UNIT - IV

6 Transfer of Prestress in Pretensioned Members

Q) Explain briefly about Transmission of Prestressing force by Bond. (5m)

Transmission of Prestressing Force by Bond:

→ The transmission of prestressing force from steel to concrete is generally through a bond comprising (i) adhesion (ii) friction and (iii) shearing resistance.

→ At intermediate points along the length of a beam, the bond stress is resisted by adhesion.

→ In the transfer zone the tendons invariably slip and sink into the concrete, destroying most of the adhesion. Consequently, the bond stresses are due to the friction and shearing resistance.

→ When the bond stress is zero, the stress in steel and concrete reach their maximum values, and uniform stress distribution is prevalent from this section. The length needed for achieving this is termed as transmission length.

→ When a wire is released from it's temporary anchorage on the prestressing bed, the end of wire swells as a result of the recovery. This is to enable the prestressing force to become zero at the end. This is generally referred to as Hoyer's effect.

→ The swelling of the wire is only a few thousandths of a millimetre, but, it produces considerable radial pressure on the concrete.

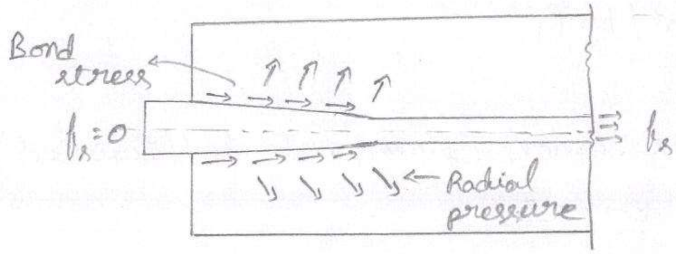


Fig. 4.1: Hoyer's effect

Q) What is meant by Transmission Length. How do you find it. (10 m)

Transmission Length :

→ The length required at the ends of a pretensioned member for the build up of maximum stress in concrete is called Transmission length.

→ It is of great importance, particularly in short pretensioned units, since it controls working B.M & S.F allowable on the section.

→ When the bond stress is zero, stress in steel & concrete reach their maximum values and uniform stress distribution is prevalent from this section. The length needed for achieving maximum & uniform stresses in concrete & steel is called Transmission Length.

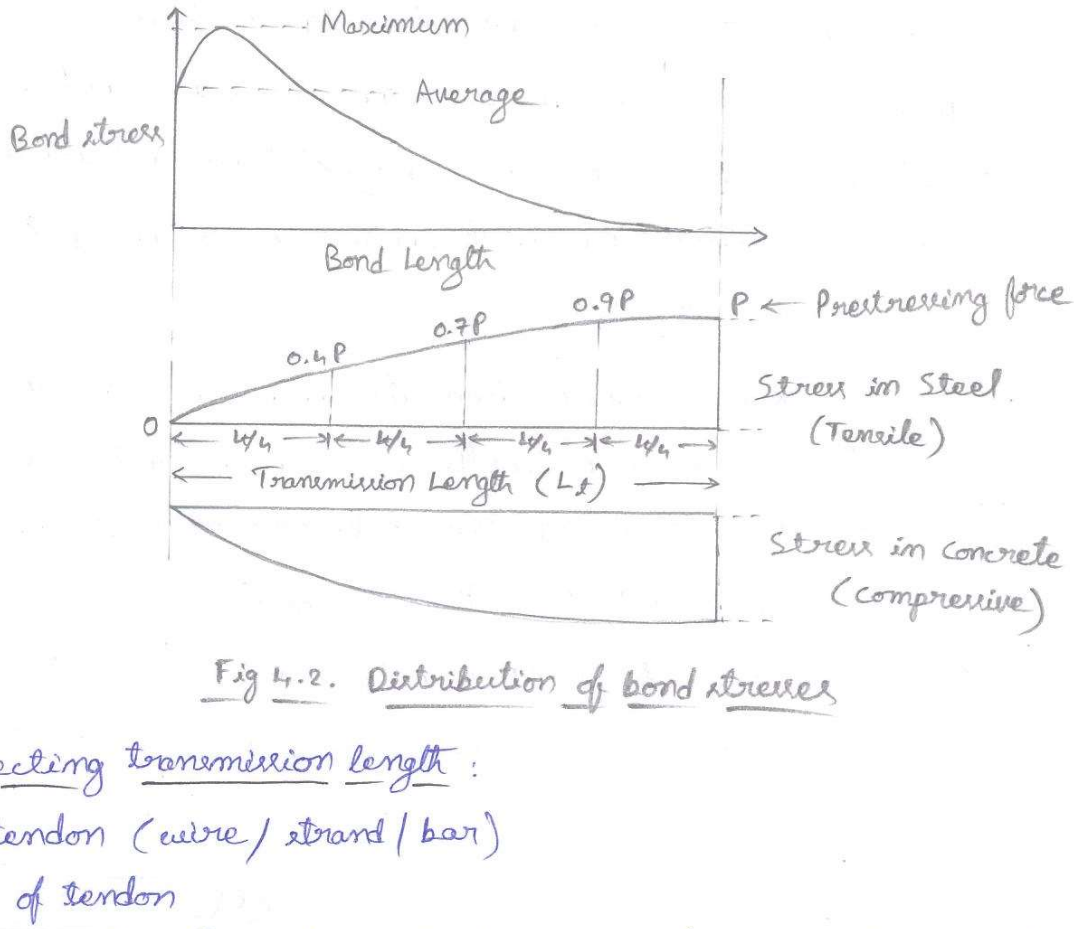


Fig 4.2. Distribution of bond stresses

Factors affecting transmission length :

→ Type of tendon (wire / strand / bar)

→ Diameter of tendon

→ Surface characteristics of the tendon (plain / indented / twisted)

→ Elastic properties of steel & concrete.

→ Strength of concrete at transfer

→ Coefficient of friction between steel and concrete.

Calculation of Transmission Length :

1) Hoyer's expression :

Based on Wedge action, Hoyer developed an expression for transmission length (L_t) as follows,

$$\boxed{L_t = \frac{\phi}{2\mu}(1+\nu_c)\left[\frac{\alpha_e}{\nu_s} - \frac{f_{pi}}{E_c}\right]\left[\frac{f_{pe}}{2f_{pi} - f_{pe}}\right]}$$

where, ϕ = diameter of wire

μ = Coefficient of friction between steel & concrete.

ν_c = Poissons ratio of concrete

ν_s = Poissons ratio of steel

α_e = modular ratio = $\left(\frac{E_s}{E_c}\right)$

E_c = Young's modulus of concrete.

f_{pi} = Initial prestress in steel.

f_{pe} = effective prestress in steel after losses.

→ Under normal range of values, the L_t varies from $\boxed{80 \text{ to } 160\,\phi}$

2) Marshall and Krishna Murthy's expression :

$$\boxed{L_t = \sqrt{\frac{\sqrt{f_{cu}} \times 10^3}{\beta}}}$$

f_{cu} = cube strength of concrete. N/mm^2.

β = constant, depending upon the details of strand & wire.

Reference Table: (no need to write in examination)

Details of wire (or) strand		β
1) 2 mm dia wire	$\longrightarrow$	0.144
2) 5 mm dia wire	$\longrightarrow$	0.0235
3) 7 mm dia wire	$\longrightarrow$	0.0174
4) 10 mm dia, 7 wire strand	$\longrightarrow$	0.144
5) 12.5 mm dia, 7 wire strand	$\longrightarrow$	0.058
6) 18 mm dia, 19 wire strand	$\longrightarrow$	0.0235
7) 19 mm dia, 7 wire strand	$\longrightarrow$	0.0235
8) Twin twisted wires (6.25 mm dia), 7 wire strand	$\longrightarrow$	0.077.

Q) Calculate the length of transmission at the end of a pretensioned beam as per Hoyer's method using the following data.

Span of beam = 50 m

Diameter of wires used = 7 mm

Coefficient of friction between steel and concrete (μ) = 0.1

Poisson's ratio for steel, $\nu_s = 0.3$

Poisson's ratio for concrete, $\nu_c = 0.15$

$E_s = 210\ kN/mm^2$, $E_c = 30\ kN/mm^2$.

Ultimate tensile strength of steel wire, $f_{pu} = 1500\ N/mm^2$.

Initial stress in steel, $f_{pi} = 0.7 f_{pu} = 1050\ N/mm^2$

Effective stress in steel, $f_{pe} = 0.6 f_{pu} = 900\ N/mm^2$.

Sol)
$$L_t = \frac{\phi}{2\mu}(1+\nu_c)\left(\frac{\alpha_e}{\nu_s} - \frac{f_{pi}}{E_c}\right)\left(\frac{f_{pe}}{2f_{pi} - f_{pe}}\right)$$

$$L_t = \frac{7}{2(0.1)}(1+0.15)\left(\frac{7}{0.3} - \frac{1050}{30\times10^3}\right)\left(\frac{900}{(2\times1050)-900}\right)$$

$\Rightarrow L_t = \boxed{700}$ mm

→ The beam is simply supported over a span of 50 m

→ There should be atleast 700 mm of the beam projection beyond the centre of supports at each end (development length)

∴ Total length of beam required $= 50 + (2 \times 0.7)$

$= \boxed{51.4}$ m

Q) Estimate the transmission length at the ends of a pretensioned beam prestressed by 7 mm - dia wires. Assume the cube strength of concrete at transfer as 42 N/mm². (Adopt Krishna Murthy's expression)

sol) Transmission length, $L_t = \sqrt{\dfrac{\sqrt{f_{cu}} \times 10^3}{\beta}}$

For 7 mm diameter smooth wires, $\beta = 0.0174$

$f_{cu} = 42$ N/mm².

$$L_t = \sqrt{\frac{\sqrt{42} \times 10^3}{0.0174}} = 610 \text{ mm} = \frac{610}{7}\phi$$

$$= \boxed{87\phi}$$

If on the other hand, 5 mm diameter wires are used, $\beta = 0.0235$

$$L_t = \sqrt{\frac{\sqrt{42} \times 10^3}{0.0235}} = 525 \text{ mm} = \frac{525}{5}\phi$$

$$= \boxed{105\phi}$$

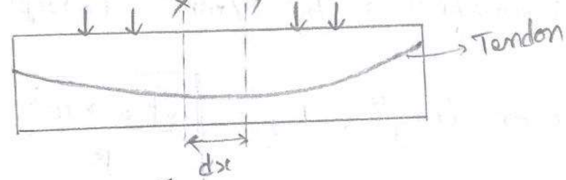
∴ As the diameter of wire decreases, transmission length increases.

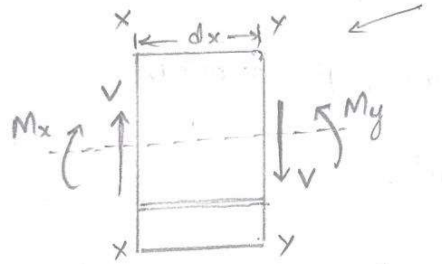
Q) Explain briefly about Flexural Bond Stresses in Prestressed Concrete members. (10 m)

Flexural Bond Stresses:

Pre tensioned and Post tensioned beams with bonded tendons will develop bond stresses between steel and concrete when the sections are subjected to transverse Shear Force due to rate of change of moment along the beam length.

In case of uncracked members, flexural Bond stresses are computed by considering complete section.



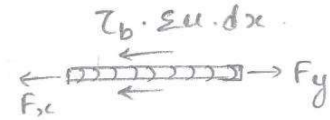


Un cracked member

Let, $M_y - M_x = M$

$\therefore$ Balancing S.F × dist.

$= V \times dx$



Flexular Bond Stresses between steel and concrete,

$$\boxed{\tau_b = \left[\frac{\alpha_e \cdot A_s \cdot y \cdot V}{I \cdot \Sigma u}\right]}$$

where V = Shear Force.

M_x & M_y = Moments at sections XX and YY

Σu = Total perimeter of tendons

y = Distance of tendon from centroidal axis

I = Moment of Inertia

$\alpha_e = \dfrac{E_s}{E_c}$, A_s = Area of steel.

For round wires,

$$\frac{A_s}{\Sigma u} = \frac{n \cdot \frac{\pi}{4} d^2}{n(\pi d)} = \frac{d}{4} = \frac{\phi}{4}.$$

$$\therefore \boxed{\tau_b = \left[\frac{V_y \cdot \alpha_e \cdot \phi}{4I}\right]}$$

In case of cracked flexural members, bond stresses change suddenly at the cracks due to abrupt transfer of tension from concrete to steel at the location of cracks.

The local bond stress can be evaluated in cracked section by using conventional linear theory of RCC sections.

$$\boxed{\tau_b = \frac{V}{Z \cdot \Sigma u}}$$

Z = Lever arm between the Compressive Force in concrete and Tensile Force in steel.

Q) A prestressed beam of rectangular section, 200 mm wide by 500 mm deep is pretensioned by 5 high tensile wires of 7 mm diameter located at an eccentricity of 150 mm. The maximum S.F at quarter span section is 200 kN. If the modular ratio is 6, compute the bond stress developed (a) when the section is uncracked (b) the section is cracked.

Sol) Given Data:

$V = 200$ kN, $b = 200$ mm, $D = 500$ mm

$e = 150$ mm, $\phi = 7$ mm, $\alpha_e = 6$, $I = \left[\frac{bD^3}{12}\right] = \frac{200 \times 500^3}{12}$

$d = 400$ mm $Z = 125 + 150 = 275$ $= 21 \times 10^8$ mm^4.

$\Sigma u = 5(\pi(7)) = 109.9$ mm

(a) Section Uncracked :

$$\tau_b = \left[\frac{V_y \cdot \alpha_e \cdot \phi}{4I}\right] = \left[\frac{200 \times 10^3 \times 150 \times 6 \times 7}{4 \times 21 \times 10^8}\right]$$

$$= \boxed{0.15} \text{ N/mm}^2.$$

(b) Section Cracked:

$$\tau_b = \left[\frac{V}{Z \cdot \Sigma u}\right] = \left[\frac{200 \times 10^3}{275 \times 109.9}\right]$$

$$= \boxed{6.61} \text{ N/mm}^2.$$

Stress Distribution in End Block of Post tensioned members.

Q) Explain briefly about stress distribution in the end block of post tensioned members.

Stress Distribution in End Block :

→ In most post-tensioned members, the prestress is induced by anchorages at end faces. As a result of this, large forces, concentrated over relatively small areas, are applied on the end blocks.

→ These highly discontinuous forces which are applied at the end, while changing progressively to continuous linear distribution, develop transverse and shear stresses.

→ The zone between the end of the beam and the section where only longitudinal stress exists is referred to as the anchorage zone (or) end block.

→ The transverse stresses developed in anchorage zone are tensile in nature, since concrete is weak in tension, sufficient reinforcement should be provided.

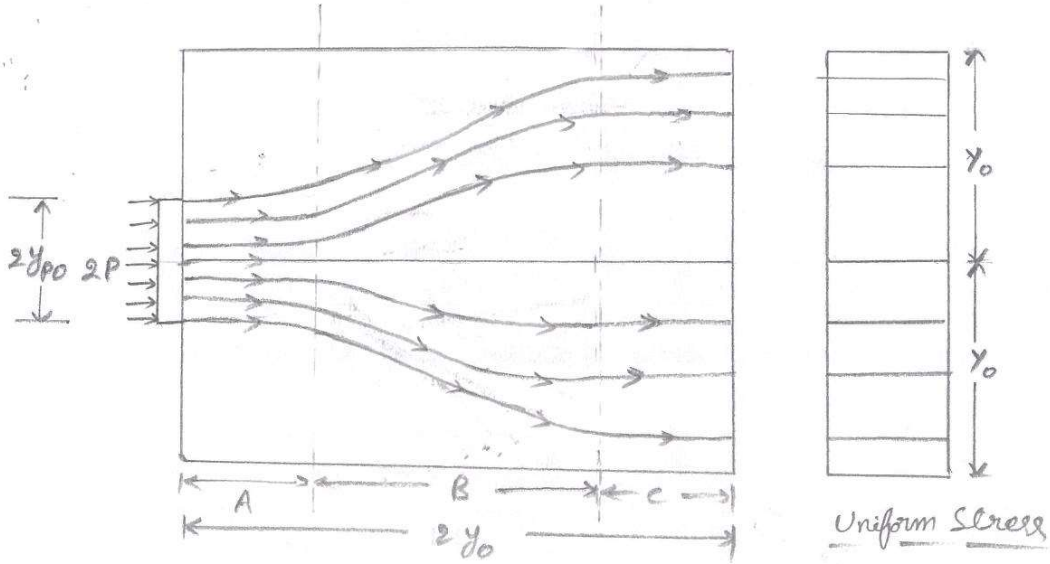


Fig. 4.3 Stress Distribution in End Block using Single anchor plate

→ In Zone A, the curvature of force line being convex towards the centre line of the block, induces compressive stresses in Zone A.

→ In Zone B, the curvature is reversed in direction and struts tend to deflect outwards, separating from each other and consequently developing transverse tensile stresses.

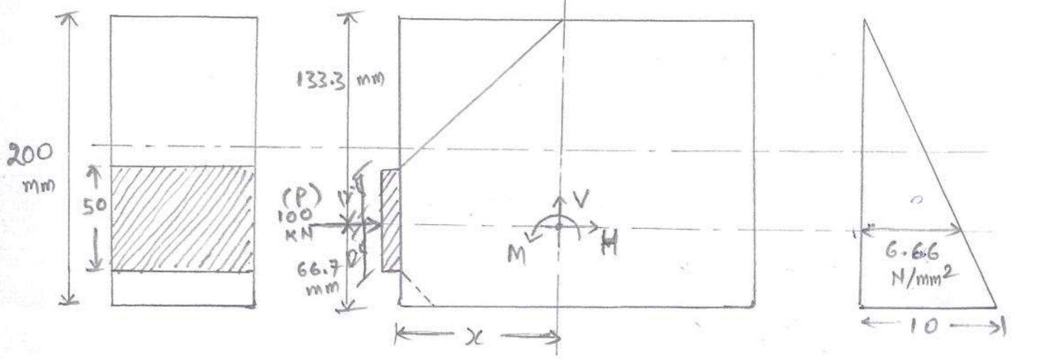
→ In Zone C, the struts are straight and parallel so that no transverse stresses are induced; only longitudinal stresses develop in this zone.

Magnel's Method

Q) The end block of a prestressed concrete beam, 100 mm wide and 200 mm deep, supports an eccentric prestressing force of 100 kN, the line of action of which coincides with the bottom kern of the section. The depth of the anchor plate is 50 mm. Estimate the magnitude and position of the principle tensile stress on a horizontal plane passing through the centre of anchorage plate. Refer the figure given below.

(For $\frac{x}{h} = 0.5$, $K_1 = -5$, $K_2 = 2$, $K_3 = 1.25$). Use Magnel's Method.

(2)



Direct Stress

$\frac{100 \times 10^3}{200 \times 100}$

Fig. 4.4 Forces acting on the end block

sd) $M = \left[\left(\frac{1}{2} \times 6.66 \times 133.3 \times 100\right)\left[\frac{1}{3} \times 133.33\right]\right] - \left[(50 \times 10^3)(12.5)\right]$

$= \boxed{1345 \times 10^3}$ N-mm

P = 100 KN

25 25

$R_A = 50$ $R_B = 50$

$M = \left(\frac{W}{2}\right)\left(\frac{L}{4}\right)$

$= \frac{100 \times 10^3}{2} \times \frac{50}{4}$

$\cancel{H}\ V = \left[\left(\frac{1}{2} \times 6.66 \times 133.3 \times 100\right) - (50 \times 10^3)\right]$

$= \boxed{-5612}$ N

$H = 0$

The principal tensile stress is critical at $x = 0.5h$ from end face.

$\therefore\ x = 0.5(200)$

$\boxed{x = 100 \text{ mm}}$ from the end face.

Stresses, $f_h = +6.66$ N/mm².

According to Magnel's Method, $\boxed{f_v = K_1\left[\frac{M}{bh^2}\right] + K_2\left[\frac{H}{bh}\right]}$

$\cancel{K_1 = K}\quad = -5\left[\frac{1345 \times 10^3}{100 \times 200^2}\right] + 0$

$\boxed{f_v = -1.66}$ N/mm².

According to Magnel's Method,

$$\boxed{\tau = k_3 \left[\frac{V}{bh}\right]}$$

$$\tau = 1.25 \left[\frac{-5612}{100 \times 200^2}\right] = \boxed{-0.35} \; N/mm^2 .$$

Principal tensile stress,

$$f_{min} = \left[\frac{f_h + f_v}{2}\right] \pm \frac{1}{2}\sqrt{(f_h - f_v)^2 + 4\tau^2}$$

$$= \left[\frac{+6.66 - 1.66}{2}\right] - \frac{1}{2}\sqrt{(6.66 + 1.66)^2 + 4(-0.35)^2}$$

$$f_{min} = \boxed{-1.7} \; N/mm^2 .$$

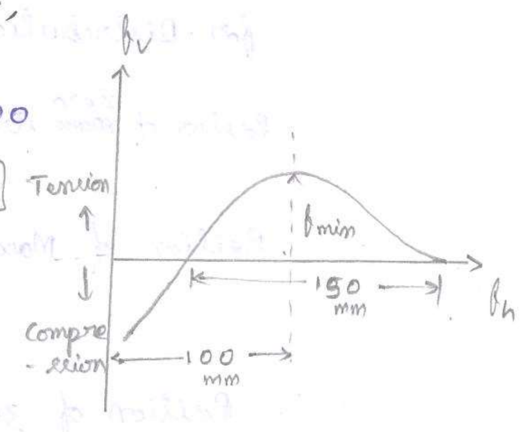
As per Magnel's Distribution of tensile stresses,

Bursting force, $F_{bst} = \left[\frac{2}{3} \times 150 \times f_{min}\right] 100$

[Area of curve × distance]

$$= \left[\frac{2}{3} \times 150 \times 1.7\right] 100$$

$$F_{bst} = \boxed{1700 \; N.}$$



Distribution of Tensile stress

Guyon's Method

Bursting Tensile Force,

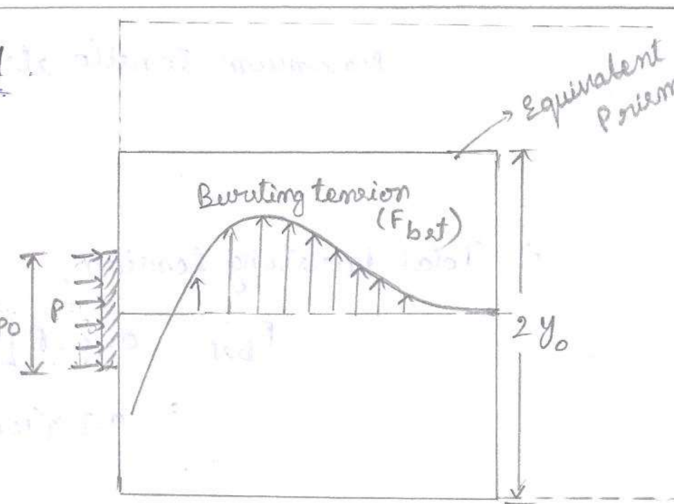
$$\boxed{F_{bst} = 0.3\,P\left[1 - \left(\frac{y_{po}}{y_o}\right)^{0.58}\right]}$$

where, P = anchorage force,

y_{po}/y_o = distribution ratio.

$2y_{po}$ = depth of the anchorage plate.

$2y_o$ = depth of the equivalent prism.



Force Distribution (Guyon)

Q) The end block of a prestressed concrete beam, 100 mm wide and 200 mm deep, supports an eccentric prestressing force of 100 KN, as detailed in fig. 4.4. Compute the maximum tensile stress and total splitting tension using Guyon's method.

sol) $P = 100$ KN.

$2y_{po} = 50$ mm

$2y_o = 133$ mm

$\therefore$ Distribution ratio, $\frac{y_{po}}{y_o} = \frac{50}{133} = \boxed{0.375}$

From the guyon's tables,

for Distribution ratio of $\boxed{0.375}$,

Position of ~~max.~~ Zero stress, $\boxed{\frac{x}{2y_o} = 0.175}$ ~~and $\frac{\text{Max stress}}{\text{Avg stress}} =$~~

Position of Max stress, $\boxed{\frac{x}{2y_o} = 0.382}$, $\boxed{\frac{\text{Max. tensile stress}}{\text{Avg stress}} = 0.285}$

$\therefore$ Position of zero stress, $x = 0.175(133) = 23.5$ mm.

Position of Max. stress, $x = 0.382(133) = 51$ mm.

Maximum tensile stress $= 0.285\left[\frac{P}{A}\right]$

$= 0.285\left[\frac{100 \times 10^3}{100 \times 133}\right] = 2.13$ N/mm2.

$\therefore$ Total bursting tension,

$F_{bst} = 0.3 . P\left[1 - \left(\frac{y_{po}}{y_o}\right)^{0.58}\right]$

$= 0.3 \times (100 \times 10^3)\left[1 - (0.375)^{0.58}\right] = \boxed{13015 \text{ N}}$

Zielinski and Rowe's Method

The recommended equations are,

Max. Tensile stress, $$\boxed{f_{v(max)} = f_c \left[0.98 - 0.825\left(\frac{y_{po}}{y_o}\right)\right]}$$

Valid for ratio of $\left(\frac{y_{po}}{y_o}\right) = \boxed{0.3 \text{ to } 0.7}$.

Bursting tension, $$F_{bst} = P_K \left[0.48 - 0.4\left(\frac{y_{po}}{y_o}\right)\right]$$

If the allowance is made for tension taken by concrete, the corrected value of the bursting tension is given by.

$$F_{bst\,(corrected)} = F_{bst}\left[1 - \left(\frac{f_t}{f_{v(max)}}\right)^2\right]$$

where, $2y_o$ = Side of equivalent prism.

$2y_{po}$ = Side of loaded (or) punching area.

y_{po}/y_o = ratio of sides of loaded to bearing area of prism

f_v = transverse tensile stress.

f_c = average compressive stress in the prism.

P_K = applied compressive force on the end block.

F_{bst} = bursting tension

$f_{v(max)}$ = maximum transverse tensile stress.

f_t = permissible tensile strength of concrete

The reinforcement required to resist the bursting tension is to be arranged between $0.2\,y_o$ and $2\,y_o$ where intensity of stress is maximum.

Q) The end block of a prestressed beam, 200 mm wide and 300 mm deep, has two Freyssinet anchorages (100 mm diameter) with their centres at 75 mm from the top and bottom of the beam. The force transmitted by each anchorage being 200 KN, estimate the maximum tensile stress and the bursting tension developed using Zielinski method.

sol) Anchorage diameter = 100 mm

Equivalent side of square, $2y_{po} = \sqrt{\frac{\pi}{4} \times 100^2}$

$= 89$ mm.

So Prism Square Area = circle area.

$(2y_{po})^2 = \frac{\pi}{4}(100)^2$.

Side of the surrounding prism, $2y_o = 150$ mm.

$$\frac{y_{po}}{y_o} = \frac{89}{150} = 0.593$$

Average compressive stress, $f_c = \left[\frac{200 \times 10^3}{150 \times 150}\right]$

$= 8.9 \ N/mm^2$.

Tensile stress, $f_{v(max)} = f_c\left[0.98 - 0.825\left(\frac{y_{po}}{y_o}\right)\right]$

$= 8.9\left[0.98 - 0.825(0.593)\right]$

$\boxed{f_{v(max)} = 4.45 \ N/mm^2}$

Transverse tension, $F_{bst} = P_K\left[0.48 - 0.4\left(\frac{y_{po}}{y_o}\right)\right]$

$= 200 \times 10^3\left[0.48 - 0.4(0.593)\right]$

$\doteq 5000$ N $= \boxed{50 \text{ KN.}}$

Indian Code Provisions

Q) Write a short notes on Codal provisions for Bond and Transmission length. (5m)

IS : 1343 provisions for Bond and Transmission length.

→ The general provisions in the IS : 1343 for transmission length are expressed in terms of the wire, bar (or) strand, taking into consideration the surface characteristics of the tendons.

→ In the absence of values based on actual test results, the following values are recommended for the transmission length.

1) Plain and indented wires   100 ϕ

2) Crimped wires   65 ϕ

3) Strands   30 ϕ

→ The concrete strength at transfer should not be less than 35 N/mm^2. and the tendons are released gradually.

→ The recommended values of transmission length can be applied to the wires of diameter not exceeding 5 mm and strands of diameter not exceeding 18 mm.

→ It is recommended that one-half of the transmission length shall overhang the support in simply supported beams and the whole of the transmission length should extend beyond the supports in the case of fixed ends.

Q) Write a short notes on Indian Code Provisions on Anchorage zone Reinforcement (5m)

IS : 1343 Provisions for Anchorage Zone Reinforcement:

→ The provisions of main reinforcement in the anchorage zone are made to withstand the bursting tension.

→ The bursting tension is determined by the transverse stress distribution on the critical axis, usually coinciding with line of action of the largest individual force.

→ Mats, helics, loops (or) links are generally provided in perpendicular directions. Tests by Zielinski and Rowe have shown that helical reinforcement is more efficient than mat reinforcement.

→ In case of short available bond lengths, loops, hooks (or) right angle bends are necessary, (even with deformed bars)

→ In cases where spalling (or) secondary tension develops at the corners, suitable steel in the form of hair pin bars should be provided to prevent the failure of corner zones.

→ The computation of the bursting tensile force in the end blocks as per IS:1343 is based on the work of Zielinski and Rowe.

Bursting tensile force, $\boxed{F_{bst} = P_K \left[0.32 - 0.3\left(\frac{y_{po}}{y_o}\right)\right]}$

where, P_K = tendon jacking force.

$\left(\frac{y_{po}}{y_o}\right)$ = distribution ~~ratio~~ ratio

→ The reinforcement is designed to resist the bursting tension and it is assumed to act at it's design strength $0.87 f_y$.

→ The designed reinforcement is distributed in the zone of $(0.2\,y_o)$ to (y_o) from the loaded face of end block.

→ If multiple anchorages are used, the end block is divided into a series of symmetrically loaded prisms and each prism is analyzed for bursting tensile forces.

Q) The end block of a prestressed concrete girder is 200 mm wide by 300 mm deep. The beam is post-tensioned by two Freyssinet anchorages each of 100 mm diatmeter with their centres located at 75 mm from the top and bottom of the beam. The force transmitted by each anchorage being 2000 KN. Compute the bursting force and design suitable reinforcements according to IS:1343 code provisions

sol) Anchorage diameter = 100 mm.

Equivalent side of the square $= 2y_{po} = \sqrt{\frac{\pi}{4} \times 100^2}$

$= 89$ mm

Side of the surrounding prism $= 2y_o = 150$ mm.

Distribution ratio $= \left(\frac{2y_{po}}{2y_o}\right) = \left(\frac{89}{150}\right) = 0.593$

Bursting tensile force, $F_{bst} = P_K \left[0.32 - 0.3\left(\frac{y_{po}}{y_o}\right)\right]$

$= 2000\left[0.32 - 0.3(0.593)\right] = \boxed{286 \text{ kN}}$

Using 10 mm diameter mild steel link with $f_y = 260$ N/mm^2.

Number of bars required $= \left[\frac{286 \times 1000}{0.87 \times 260 \times 79}\right] = 16.$

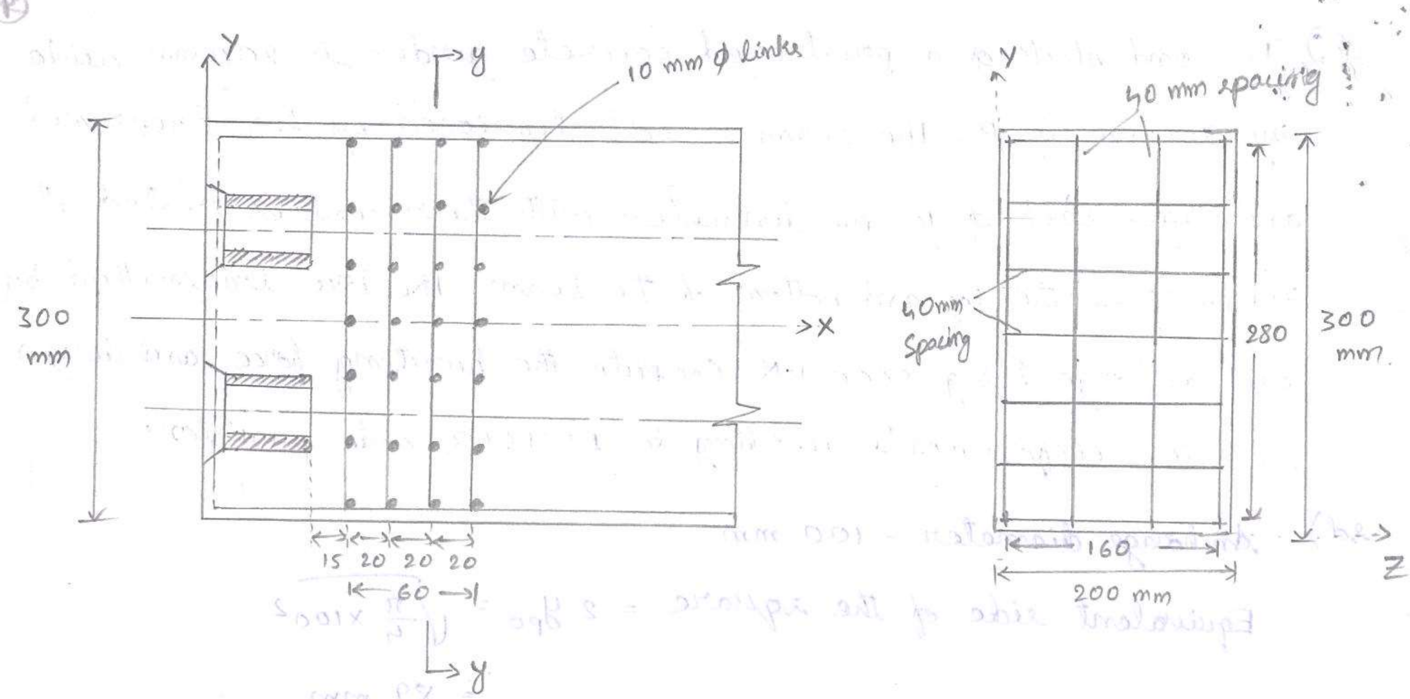
The R/F is arranged between $0.2y_o$ and y_o.

$0.2y_o = [0.2 \times 75] = 15$ mm and $y_o = 75$ mm

$\therefore$ Adopting 4 bars in Z-direction, spacing $= \frac{200 - \overset{\text{cover}}{40}}{4} = 40$ mm

→ Adopting 4 bars in y-direction for each anchorage in which bars along centre line are common to both anchorages, thus total no. of bars in y-direction $= 3+1+3 = 7$, spacing $= \frac{300 - \overset{\text{cover}}{20}}{7} = 40$ mm.

→ Providing the reinforcement of Y-Z mesh for each cross section (Y-Z) at an interval of 20 mm (between, 15 mm & 75mm i.e, 60 mm)



Longitudinal section

cross section

Arrangement of Anchorage Zone Reinforcement

Objective type Questions

1) Transfer of prestress in pretensioned members is due to

Ans) Bond b/w Steel & concrete

2) The transmission length at the end of pretensioned member depends upon diameter of the high tensile wire

3) The bond stress between steel wire and concrete is maximum at end face.

4) At the end face of a pretensioned beam the tensile stress in steel is zero

5) The transmission length in a pretensioned beam is inversely proportiona to coefficient of friction between steel and concrete

6) The transmission length as per IS:1343 is 30ϕ

7) Very near the end face of a pretensioned beam, the stresses developed in concrete are tensile.

8) At a distance equal to the transmission length from the end face of a pretensioned beam, the force in the tendon is equal to the initial prestressing force.

9) In the anchorage zone of a post tensioned PSC beam, the stress distribution is ~~Biaxial~~ triaxial.

10) The anchorage zone in a post-tensioned PSC beam extends over a length of depth of the beam

11) In the anchorage zone of a post tensioned beam splitting cracks due to bursting tension develop in the direction of horizontal axis of beam

12) The transverse tensile stress in the anchorage zone depends mainly on the ratio of anchorage depth to overall depth

13) The concept of equivalent prism was developed by Rowe & Zeilinski

14) Experimental investigations on end blocks of concrete prisms were conducted by Rowe & Zeilinski

15) The bursting tension in the anchorage zone is a function of the ratio of depth of anchorage to depth of equivalent prism

7) Very near the end face of a pretensioned beam, the stresses developed in concrete are tensile

8) At a distance equal to the transmission length from the end face of a pretensioned beam the force in the tendon is equal to the initial prestressing force

9) In the anchorage zone of a post tensioned PSC beam, the stress distribution is [illegible]

10) The anchorage zone in a post-tensioned PSC beam extends over a length of depth of the beam

11) In the anchorage zone of a post tensioned beam splitting cracks due to bursting tension develop in the direction of horizontal axis of beam

12) The transverse tensile stress in the anchorage zone depends mainly on the ratio of anchorage depth to prism depth

13) The concept of equivalent prism was developed by Rowe & Zielinski

14) Experimental investigations on end blocks of concrete prisms were conducted by Rowe & Zielinski

15) The bursting tension in the anchorage zone is a function of the ratio of depth of anchorage to depth of equivalent prism