UNIT - II

3

Losses of Prestress

Q) List the various types of loss of prestress in pre-tensioned and post-tensioned members. (3 m)

Types of Losses of Prestress

Pre-tensioning	Post-tensioning
1) Elastic deformation of concrete.	1) No loss due to elastic deformation if all the wires are simultaneously tensioned. If the wires are successively tensioned, there will be loss of prestress due to elastic deformation of concrete.
2) Relaxation of stress in steel.	2) Relaxation of stress in steel
3) Shrinkage of Concrete	3) Shrinkage of concrete.
4) Creep of concrete	4) Creep of concrete.
	5) Friction
	6) Anchorage slip.

Q) How do you compute loss ~~due to~~ of stress due to elastic deformation of concrete. (3 m)

Loss of stress due to Elastic deformation of concrete:

The loss of prestress due to elastic deformation of concrete depends on the modular ratio and the average stress in concrete at the level of steel.

Modular ratio, $\alpha_e = \dfrac{E_s}{E_c}$

If f_c = prestress in concrete at the level of steel.
E_s = modulus of elasticity of steel.
E_c = modulus of elasticity of concrete

Strain in concrete at the level of steel $= \left[\dfrac{f_c}{E_c}\right]$

Loss of stress in steel corresponding to this strain $= \left[\dfrac{f_c}{E_c}\right] E_s = \boxed{\alpha_e f_c}$

→ If the initial stress in steel (i.e, transferred to concrete) is known, the % loss of stress in steel can be computed. $\boxed{\% \text{ Loss of stress} = \dfrac{\text{Loss of stress}}{\text{Initial stress}} \times 100}$

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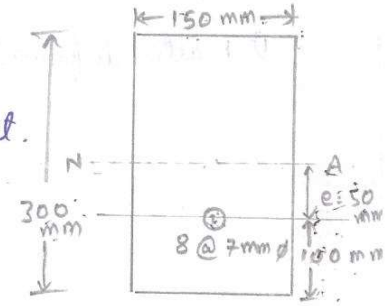
Q) A pretensioned concrete beam of rectangular c/s, 150 mm wide & 300 mm deep, is prestressed by 8 hight tensile wires of 7 mm dia located at 100 mm from the soffit (bottom) of the beam. If the wires are tensioned to a stress of 1100 N/mm², calculate the % loss of stress due to elastic deformation of concrete assuming modulus of elasticity of concrete and steel as 31.5 and 210 KN/mm². (5m)

sol) Given:



Size of the beam = 150 mm × 300 mm.

No. of wires = 8 @ 7 mm dia located at 100 mm from soffit.

Initial prestress = 1100 N/mm².

$E_c = 31.5\ KN/mm^2$, $E_s = 210\ KN/mm^2$.

→ Area of 8 steel wires = $A_p = \left(8 \times \frac{\pi}{4}(7^2)\right) = 308\ mm^2$.

Prestressing force, $P = (1100 \times 308) = 338.8 \times 10^3\ N$.

Area of concrete section, $A = (150 \times 300) = 45000\ mm^2$.

Moment of Inertia of concrete section, $I = \frac{bd^3}{12} = \frac{150 \times 300^3}{12}$

$= 33.75 \times 10^7\ mm^4$.

Modular ratio, $\alpha_e = \frac{E_s}{E_c} = \frac{210}{31.5} = 6.66$

Eccentricity, e = 50 mm

Distance of N.A from the bottom steel, y = 50 mm

→ Stress in concrete at the level of steel,

$$f_c = \left[\frac{P}{A}\right] + \left[\frac{P.e}{I} \cdot y\right]$$

$$= \left[\frac{338.8 \times 10^3}{45000}\right] + \left[\frac{338.8 \times 10^3 \times 50 \times 50}{33.75 \times 10^7}\right] = \boxed{10.02}\ N/mm^2$$

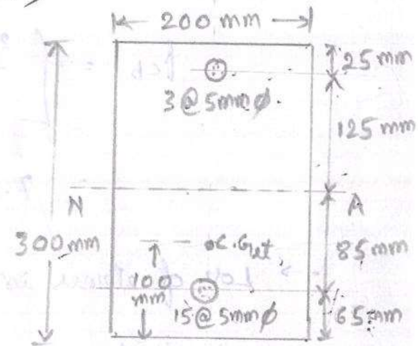
→ Loss of stress in steel due to elastic deformation of concrete, $= (\alpha_e \times f_c)$

$= (6.66 \times 10.2) = \boxed{66.73}\ N/mm^2$

→ % loss of stress in steel = $\frac{\text{Loss of stress}}{\text{Initial stress}} \times 100 = \frac{66.73}{1100} \times 100 = \boxed{6.06\%}$

Q) A rectangular concrete beam, 300 mm deep and 200 mm wide is prestressed by means of 15 (5 mm) diameter wires located at 65 mm from the bottom of the beam and 3 (5 mm) diameter wires, located 25 mm from the top of the beam. If the wires are initially tensioned to a stress of 840 N/mm², calculate the % loss of stress in steel immediately after transfer of prestress, allowing for the loss due to elastic deformation of concrete only. (5m) ($E_s = 210$ kN/mm², $E_c = 31.5$ kN/mm²)

Sol) Given:



→ Size of the beam = 200 mm × 300 mm

Prestressing wires = 15 no.s @ 5 mm diameter at bottom and 3 no.s @ 5mm dia. at top

Initial prestress = 840 N/mm².

$E_s = 210$ kN/mm²

$E_c = 31.5$ kN/mm².

→ Prestressing force, $P = 840\left[18 \times \frac{\pi}{4}(5)^2\right] = 296.88 \times 10^3$ N

Area of concrete section, $A = (200 \times 300) = 6 \times 10^4$ mm².

Position of centroid of the wires from the bottom of the beam.

$$C.G_{st.} = \left[\frac{15\left(\frac{\pi}{4}(5)^2\right) \times 65 + \left(3\left(\frac{\pi}{4}(5)^2\right) \times 275\right)}{15\left(\frac{\pi}{4}(5)^2\right) + 3\left(\frac{\pi}{4}(5)^2\right)}\right]$$

$= 100$ mm from bottom.

∴ Eccentricity, $e = (150 - 100) = 50$ mm

$y_{top} = 125$ mm

$y_{bottom} = 85$ mm

Moment of Inertia of beam c/s, $I = \frac{bd^3}{12} = \frac{200 \times 300^3}{12} = 45 \times 10^7$ mm⁴.

Modular ratio, $\alpha_e = \frac{E_s}{E_c} = \frac{210}{31.5} = 6.66$

Stress in concrete :

At the level of top wires $= \left[\frac{P}{A}\right] - \left[\frac{P.e}{I}.y_t\right]$

$$f_{ct} = \left[\frac{296.88 \times 10^3}{6 \times 10^4}\right] - \left[\frac{296.88 \times 10^3 \times 50 \times 125}{45 \times 10^7}\right]$$

$$= \boxed{0.83} \ N/mm^2$$

At the level of bottom wires $= \left[\frac{P}{A}\right] + \left[\frac{P.e}{I}.y_b\right]$

$$f_{cb} = \left[\frac{296.88 \times 10^3}{6 \times 10^4}\right] + \left[\frac{296.88 \times 10^3 \times 50 \times 85}{45 \times 10^7}\right]$$

$$= \boxed{7.85} \ N/mm^2$$

→ Loss of stress in wires at top $= (\alpha_e \times f_{ct}) = (6.66 \times 0.83) = \boxed{5.52} \ N/mm^2$

Loss of stress in wires at the bottom $= (\alpha_e \times f_{cb}) = (6.66 \times 7.85) = \boxed{52.2} \ N/mm^2$

→ Percentage loss of stress :

For wires at top $= \left[\frac{5.52}{840} \times 100\right] = \boxed{0.65 \ \%}$

For wires at bottom $= \left[\frac{52.2}{840} \times 100\right] = \boxed{6.21 \ \%}$

Q) A post-tensioned concrete beam, 100 mm wide and 300 mm deep is prestressed by three cables, each wire with c/s area of 50 mm² and with an initial prestress of 1200 N/mm². All the 3 cables are located 100 mm from the soffit of the beam. If the modular ratio is 6, calculate the loss of stress in 3 cables due to ~~due to~~ elastic deformation of concrete for only the following cases

a) Simultaneous tensioning and anchoring of all the 3 cables and

b) Successive tensioning of the 3 cables, one at a time. (10 m) sd). Given :

→ Size of the beam c/s, 100 mm × 300 mm.

No. of prestressing wires : 3 (straight)

c/s Area of each wire = 50 mm^2.

Initial prestress in wires = 1200 N/mm^2.

Location of the wires, = 100 mm from the soffit.

Modular ratio, $\alpha_e = 6$.

→ Force in each cable, $P = (1200 \times 50) = 60 \times 10^3$ N.

Area of c/s of concrete beam, $A = 100 \times 300 = 3 \times 10^4$ mm^2.

Eccentricity, $e = 50$ mm.

Distance of N.A from the position of steel, $y = 50$ mm

Moment of Inertia, $I = \frac{bd^3}{12} = \frac{100 \times 300^3}{12} = 225 \times 10^6$ mm^4.

Stress in concrete at the level of steel, $f_c = \left[\frac{P}{A}\right] + \left[\frac{P.e}{I}.y\right]$

$$f_c = \left[\frac{60 \times 10^3}{3 \times 10^4}\right] + \left[\frac{60 \times 10^3 \times 50 \times 50}{225 \times 10^6}\right]$$

$$= \boxed{2.66} \text{ N/mm}^2.$$

Case (a) : Simultaneous tensioning and anchoring of all 3 cables :

Under simultaneous tensioning and anchoring of all 3 cables, there will be no loss of prestress due to elastic deformation of concrete.

Case (b) : ~~Simulta~~ Successive tensioning of 3 cables, one at a time :

When the cables are successively tensioned :

* Cable 1 is tensioned and anchored → no loss due to elastic deformatio

* Cable 2 is tensioned and anchored :

→ Loss of stress in cable 1 = $\alpha_e . f_c = (6 \times 2.66) = 15.96$ N/mm^2.

→ No loss in Cable 2.

* Cable 3 is tensioned and anchored:

→ Loss of stress in Cable 1 = $(6 \times 2.66) = 15.96 \ N/mm^2$.

→ Loss of stress in Cable 2 = $(6 \times 2.66) = 15.96 \ N/mm^2$.

→ No Loss of stress in Cable 3.

∴ Total loss of stress due to elastic deformation of concrete in,

Cable 1 = $(15.96 + 15.96) = 32 \ N/mm^2$.

Cable 2 = $15.96 \ N/mm^2 \approx 16 \ N/mm^2$

Cable 3 = $0 \ N/mm^2$.

Average loss of stress considering all 3 cables = $\frac{32+16+0}{3} = \boxed{16} \ N/mm^2$

Successive Tensioning of curved Cables

Q) A post tensioned concrete beam, 100 mm x 300 mm, spanning over 10 m is stressed by successive tensioning and achoring of 3 cables 1, 2 & 3 respectively. The c/s area of each cable is 200 mm² and the initial stress in cable is 1200 N/mm², $\alpha_e = 6$. The first cable is parabolic with an eccentricity of 50 mm below the centroidal axis at centre of span and 50 mm above the centroidal axis at the support sections. The second cable is parabolic with zero eccentricity at the supports and an eccentricity of 50 mm at the centre of the span. The 3rd cable is straight with a uniform eccentricity of 50 mm below the centroidal axis. Estimate the percentage loss of stress in each of the cables, if they are successively tensioned and anchored.

sol) Given:

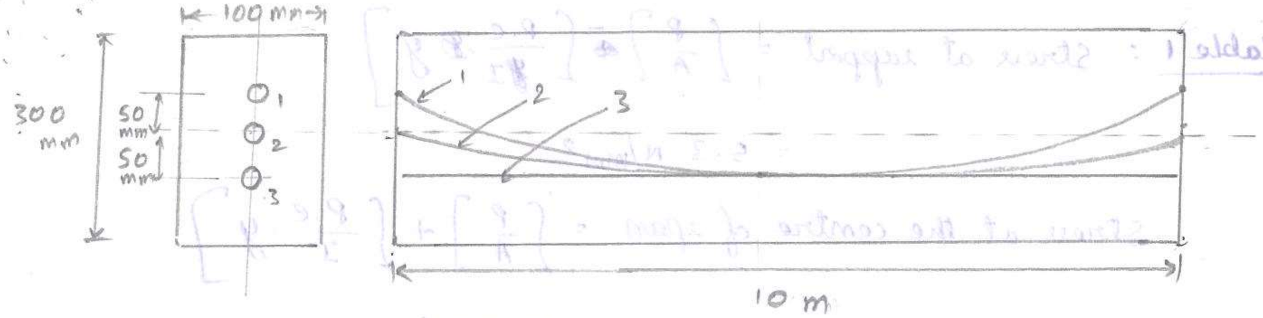
Size of the section = 100 mm x 300 mm

Span of the beam = 10 m.

Initial stress in each cable = $1200 \ N/mm^2$.

c/s area of each cable, A_p = ~~120~~ 200 N/mm².

~~Excess~~ Modular ratio, $\alpha_e = 6$.



→ Force in each cable, $P = (1200 \times 200) = 240 \times 10^3$ N

Area of c/s of beam, $A = 100 \times 300 = 3 \times 10^4\ mm^2$.

Moment of Inertia of the section $= I = \frac{100 \times 300^3}{12} = 225 \times 10^6\ mm^4$.

→ When Cable 1 is tensioned:

No Loss of prestress

→ When Cable 2 is tensioned:

* Loss of stress in cable 1:

Stress at support section $= \left[\frac{P}{A}\right] - \left[\frac{P.e}{I}.y\right] = \left[\frac{240 \times 10^3}{3 \times 10^4}\right] - \left[\frac{240 \times 10^3 \times 50 \times 50}{225 \times 10^6}\right]$

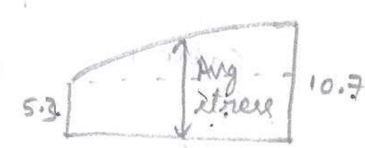
$= \boxed{5.3}\ N/mm^2$.

Stress at centre of span ~~support section~~ $= \left[\frac{P}{A}\right] + \left[\frac{P.e}{I}.y\right]$

$= \left[\frac{240 \times 10^3}{3 \times 10^4}\right] + \left[\frac{240 \times 10^3 \times 50 \times 50}{225 \times 10^6}\right] = \boxed{10.7}\ N/mm^2$.

Average stress in concrete $= \left[5.3 + \frac{2}{3}(10.7 - 5.3)\right]$

$= 8.9\ N/mm^2$



Loss of stress in concrete $= 6 \times 8.9 = 53.4\ N/mm^2$.

* Loss of stress in cable 2:

No loss of pre stress.

→ When cable 3 is tensioned :

Cable 1 : Stress at support $= \left[\frac{P}{A}\right] - \left[\frac{P.e}{I} . y\right]$

$= 5.3 \ N/mm^2.$

Stress at the centre of span $= \left[\frac{P}{A}\right] + \left[\frac{P.e}{I} . y\right]$

$= 10.7 \ N/mm^2.$

Average stress in contrete $= 5.3 + \frac{2}{3}(10.7) = \boxed{8.9} \ N/mm^2$

Loss of stress in cable 1 $= 6 \times 8.9$

$= \boxed{53.4} \ N/mm^2$

Cable 2 : Stress at support $= \frac{P}{A} = \left[\frac{240 \times 10^3}{3 \times 10^4}\right] = 8 \ N/mm^2.$

Stress at centre of span $= \left[\frac{P}{A}\right] + \left[\frac{P.e.y}{I}\right]$

$= \left[\frac{240 \times 10^3}{3 \times 10^4}\right] + \left[\frac{240 \times 10^3 \times 50 \times 50}{225 \times 10^6}\right]$

$= 10.7 \ N/mm^2.$

Average ~~loss of~~ stress in concrete $= \left[8 + \frac{2}{3}(2.7)\right] = \boxed{9.8} \ N/mm^2$

Loss of stress in cable 2 $= \alpha_e \times f_c = 6 \times 9.8 = \boxed{58.8} \ N/mm^2.$

Cable	Total Losses (N/mm^2)	% loss of prestress
Cable 1	$(53.4 + 53.4) = 106.8 \ N/mm^2$	$\frac{106.8}{1200} \times 100 = 8.9\%$
Cable 2	$58.8 \ N/mm^2$	$\frac{58.8}{1200} \times 100 = 4.9\%$
Cable 3	No loss of prestress	0 %

Loss due to Shrinkage of Concrete

Q) How do you compute loss due to shrinkage of concrete as per IS:1343. (5m)

Loss of stress in steel due to shrinkage of concrete:

→ The shrinkage of concrete in prestressed members results in shortening of tensioned wires and hence contributes to the loss of stress.

→ The rate of shrinkage is higher at the surface than interior, resulting in differential shrinkage between interior and surface leading to surface cracking.

→ In case of pre-tensioned members, generally moist curing is adopted in order to prevent shrinkage until the time of transfer of prestress.

→ The total residual shrinkage strain will be larger in pretensioned member than post tensioned member as a portion of shrinkage will have already taken place by the time of transfer of prestress in post tensioning.

→ Recommendations made by the Indian Standard Code (IS:1343) for the loss of prestress due to shrinkage of concrete are as follows.

Total residual shrinkage strain,

$$\boxed{\varepsilon_{cs} = 300 \times 10^{-6}} \quad \text{for pretensioning}$$

$$\boxed{\varepsilon_{cs} = \left[\frac{200 \times 10^{-6}}{\log_{10}(t+2)}\right]} \quad \text{for post tensioning}$$

where t = age of concrete at transfer in days.

* ε_{cs} for post tensioning may be increased by 50% in dry atmospheric conditions, subjected to a maximum value of 300×10^{-6}.

→ The loss of stress in steel due to shrinkage of concrete is estimated as

$$\boxed{\text{Loss of stress} = (\varepsilon_{cs} \times E_s)}$$

where E_s = modulus of elasticity of steel

Q) A concrete beam is prestressed by a cable carrying an initial prestressing force of 300 kN. The cross-sectional area of the wires in the cable is 300 mm^2. Calculate the % loss of stress in the cable only due to shrinkage of concrete using IS:1343 recommendations assuming the beam to be —
a) Pre-tensioned b) Post tensioned. {Assume E_s = 210 kN/mm^2 and age of concrete at transfer = 8 days. (3m)

Ans) Given:

Initial prestressing force, $P = 300 \times 10^3$ N.

Cross area of wires, $A_{st} = 300\ mm^2$.

$E_s = 210$ kN/mm^2

Age of concrete at transfer $t = 8$ days.

→ Initial stress in wires $= \frac{P}{A_{st}} = \frac{300 \times 10^3}{300} = 1000\ N/mm^2$.

(a) If the beam is pre-tensioned,

Loss of stress: As per IS 1343, Total residual shrinkage strain,

$$\varepsilon_{cs} = 300 \times 10^{-6}$$

$\therefore$ Loss of stress $= \varepsilon_{cs} \times E_s = (300 \times 10^{-6})(210 \times 10^3)$

$= \boxed{63}\ N/mm^2$.

$\therefore$ Percentage loss of stress $= \left[\frac{63}{1000} \times 100\right] = \boxed{6.3\ \%}$

(b) If the beam is post-tensioned,

As per IS 1343, Total residual shrinkage strain,

$$\varepsilon_{cs} = \left[\frac{200 \times 10^{-6}}{\log_{10}(t+2)}\right] = \left[\frac{200 \times 10^{-6}}{\log_{10}(8+2)}\right] = 200 \times 10^{-6}$$

$\therefore$ Loss of stress $= \varepsilon_{cs} \times E_s = (200 \times 10^{-6})(210 \times 10^3) = \boxed{42}\ N/mm^2$.

$\therefore$ Percentage loss of stress $= \left[\frac{42}{1000} \times 100\right] = \boxed{4.2\ \%}$

Loss due to Creep of concrete:

Q) How do you compute loss due to creep of concrete. (5m)

Loss of stress in steel due to creep of concrete:

→ The sustained prestress in the concrete results in creep of concrete which effectively reduces the stress in steel.

→ The loss of stress in steel due to creep of concrete can be estimated if the magnitude of ultimate creep strain (or) creep coefficient is known.

1) Ultimate creep strain method:

The loss of stress in steel due to creep of concrete = $\boxed{\varepsilon_{cc} \cdot f_c \cdot E_s}$

where, ε_{cc} = Ultimate creep strain for a sustained unit stress

f_c = Compressive stress in concrete at the level of steel.

E_s = Modulus of elasticity of steel

2) Creep coefficient method:

$$\text{creep coefficient} = \frac{\text{creep strain}}{\text{elastic strain}}$$

$$\boxed{\phi = \frac{\varepsilon_c}{\varepsilon_e}}$$

$$\varepsilon_c = \phi\,\varepsilon_e = \phi\left[\frac{f_c}{E_c}\right]$$

$$\text{Loss of stress in steel} = \varepsilon_c E_s = \phi\left[\frac{f_c}{E_c}\right]E_s = \boxed{\phi f_c \cdot \alpha_e}$$

where, ϕ = creep coefficient

ε_c = creep strain

ε_e = elastic strain

α_e = modular ratio

f_c = stress in concrete.

E_c = modulus of elasticity of concrete.

E_s = modulus of elasticity of steel.

→ The magnitude of creep coefficient, ϕ varies depending upon humidity, concrete quality, duration of applied loading and the age of the concrete when loaded.

→ The general values recommended for ϕ vary from 1.5 for watery situations to 4 for dry conditions with relative humidity of 35 %.

Age of Concrete	Creep Coefficient (ϕ)
7 days →	2.2
28 days →	1.6
1 year →	1.1

Q) A concrete beam of rectangular section, 100 mm × 300 mm is prestressed by 5 wires of 7 mm diameter located at an eccentricity of 50mm below the ~~NA~~ centroidal axis. The initial stress in the wires is 1200 N/mm². Estimate the loss of stress in steel due to creep of concrete using Ultimate creep strain method and creep coefficient method (IS: 1343). Take $E_s = 210$ KN/mm², $E_c = 35$ KN/mm², Ultimate creep strain $\varepsilon_{cc} = 41 \times 10^{-6}$ mm/mm per N/mm². creep coefficient (ϕ) = 1.6. (3m)

sol) Given:

Size of the beam cross section = 100 mm × 300 mm

Prestressing wires → 5 no.s @ 7 mm dia

Eccentricity = 50 mm.

Initial stress = 1200 N/mm².

$E_s = 210$ KN/mm²

$E_c = 35$ KN/mm².

Ultimate creep strain, $\varepsilon_{cc} = 41 \times 10^{-6}$ mm/mm per N/mm².

creep coefficient, $\phi = 1.6$.

→ Area of c/s of beam $= 100 \times 300 = 3 \times 10^4$ mm², $I = \frac{100 \times 300^3}{12} = 225 \times 10^6$ mm⁴

→ Prestressing force, P = stress × A_{st} $= 1200\left(5 \times \frac{\pi}{4} 7^2\right) = 23 \times 10^4$ N.

→ Stress in concrete at the level of steel,

$$f_c = \left[\frac{P}{A}\right] + \left[\frac{P.e}{I} \cdot y\right] = \left[\frac{23 \times 10^4}{3 \times 10^4}\right] + \left[\frac{23 \times 10^4 \times 50 \times 50}{225 \times 10^6}\right] = 10.2 \ N/mm^2$$

$$\boxed{f_c = 10.2 \ N/mm^2}$$

$$\alpha_e = \frac{E_s}{E_c} = \frac{210}{35} = 6.$$

Ultimate creep strain method :

Loss of stress in steel $= \varepsilon_{cc} \cdot f_c \cdot E_s$

$= (41 \times 10^{-6})(10.2)(210 \times 10^3)$

$= \boxed{88} \ N/mm^2$.

Creep coefficient method :

Loss of stress in steel $= \phi \cdot f_c \cdot \alpha_e$.

$= (1.6 \times 10.2 \times 6)$

$= \boxed{97.92} \ N/mm^2$

Q) A post tensioned concrete beam of rectangular section, 100 mm X 300 mm, is stressed by a parabolic cable with zero eccentricity at the supports and 50 mm eccentricity at centre of span. The area of cable is 200 mm². and initial stress in the cable is 1200 N/mm². If the ultimate creep strain is 30×10^{-6} mm/mm per N/mm² of stress and modulus of elasticity of steel is 210 $\frac{kN}{mm^2}$, compute loss of stress only due to creep of concrete.

sol) Given :

Size of the beam c/s = 100 mm X 300 mm

Eccentricity at support = 0

at centre of span = 50 mm.

Area of cable = 200 mm² = A_{st}.

Initial stress in the cable = 1200 N/mm².

Ultimate creep strain, $\varepsilon_{cc} = 30 \times 10^{-6}$ mm/mm per N/mm² of stress.

$E_s = 210$ mm².

→ Area of beam c/s $= 100 \times 300$ → $I = \frac{bd^3}{12} = \frac{100 \times 300^3}{12} = 225 \times 10^6$ mm⁴.

$A = 3 \times 10^4$ mm².

Prestressing force, P = initial stress × Ast

$= 1200 \times 200 = 240 \times 10^3$ N.

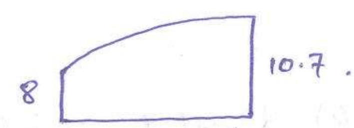
→ Stress in concrete at the level of steel:

At support section $= \left[\frac{P}{A}\right] = \left[\frac{240 \times 10^3}{3 \times 10^4}\right] = 8$ N/mm².

At centre of span $= \left[\frac{P}{A}\right] + \left[\frac{Pe}{I} \cdot y\right]$

$$= \left[\frac{240 \times 10^3}{3 \times 10^4}\right] + \left[\frac{240 \times 10^3 \times 50 \times 50}{225 \times 10^6}\right] = 10.7 \text{ N/mm}^2$$

∴ Average stress at the level of steel,



$$f_c = \left[8 + \frac{2}{3}(10.7 - 8)\right]$$

$= 9.8$ N/mm².

∴ Loss of stress in the cable due to creep of concrete,

$= \varepsilon_{cc} \cdot f_c \cdot E_s$

$= (30 \times 10^{-6})(9.8)(210 \times 10^3)$

$= \boxed{62}$ N/mm²

<u>Loss due to Relaxation of stress in steel.</u>

Q) Discuss briefly about loss of stress in steel due to relaxation of stress. (3m)

<u>Loss due to Relaxation of stress in steel:</u>

→ Most of the codes provide the loss of stress due to relaxation of steel as a percentage of the initial stress in steel.

→ Over a period of time, there will be relaxation of stress in steel at constant strain due to molecular re-arrangement.

→ The Indian Standard Code, IS:1343, recommends the loss of stress due to relaxation of steel as percentage of initial stress in wires varying from 0.5 f_{pu} to 0.8 f_{pu} as shown in the following table. (where f_{pu} = ultimate tensile strength).

Relaxation losses for prestressing steel at 1000 hr at 27 ± 2 °c (IS:1343)

S. No.	Initial Stress	Relaxation Loss (%)	
		Normal	Low.
1	0.5 f_{pu}	0	0
2	0.6 f_{pu}	0.3 %	1 %
3	0.7 f_{pu}	5 %	2.5 %
4.	0.8 f_{pu}	8 %.	4.5 %.

Loss of Stress due to Friction

Q) Explain briefly about loss of stress due to friction.

Loss of stress due to Friction:

→ On tensioning the curved tendons, loss of stress occurs in the post-tensioned members due to friction between the tendons and the surrounding concrete ducts.

→ The magnitude of this loss is of the following types.

1) Loss of stress due to the curvature effect, which depends upon the tendon alignment which generally follows a curved profile along the length.

2) Loss of stress due to the wobble effect, which depends upon the local deviations in the alignment of the cable. The wobble (or) wave effect is the result of accidental (or) unavoidable misalignment as ducts (or) sheath cannot be perfectly aligned as predetermined.

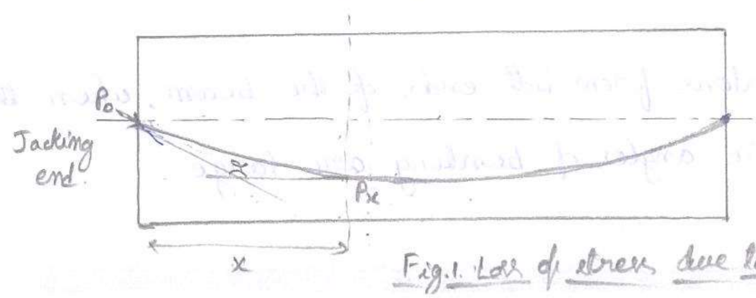


Fig.1. Loss of stress due to friction

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→ Referring to the Fig. 1, the magnitude of prestressing force, P_x at a distance 'x' from the tensioning end follows an exponential function,

$$\boxed{P_x = P_0 \cdot e^{-(\mu\alpha + kx)}}$$

$$\boxed{P_x = P_0\left[1 - (\mu\alpha + kx)\right]} \rightarrow \text{for small value of } (\mu\alpha + kx)$$

where, P_0 = Prestressing force at the jacking end.

μ = coefficient of friction between cable and duct.

α = the cumulative angle (in radians) between tangents of any two points under consideration

K = friction coefficient for 'wave' effet.

e = 2.7183.

The Indian standard code recommends following values for μ and k.

Values for μ:

	μ
For steel moving on smooth concrete →	0.55
For steel moving on steel fixed to duct →	0.35
For steel moving on steel fixed to concrete →	0.25
For steel moving on lead →	0.25.
For wires in rigid rectangular steel sheaths →	0.18 - 0.3
For wires with spacer plates providing seperation →	0.15 to 0.25.

Values for k:

For normal conditions → 0.15 /100 m.

For thin walled ducts under heavy vibrations → 1.5 /100 m.

→ Friction losses can be reduced by

1) Overtensioning the tendons by an amount equal to maximum frictional loss

2) Jacking the tendons from both ends of the beam, when the tendons are long (or) when the angles of bending are large.

Q) A concrete beam of 10 m span, 100 mm wide and 300 mm deep, is prestressed by 3 cables. The area of each cable is 200 mm² and the initial stress in cable is 1200 N/mm². Cable 1 is parabolic with an eccentricity of 50 mm above the centroid at supports and 50 mm below at the centre of span. Cable 2 is also parabolic with zero eccentricity at supports and 50 mm below the centroid at the centre of span. Cable 3 is straight with uniform eccentricity of 50 mm below the centroid. If the cables are tensioned at one end only, estimate the percentage loss of stress in each cable due to friction. Assume $\mu = 0.35$ and $K = 0.0015$ per meter. Equation of a parabola is $y = \left(\frac{4e}{L^2}\right) x (L-x)$ (5m)

Sol) Given: Length of beam = 10 m

Area of cross section of beam, $A = 100 \text{ mm} \times 300 \text{ mm}$

$= 3 \times 10^4 \text{ mm}^2$.

Initial stress $= 1200 \text{ N/mm}^2$

$\mu = 0.35$

$K = 0.0015 \text{ /m}$

Equation of parabola, $y = \left[\frac{4e}{L^2}\right] x (L-x)$

$$\text{Slope} = \frac{dy}{dx} = \left[\frac{4e}{L^2}\right](L-2x)$$

$$\text{At supports } \underset{(x=0)}{\frac{dy}{dx}} = \frac{4e}{L} \times L^2 = \frac{4e}{L}$$

For Cable 1:

$$\text{Slope at one end} = \left[\frac{4 \times 100}{10 \times 10^3}\right] = 0.04$$

$\therefore$ cumulative angle between tangents of 2 ends, $\alpha = 2(0.04)$

$= \boxed{0.08}$ radians

For Cable 2:

$$\text{Slope at one end} = \left[\frac{4 \times 50}{10 \times 1000}\right] = 0.02.$$

$\therefore \alpha = 2(0.02) = \boxed{0.04}$ radians

→ Initial prestressing force in each cable, $P_o = 200 \times 1200$
$= 240 \times 10^3$ N.

Let P_x be the prestressing force in the cable at other end.

$\therefore P_x = P_o \cdot e^{-(\mu\alpha + kx)}$

where $x = 10$ m

For small values of $(\mu\alpha + kx)$

$P_x = P_o(1 - (\mu\alpha + kx))$

Loss of prestressing force, $P_o - P_x = P_o(\underbrace{\mu\alpha}_{\text{curvature effect}} + \underbrace{kx}_{\text{wobble effect}})$

For cable 1, $P_o - P_x = 240 \times 10^3 (0.35 \times 0.08 + 0.0015 \times 10) = 10.3 \times 10^3$ N

For cable 2, $P_o - P_x = 240 \times 10^3 (0.35 \times 0.04 + 0.0015 \times 10) = 39.36 \times 10^3$ N.

For cable 3, $P_o - P_x = 240 \times 10^3 (0 + 0.015 \times 10) = 3.6 \times 10^3$ N.

Cable No.	Loss of stress $= \frac{P_x - P_o}{1200}$ N/mm²	Percentage Loss
1.	51.6	4.3
2.	34.8	2.9
3.	18	1.5

Q) How do you compute loss due to Anchorage Slip. (2 m)

Loss due to Anchorage Slip:

In most of the post-tensioning systems, when the cable is tensioned and the jack is released, the friction wedges slip over a small distance before the wires are firmly housed between the wedges. The magnitude of slip depends upon the type of wedge and the stress in the wires.

The magnitude of the loss of stress due to the slip in anchorage is computed as follows:

$$\boxed{\text{Loss of stress} = \left[\frac{P}{A}\right] = \left[\frac{E_s \Delta}{L}\right]}$$

where Δ = Slip of anchorage, mm
L = Length of cable, mm
A = cross-sectional area of cable, mm²
E_s = modulus of elasticity of steel, N/mm²
P = Prestressing force in the cable, N.

Q) A concrete beam is post-tensioned by a cable carrying an initial stress of 1000 N/mm^2. The slip at the jacking end was observed to be 5 mm. The modulus of elasticity of steel is 210 KN/mm^2. Estimate the percentage loss of stress due to anchorage slip if the length of the beam is a) 30 m and b) 3 m. (2m)

sd) Loss of stress due to anchorage slip = $\left[\frac{E_s \Delta}{L}\right]$

a) For a 30 m long beam :

Loss of stress due to anchorage slip = $\frac{210 \times 10^3 (5)}{30 \times 1000} = 35 \ N/mm^2$.

$\therefore$ Percentage loss of stress = $\left[\frac{35}{1000} \times 100\right] = 3.5\%$.

b) For a 3 m long beam,

Loss of stress = $\frac{210 \times 10^3 (5)}{3 \times 1000} = 350 \ N/mm^2$.

$\therefore$ Percentage loss of stress = $\left[\frac{350}{1000} \times 100\right] = 35\%$.

Total Losses in Pre Stressing.

Q) A pretensioned beam, 200 mm wide and 300 mm deep is prestressed by 10 wires of 7 mm diameter, initially stressed to 1200 N/mm^2, with their centroids located 100 mm from the soffit. Find the maximum stress in concrete immediately after transfer, allowing only for elastic shortening of concrete.

If the concrete undergoes a further shortening due to creep and shrinkage while there is a relaxation of 5% of steel stress, estimate the final percentage loss of stress in the wires using the IS:1343 regulations, and the following data.
$E_s = 210 \ KN/mm^2$, $E_c = 5700\sqrt{f_{cu}}$, $f_{cu} = 42 \ N/mm^2$, $\phi = 1.6$, $\mathcal{E}_s = 3 \times 10^4$.

sd) $E_c = 5000\sqrt{f_{cu}} = 5000\sqrt{42} = 32403.7 \ N/mm^2$

Prestressing force, P = Stress $\times$ Area of Steel = $1200\left[10 \times \frac{\pi}{4} 7^2\right] = 461.8 \times 10^3 \ N/mm^2$

Area of cross section of beam, $A = 200 \times 300 = 6 \times 10^4 \ mm^2$.

eccentricity $e = 50$

(10)

Moment of Inertia, $I = \frac{bd^3}{12} = \frac{200 \times 300^3}{12} = 450 \times 10^6 \text{ mm}^4$

Stress in concrete at the level of steel,

$$f_c = \frac{P}{A} + \frac{Pe}{I} \cdot y$$

$$= \left[\frac{461.8 \times 10^3}{6 \times 10^4}\right] + \left[\frac{461.8 \times 10^3 \times 50 \times 50}{450 \times 10^6}\right] = 10.26 \text{ N/mm}^2.$$

Modular ratio, $\alpha_e = \frac{E_s}{E_c} = \frac{210 \times 10^3}{32403.7} = 6.48$

Loss of stress due to elastic shortening of Concrete:

$$= f_c \cdot \alpha_e = 10.26 \times 6.48 = \boxed{66.48} \text{ N/mm}^2.$$

Force in cable after this loss $= [1200 - 66.48] \times \left[10 \times \frac{\pi}{4} 7^2\right]$

$$P' = 436.2 \times 10^3 \text{ N}.$$

Stress in ~~cable~~ concrete at the level of steel.

$$f_c' = \frac{P'}{A} + \frac{P'.e}{I} \cdot y$$

$$= \frac{436.2 \times 10^3}{6 \times 10^4} + \frac{436.2 \times 10^3 \times 50}{450 \times 10^6} \times 50 = \boxed{9.69} \text{ N/mm}^2.$$

Loss of stress due to creep of concrete:

Loss of stress $= \phi \cdot f_c \cdot \alpha_e$.

$$= 1.6\,(9.69)\,(6.48) = \boxed{100.46} \text{ N/mm}^2.$$

Loss due to shrinkage of concrete:

Loss of stress $= \varepsilon_{cs} \times E_s$

$$= [3 \times 10^{-4}]\,(210 \times 10^3)$$

$$= \boxed{63} \text{ N/mm}^2.$$

Loss due to relaxation of stress in steel:

$= 5\%$ of initial stress

$$= \frac{5}{100}(1200) = \boxed{60} \text{ N/mm}^2.$$

$\Rightarrow$ Total loss of stress $= 66.48 + 100.46 + 63 + 60$

$= \boxed{289.94}\ N/mm^2$.

Percentage loss of stress $= \frac{289.94}{1200} \times 100$

$= \boxed{24.16\ \%}$

Objective Type Questions :

1) In a pretensioned beam, there will be loss of stress due to (c)

   a) Friction b) Anchorage Slip c) Elastic deformation of concrete.

2) In a post-tensioned beam, there will be loss of stress due to (d.)

   a) Elastic deformation of concrete b) Shrinkage of concrete.

   c) Friction d) All the above.

3) Loss of stress due to elastic deformation of concrete depends upon (c)

   a) Relaxation of steel b) Friction and anchorage slip c) Modular ratio

4) Loss of prestress due to shrinkage of concrete depends upon. Shrinkage strain

5) Loss of stress due to creep of concrete is influenced by creep coefficient

6) Loss of stress due to relaxation of steel is influenced by. Initial stress in steel.

7) Loss of stress due to friction depends upon. coefficient of friction.

8) Loss of stress due to curvature effect depends upon. tendon alignment

9) Loss of stress due to wobble effect depends upon the local deviations in the alignment of the cable.

10) IS 1343 recommends loss of stress due to relaxation of stress in steel as percentage of. Initial stress