined as zone for placing the c.G of tendons such that the tensile stresses in extreme edges are within allowable values

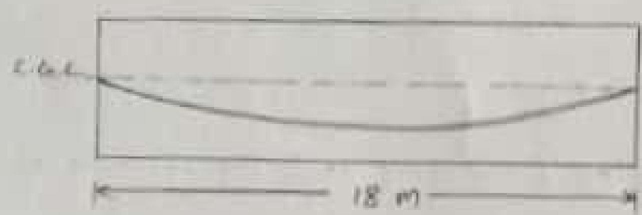
→ Tension is allowed at transfer and under service loads, but below the cracking stress

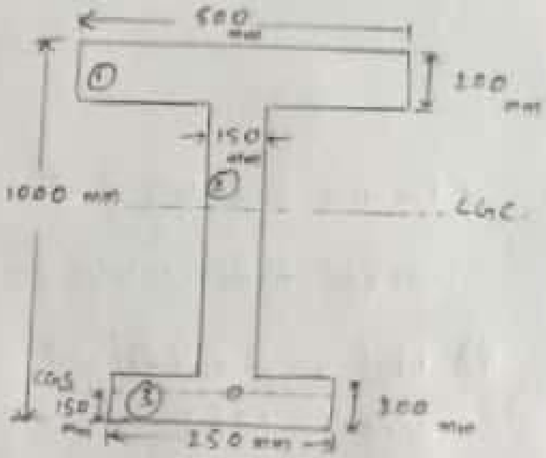
3) Type 3 - Partially prestressed member .

→ For this type limiting zone is defined as zone for placing c.G of tendons such that the cracking in the extreme edges is within allowable limiting .

→ Tension and cracking is allowed, but crack width should be within allowable limits.

Q) For the post tensioned beam with a flanged section as shown in the figure. The profile of the C.G.S is parabolic with no eccentricity at ends. The live load moment due to service loads at mid span is 648 kN-m. The prestress at transfer is 1600 kN. Assume 15% loss at service, Grade of concrete as M30.





C/s at mid span

Evaluate

a) Kern levels

b) Cracking moment

c) Location of pressure line at mid span under transfer and service loads

d) Stresses at the top & bottom fibres at transfer and service

For compression, f_{cc} allowable $= 18\ N/mm^2$

For tension, f_{ct} allowable $= -1.5\ N/mm^2$ (10 m)

Ad) Calculation of Geometric properties.

Area of cross section, $A = A_1 + A_2 + A_3$

$= (500 \times 200) + (150 \times 600) + (250 \times 200)$

$= 240 \times 10^3\ mm^2$

C.G.C, $\bar{y}$ (from soffit) $= \dfrac{(500 \times 200)(900) + (150 \times 600)(500) + (250 \times 200)(100)}{(500 \times 200) + (150 \times 600) + (250 \times 200)}$

$= 583.3\ mm$

$\therefore\ y_b = \boxed{583.3}\ mm, \quad y_t = 1000 - 583.3 = \boxed{416.7}\ mm$

Eccentricity of C.G.S from C.G.C, $e = \bar{y} - 150 = \boxed{433.3}\ mm$

$I_G + A_1 h_1^2 = \dfrac{500 \times 200^3}{12} + (500 \times 200)(416.7 - 100)^2 = 1.036 \times 10^{10}\ mm^4$

$$I_{G2} + A_2 \cdot h_2^2 = \frac{(150)\times 600^3}{12} + (150\times 600)(583.3 - 500)^2 = 3.32\times 10^9 \text{ mm}^4$$

$$I_{G3} + A_3 \cdot h_3^2 = \frac{(250)\times 200^3}{12} + (250\times 200)(583.3 - 100)^2 = 1.184\times 10^{10} \text{ mm}^4$$

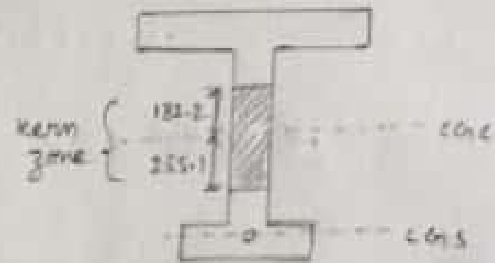
$$I = (I_{G1} + A_1 h_1^2) + (I_{G2} + A_2 h_2^2) + (I_{G3} + A_3 \cdot h_3^2) = \boxed{2.55\times 10^{10}} \text{ mm}^4$$

$$r^2 = \frac{I}{A} = \frac{2.55\times 10^{10}}{240\times 10^3} = \boxed{1.063\times 10^5} \text{ mm}^2$$

(a) Kern Levels of the section:

$$K_t = \frac{r^2}{y_b} = \frac{1.063\times 10^5}{583.3} = 182.2 \text{ mm}$$

$$K_b = \frac{r^2}{y_t} = \frac{1.063\times 10^5}{416.7} = 255.1 \text{ mm}$$



(b) Cracking Moment:

Modulus of rupture, $f_{cr} = 0.7\sqrt{f_{ck}}$

$= 0.7\sqrt{30} = 3.83 \text{ N/mm}^2$

Cracking Moment, $M_{cr} = P(e + e_c)$

$= P(e + K_t + \Delta z)$

$= P.e + P.\frac{r^2}{y_b} + \frac{f_{cr}\cdot I}{y_b}$

$= \left[0.85(1600\times 10^3)433\right] + \left[0.85(1600\times 10^3)182.2\right]$

$+ \left[\frac{3.83\times 2.55\times 10^{10}}{583.3}\right] = \boxed{970.1} \text{ kN/m}$

Dead load Moment, $M_d = \frac{w_d \cdot L^2}{8} = \frac{(24\times 0.24)\,18^2}{8} = \boxed{233.3}$ kN-m

Live Load Moment, $M_L = \boxed{648}$ kN-m

Live load Moment required for cracking $= 970.1 - 233.3 = \boxed{736.8}$ kNm

$\Rightarrow$ L.L given, $648 < 736.8$

$\therefore$ Section is uncracked

(c) Location of pressure line at mid span :

At transfer, $z = \frac{M_D}{C}$ (where lever arm, $z = e + e_c$)

$= \frac{233.3 \times 10^6}{1600 \times 10^3}$

$= 145.8$ mm

$e_c = z - e = 145.8 - 433.3 = \boxed{-287.5}$ mm

∴ Pressure Line is below C.G.C.

∵ $287.5 > K_b$. there will be tension at the top

At service, $z = \frac{M_{D+L}}{C} = \frac{(233.3 + 648) \times 10^6}{0.85\,(1600) \times 10^3} = \boxed{648}$ mm

$e_c = z - e = 648 - 433 = \boxed{214.7}$ mm

∴ Pressure line is above C.G.C

∵ $e_c > K_t$, there will be tension at bottom

(d) Calculation of Stresses :

At transfer,

At top : $\frac{P}{A} = \frac{1600 \times 10^3}{240 \times 10^3} = 6.67$ N/mm²

$\frac{P.e}{I} y_t = \frac{(1600 \times 10^3)(433.3)}{2.553 \times 10^{10}} \times 416.7 = 11.32$ N/mm²

$\frac{M_D}{I} \cdot y_t = \frac{233.3 \times 10^6}{2.553 \times 10^{10}} (416.7) = 3.81$ N/mm²

∴ Stress at top $= 6.67 - 11.32 + 3.81 = \boxed{-0.84}$ N/mm² . (tensile)

At bottom : $\frac{P}{A} = 6.67$

$\frac{P.e}{I} y_b = \frac{(1600 \times 10^3)(433.3)}{2.553 \times 10^{10}} \times 583.3 = 15.85$ N/mm²

$\frac{M_D}{I} \cdot y_b = \frac{233.3 \times 10^6}{2.553 \times 10^{10}} (583.3) = 5.33$ N/mm²

∴ Stress at bottom $= 6.67 + 15.85 - 5.33 = \boxed{17.19}$ N/mm² (compressive)

At Service,

At top : $\frac{P}{A} = \frac{0.85\,(1600\times10^3)}{240\times10^3} = 5.67\ N/mm^2$

$\frac{P.e}{I}\cdot y_t = \frac{0.85\,(1600\times10^3)\,(433.3)}{2.553\times10^{10}} \times 416.7 = 9.62\ N/mm^2$

$\frac{M_D}{I}\cdot y_t = 3.81\ N/mm^2$

$\frac{M_L}{I}\cdot y_t = \frac{648\times10^6\times416.7}{2.55\times10^{10}} = 10.58\ N/mm^2$

$\therefore$ Stress at top $= 5.67 - 9.62 + 3.81 + 10.58 = \boxed{10.44}\ N/mm^2$ (compressive)

At bottom : $\frac{P}{A} = 5.67\ N/mm^2$

$\frac{P.e}{I}\cdot y_b = \frac{0.85\,(1600\times10^3)\,(583.3)\,(433.3)}{2.55\times10^{10}} = 13.47\ N/mm^2$

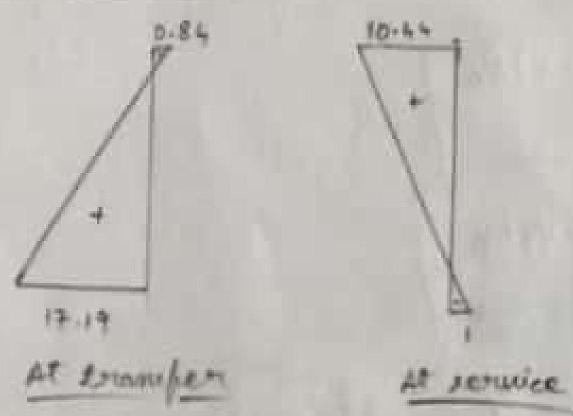
$\frac{M_D}{I}\cdot y_b = 5.33\ N/mm^2$

$\frac{M_L}{I}\cdot y_b = \frac{648\times10^6\times583.3}{2.55\times10^{10}} = 14.82\ N/mm^2$

$\therefore$ Stress at bottom $= 5.67 + 13.47 - 5.33 - 14.82$

$= \boxed{-1}\ N/mm^2$ (tensile)

Stress distribution



f_{cc} allow in compression $= 18\ N/mm^2$

f_{ct} allow in tension $= 1.5\ N/mm^2$

$\therefore$ Stresses are within allowable limits

Design of Sections for Flexure

PSC sections under the action of flexure should satisfy the permissible stress limits at transfer of prestress and at service loads.

Let loss ratio, $\eta = \dfrac{\text{final stress after losses}}{\text{Initial stress}}$

Conditions for stresses at transfer:

Top fibres $\rightarrow$ $f_{sup} = \left(\dfrac{P}{A} - \dfrac{Pe}{Z_t}\right) \boxed{+ \dfrac{M_D}{Z_t}}$

$$\left[\frac{M_D}{Z_t} + f_{sup}\right] \geq f_{t\,allowable} \; (-ve)$$

Bottom fibres $\rightarrow$ $f_{inf} = \left(\dfrac{P}{A} + \dfrac{Pe}{Z_b}\right) \boxed{- \dfrac{M_D}{Z_b}}$

$$\left[- \frac{M_D}{Z_b} + f_{inf}\right] \leq f_{c\,allowable} \; (+ve)$$

Conditions for stresses at service loads:

Top fibres $\rightarrow$ $\left[\eta\,(f_{sup}) + \dfrac{M_D + M_L}{Z_t}\right] \leq f_{c\,allowable} \; (+ve)$.

Bottom fibres $\rightarrow$ $\left[\eta\,(f_{inf}) - \dfrac{(M_D + M_L)}{Z_b}\right] \geq f_{t\,allowable} \; (-ve)$

Minimum Section Modulus:

$$Z_{t\,(min)} \geq \left[\frac{M_L + (1-\eta) M_D}{f_{top\,range}}\right]$$

$$Z_{b\,(min)} \geq \left[\frac{M_L + (1-\eta) M_D}{f_{bottom\,range}}\right]$$

where $f_{top\,range} = f_{tr} = (f_{c\,allow} - \eta f_{t\,allow})$

$f_{bottom\,range} = f_{br} = (\eta f_{c\,allow} - f_{t\,allow})$

Minimum Prestressing force:

$$P_{(min)} = A\left[\frac{Z_t \cdot f_{sup} + Z_b f_{inf}}{Z_t + Z_b}\right]$$

Minimum Eccentricity:

$$e_{(min)} = \frac{Z_t \cdot Z_b \,(f_{inf} - f_{sup})}{A\left[f_{sup}(Z_t) + f_{inf}(Z_b)\right]}$$

Q) A post tensioned PSC beam of rectangular section, 250 mm wide is to be designed for an imposed load of 12 kN/m (UDL) on a span of 12 m. The stress in the concrete must not exceed 17 N/mm² in compression (or) 1.4 N/mm² in tension at any time. Loss of prestress may be assumed to be 15%.

Calculate a) the minimum possible depth of the beam

b) For the section provided, the minimum prestressing force and corresponding eccentricity (10 m)

sol) Live load, $\omega_L = 12$ kN/m

Loss coefficient, $\eta = 0.85$

width, $b = 250$ mm.

$f_{callowable} = 17$ N/mm².

$f_{tallowable} = -1.4$ N/mm².

Live load moment, $M_L = \frac{12 \times 12^2}{8} = 216$ kN-m.

Dead load moment, $M_D = \left(\frac{bh}{10^6}\right) \times \frac{24 \times 12^2}{8} = \frac{432\, bh}{10^6}$ kN-m

where b & h are in mm.

Range of bottom stresses, $f_{br} = (\eta f_{callow} - f_{tallow})$

$= 0.85(17) - (-1.4) = \boxed{15.85}$ N/mm²

(a) Minimum section modulus :

$$Z_b = \left[\frac{M_L + (1-\eta) M_D}{f_{br}}\right]$$

$$\left(\frac{bh^2}{6}\right) = \left[\frac{(216 \times 10^6) + (1-0.85)\,432\,bh}{15.85}\right]$$

substituting $b = 250$ mm

$$h^2 - 24.5h - 325000 = 0$$

$$\boxed{h = 580 \text{ mm}}$$

(b) For the section, 250×580 mm

$$A = (145 \times 10^3)\ mm^2$$

$$Z_b = Z_t = \frac{b \cdot h^2}{6} = \frac{250 \times 580^2}{6} = 14 \times 10^6\ mm^3$$

Self wt. moment, $M_D = 625 \times 10^5$ N-mm

$$(M_D + M_L) = (62.5 \times 10^6) + (216 \times 10^6)$$

$$= \boxed{278.5 \times 10^6} \text{ N-mm}$$

$$f_{sup} = \left[f_{t\,allow} - \frac{M_D}{Z_t}\right] = \left[-1.4 - \frac{62.5 \times 10^6}{14 \times 10^6}\right] = -5.9\ N/mm^2.$$

$$f_{inf} = \left[\frac{f_{t\,allow}}{\eta} + \frac{(M_D + M_L)}{\eta Z_b}\right] = \left[\frac{-1.4}{0.85} + \frac{278.5 \times 10^6}{0.85 \times 14 \times 10^6}\right] = 22\ N/mm^2$$

Minimum prestressing force :

$$P_{min} = \left[\frac{A(f_{inf} \cdot Z_b + f_{sup} \cdot Z_t)}{Z_b + Z_t}\right] = \left[\frac{145 \times 10^3 (22 - 5.9) \times 14 \times 10^6}{28 \times 10^6}\right]$$

$$= \boxed{1170 \text{ kN}}$$

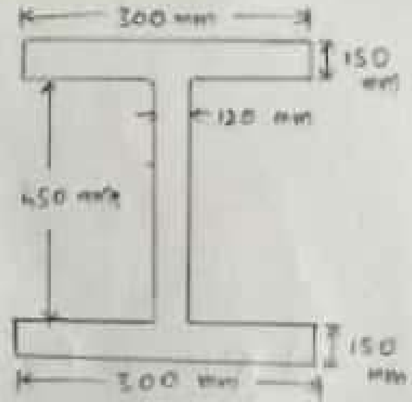
(c) Corresponding eccentricity :

$$e = \left[\frac{Z_t \cdot Z_b (f_{inf} - f_{sup})}{A(f_{sup} \cdot Z_t + f_{inf} \cdot Z_b)}\right] = \left[\frac{(14 \times 10^6)(14 \times 10^6)(22 - (-5.9))}{(145 \times 10^3)(22 - 5.9)\,14 \times 10^6}\right] = \boxed{167.5 \text{ mm}}$$

Q) A PSC girder has to be designed to cover a span of 12 m, to support an U.D.L of 15 kN/m. M-45 grade concrete is used. The permissible stresses in compression may be assumed as 14 N/mm² & 1.4 N/mm² in tension. Assume 15% loss in prestress at service loads. The preliminary section proposed is as shown in the figure.

a) Check the adequacy of the section provided to resist the service loads.

b) Design the minimum prestressing force required and corresponding eccentricity. (10 m)



Sol) Length of the span, $L = 12$ m

Loss ratio, $\eta = 0.85$

$f_{c\,allow} = 14$ N/mm²

$f_{t\,allow} = 1.4$ N/mm²

Live Load = 15 kN/m

$y_t = y_b = 375$ mm

Area of c/s, $A = [2(300 \times 150) + (120 \times 450)] = 144000$ mm²

$$I = \left[\frac{300 \times 750^3}{12} - \frac{180 \times 450^3}{12}\right] = (918 \times 10^7) \text{ mm}^4$$

$$Z = Z_t = Z_b = \left[\frac{I}{y}\right] = \left[\frac{918 \times 10^7}{375}\right] = (24.48 \times 10^6) \text{ mm}^3$$

Self wt. of girder, $w_d = \left[\frac{144000}{375}\right] 24 = 3.456$ kN/m

Dead Load moment, $M_d = \left[\frac{w_d L^2}{8}\right] = \left[\frac{3.456 \times 12^2}{8}\right] = 62.2$ kN-m

Live load moment, $M_L = \left[\frac{w_L \cdot L^2}{8}\right] = \left[\frac{15 \times 12^2}{8}\right] = 270$ kN-m

$f_{bottom\ range} = 0.85(14) + 1.4 = 13.3\ N/mm^2.$

(a) Check for adequacy of section:

$$Z_{b(min)} = \left[\frac{M_L + (1-\eta)M_d}{f_{b\,range}}\right] = \left[\frac{(270\times10^6) + (1-0.85)(62.208\times10^6)}{13.3}\right]$$

$$= (21\times10^6)\ mm^3.$$

$Z_{provided} > Z_{min}.$

∴ Section is adequate to resist service loads

(b) Minimum Prestressing force:

$$f_{top} = \left[f_{tallow} - \frac{M_d}{Z_t}\right] = \left[-1.4 - \frac{(62.208\times10^6)}{(24.48\times10^6)}\right] = -3.94\ N/mm^2$$

$$f_{bottom} = \left[\frac{f_{tallow}}{\eta} + \frac{M_d+M_L}{\eta Z_b}\right] = \left[\frac{-1.4}{0.85} + \frac{(62.2+270)10^6}{0.85(24.48\times10^6)}\right] = 14.36\ N/mm^2$$

$$P_{min} = \frac{A(f_{ptop}Z_t + f_{pbottom}Z_b)}{(Z_t+Z_b)} = \left[\frac{144000(-3.94+14.36)24.48\times10^6}{2\times24.48\times10^6}\right]$$

$$= \boxed{747.28}\ kN.$$

$$e = \left[\frac{Z_t.Z_b(f_b - f_t)}{A(f_t.Z_t + f_b.Z_b)}\right]$$

$$= \left[\frac{(24.48\times10^{12})(14.36-(-3.94))}{144000(-3.94+14.36)\ 24.48\times10^6}\right]$$

$$= \boxed{298.5}\ mm$$

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Objective Questions :

1) Concentric tendons in a concrete beam section induces Uniform Compressive stress

2) Eccentric tendons in a concrete beam section induce Direct & bending stress

3) Resultant stress in the cross section of a prestressed beam comprises of prestress + D.L stress + L.L stress.

4) In a prestressed concrete beam subjected to prestress only, pressure line ~~shifts~~ coincides with the cable line.

5) In a concrete beam subjected to prestress, dead and live loads, the pressure line shifts towards the top of beam as load increases

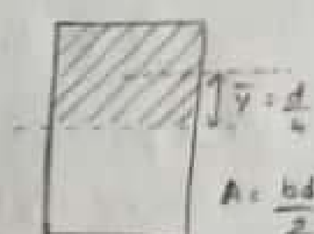
6) In a prestressed concrete beam, the applied loads are resisted by shift in pressure line.

7) A concentrated live load at centre of span of a prestressed concrete beam can be counter balanced by selecting Linearly varying profile with zero eccentricity at centre of span.

8) Uniformly distributed load on a concrete beam can be effectively counter balanced by selecting a parabolic cable.

Shear and Principal Stresses

The shear stress at a point, $\tau_v = \left(\dfrac{V \cdot A \cdot \bar{y}}{I \cdot b}\right)$



where, τ_v = shear stress due to transverse loads

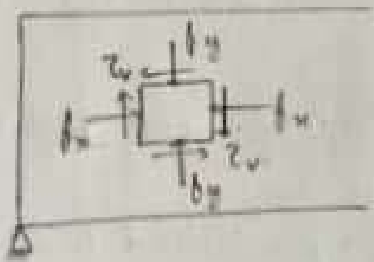
V = Shear Force.

A = Area of c/s in which shear ~~f~~ stress is calculated.

I = M.I about it's centroid

b = breadth of the section at the given point

The shear stress induces principal tensile stresses on diagonal planes.



Both direct stresses f_x & f_y are compressive. Therefore magnitude of principal tensile stress is considerably reduced and some times '0'. Thus under service loads both major & minor principal stresses are compressive, eliminating the risk of diagonal tension cracks.

The max. & min. principal stresses are

$$f_{max/min} = \left[\left(\frac{f_x + f_y}{2}\right) \pm \frac{1}{2}\sqrt{(f_x - f_y)^2 + 4\tau_v^2}\right]$$

Q) What are the different methods of improving shear resistance by prestressing techniques. (2 m)

In general, there are 3 ways of improving the shear resistance of structural concrete members by prestressing techniques.

1) Horizontal (or) axial prestressing
2) Prestressing by inclined (or) sloping cables
3) Vertical (or) transverse prestressing.

Q) A prestressed concrete beam (span = 10 m) of rectangular section, 120 mm × 300 mm is axially prestressed by a cable carrying an effective force of 180 kN. The beam supports a total U.D.L of 5 kN/m which includes self weight of the member. Compare the magnitude of principal tension developed in the beam with and without the axial prestress. (5 m) sol) Area of c/s, $A = (120 \times 300) = 36 \times 10^3\ mm^2$

$$I = \frac{120 \times 300^3}{12} = 27 \times 10^7\ mm^4$$

$$w_{d+L} = 5\ kN/m$$

Max shear force occurring at the support,

$$V_{(max)} = \frac{wL}{2} = \frac{5 \times 10}{2} = 25\ kN$$

Max. shear stress a support.

$$\tau_v = \frac{V.A\bar{y}}{I \cdot b} = \frac{V\left(\frac{bd}{2}\right)\left(\frac{d}{4}\right)}{\frac{bd^3}{12} \cdot b} = \frac{3V}{2bd}$$

$$= \left(\frac{3}{2} \times \frac{25 \times 10^3}{120 \times 300}\right) = 1.05\ N/mm^2$$

Principal stresses without prestress

$$= \pm \frac{1}{2}\sqrt{4\tau_v^2} = \pm \tau_v = \pm 1.05\ N/mm^2$$

(C & T)

Axial prestress, $f_x = \left(\frac{180 \times 10^3}{36 \times 10^3}\right) = 5\ N/mm^2$.

Max & Min. principal stress $= \left(\frac{f_x}{2}\right) \pm \frac{1}{2}\sqrt{f_x^2 + 4\tau_v^2}$

$$= \left(\frac{5}{2}\right) \pm \frac{1}{2}\sqrt{5^2 + (4 \times 1.05^2)} = 2.5 \pm 2.73$$

$= +5.23\ N/mm^2$ (C) & -0.23 (T)

Hence with axial prestress, the principal tension is reduced by

$$\left(\frac{1.05 - 0.23}{1.05}\right) \times 100 = \boxed{78\ \%}$$

Q) A prestressed concrete beam (span = 10 m) of rectangular section, 120 mm × 300 mm is prestressed by a curved cable having an eccentricity of 100 mm at the centre of span and reducing to zero at supports. Estimate the % reduction in the principal tension in comparison with the case of axial prestressing as given in the above problem.

sd) Slope $= \frac{4e}{L} = \frac{4 \times 100}{10 \times 1000} = 0.04$ radians

Vertical component of prestressing force $= (180 \times 0.04) = 7.2$ kN

Horizontal component of prestressing force $= 180$ kN.

$\therefore$ Net shear at support; $V = (25 - 7.2) = 17.8$ kN

Max. shear stress $= \frac{3}{2} \cdot \frac{V}{bh} = \frac{3}{2}\left(\frac{17.8 \times 10^3}{120 \times 300}\right) = 0.74$ N/mm²

Axial prestress, $f_x = \left(\frac{180 \times 10^3}{120 \times 300}\right) = 5$ N/mm²

$$f_{max/min} = \frac{5}{2} \pm \frac{1}{2}\sqrt{5^2 + (4 \times 0.74^2)} = 2.5 \pm 2.62$$

$$= 5.12 \text{ (C) N/mm}^2$$

$$= -0.12 \text{ (T) N/mm}^2$$

% Reduction in principal tensile stress, compared to axial prestressing

$$= \left(\frac{0.23 - 0.12}{0.23}\right) \times 100 = \boxed{48\%}$$

Q) An prestressed I section has the following properties $A = 55 \times 10^3$ mm², $I = 189 \times 10^7$ mm⁴. Statical moment about centroid, $A\bar{y} = 468 \times 10^4$ mm³. Thickness of web $= 50$ mm. It is prestressed horizontally by 24 wires of 5 mm dia and vertically by similar wires at 150 mm centres. All the wires carry a tensile stress of 900 N/mm². Calculate the principal stresses at the centroid, when a shear force of 80 kN acts upon the section.

sd) Shear stress, $\tau_v = \frac{V}{Ib}(A\bar{y}) = \left(\frac{80 \times 10^3 \times 468 \times 10^4}{189 \times 10^7 \times 50}\right) = 3.95$ N/mm².

Horizontal prestress at centroid $= \frac{24 \times \frac{\pi}{4}(5)^2 \times 900}{55 \times 10^3} = 7.75$ N/mm²

Vertical prestress $= \frac{\frac{\pi}{4}(5)^2 \times 900}{150 \times 50} = 2.36$ N/mm²

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$\therefore f_x = 7.75\ N/mm^2$

$f_y = 2.46\ N/mm^2$

$\tau_v = 3.95\ N/mm^2$

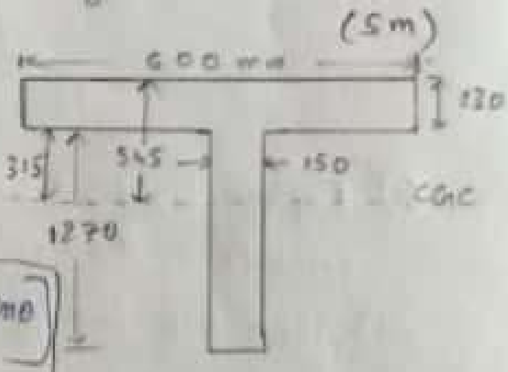
Max & Min principal stresses $= \left(\frac{f_x + f_y}{2}\right) \pm \frac{1}{2}\sqrt{(f_x - f_y)^2 + 4\tau_v^2}$

$= \left(\frac{7.75 + 2.36}{2}\right) \pm \left(\frac{1}{2}\sqrt{(7.75 - 2.36)^2 + 4(3.95)^2}\right)$

$= (5.1 \pm 4.7) = \boxed{9.8\ \&\ 0.4}\ N/mm^2$ (compression)

Shear Resistance

Q) The c/s of a bridge girder is made up of T-section as shown in the fig. The girder is used over a span of 25 m. Tendons with c/s of 2300 mm² are parabolic with an eccentricity of 650 mm at the centre of span and 285 mm at the support section. The effective prestress in the tendons is 900 N/mm² after all losses. If tensile strength of concrete is 1.6 N/mm², estimate ultimate shear resistance of the support section and maximum permissible UDL on the beam using an overall load factor of 2. ($A = 338500\ mm^2$, $I = 665 \times 10^8\ mm^4$)



sol) Ultimate shear resistance for web shear cracking is

$$\boxed{V_{cw} = \left[b_w \cdot \left(\frac{I}{A \cdot \bar{y}}\right)\sqrt{f_{t\,allow}^2 + f_{cp} \cdot f_{t\,allow}} + \eta p \sin\theta\right]}$$

$A \cdot \bar{y} = \left(600 \times 230 \times \left(315 + \frac{230}{2}\right)\right) + \left(150 \times 315 \times \left(\frac{157.5}{}\right)\right)$

$= 665 \times 10^5\ mm^3$

$\frac{I}{A \cdot \bar{y}} = \left(\frac{665 \times 10^8}{665 \times 10^5}\right) = 1000\ mm$

Compressive prestress at the centroid of section, $f_{cp} = \left(\frac{900 \times 2300}{338500}\right)$

$= 6.1\ N/mm^2$

$\eta P = (900 \times 2300) = 207 \times 10^4$ N.

Slope of parabolic cable at support, $\theta = \frac{4e}{L}$

$$= \frac{4(650-285)}{25 \times 1000}$$

$$= 0.585 \approx \sin\theta$$

$$\eta P \sin\theta = (207 \times 10^4 \times 0.585) = 122 \times 10^3 \text{ N}$$

$$\therefore V_{cw} = \left[150(1000)\sqrt{1.6^2 + (6.2 \times 1.6)} + 122 \times 10^3\right]$$

$$= 655 \times 10^3 \text{ N} = 655 \text{ kN}$$

The maximum shear force under working loads $= \frac{V_{cw}}{F.O.S} = \frac{V_{cw}}{2}$

$= 327.5$ kN

U.D.L than can be applied over 25 m,

$$V'_{cw} = \frac{wL}{2}$$

$$w = \frac{2(327.5)}{25}$$

$$\rightarrow \boxed{w = 26.2 \text{ kN/m}}$$

Q) Explain briefly about types of shear cracks and the resistance of concrete to these. (5 m)

A) There are two types of shear cracks (or) shear failures

(1) <u>Failure due to Web-shear cracks</u>

→ Web shear cracks generally start from an interior point, when the local principal tensile stress exceeds the tensile strength of concrete.

→ Web shear cracks are likely to develop in highly prestressed beams with thin webs.

→ The ultimate shear resistance to web-shear failure.

$$\boxed{V_{cw} = \left[\frac{I}{A \cdot \bar{y}} b_w \cdot \sqrt{f_{t\,allow}^2 + f_{cp} \cdot f_{t\,allow}}\right]}$$ for straight tendons

$$\boxed{V_{cw} = \left[b_w \cdot \left(\frac{I}{A \cdot \bar{y}}\right) \sqrt{f_{tallow}^2 + f_{cp} \cdot f_{tallow}} + \eta P \sin\theta \right]}$$

for parabolic (or) curved tendons

→ The value of $\left(\frac{I}{A \cdot \bar{y}}\right)$ varies from 0.67 (D) (for rectangular sections) to 0.55 (D) (for flanged sections)

where $A \cdot \bar{y}$ = First moment of area

b_w = breadth of web

f_{cp} = compressive prestress at the centroid of section

P = initial prestressing force

η = loss ratio

$\sin\theta$ = slope of cable

D = Overall depth of the section

(2) Failure due to Flexure - Shear Cracks :

→ Flexure shear cracks are first initiated by flexural cracks in the inclined direction.

→ Flexure shear cracks develop when the combined shear and flexural tensile stresses produce a principal tensile stress exceeding the tensile strength of concrete.

→ The ultimate shear resistance to Flexure - shear failure is given by

$$\boxed{V_{cf} = \left[1 - 0.55 \frac{f_{pe}}{f_p}\right] \tau_c \cdot b_w \cdot d + \left[\frac{M_0}{M}\right] V}$$

where,

f_{pe} = effective prestressing force after all losses ($\not> 0.6 f_p$)

f_p = characteristic strength of prestressing steel.

τ_c = ultimate shear stress capacity of concrete (as per IS:1343)

d = effective depth of section upto tendons

M_0 = Moment necessary to produce zero stress in extreme fibre.

$= \boxed{0.8 \left[\frac{f_{pt}(I)}{y_b}\right]}$ (f_{pt} = compressive strength at extreme tension fibre of concrete)

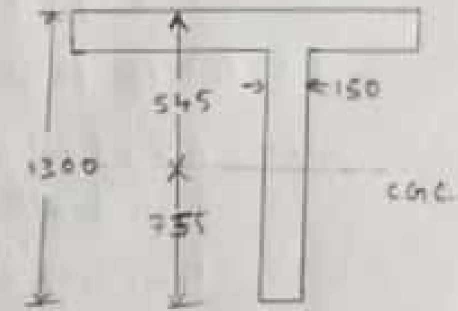
V & M = Shear Force & B.M at the section considered

Q) The cross section of PSC beam is an unsymmetrical T-section as shown in the figure. At a particular section beam is subjected to an ultimate moment, M = 2130 KN-m and shear force, V = 237 KN.

Effective depth, d = 1100 mm

strength of concrete, = 45 N/mm²

Effective prestress at the extreme tensile face of the beam, f_{pt} = 19.3 N/mm²



$I = 665 \times 10^8$ mm⁴

Area of c/s of steel = 2310 mm²

Tensile strength of tendons, f_p = 1500 N/mm²

Effective stress in tendons after all losses, f_{pe} = 890 N/mm²

sol) Moment necessary to produce zero stress in extreme bottom fibre,

$$M_o = \left[\frac{0.8 f_{pt} I}{y_b}\right] = \left[\frac{0.8 \times 19.3 \times 665 \times 10^8}{755}\right]$$

$$= 136 \times 10^7 \text{ N-mm}$$

$$\frac{100 A_{st}}{bd} = \frac{100 \times 2310}{150 \times 1100} = 1.4$$

From IS 1343, τ_c = 0.77 N/mm² (for $\frac{A_p 100}{bd} = 1.4$)

Flexural shear resistance of section is

$$V_{cf} = \left(1 - 0.55 \frac{f_{pe}}{f_p}\right) \tau_c \cdot b_w \cdot d + \frac{M_o}{M} V$$

$$= \left[\left(1 - 0.55 \times \frac{890}{1500}\right) 0.77 \times 150 \times 1100\right] + \left[\left(\frac{136 \times 10^7}{2130 \times 10^6}\right) 237 \times 10^3\right]$$

$$= \boxed{240 \text{ kN}}$$

Since, Actual shear (237 kN) < Ultimate shear (240 kN), min shear R/F is enough.

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Design of Shear Reinforcement

Q) What are IS: 1343 recommendations for shear reinforcement in PSC members. (5m)

Indian Code (IS: 1343) Recommendations:

→ At any given section, the ultimate shear resistance V_c of the concrete alone should be lesser than the values of V_{cw} and V_{cf}.

→ When V (the shear force due to the ultimate loads), is less than V_c, (the shear force which can be carried by the concrete), then minimum shear reinforcement should be provided in the form of stirrups.

Spacing of the stirrups, $S_v = \left[\dfrac{A_{sv} \cdot 0.87 f_y}{0.4\, b}\right]$ (for $V < V_c$)

where, A_{sv} = total c/s area of stirrups effective in shear

b = breadth of member (breadth of web for flanged sections)

f_y = characteristic strength of stirrups ($\ngtr 415\ N/mm^2$)

→ If $V > V_c$,

$$S_v = \left[\frac{A_{sv} \cdot 0.87 f_y \cdot d_t}{V - V_c}\right]$$

d_t = depth from the extreme compression fibre either to the Reinforcement bars (or) to the centroid of tendons, which ever is greater

→ $S_v \ngtr 0.75\, d_t$ (or) $\ngtr 4\, d_w$ } which ever is minimum.

→ When, $V > (1.8\, V_c)$ then S_v should be reduced to $(0.5\, d_t)$

PTO

Q) A prestressed girder of rectangular section 150 mm wide and 300 mm deep is to be designed to support an ultimate shear force of 130 kN. The ~~ultimate~~ uniform prestress across the section is 5 N/mm². Given the characteristic strength of concrete as 40 N/mm² and Fe 415 HYSD bars of 8 mm diameter, design suitable spacing for the stirrups conforming to IS:1343 recommendations. Assume R/F cover as 50 mm. (5m)

Sol) Given Data:

$b_w = 150$ mm $f_{ck} = 40$ N/mm²

$D = 300$ mm $f_{cp} = 5$ N/mm²

$d = 250$ mm $f_y = 415$ N/mm²

$V = 130$ kN $f_t = \boxed{0.24\sqrt{f_{ck}}}$

$= 0.24\sqrt{40} = 1.518$ N/mm².

According to IS:1343, Ultimate shear strength of the section

$$\boxed{V_{cw} = V_c = 0.67\, b_w \cdot D\sqrt{f_t^2 + 0.8 f_{cp} \cdot f_t}}$$

$$V_c = 0.67 \times 150 \times 300\sqrt{[1.518^2 + (0.8 \times 5 \times 1.518)]}$$

$$= 87260 \text{ N} = 87.26 \text{ kN}$$

Balance shear, $V - V_c = [130 - 87.26] = 42.74$ kN

$\because V > V_c$, Using 8 mm ϕ 2 legged stirrups.

$$S_v = \left[\frac{A_{sv} \cdot 0.87 f_y \cdot d}{(V - V_c)}\right] = \left[\frac{2 \times 50.26 \times 0.87 \times 415 \times 250}{42.74 \times 10^3}\right]$$

$$= \boxed{212.28} \text{ mm}$$

Maximum permissible spacing $= 0.75\, d = 0.75\,(250) = \boxed{187.5}$ mm

$\therefore$ Adopt 8 mm diameter, two legged stirrups @ 180 mm centres.