DETECTION OF RADAR SIGNALS IN NOISE

Introduction

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Response characteristics and derivation

Correlation function and cross correlation receiver

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Matched filter with non-white noise

- The two basic operations performed by radar are **(1)** ***detection*** of the presence of reflecting objects, and (2) ***extraction*** of information from the received waveform to obtain such target data as position, velocity, and perhaps size.
- In this chapter some aspects of the problem of detecting radar signals in the presence of noise will be considered. Noise ultimately limits the capability of any radar.

Matched-Filter Receiver

- A network whose frequency-response function maximizes the output peak-signal-to- mean-noise (power) ratio is called a matched filter. This criterion, or its equivalent, is used for the design of almost all radar receivers.
- The frequency-response function, denoted H(f), expresses the relative amplitude and phase of the output of a network with respect to the input when the input is a pure sinusoid.
- The magnitude |H(f)|of the frequency-response function is the receiver amplitude pass band characteristic.
- If the bandwidth of the receiver pass band is wide compared with that occupied by the signal energy, extraneous noise is introduced by the excess bandwidth which lowers the output signal-to-noise ratio. On the other hand, if the receiver bandwidth is narrower than the bandwidth occupied by the signal, the noise energy is reduced along with a considerable part of the signal energy.

- The net result is again a lowered signal-to-noise ratio. Thus there is an optimum bandwidth at which the signal-to-noise ratio is a maximum. This is well known to the radar receiver designer.
- The rule of thumb quoted in pulse radar practice is that the receiver bandwidth B should be approximately equal to the reciprocal of the pulse width τ. This is a reasonable approximation for pulse radars with conventional superheterodyne receivers. It is not generally valid for other waveforms.
- The exact specification of the optimum receiver characteristic involves the frequency- response function and the shape of the received waveform.
- The receiver frequency-response function, is assumed to apply from the antenna terminals to the output of the IF amplifier.
- The second detector and video portion of the well designed radar superheterodyne receiver will have negligible effect on the output signal-to- noise ratio if the receiver is designed as a matched filter. Narrow banding is most conveniently accomplished in the IF.
- The bandwidths of the RF and mixer stages of the normal superheterodyne receiver are usually large compared with the IF bandwidth. Therefore the frequency-response function of the portion of the receiver included between the antenna terminals to the output of the IF amplifier is taken to be that of the IF amplifier alone.

- Thus we need only obtain the frequency-response function that maximizes the signal-to-noise ratio at the output of the IF. The IF amplifier may be considered as a filter with gain.

- For a received waveform s(t) with a given ratio of signal energy E to noise energy No (or noise power per hertz of bandwidth), North showed that the frequency-response function of the linear, time-invariant filter which maximizes the output peak-signal-to-mean- noise (power) ratio for a fixed input signal-to-noise (energy) ratio is

$$H(f) = G_a S^*(f) \exp(-j2\pi f t_1)$$

where $S(f) = \int_{-\infty}^{\infty} s(t) \exp(-j2\pi f t)\, dt$ = voltage spectrum (Fourier transform) of input signal

$S^*(f)$ = complex conjugate of $S(f)$

t_1 = fixed value of time at which signal is observed to be maximum

G_a = constant equal to maximum filter gain (generally taken to be unity)

- The noise that accompanies the signal is assumed to be stationary and to have a uniform spectrum (white noise). It need not be Gaussian.

- The filter whose frequency-response function is given by Eq. above has been called the North filter, the conjugate filter, or more usually the matched filter. It has also been called the Fourier transform criterion.

- The frequency-response function of the matched filter is the conjugate of the spectrum of the received waveform except for the phase shift exp (- j2Πft1). This phase shift varies uniformly with frequency. Its effect is to cause a constant time delay.

- The frequency spectrum of the received signal may be written as an amplitude spectrum |S(f)|and a phase spectrum exp [- jφs (f)].

- The matched- filter frequency-response function may similarly be written in terms of its amplitude and phase spectra |H(f)| and exp [- jφm(f)]. Ignoring the constant Ga, Eq. above for the matched filter may then be written as

$$|H(f)| \exp[-j\phi_m(f)] = |S(f)| \exp\{j[\phi_s(f) - 2\pi f t_1]\}$$

or

$$|H(f)| = |S(f)|$$

and

$$\phi_m(f) = -\phi_s(f) + 2\pi f t_1$$

- Thus the amplitude spectrum of the matched filter is the same as the amplitude spectrum of the signal, but the phase spectrum of the matched filter is the negative of the phase spectrum of the signal plus a phase shift proportional to frequency.

- The matched filter may also be specified by its impulse response h(t), which is the inverse Fourier transform of the frequency-response function.

$$h(t) = \int_{-\infty}^{\infty} H(f) \exp(j2\pi ft)\, df$$

- Physically, the impulse response is the output of the filter as a function of time when the input is an impulse (delta function).

Since $S^*(f) = S(-f)$, we have

$$h(t) = G_a \int_{-\infty}^{\infty} S(f) \exp[j2\pi f(t_1 - t)]\, df = G_a s(t_1 - t)$$

- A rather interesting result is that the impulse response of the matched filter is the image of the received waveform; that is, it is the same as the received signal run backward in time starting from the fixed time t1.
- Figure 1 shows a received waveform s(t) and the impulse response h(t) of its matched filter. The impulse response of the filter, if it is to be realizable, is not defined for t< 0. (One cannot have any response before the impulse is applied.) Therefore we must always have t < t1.

- This is equivalent to the condition placed on the transfer function H(f) that there be a phase shift exp (-j2Πft1). However, for the sake of convenience, the impulse response of the matched filter is sometimes written simply as s(- t).

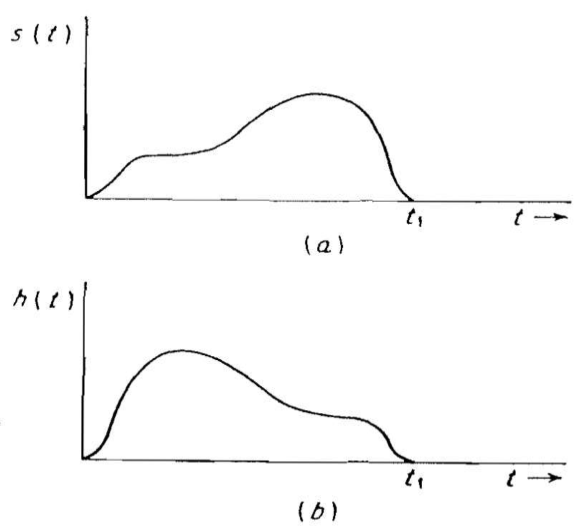


Fig.1 (a) Received waveform s(t); (b) impulse response h(t) of the matched filter.

Derivation of the matched-filter characteristic

- The frequency-response function of the matched filter has been derived by a number of authors using either the calculus of variations or the Schwartz inequality. We shall derive the matched-filter frequency-response function using the Schwartz inequality.
- We wish to show that the frequency-response function of the linear, time-invariant filter which maximizes the output peak-signal-to-mean-noise ratio is

$$H(f) = G_a S^*(f) \exp(-j2\pi f t_1)$$

- When the input noise is stationary and white (uniform spectral density). The ratio we wish to maximize is

$$R_f = \frac{|s_o(t)|^2_{\max}}{N}$$

- Where |so(t)| max = maximum value of output signal voltage and N = mean noise power at receiver output. The ratio Rf is not quite the same as the signal-to-noise ratio which has been considered in the radar equation.

- The output voltage of a filter with frequency-response function H(f) is

$$|s_o(t)| = \left| \int_{-\infty}^{\infty} S(f) H(f) \exp(j2\pi f t)\, df \right|$$

- Where S(f) is the Fourier transform of the input (received) signal. The mean output noise power is

$$N = \frac{N_o}{2} \int_{-\infty}^{\infty} |H(f)|^2\, df$$

- Where No is the input noise power per unit bandwidth. The factor appears before the integral because the limits extend from - ∞ to +∞, whereas No is defined as the noise power per cycle of bandwidth over positive values only. Assuming that the maximum value of |so(t)2| occurs at time t = t1, the ratio Rf becomes

$$R_f = \frac{\left| \int_{-\infty}^{\infty} S(f) H(f) \exp(j2\pi f t_1)\, df \right|^2}{\frac{N_0}{2} \int_{-\infty}^{\infty} |H(f)|^2\, df}$$

Schwartz's inequality states that if P and Q are two complex functions, then

$$\int P^* P\, dx \int Q^* Q\, dx \geq \left| \int P^* Q\, dx \right|^2$$

- The equality sign applies when P = kQ, where k is a constant. Letting

$$P^* = S(f)\exp(j2\pi f t_1) \quad \text{and} \quad Q = H(f)$$

and recalling that

$$\int P^* P\, dx = \int |P|^2\, dx$$

- We get, on applying the Schwartz inequality to the numerator of Eq. earlier, we get

$$R_f \le \frac{\int_{-\infty}^{\infty} |H(f)|^2\, df \int_{-\infty}^{\infty} |S(f)|^2\, df}{\frac{N_0}{2}\int_{-\infty}^{\infty} |H(f)|^2\, df} = \frac{\int_{-\infty}^{\infty} |S(f)|^2\, df}{\frac{N_0}{2}}$$

From Parseval's theorem,

$$\int_{-\infty}^{\infty} |S(f)|^2\, df = \int_{-\infty}^{\infty} s^2(t)\, dt = \text{signal energy} = E$$

Therefore we have

$$R_f \le \frac{2E}{N_0}$$

- The frequency-response function which maximizes the peak-signal-to-mean-noise ratio Rf may be obtained by noting that the equality sign in Eq. applies when P = kQ, or

$$H(f) = G_a S^*(f)\exp(-j2\pi f t_1)$$

Where the constant k has been set equal to $1/G_a$.

The matched filter and the correlation function

- The output of the matched filter may be shown to be proportional to the input signal cross-correlated with a replica of the transmitted signal, except for the time delay t1.
- The cross correlation function R(t) of two signals y(λ) and s(λ), each of finite duration, is defined as

$$R(t) = \int_{-\infty}^{\infty} y(\lambda)s(\lambda - t)\, d\lambda$$

- The output yo(t) of a filter with impulse response h(t) when the input is yin(t) = s(t) + n(t) is

$$y_0(t) = \int_{-\infty}^{\infty} y_{in}(\lambda)h(t - \lambda)\, d\lambda$$

- If the filter is a matched filter, then h(λ) = s(t1 - λ) and Eq. above becomes

$$y_0(t) = \int_{-\infty}^{\infty} y_{in}(\lambda)s(t_1 - t + \lambda)\, d\lambda = R(t - t_1)$$

- Thus the matched filter forms the cross correlation between the received signal corrupted by noise and a replica of the transmitted signal.
- The replica of the transmitted signal is "built in" to the matched filter via the frequency-response function.
- If the input signal yin (t) were the same as the signal s(t) for which the matched filter was designed (that is, the noise is assumed negligible), the output would be the autocorrelation function.

Cross correlation receiver(correlation detection)

$$y_0(t) = \int_{-\infty}^{\infty} y_{in}(\lambda) s(t_1 - t + \lambda)\, d\lambda = R(t - t_1)$$

- Equation above describes the output of the matched filter as the cross correlation between the input signal and a delayed replica of the transmitted signal.
- This implies that the matched-filter receiver can be replaced by a cross-correlation receiver that performs the same mathematical operation as shown in Fig.

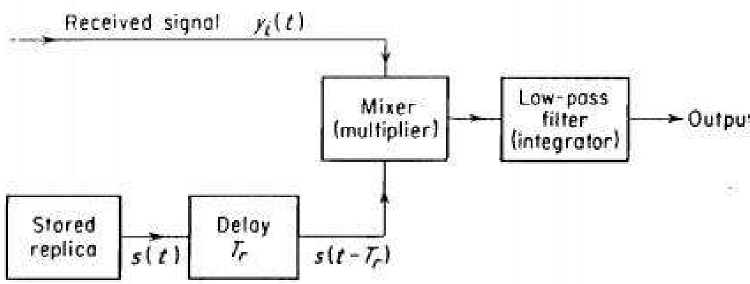
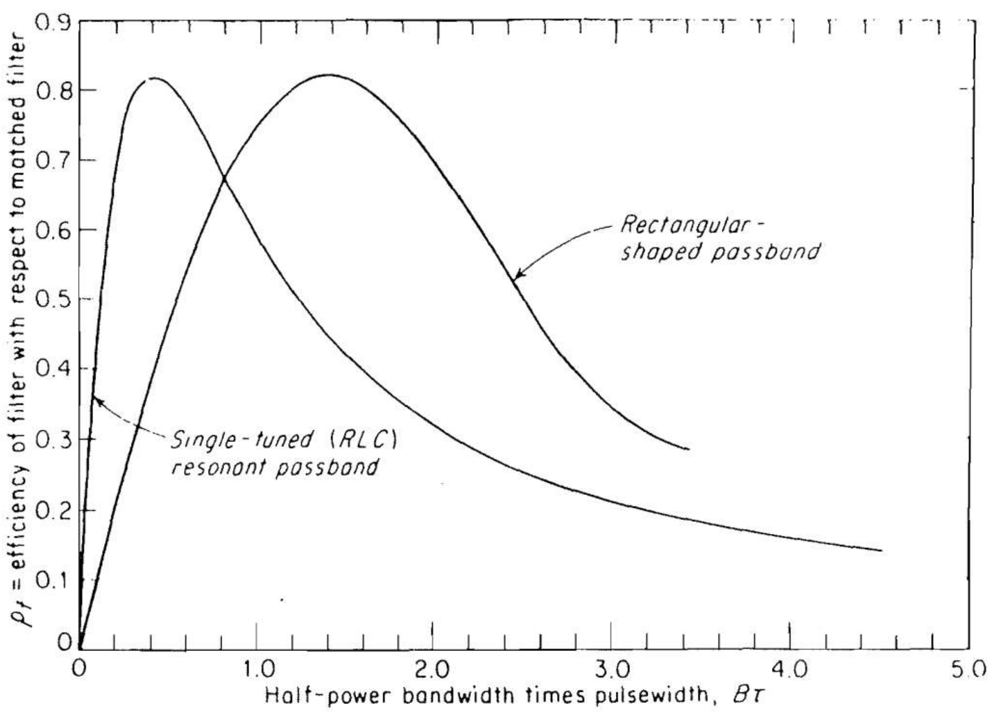


Fig: Block diagram of a cross-correlation receiver

- The input signal y (t) is multiplied by a delayed replica of the transmitted signal s(t - Tr), and the product is passed through a low-pass filter to perform the integration.
- The cross-correlation receiver of above Fig tests for the presence of a target at only a single time delay Tr. Targets at other time delays, or ranges, might be found by varying Tr. However, this requires a longer search time.
- The cross-correlation receiver and the matched-filter receiver are equivalent mathematically, the choice as to which one to use in a particular radar application is determined by which is more practical to implement.
- The matched-filter receiver, or an approximation, has been generally preferred in the vast majority of applications.

Efficiency of non-matched filters

- In practice the matched filter cannot always be obtained exactly. It is appropriate, therefore, to examine the efficiency of non matched filters compared with the ideal matched filter.
- The measure of efficiency is taken as the peak signal-to-noise ratio from the non matched filter divided by the peak signal-to-noise ratio (2E/No) from the matched filter.
- Figure. Plots the efficiency for a single-tuned (RLC) resonant filter and a rectangular-shaped filter of half-power bandwidth $B\tau$ when the input is a rectangular pulse of width τ
- The maximum efficiency of the single-tuned filter occurs for $B\tau \approx 0.4$. The corresponding loss in signal-to-noise ratio is 0.88 dB as compared with a matched filter. .



- Table lists the values of Bτ which maximize the signal-to-noise ratio (SNR) for various combinations of filters and pulse shapes. It can be seen that the loss in SNR incurred by use of these non-matched filters is small.

Input signal	Filter	Optimum $B\tau$	Loss in SNR compared with matched filter, dB
Rectangular pulse	Rectangular	1.37	0.85
Rectangular pulse	Gaussian	0.72	0.49
Gaussian pulse	Rectangular	0.72	0.49
Gaussian pulse	Gaussian	0.44	0 (matched)
Rectangular pulse	One-stage, single-tuned circuit	0.4	0.88
Rectangular pulse	2 cascaded single-tuned stages	0.613	0.56
Rectangular pulse	5 cascaded single-tuned stages	0.672	0.5

Table: Efficiency of nonmatched filters compared with the matched filter

Matched filter with nonwhite noise

- In the derivation of the matched-filter characteristic, the spectrum of the noise accompanying the signal was assumed to be white; that is, it was independent of frequency.
- If this assumption were not true, the filter which maximizes the output signal-to-noise ratio would not be the same as the matched filter.
- It has been shown that if the input power spectrum of the interfering noise is given by [Ni(f)]2, the frequency-response function of the filter which maximizes the output signal-to-noise ratio is

$$H(f) = \frac{G_a S^*(f) \exp(-j2\pi f t_1)}{[N_i(f)]^2}$$

- When the noise is nonwhite, the filter which maximizes the output signal-to-noise ratio is called the NWN (nonwhite noise) matched filter.

- For white noise [Ni(f)]2 = constant and the NWN matched-filler frequency-response function of Eq. above reduces to that of Eq. discussed earlier in white noise. Equation above can be written as

$$H(f) = \frac{1}{N_i(f)} \times G_a \left(\frac{S(f)}{N_i(f)} \right)^* \exp(-j2\pi f t_1)$$

- This indicates that the NWN matched filter can be considered as the cascade of two filters.

- The first filter, with frequency-response function l/Ni (f), acts to make the noise spectrum uniform, or white. It is sometimes called the whitening filter.

- The second is the matched filter when the input is white noise and a signal whose spectrum is $S(f)/N_i(f)$.

RADAR RECEIVERS

Noise figure and Noise Temperature

Displays-Types

Duplexers-Branch type and Balanced Type

Circulators as duplexers

Phased array Antennas Introduction, Basic concepts, Radiation Pattern

Beam steering and Beam width changes

Series vs parallel feeds

Applications, Advantages and Limitations

Radar Receivers

- The receiver section is connected to the antenna by duplexer. The receiver section is super heterodyne type and it consists of mixer, IF amplifier, detector and video amplifier.
- The first stage of the receiver is usually an RF amplifier with very low noise properties. The RF amplifier is followed by a mixer of fairly low noise figure.
- The mixer is a crystal diode which is fed to IF amplifier. The IF amplifier operates at 30 or 60 MHz or at nearby frequency.
- IF stage provides most of the receiver gain. The IF amplifier should be a low noise device to ensure that the overall noise figure of the receiver does not deteriorate to great extent.
- The down conversion from microwave frequency to IF frequency may be done in number of stages to ensure adequate image frequency suppression.
- The detector is crystal diode whose output is amplified by video amplifier having the same bandwidth as the IF amplifier.
- The output signal is then fed to a display unit which may be a CRT.

Characteristics of radar receiver

- The function of the radar receiver is to detect desired echo signals in the presence of noise, interference, or clutter.
- It must separate wanted from unwanted signals, and amplify the wanted signals to a level where target information can be displayed to an operator or used in an automatic data processor.
- The design of the radar receiver will depend not only on the type of waveform to be detected, but on the nature of the noise, interference, and clutter echoes with which the desired echo signals must compete.
- Noise can enter the receiver via the antenna terminals along with the desired signals, or it might be generated within the receiver itself. At the microwave frequencies usually used for radar, the external noise which enters via the antenna is generally quite low. The measure of receiver internal noise is the noise-figure.
- Good receiver design is based on maximizing the output signal-to-noise ratio. To maximize the output signal-to-noise ratio, the receiver must be designed as a matched filter, or its equivalent. The matched filter specifies the frequency response function of the IF part of the radar receiver.
- Obviously, the receiver should be designed to generate as little internal noise as possible, especially in the input stages where the desired signals are the weakest.

- Receiver design also must be concerned with achieving sufficient gain, phase, and amplitude stability, dynamic range, tuning, ruggedness, and simplicity.
- Protection must be provided against overload or saturation, and burnout from nearby interfering transmitters. Timing and reference signals are needed to properly extract target information.
- Timing and reference signals are needed to properly extract target information. Specific applications such as MTI radar, tracking radar, or radars designed to minimize clutter place special demands on the receiver.
- Receivers that must operate with a transmitter whose frequency can drift need some means of automatic frequency control (AFC).
- Radars that encounter hostile counter-measures need receivers that can minimize the effects of such interference.
- Thus there can be many demands placed upon the receiver designer in meeting the requirements of modern high-quality radar systems.
- Although the super regenerative, crystal video and tuned radio frequency (TRF) receivers have been employed in radar systems, the superheterodyne has seen almost exclusive application because of its good sensitivity, high gain, selectivity, and reliability. No other receiver type has been competitive to the superheterodyne.

Receiver Noise Figure

Noise figure:

Noise figure of a receiver is a measure of the noise produced by a practical receiver as compared with the noise of an ideal receiver. The noise figure F_n may be defined as:

$$F_n = \frac{S_{in}/N_{in}}{S_{out}/N_{out}} = \frac{N_{out}}{kT_0 B_n G} \qquad \text{Eq.(1)}$$

Where S_{in} = available input signal power

N_{in} = available input noise power (equal to kT_oB)

S_{out} = available output signal power

N_{out} = available output noise power

"Available power" refers to the power which would be delivered to a matched load. The available gain G is equal to S_{out}/S_{in},

k = Boltzmann's constant = **1.38×10^{-23} J/deg**, T_o = standard temperature of **290 K** (approximately room temperature) and B_n is the noise bandwidth (defined earlier).

The product $kT_o \approx$ **4×10^{-21} W/Hz.** The purpose of defining a standard temperature is to refer any measurements to a common basis of comparison.

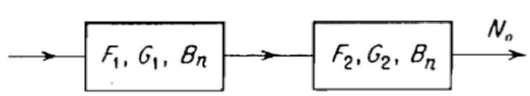
- Equation above permits two different but equivalent interpretations of noise figure.
- It may be considered as the degradation of the signal-to-noise ratio caused by the network (receiver), or it may be interpreted as the ratio of the actual available output noise power to the noise power which would be available if the network merely amplified the thermal noise.
- The noise figure may also be written

$$F_n = \frac{kT_0 B_n G + \Delta N}{kT_0 B_n G} = 1 + \frac{\Delta N}{kT_0 B_n G}$$

- where ΔN is the additional noise introduced by the network itself. The noise figure is commonly expressed in decibels, that is, 10 log Fn . The term noise factor is also used at times instead of noise figure. The two terms are synonymous.

Noise figure of networks in cascade

- Consider two networks in cascade, each with the same noise bandwidth Bn but with different noise figures and available gain (Fig).
- Let F1, G1 be the noise figure and available gain, respectively, of the first network, and F2, G2 be similar parameters for the second network.
- The problem is to find Fo, the overall noise-figure of the two circuits in cascade.



- From the definition of noise figure the output noise No of the two circuits in cascade is

N_o= $F_oG_1G_2kT_0B_n$ =Noise from network **1 at output of network2+ Noise** ***ΔN₂ introduced by network2***

$$N_o = kT_0 B_n F_1 G_1 G_2 + \Delta N_2 = kT_0 B_n F_1 G_1 G_2 + (F_2 - 1) kT_0 B_n G_2$$

$$F_o = F_1 + \frac{F_2 - 1}{G_1}$$

- The contribution of the second network to the overall noise-figure may be made negligible if the gain of the first network is large. This is of importance in the design of multistage receivers.
- It is not sufficient that only the first stage of a low-noise receiver have a small noise figure.
- The succeeding stage must also have a small noise figure, or else the gain of the first stage must be high enough to swamp the noise of the succeeding stage if the first network is not an amplifier but is a network with loss (as in a crystal mixer), the gain GI should be interpreted as a number less than unity.
- Extending the same method, the noise figure of ***N*** networks in cascade may be shown to be:

$$F_o = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \cdots + \frac{F_N - 1}{G_1 G_2 \cdots G_{N-1}}$$

Noise temperature

- The noise introduced by a network may also be expressed as an ***effective*** **Noise *temperature,*** T_e defined as that (fictional) temperature at the input of the network which would account for the noise **ΔN** at the output. Therefore **ΔN** = $\mathbf{kT_e\ B_0\ G}$ and

$$F_n = 1 + \frac{T_e}{T_0}$$

$$T_e = (F_n - 1)T_0$$

- The system noise temperature Ts is defined as the effective noise temperature of the receiver system including the effects of antenna temperature Ta. (It is also sometimes called the system operating noise temperature) If the receiver effective noise temperature is Te, then

$$T_s = T_a + T_e = T_0 F_s$$

- where Fs is the system noise-figure including the effect of antenna temperature. The effective noise temperature of a receiver consisting of a number of networks in cascade is

$$T_e = T_1 + \frac{T_2}{G_1} + \frac{T_3}{G_1 G_2} + \cdots$$

where Ti and Gi , are the effective noise temperature and gain of the ith network.

- The effective noise temperature and the noise figure both describe the same characteristic of a network. In general, the effective noise temperature has been preferred for describing low-noise devices, and the noise figure is preferred for conventional receivers.

Displays-Types

- The purpose of the display is to visually present the information contained in the radar echo signal in a form suitable for operator interpretation and action.
- When the display is connected directly to the video output of the receiver, the information displayed is called raw video. This is the "traditional” type of radar presentation.
- When the receiver video output is first processed by an automatic detector or automatic detection and tracking processor (ADT), the output displayed is sometimes called synthetic video.
- The cathode-ray tube (CRT) has been almost universally used as the radar display. There are two basic cathode-ray tube displays. One is the dejection-modulated CRT, such as the A-scope, in which a target is indicated by the deflection of the electron beam.
- The other is the intensity modulated CRT, such as the PPI, in which a target is indicated by intensifying the electron beam and presenting a luminous spot on the face of the CRT.

- In general, deflection-modulated displays have the advantage of simpler circuits than those of intensity- modulated displays, and targets may be more readily discerned in the presence of noise or interference.
- On the other hand, intensity-modulated displays have the advantage of presenting data in a convenient and easily interpreted form.
- The deflection of the beam or the appearance of an intensity-modulated spot on a radar display caused by the presence of a target is commonly referred to as a blip.

Types of display presentations: The various types of CRT displays which might be used for surveillance and tracking radars are defined as follows:

- **A-scope:** A deflection-modulated display in which the vertical deflection is proportional to target echo strength and the horizontal coordinate is proportional to range.
- **B-scope:** An intensity-modulated rectangular display with azimuth angle indicated by the horizontal coordinate and range by the vertical coordinate.
- **C-scope:** An intensity-modulated rectangular display with azimuth angle indicated by the horizontal coordinate and elevation angle by the vertical coordinate.

- **D-scope:** A C-scope in which the blips extend vertically to give a rough estimate of distance.
- **E-scope:** An intensity-modulated rectangular display with distance indicated by the horizontal coordinate and elevation angle by the vertical coordinate. Similar to the RHI in which target height or altitude is the vertical coordinate.
- **F-Scope:** A rectangular display in which a target appears as a centralized blip when the radar antenna is aimed at it. Horizontal and vertical aiming errors are respectively indicated by the horizontal and vertical displacement of the blip.
- **G-Scope:** A rectangular display in which a target appears as a laterally centralized blip when the radar antenna is aimed at it in azimuth, and wings appear to grow on the pip as the distance to the target is diminished; horizontal and vertical aiming errors are respectively indicated by horizontal and vertical displacement of the blip.
- **H-scope:** A B-scope modified to include indication of angle of elevation. The target appears as two closely spaced blips which approximate a short bright line, the slope of which is in proportion to the sine of the angle of target elevation.

- **I-scope:** A display in which a target appears as a complete circle when the radar antenna is pointed at it and in which the radius of the circle is proportional to target distance; incorrect aiming of the antenna changes the circle to a segment whose arc length is inversely proportional to the magnitude of the pointing error, and the position of the segment indicates the reciprocal of the pointing direction of the antenna.
- **J-scope:** A modified A-scope in which the time base is a circle and targets appear as radial deflections from the time base.
- **K-scope:** A modified A-scope in which a target appears as a pair of vertical deflections. When the radar antenna is correctly pointed at the target, the two deflections are of equal height, and when not so pointed, the difference in deflection amplitude is an indication of the direction and magnitude of the pointing error.
- **L-scope:** A display in which a target appears as two horizontal blips, one extending to the right from a central vertical time base and the other to the left; both blips are of equal amplitude when the radar is pointed directly at the target, any inequality representing relative pointing error, and distance upward along the baseline representing target distance.

- **M-scope:** A type of A-scope in which the target distance is determined by moving an adjustable pedestal signal along the baseline until it coincides with the horizontal position of the target signal deflections; the control which moves the pedestal is calibrated in distance.
- **N-scope**: A K-scope having an adjustable pedestal signal, as in the M-scope, for the measurement of distance.
- **O-scope:** An A-scope modified by the inclusion of an adjustable notch for measuring distance.
- **PPI or Plan Position Indicator (also called P-scope):** An intensity-modulated circular display on which echo signals produced from reflecting objects are shown in plan position with range and azimuth angle displayed in polar (rho-theta) coordinates, forming a map-like display. An offset, or off center, PPI has the zero position of the time base at a position other than at the center of the display to provide the equivalent of a larger display for a selected portion of the service area. A delayed PPI is one in which the initiation of the time base is delayed.
- **R-scope:** An A-scope with a segment of the time base expanded near the blip for greater accuracy in distance measurement.
- **RHI or Range-Height Indicator:** An intensity modulated display with height (altitude) as the vertical axis and range as the horizontal axis.
- **PPI, A-scope, B-scope,** and **RHI** are among the more usual displays employed in radar.

Duplexers

The duplexer is the device that 1) switches the radar antenna to either transmitter or the receiver 2) serves to protect the receiver from burnout or damage during the transmission.

Branch type Duplexer

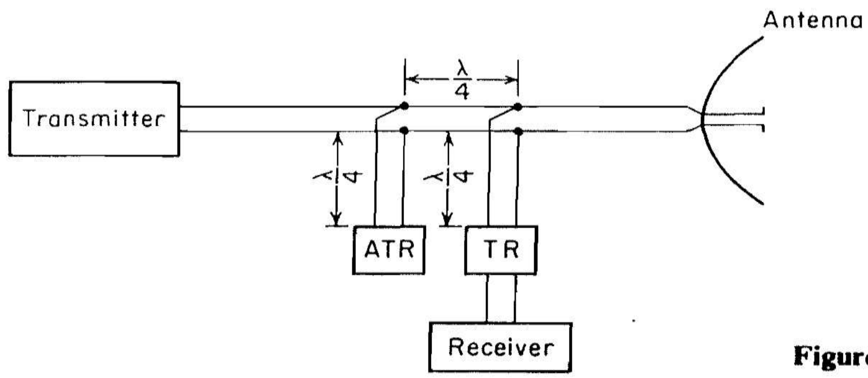


Figure : Principle of branch-type duplexer

- The branch-type duplexer, shown in above Fig is one of the earliest duplexer configurations. It consists of a TR (transmit-receive) switch and an ATR (anti-transmit receive) switch, both of which are gas-discharge tubes.
- When the transmitter is turned on, the TR and the ATR tubes ionize; that is, they break down, or fire. The TR in the fired condition acts as a short circuit to prevent transmitter power from entering the receiver.
- Since the TR is located a quarter wavelength from the main transmission line, it appears as a short circuit at the receiver but as an open circuit at the transmission line so that it does not impede the flow of transmitter power.
- Since the ATR is displaced a quarter wavelength from the main transmission line, the short circuit it produces during the fired condition appears as an open circuit on the transmission line and thus has no effect on transmission.
- During reception, the transmitter is off and neither the TR nor the ATR is fired.
- The open circuit of the ATR, being a quarter wave from the transmission line, appears as a short circuit across the line.
- Since this short circuit is located a quarter wave from the receiver branch line, the transmitter is effectively disconnected from the line and the echo signal power is directed to the receiver.
- The branch-type duplexer is of limited bandwidth and power handling capability, and has generally been replaced by the balanced duplexer and other protection devices. It is used, inspite of these limitations, in some low-cost radars.

- **Balanced duplexer**

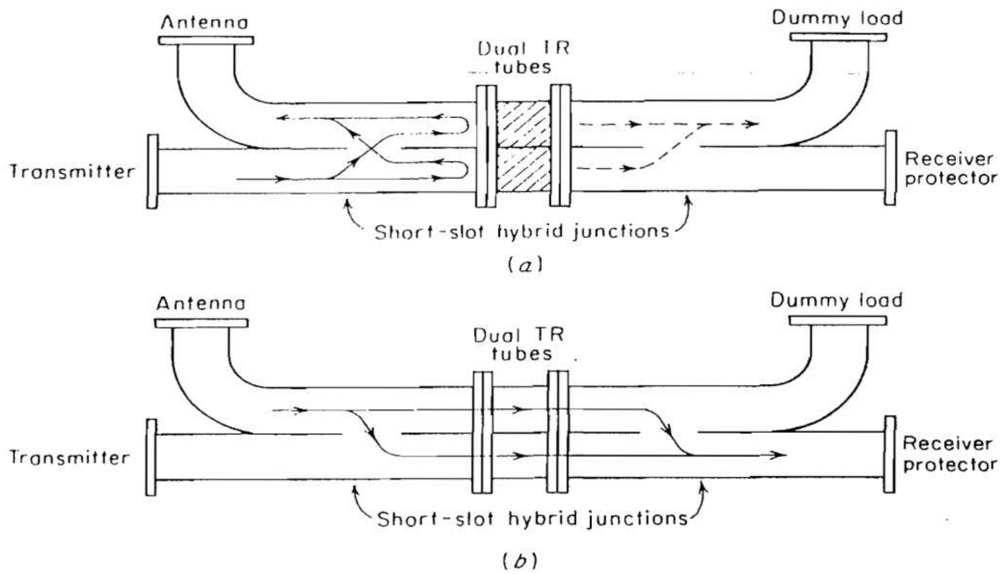
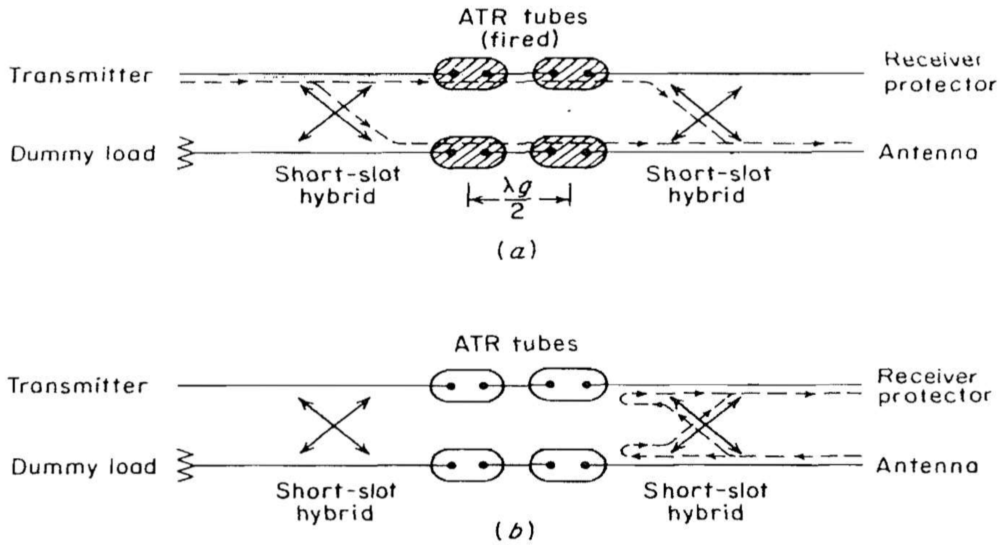


Figure : Balanced duplexer using dual TR tubes and two short-slot hybrid junctions. (a) Transmit Condition (b) Receive condition.

- The balanced duplexer is based on the short-slot hybrid junction which consists of two sections of waveguides joined along one of their narrow walls with a slot cut in the common narrow wall to provide coupling between the two.
- The short-slot hybrid may be considered as a broadband directional coupler with a coupling ratio of 3 dB.
- In the transmit condition (Figure a) power is divided equally into each waveguide by the first short slot hybrid junction. Both TR tubes break down and reflect the incident power out the antenna arm as shown.
- *The short-slot hybrid has the property that each time the energy passes through the slot in either direction, its phase is advanced 90^0*. Therefore, the energy travels as indicated by the solid lines.
- Any energy which leaks through the TR tubes (shown by the dashed lines) is directed to the arm with the matched dummy load and not to the receiver. In addition to the attenuation provided by the TR tubes, the hybrid junctions provide an additional 20 to 30 dB of isolation.
- On reception the TR tubes are unfired and the echo signals pass through the duplexer and into the receiver as shown in Fig.6b. The power splits equally at the first junction and because of the 90^0 phase advance on passing through the slot, the energy recombines in the receiving arm and not in the dummy-load arm.
- The power-handling capability of the balanced duplexer is inherently greater than that of the branch-type duplexer and it has wide bandwidth, over ten percent with proper design. *A receiver protector,* is usually inserted between the duplexer and the receiver for added protection.



Balanced duplexer using ATR tubes.
(a) Transmit condition (b) receive condition

CIRCULATORS AS DUPLEXERS
CIRCULATOR AND RECEIVER PROTECTOR

- The ferrite circulator is a three or four-port device that can in principle, offer separation of the transmitter and receiver without the need for the conventional duplexer configurations explained earlier.

- The circulator does not provide sufficient protection by itself and requires a receiver protector like duplexers.

- The isolation between the transmitter and receiver ports of a circulator is seldom sufficient to protect the receiver from damage.

- However, it is not the isolation between transmitter and receiver ports that usually determines the amount of transmitter power at the receiver, but the impedance mismatch at the antenna which reflects transmitter power back into the receiver.

- The VSWR is a measure of the amount of power reflected by the antenna. For example, a VSWR of 1.5 means that about 4 percent of the transmitter power will be reflected by the antenna mismatch in the direction of the receiver, which corresponds to an isolation of only 14 dB. About 11 percent of the power is reflected when the VSWR is 2.0, corresponding to less than 10 dB of isolation. Thus, a receiver protector is almost always required.

- It also reduces to safe level radiations from nearby transmitters.
- The receiver protector might use solid-state diodes for an all solid-state configuration, or it might be a passive TR-limiter consisting of a radioactive primed TR-tube followed by a diode limiter.
- The ferrite circulator with receiver protector is attractive for radar applications because of its long life, wide bandwidth, and compact design.

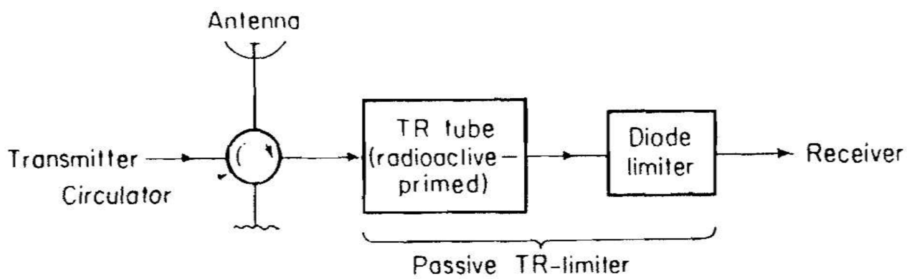


Figure : circulator and receiver protector. A four-port circulator is shown with the fourth port terminated in a matched load to provide greater isolation between the transmitter and the receiver than provided by a three-port circulator

Introduction to phased array antennas

- The phased array is a directive antenna made up of individual radiating antennas, or elements, which generate a radiation pattern whose shape and direction is determined by the relative phases and amplitudes of the currents at the individual elements.
- By properly varying the relative phases it is possible to steer the direction of the radiation.
- The radiating elements might be dipoles open-ended waveguides, slots cut in waveguide, or any other type of antenna.
- Became of interest to Radar due to the inherent flexibility it has offered in steering the beam by means of electronic control rather than by physical movement of the antenna.
- It has been considered in those radar applications where it is necessary to shift the beam rapidly from one position in space to another, or where it is required to obtain information about many targets at a flexible, rapid data rate.
- The full potential of a phased-array antenna requires the use of a computer that can determine in real time, on the basis of the actual operational situation, how best to use the capabilities offered by the array.

- Initially In World War 2**,** the United States, Great Britain, and Germany used radar with fixed phased-array antennas in which the beam was scanned by mechanically actuated phase shifters.
- A major advance in phased array technology was made in the early 1950s with the replacement of mechanically actuated phase shifters by electronic phase shifters.
- Frequency scanning in one angular coordinate was the first successful electronic scanning technique to be applied.
- The introduction of digitally switched phase shifters employing either ferrites or diodes in the early 1960s made a significant improvement in the practicality of phased arrays that could be electronically steered in two orthogonal angular coordinates.

Basic concept

- An array antenna consists of a number of individual radiating elements suitably spaced with respect to one another.
- Two common geometrical forms of array antennas used in radar are the linear array and the planar array.

Linear array

- A *linear array* consists of elements arranged in a straight line in one dimension.
- The linear array generates a fan beam when the phase relationships are such that the radiation is perpendicular to the array.
- When the radiation is at some angle other than broadside, the radiation pattern is a conical-shaped beam.
- The linear array can also act as a feed for a parabolic-cylinder antenna.

Planar array

- A *planar array* is a two dimensional configuration of elements arranged to lie in a plane. The planar array may be thought of as a linear array of linear arrays.
- The two-dimensional planar array is the most commonly used in radar applications since it is fundamentally the most versatile of all radar antennas.
- A rectangular aperture can produce a fan shaped beam. A square or a circular aperture produces a pencil beam.
- The array can be made to simultaneously generate many search and/or tracking beams with the same aperture.

Broadside array

- A *broadside array* is one in which the direction of maximum radiation is perpendicular, or almost perpendicular to the line (or plane) of the array.
- The broadside linear-array antenna may be used where broad coverage in one plane and narrow beam width in the orthogonal plane are desired.

End fire

- An *end fire* array has its maximum radiation parallel to the array.
- The end fire array is a special case of the linear or the planar array when the beam is directed along the array.
- End fire linear arrays have not been widely used in radar applications.
- They are usually limited to low or medium gains since an end fire linear antenna of high gain requires an excessively long array.
- Small end fire arrays are sometimes used as the radiating elements of a broadside array if directive elements are required.

Other types of array antennas

- An array whose elements are distributed on a non planar surface is called a *conformal array.*
- An array in which the relative phase shift between elements is controlled by electronic devices is called an *electronically scanned array.*
- In an electronically scanned array the antenna elements, the transmitters, the receivers, and the data-processing portions of the radar are often designed as a unit.

Radiation pattern

- Consider a linear array made up of **N** elements equally spaced a distance ***d*** apart as shown in ***Fig.***

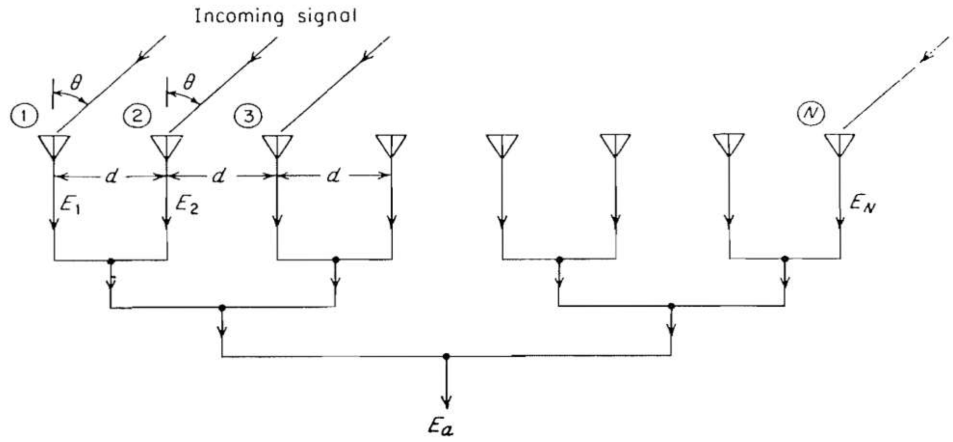


Figure : N-element linear array.

- The elements are assumed to be isotropic point sources radiating uniformly in all directions with equal amplitude and phase.

- The outputs of all the elements are summed via lines of equal length to give a sum output voltage E_a.
- Element **1** will be taken as the reference signal with zero phase. The difference in the phase of the signals in adjacent elements is $\Psi = 2\pi\,(d/\lambda)\,\sin\theta$, where θ is the direction of the incoming radiation.
- It is further assumed that the amplitudes and phases of the signals at each element are weighted uniformly. Therefore the amplitudes of the voltages in each element are the same and, for convenience, will be taken to be unity.
- The sum of all the voltages from the individual elements, when the phase difference between adjacent elements is **Ψ**, can be written as

$$E_a = \sin \omega t + \sin(\omega t + \psi) + \sin(\omega t + 2\psi) + \cdots + \sin[\omega t + (N-1)\psi]$$

where ω is the angular frequency of the signal. The sum can be written as

$$E_a = \sin\left[\omega t + (N-1)\frac{\psi}{2}\right]\frac{\sin(N\psi/2)}{\sin(\psi/2)}$$

- The first factor is a sine wave of frequency ω with a phase shift **(N - 1) ψ/2.** The second term represents the amplitude factor of the form **sin (Nψ/2)/sin (ψ/2).** The field intensity pattern is the magnitude of the equation **2** , or

$$|E_a(\theta)| = \left| \frac{\sin [N\pi(d/\lambda) \sin \theta]}{\sin [\pi(d/\lambda) \sin \theta]} \right|$$

- The pattern has nulls whenever the numerator is zero.
- $N\Pi(d/\lambda)\sin\theta = 0, \pm \Pi, \pm 2\Pi..., \pm n\Pi$, where n = integer. The denominator, however, is zero when $\Pi\ (d/\lambda) \sin\theta, = 0, \pm \Pi, \pm 2\Pi..., \pm n\Pi$. Note that when the denominator is zero, the numerator is also zero.
- The value of the field intensity pattern is indeterminate when both the denominator and numerator are zero. However, by applying L'Hopital's rule (differentiating numerator and denominator separately) it is found that $|Ea|$ is a maximum whenever $\sin\theta = \pm n\lambda/d$.
- These maxima all have the same value and are equal to N. The maximum at $\sin\theta = 0$ defines the main beam. The other maxima are called grating lobes. They are generally undesirable and are to be avoided.

- The radiation pattern is equal to the normalized square of the amplitude, or

$$G_a(\theta) = \frac{|E_a|^2}{N^2} = \frac{\sin^2 [N\pi(d/\lambda) \sin \theta]}{N^2 \sin^2 [\pi(d/\lambda) \sin \theta]}$$

- When directive elements are used, the resultant array antenna radiation pattern is

$$G(\theta) = G_e(\theta) \frac{\sin^2 [N\pi(d/\lambda) \sin \theta]}{N^2 \sin^2 [\pi(d/\lambda) \sin \theta]} = G_e(\theta) G_a(\theta)$$

- where Ge (θ) is the radiation pattern of an individual element. The resultant radiation pattern is the product of the element factor Ge(θ) and the array factor Ga(θ).
- Grating lobes caused by a widely spaced array may therefore be eliminated with directive elements which radiate little or no energy in the directions of the undesired lobes.
- For example, when the element spacing $d = 2\lambda$, grating lobes occur at $\theta = \pm 30°$ and $\pm 90°$ in addition to the main beam at $\theta = 0°$. If the individual elements have a beamwidth somewhat less than 60°, the grating lobes of the array factor will be suppressed.

- In a two-dimensional, rectangular planar array, the radiation pattern may sometimes be written as the product of the radiation patterns in the two planes which contain the principal axes of the antenna.
- If the radiation patterns in the two principal planes are $\mathbf{G_1(\theta_e)}$ and $\mathbf{G_2(\theta_a)}$ the two-dimensional antenna pattern is

$$G(\theta_e, \theta_a) = G_1(\theta_e)G_2(\theta_a)$$

- Thus, the normalized radiation pattern of a uniformly illuminated rectangular array is

$$G(\theta_e, \theta_a) = \frac{\sin^2 [N\pi(d/\lambda) \sin \theta_a] \quad \sin^2 [M\pi(d/\lambda) \sin \theta_e]}{N^2 \sin^2 [\pi(d/\lambda) \sin \theta_a] \, M^2 \sin^2 [\pi(d/\lambda) \sin \theta_e]}$$

- Where **N** = number of radiating elements in $\boldsymbol{\theta_a}$ dimension with spacing **d** and **M** the number in $\boldsymbol{\theta_e}$ dimension.

Beam steering phased array antennas

- The beam of an array antenna may be steered rapidly in space without moving large mechanical masses by properly varying the phase of the signals applied to each element.
- Consider an array of equally spaced elements. The spacing between adjacent elements is **d**, and the signals at each element are assumed to be of equal amplitude.
- If the same phase is applied to all elements, the relative phase difference between adjacent elements is zero and the position of the main beam will be broadside to the array at an angle $\boldsymbol{\theta = 0}$.
- The main beam will point in a direction other than broadside if the relative phase difference between elements is other than zero.
- The direction of the main beam is at an angle $\boldsymbol{\theta_0}$**,** when the phase difference is $\boldsymbol{Ø = 2\pi\ (d/\lambda) \sin \theta_0}$**.** The phase at each element is therefore $\boldsymbol{(Ø_c + m\,Ø)}$ where **m = 0, 1,2. . . (N - I)** and $\boldsymbol{Ø_c}$ is any constant phase applied to all elements.

- The normalized radiation pattern of the array when the phase difference between adjacent elements is **Ø** is given by:

$$G(\theta) = \frac{\sin^2 [N\pi(d/\lambda)(\sin\theta - \sin\theta_0)]}{N^2 \sin^2 [\pi(d/\lambda)(\sin\theta - \sin\theta_0)]}$$

- The maximum of the radiation pattern occurs when **$\sin\theta = \sin\theta_0$**
- The above Equation states that the main beam of the antenna pattern may be positioned to an angle **θ_0** by the insertion of the proper phase shift **Ø** at each element of the array. If variable, rather than fixed, phase shifters are used, the beam may be steered as the relative phase between elements is changed

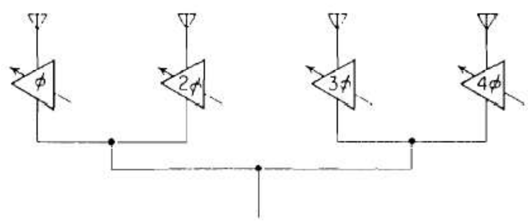


FIG: Steering of an antenna beam with variable phase shifters (parallel-fed array).

- Using an argument similar to the non scanning array described previously, grating lobes appear at an angle **θ_g** whenever the denominator is zero, or when

$$\pi \frac{d}{\lambda}(\sin\theta_g - \sin\theta_0) = \pm n\pi$$

or

$$|\sin\theta_g - \sin\theta_0| = \pm n \frac{\lambda}{d}$$

- If a grating lobe is permitted to appear at -90^0 when the main beam is steered to +90°, it is found from the above that **d = λ/2.**

- Thus the element spacing must not be larger than a half wavelength if the beam is to be steered over a wide angle without having undesirable ***grating*** lobes appear.

- Practical array antennas do not scan +/- 90°. If the scan is limited to +/- 60^0 the element spacing should be less than **0.54λ**.

Change of beam width with steering angle

- The half-power beam width in the plane of scan increases as the beam is scanned off the broadside direction. The beam width is approximately inversely proportional to **cos θ_0**, where θ_0 is the angle measured from the normal to the antenna.
- This may be proved by assuming that the sine in the denominator of **G(θ)** discussed earlier can be replaced by its argument, so that the radiation pattern is of the form$(\sin^2 u)/u^2$, where $u = N\Pi (d/\lambda)(\sin\theta - \sin\theta o)$.
- The $(\sin^2 u)/u^2$ antenna pattern is reduced to half its maximum value when $u = \pm 0.443\Pi$. Denote by θ+ the angle corresponding to the half-power point when $\theta > \theta o$, and θ- , the angle corresponding to the half-power point when $\theta < \theta o$; that is, θ+ corresponds to $u = +0.443\Pi$ and θ- to $u = -0.443\Pi$.
- The sinθ - sinθo, term in the expression for u can be written

$$\sin\theta - \sin\theta_0 = \sin(\theta - \theta_0)\cos\theta_0 - [1 - \cos(\theta - \theta_0)]\sin\theta_0$$

- The second term on the right-hand side of Eq. above can be neglected when θo is small (beam is near broadside), so that

$$\sin\theta - \sin\theta_0 \approx \sin(\theta - \theta_0)\cos\theta_0$$

- Using the above approximation, the two angles corresponding to the 3-dB points of the antenna pattern are

$$\theta_+ - \theta_0 = \sin^{-1}\frac{0.443\lambda}{Nd\cos\theta_0} \approx \frac{0.443\lambda}{Nd\cos\theta_0}$$

$$\theta_- - \theta_0 = \sin^{-1}\frac{-0.443\lambda}{Nd\cos\theta_0} \approx \frac{-0.443\lambda}{Nd\cos\theta_0}$$

- The half-power beamwidth is

$$\theta_B = \theta_+ - \theta_- \approx \frac{0.886\lambda}{Nd\cos\theta_0}$$

- Therefore, when the beam is positioned an angle θo off broadside, the beamwidth in the plane of scan increases as $(\cos\theta o)^{-1}$.

- The variation of the beam shape with scan angle is graphically shown in **Fig below**.

- The antenna radiation pattern is plotted in spherical coordinates as a function of the two direction cosines, **cos** $\boldsymbol{\alpha_x}$ and **cos** $\boldsymbol{\alpha_y}$ of the radius vector specifying the point of observation.

- The angle **Ø** is measured from the **cos** $\boldsymbol{\alpha_x}$ axis, and $\boldsymbol{\theta}$ is measured from the axis perpendicular to the **cos** $\boldsymbol{\alpha_x}$ and **cos** $\boldsymbol{\alpha_y}$ axes.

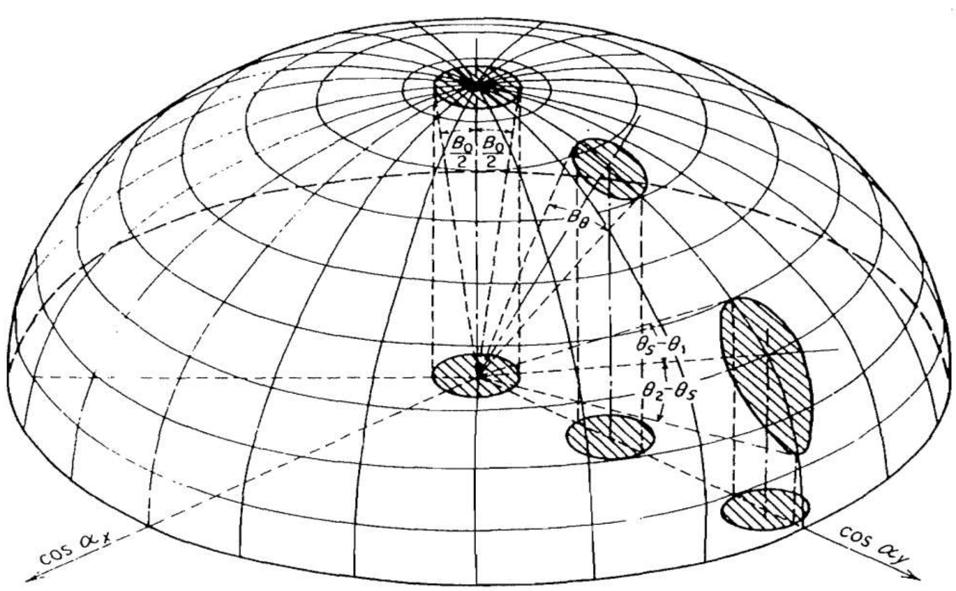
- In **Fig. Ø** is taken to be a constant value of $\mathbf{90^0}$ and the beam is scanned in the $\boldsymbol{\theta}$ coordinate.

- At $\boldsymbol{\theta}$= 0 (beam broad side to the array) a symmetrical pencil beam of half-power width $\mathbf{B_0}$ is assumed.

- The shape of the beam at the other angular positions is the projection of the circular beam shape on the surface of the unit sphere.

- It can be seen that as the beam is scanned in the $\boldsymbol{\theta}$ direction, it broadens in that direction, but is constant in the **Ø** direction.

- For $\boldsymbol{\theta \neq 0}$, the beam shape is not symmetrical about the center of the beam, but is eccentric.



Beam width and eccentricity of the scanned beam

Applications of phased array antennas

The phased array antenna has seen application in radar for a wide variety of purposes:

- Aircraft surveillance from on board ship
- Satellite surveillance
- Ballistic missile defense
- Air defense
- Aircraft landing systems
- Mortar and artillery location
- Tracking of ballistic missiles and Airborne bomber radar (EAR).
- Many developmental array radars have been developed and built in USA. Although much effort and funds have been spent on this activity, ***except for limited-scan arrays*** there has not been any large serial production of such radars compared to the serial production of radars with mechanically rotating reflector antennas.

Advantages phased array antennas.

- **Inertialess rapid beam-steering:** The beam from an array can be scanned, or switched from one position to another, in a time limited only by the switching speed of the phase shifters. Typically, the beam can be switched in several microseconds, but it can be considerably shorter if desired.
- **Multiple, independent beams:** A single aperture can generate many simultaneous independent beams. Alternatively, the same effect can be obtained by rapidly switching a single beam through a sequence of positions.
- **Potential for large peak and / or average power:** If necessary, each element of the array can be fed by a separate high-power transmitter with the combining of the outputs made in space to obtain a total power greater than can be obtained from a single transmitter.

- **Control of the radiation pattern**. A particular radiation pattern may be more readily obtained with the array than with other microwave antennas since the amplitude and phase of each array element may be individually controlled. Thus, radiation patterns with extremely low side lobes or with a shaped main beam may be achieved conveniently. Separate monopulse sum and difference patterns, each with it's own optimum shape, can also be generated.

- **Graceful degradation**. The distributed nature of the array means that it can fail only gradually and not l at once (catastrophically).

- **Convenient aperture shape**. The shape of the array permits flush mounting and it can be strengthened to resist blast.

- **Electronic beam stabilization**. The ability to steer the beam electronically can be used to stabilize the beam direction when the radar is on an unstable platform, such as a ship or aircraft that is subject to roll, pitch, and yaw disturbances.

Limitations of phased array antennas

1. The major limitation that has limited the widespread use of the conventional phased array in radar is its high cost, which is due in large part to its complexity.

2. When graceful degradation has gone too far a seperate maintenance is needed.

3. When a planar array is electronically scanned, the change of mutual coupling that accompanies a change in beam position makes the maintenance of low sidelobes more difficult.

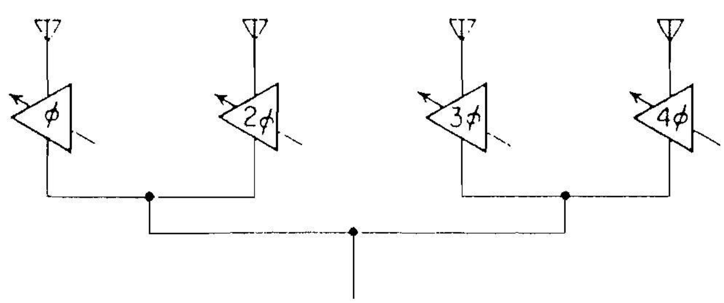
4. Although the array has the potential for radiating large power, it is seldom that an array is required to radiate more power than can be radiated by other antenna types or to utilize a total power which cannot possibly be generated by current high-power microwave tube technology that feeds a single transmission line.

Series Vs Parallel feeds

- The phase relationship between the adjacent elements of the array can be obtained with either series fed or parallel fed arrangement.
- In series fed arrangement, the energy may be transmitted from one end to the line or it may be fed from the center out to each end.
- The adjacent elements are connected by a phase shifter with phase ϕ.
- All the phase shifters are identical and introduce the same amount of phase shift which is less than 2Π radians.
- In parallel fed the energy to be radiated is divided between the elements by a power splitter.
- When a series of power splitters are used to create a tree like structure is called a corporate feed.
- Equal lengths of line transmit the energy to each element so that no unwanted phase difference are introduced by the lines themselves.
- The maximum phase change required of each phase shifter in the parallel-fed array is many times ***2*Π *radians.***

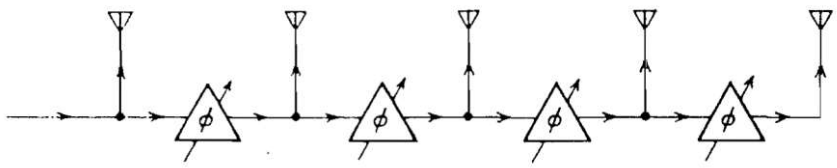
- In a series-fed array containing N phase shifters, the signal suffers the insertion loss of a single phase shifter N times. In a parallel-fed array the insertion loss of the p h s e shifter is introduced effectively but once.

- Since each phase shifter in the series-fed linear array of below Fig ha**s the same value of** phase shift, only a single control signal is needed to steer the beam. The N-element parallel-fed linear array similar to that of below Fig requires a separate control signal for each phase shifter or N - **1 total.**

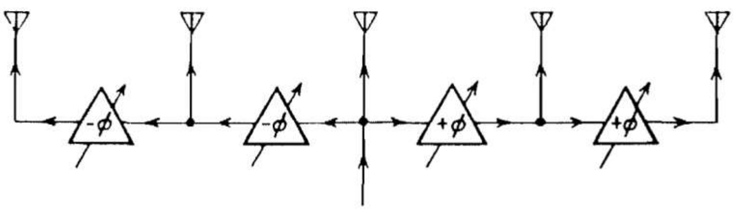


Parallel Feed

Series arrangements for applying phase relationships in an array.
(a) fed from one end; ***(b) center-fed.***



(a)



(b)