UNIT - I

Sensors / Transducer :-

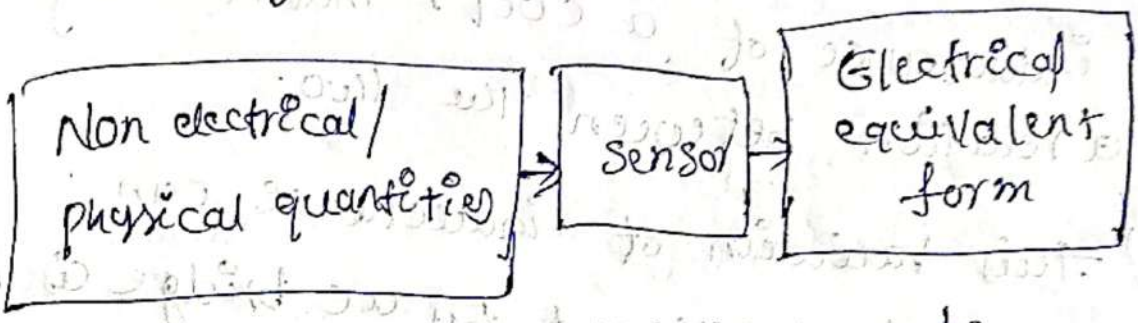
Sensor :- A sensor is a device which converts physical quantities in to electrical form or converts non electrical quantities in to electrical equivalent.

Transducer :- Transducer is a device which used to convert one form of energy to another form.

⇒ In another word transducer which converts a non electrical quantity that means are a physical quantity in to an electrical quantity.

Introduction:-

Sensors :- A sensor is a device which converts physical quantity in to electrical form or converts non electrical quantities in to electrical quantity.



⇒ The physical quantity can be temperature, pressure, force, humidity, motion and displacement light, flow, etc.

⇒ These physical quantities are converted in to electrical form i.e. change in resistance, inductance, capacitance etc.

⇒ These are then converted in to voltage or current signals with in a specified range by the sensor for measurement purpose.

Classification:-

~~Active and~~ To classify the different types of sensors according to different parameter.

⇒ 1) Active and passive sensors:-

Active sensor:- Active sensor are those which require an external excitation signal or a power signal.

Passive sensor:-

Passive sensor, on the other hand, do not require any external power signal and directly generates output response.

2) classification is based on the means of detection used in the sensor.

=) Some of the means of detection are electric, Biological, Chemical, Radio active etc.

3) classification is based on conversion phenomenon i.e., the input and the output.

=) Some of the common conversion phenomena are photoelectric, Thermoelectric, Electrochemical, Electromagnetic, Thermo optic, etc.

4) Analog and digital sensor :-

Analog sensors produce an analog output i.e, a continuous output signal (usually voltage but sometimes other quantities like Resistance etc). with respect to the quantity being measured.

Characteristics :-

sensor characteristics affect their measurement capabilities and define the suitability of a sensor for a particular application.

1) Static
2) Dynamic

1) Static Characteristics :-

(a) Accuracy
(b) Precision
(c) Resolution
(d) Sensitivity
(e) Linearity
(f) Hysteresis

2) Dynamic characteristics :-

The properties of the system transient response to an input.

→ zero order systems
→ First order systems
→ Second order system.

⇒ Static characteristics :-

(a) Accuracy :-

→ Accuracy is defined in terms of closeness to the true value.

→ Error is defined as the difference between the measured value and true value.

→ Smaller the Error more accurate the instrument.

→ Accuracy is measured by the absolute and relative errors.

Absolute Error = Result − True value

$$\text{Relative error} = \frac{\text{Absolute Error}}{\text{True value}}$$

b) Precision :- Is the capacity of a measuring instrument to give the same reading when repetitively measuring the same quantity under the same prescribed conditions.

→ Precision implies agreement successive readings. NOT closeness to the true value.

→ Precision is related to the Variance of a set of measurements.

→ Precision is a necessary but not sufficient condition for accuracy.

Two terms closely related to precision

Re. Repetability and Reproducibility.

Repetability :- is the Precision of a set of measurements taken over a short time interval.

Reproducibility : is the precision of a set of measurements. but,

→ Taken over a long time interval or

→ Performed by different operators or

→ With different instruments or

→ In different laboratories.

(c) Resolution :- Resolution is the smallest increment that the sensor or measurement system can accurately and reliably detect.

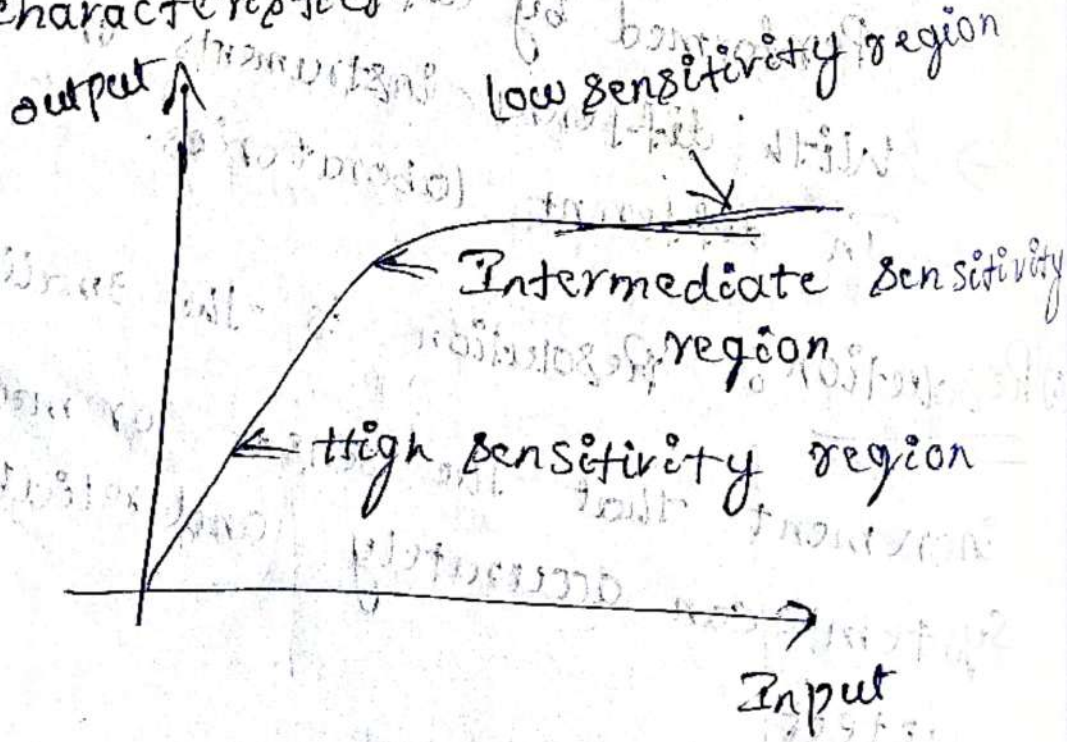
→ In case of analog sensors the quality of the primary transducer decided the resolution.

→ In case of digital sensor the on board signal conditioning circuitry is as important as the transducer itself.

d) Sensitivity :-

⇒ Sensitivity of a sensor is defined in terms of the change in output with respect to the change in input on a per unit basis.

⇒ For analog sensors it is the slope of the transducer input output characteristics.



(e) Linearity :-

→ Most sensor manufacturers specific the accuracy in terms of the sensors range of measurement.

→ This is to account for errors due to any non linearity in the sensor input / output characteristics.

→ Linearity is the maximum deviation between the measured values of a sensor from ideal curve.

(f) Hysteresis :- It is the difference in output when input is varied in two ways increasing and decreasing.

→ Some sensors do not return to the same output when input is increased or decreased.

→ The width of expected error in terms of measured quantity is known as Hysteresis.

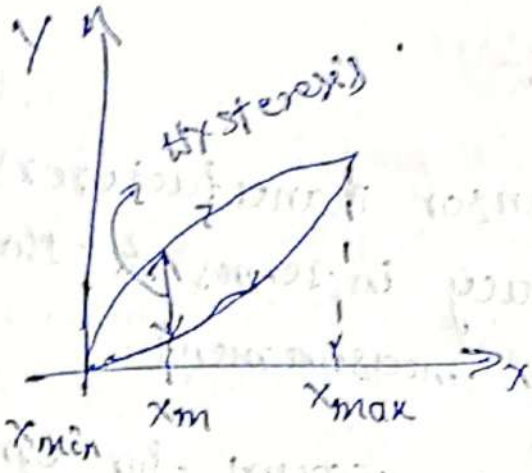


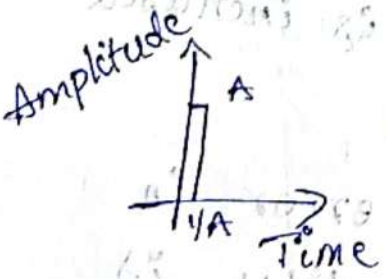
fig - Hysteresis curve.

Dynamic characteristics :-

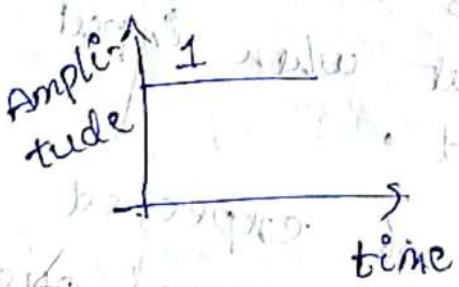
→ The sensor Response to a Variable input is different from that exhibited when the input signals are Constant.

→ The reason for dynamic characteristics is the Presence of energy-storing elements. ~~Intetial~~ :

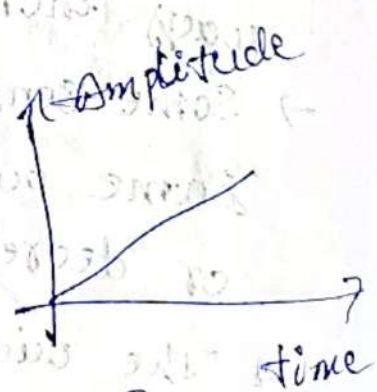
→ Dynamic characteristics are determined by analyzing the response of the sensor to a family of Variable input waveforms.



Impulse

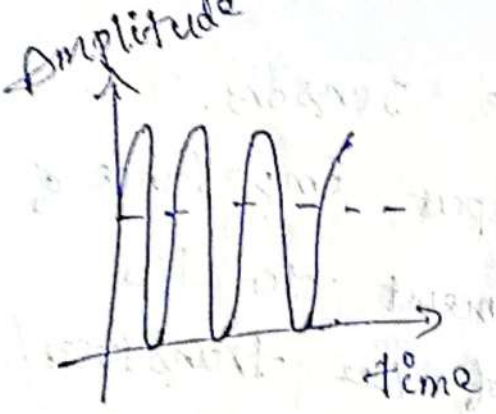


step

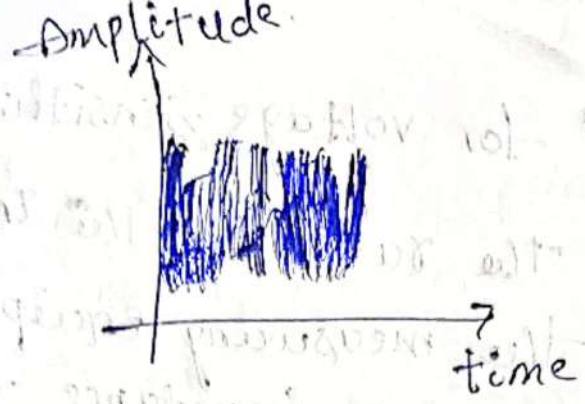


Ramp

~~Ramp~~



Sinusoidal



White noise

Characterization :-

→ Characterization of the sensors can be done in many ways depending on the types of sensors, specifically microsensors.

→ These are electrical, Mechanical, optical, thermal, chemical, biological, and so on.

~~Electro~~

Electrical characterization :-

→ It consists of evaluation of electrical parameter like.

(a) impedance, voltage and currents,
(b) break down voltages and fields.
(c) leakage currents
(d) noise. (e) cross talk and so on.

(a) Impedance, voltage and currents :-

→ The sensor 'output impedance' is very important for coupling the measuring equipment to it.

→ for voltage sensitive sensors.

→ the ratio of the input impedance of the measuring equipment to the output impedance of the transducer/sensor should be very high. while for current sensitive sensors, reverse is true.

(B) Break down voltages and fields :-

→ Break down of the insulating part of the sensor is very critical as the health of the system depends on it.

→ For Metal insulator - semiconductor (or) Metal insulator - metal (MIM) structures

→ Three different types of break down are of (i) dielectric strength
(ii) wear out and
(iii) current induced breakdown.

→ tests are also different and are performed with a particular independent parameter for a particular case.

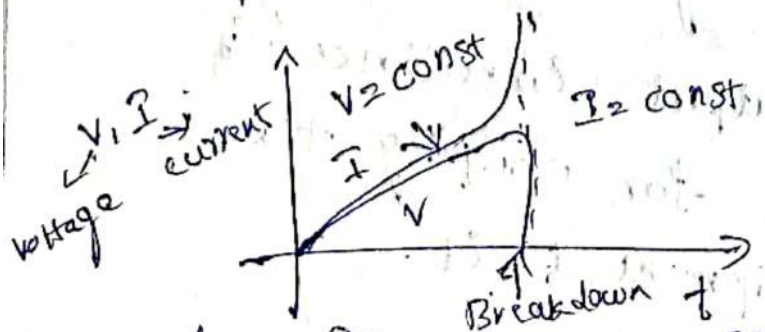


fig (a) - voltage, current.

(b)

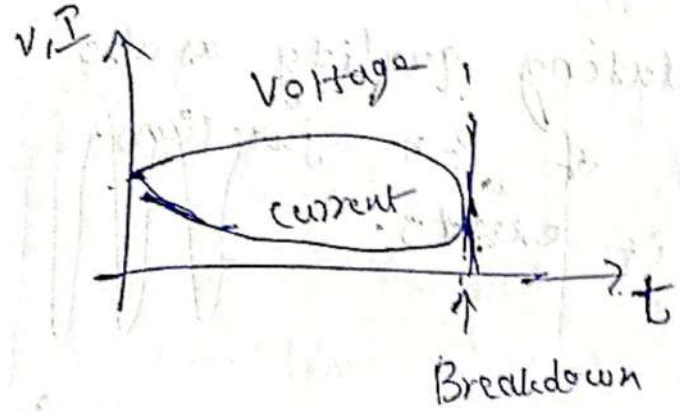
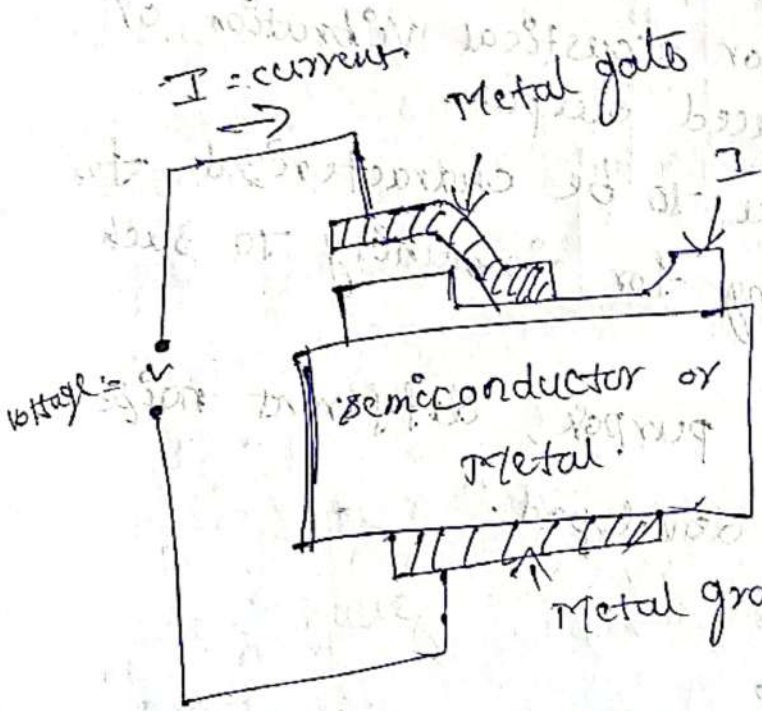


fig:- Semiconductor or metal



(C) ~~Leakage~~ currents:-

→ Break down generally implies a sudden or avalanche change in the voltage or current - voltage droping to a negligible value and current rising to a very high value.

(C) → Leakage currents :- ~~measurents~~

Leakage current measurements specifies the sensor quality.

→ Its Insulating quality as also the quality of p-n junctions wherever it exists.

(d) Noise :-

→ Noise comes from electromagnetic interference, AC magnetic fluctuation, 50Hz supply pick up,

→ Mechanical or acoustical vibration or photon induced output.

→ Sensors are to be characterized for noise testing. for immunity to such noise.

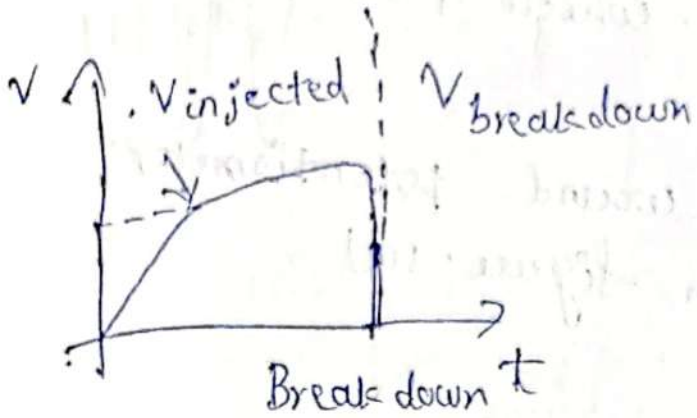
→ for testing purpose, different noise sources are developed.

(e) Cross talk :-

→ cross talk may occur due to overlapping of signals between the two adjacent transducer elements.

→ A signal transducer system because of inductive or capacitive coupling or coupling through the common voltage source during transduction inside the element.

(c) leakage current :-



Electromechanical sensors :-

Introduction :

→ Electromechanical sensors : to convert the input form into an electrical output form : for convenience of processing and display :

→ ~~Me~~ · Electromechanical devices are ones which have both electrical and mechanical processes.

→ operated switch is an electromechanical component due to the mechanical movement causing an electrical output.

Resistive potentiometer :-

→ Resistive potentiometer is a kind of Variable resistance transducer.

→ These are strain gauges, RTD, Thermistor, wire anemometer, piezoresistor and many more.

→ This is a precision wire wound potentiometer which is used as a ~~tension~~ sensor.

→ The wire wound potentiometer is shown in figure (a)

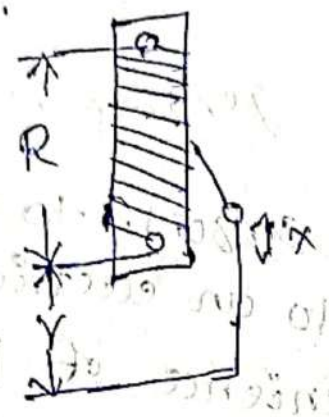


fig (a) wire wound potentiometer.

→ for a voltage supply 'V' to the potentiometer the voltage Resolution would be.

$$\Delta V = \frac{V}{n}$$

→ The solid line stairs show the o/p voltage steps each of which is equivalent to a value V/n.

→ Two adjacent wires are likely to be shorted as ~~sho~~ shown in fig. (b)

→ A major ~~so~~ resolution pulse of magnitude

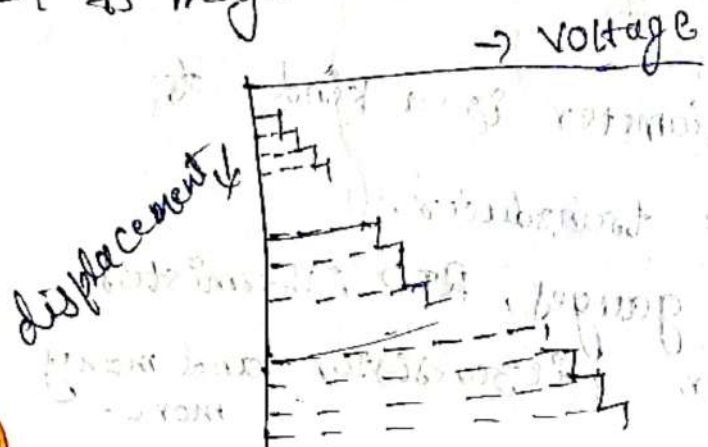


fig (b)

$$\Delta V_m = V_P\left[\frac{1}{n-1} - \frac{1}{n}\right]$$

This is an actual resolution value.

$$\Delta V - \Delta V_m = \frac{V}{n} - V_P\left[\frac{1}{n-1} - \frac{1}{n}\right]$$

→ The ratio of jockey radius to wire radius and geometry of wire winding should be considered for reducing ΔV_m.

→ Resistance wire and jockey are equally important, particularly from the wearing point of view and noise.

→ A few types of jockeys are shown in fig.

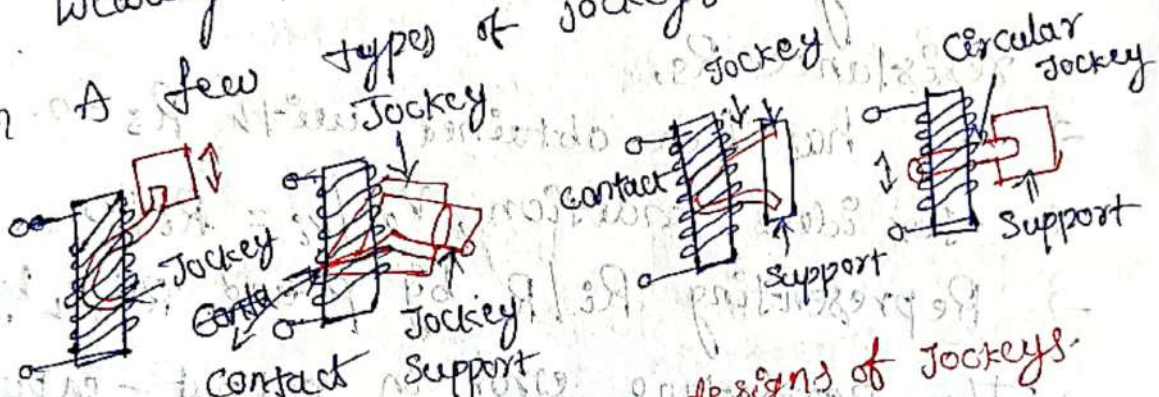
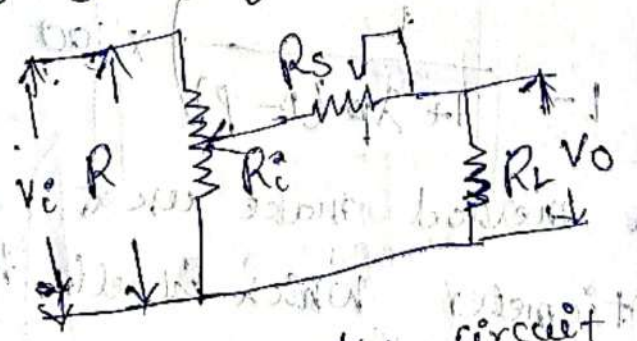


fig:- different designs of Jockeys.

→ The performance of the potentiometer changes in the loaded condition.

→ Its linearity is badly affected.



Considering the circuit of fig.

→ if R_i is the load resistance,

→ V_i and V_o are input and output voltages.

→ R_i is the instantaneous tapped resistance across which V_o is obtained.

→ The minimum $R_i = 0$ and maximum $R_i = R$, then,

$$\frac{V_o}{V_i} = \frac{R_i/R}{1 + \frac{R_i}{R_L}\left(1 - \frac{R_i}{R}\right)}$$

→ The figure shows a variable series resistance R_s,

→ It has been obtained with $R_s = 0$.

→ For ideal condition, $V_{oi}/V_i = R_i/R$.

→ Representing R_i/R by p and R/R_L by λ.

→ The percentage error in output - input voltage ratio is given as

$$\epsilon = \frac{(V_{oi}/V_i - V_o/V_i)}{(V_{oi}/V_i)} \times 100$$

$$= \left(1 - \frac{1}{1 + \lambda p(1-p)}\right) \times 100$$

→ Alternate method make use of

(i) a potentiometer which itself has nonlinear characteristics or

(ii) A nonlinear variable resistance R_s, in series with the load.

→ The first method, This non linear Profile is such that the resistance ratio R_i/R curve drawn against, the Jockey movement is complementary to that of V_o/V_i.

→ In this second method, R_s is also variable.

→ A double Jockey system one for R and the other for R_s with equal length to move should be used. A resistance $R_s = R/4$ with Parabolic resistance.

Strain Gauge :-

→ The basic principle of change in resistance of a metalic wire.

→ To strain produced in it was known as early as the mid-nineteenth century.

→ strain gauges are of two types. namely the resistance type and the semiconductor type.

Resistance strain Gauge :-

Resistance strain gauges can be devided into two categories.

(a) unbonded and

(b) bonded.

(a). Unbonded Strain Gauge :-

→ The unbonded strain gauge consists of a piece of wire stretched in multiple folds.

→ Between a pair or more of insulated pins fixed to movable members of a body or even a single flexible member.

→ These occurs a relative motion between the two members on strain and the wire gets strained as well with a corresponding change in its resistance value.

→ The scheme of such a system is shown in figure.

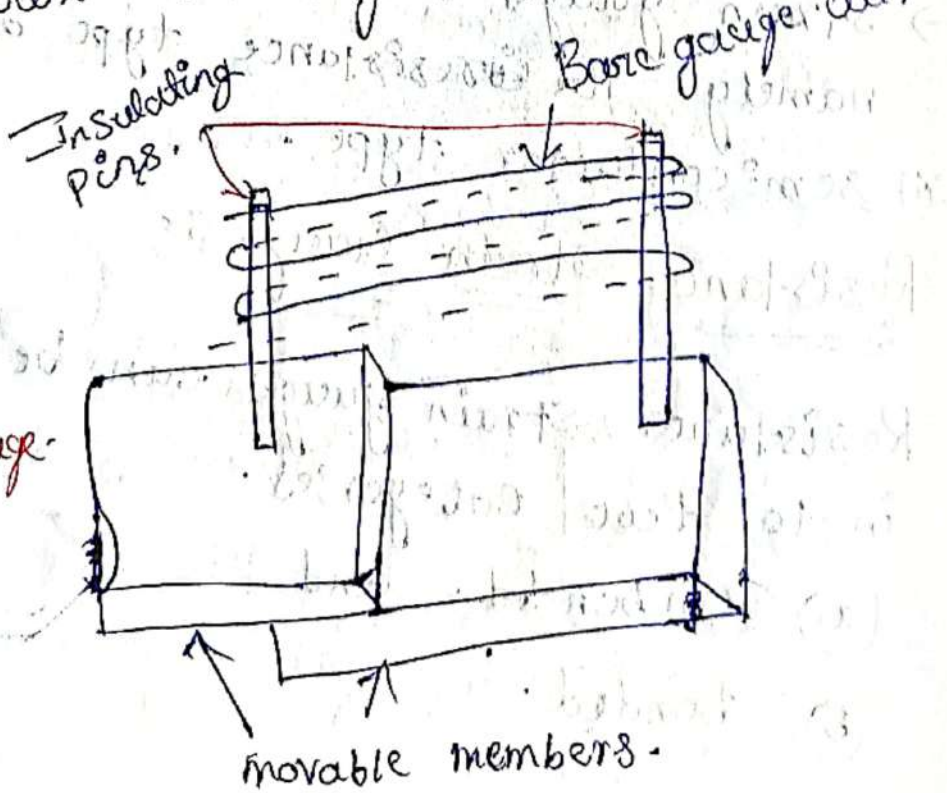


fig :- unbonded strain gauge.

(b) Bonded strain gauge :-

→ The bonded type is more common and in its simplest form consists of wire / strip of resistance material arranged usually in the form of a grid for larger length and resistance value.

→ The grid is bonded to the test specimen with an insulation layer between the gauge material and the specimen as shown in fig.

→ If the insulating and the bonding material thickness is h which also is the height of the wire above the specimen surface and H is the distance of the natural axis of the specimen from its surface.

→ The actual strain ε, in terms of measured strain ε_m, is given by

$$\boxed{\varepsilon = \varepsilon_m \frac{H}{h+H}}$$

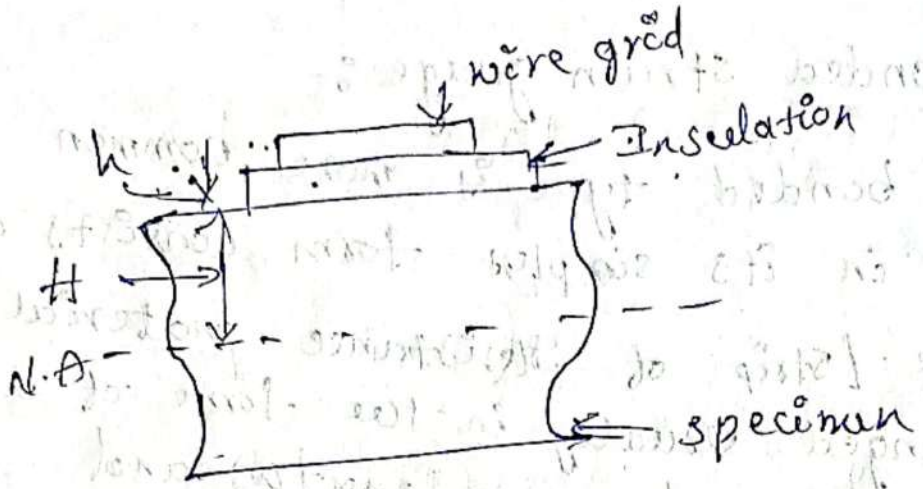


fig :- Bonded strain gauge.

→ depending upon the implementation, the resistance gauges can be classified as

(a) unbonded metal wire.
(b) Bonded metal wire.
(c) Bonded metal foil,
(d) Thin metal film by vacuum deposition and
(e) Thin metal film by sputter deposition.

Considering a circular cross section metal resistance wire of length 'l' and cross sectional area 'A', with Resistivity 'ρ' of the material.

→ The unstrained ~~of the~~ resistance of the wire is given by.

$$R = \frac{\rho l}{A} \quad \text{— (1)}$$

If the wire is uniformly stressed along its length figure.

→ If the stress is given by σ,

then $\frac{dR}{d\sigma} = \frac{d}{d\sigma}\left(\rho \frac{l}{A}\right)$ — (2)

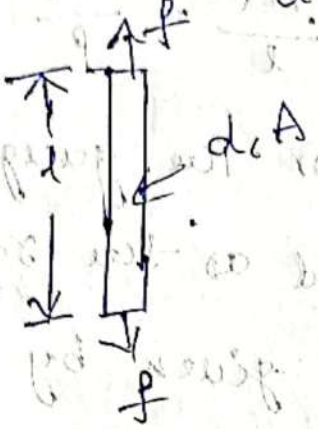


fig: Strain of an elastic member.

which gives,

$$\left(\frac{1}{R}\right)\frac{dR}{d\sigma} = \left(\frac{1}{l}\right)\left(\frac{\partial l}{\partial \sigma}\right) - \left(\frac{1}{A}\right)\left(\frac{\partial A}{\partial \sigma}\right) + \left(\frac{1}{\rho}\right)\left(\frac{\partial \rho}{\partial \sigma}\right)$$

Eliminating all σ terms, we get,

$$\frac{dR}{R} = \frac{\partial l}{l} - \frac{\partial A}{A} + \frac{\partial \rho}{\rho} \quad \text{— (3)}$$

If the wire ~~di~~ has a diameter d then the lateral contraction of the wire,

$\Delta d/d = (1/2)(\Delta A/A)$.

to the functional extension of the length $\therefore \varepsilon = \Delta l/l$ by the poisson's ratio μ as.

$$\frac{\Delta d}{d} = -\frac{\mu \Delta l}{l} \quad \text{--- (4)}$$

So that eq'n (3)

$$\frac{\Delta R}{R} = (1+2\mu)\frac{\Delta l}{l} + \frac{\Delta \rho}{\rho} \quad \text{--- (5)}$$

The strain sensitivity or the gauge factor λ is ~~now~~ defined as the ratio $\left(\frac{\Delta R}{R}\right) / \left(\frac{\Delta l}{l}\right)$. and is given by

$$\lambda = \frac{\Delta R/R}{\Delta l/l} = 1 + 2\mu + \frac{\Delta \rho/\rho}{\Delta l/l} \quad \text{--- (6)}$$

It is generally assumed that resistivity of a metallic material is usually constant. ~~that~~ gauge factor λ is constant.

⇒ un bonded strain gauges are used in Preloaded conditions not to allow the strings to go slack.

⇒ A bonded strain gauges are of a few types. (i) flat grid type, (ii) ~~wor~~ wrap around type, and (iii) woven type.

→ The gauge consists of the resistance element of proper design/shape,

→ The gauge backing, cement, connection leads, and often protective coating or other protective.

→ The construction of the flat grid bonded strain gauge is shown in fig.

→ Such a construction has the advantage of better strain transmission from the member to the wire grid.

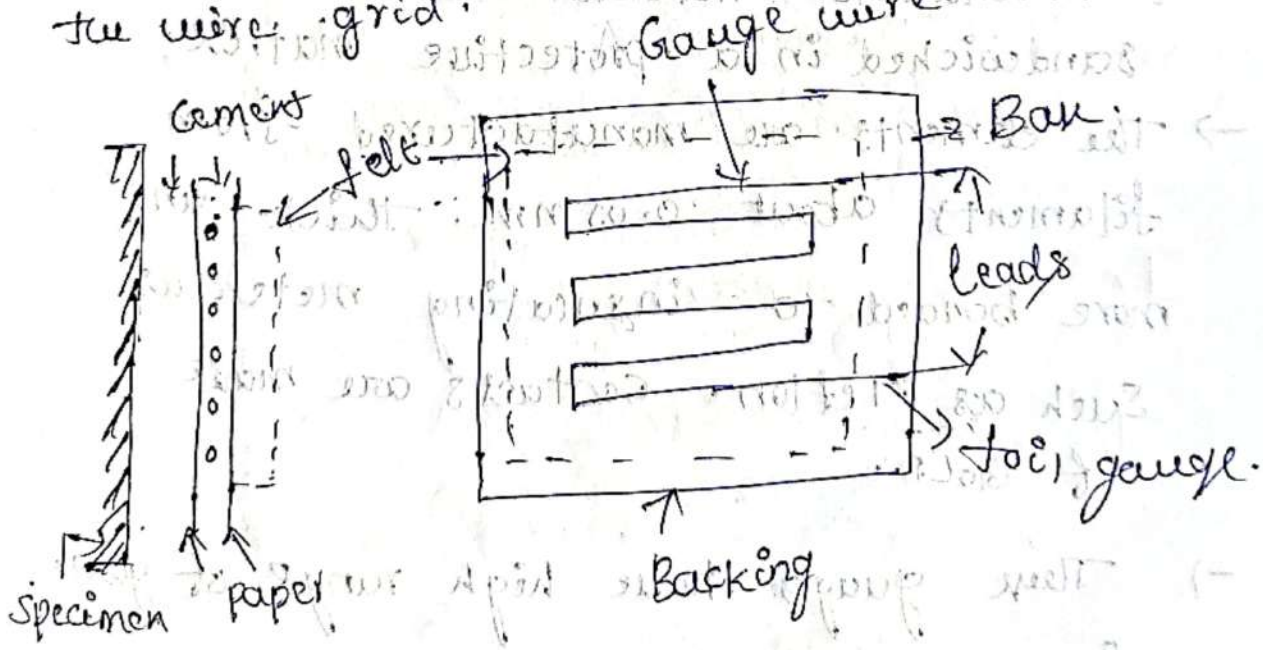


fig :- Grid type gauge.

→ The foil gauges are etched out from deposited films or sheets and have higher surface area to cross section ratio than wire gauges.

Semiconductor Strain gauges :-

→ These are the gauges manufactured from semiconducting meterials like silicon and germanium with additives like boron.

→ These gauges depends for their action of pizeoresistive effect i.e maximum resistive change due to change in resistivity of material.

→ It consists of the strain sensitive semiconductor meterial and leads sandwiched in a protective matrix.

→ The elements are manufactured from filaments about 0.05mm thick and more bonded to insulating meterials such as Teflon. Contacts are made of GOLD.

→ These gauges have high range of gauge factor of 100 - 200 Good response, long life but highly sensitive to temperature

and have poor linearity. These are brittle and not suitable for large strain measurement.

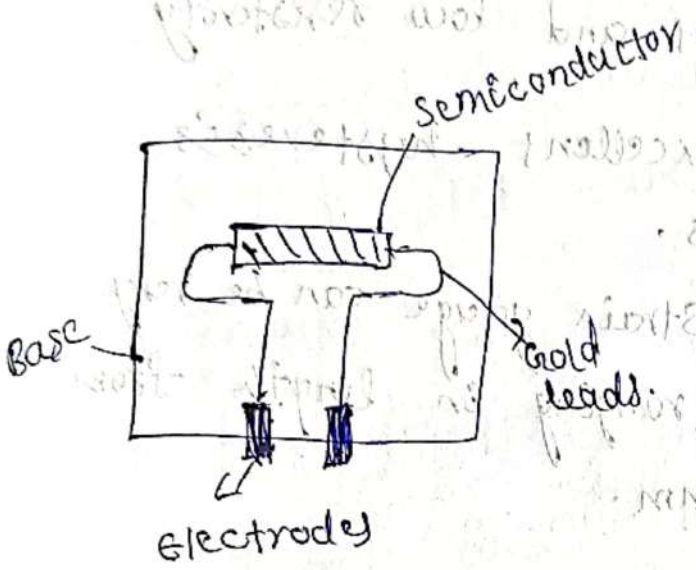
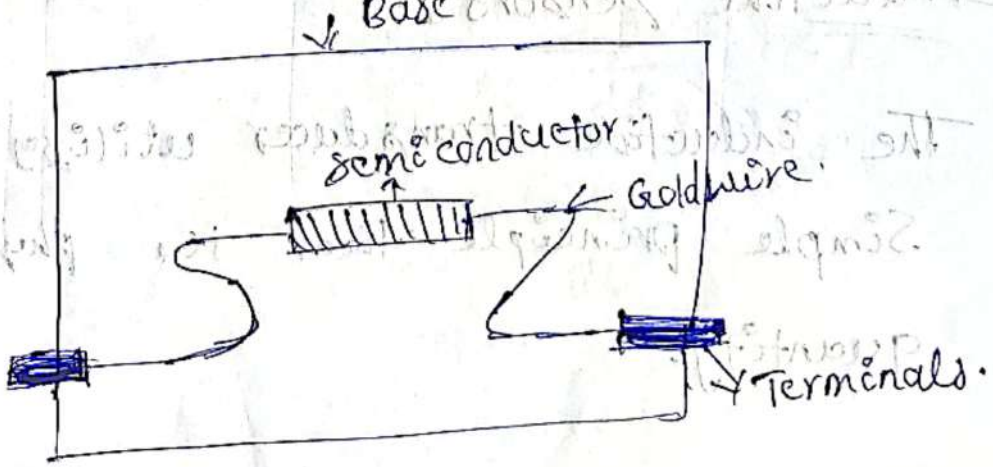


fig:- semiconductor strain gauge

→ Gold leads are generally used for making constant.

→ semiconductor strain gauge is as the metallic type, but has a much higher output.

→ Simple temperature compensation can be applied to semiconductor strain gauges, so that small values of strain.



Advantage:-

→ Hige gauge factor

→ chemically inert and loco resistivity.

→ Linearity & Excellent hysteresis characteristics.

→ semi conductor strain gauge can be very small in size ranging in length from 0.7 to 7.0mm.

Dis-advantages:-

⇒ They are very sensitive to changes in temperature.

⇒ Linearity of semiconductor strain gauge is poor.

⇒ ~~Here~~ They are more expensive.

Capacitive Sensor :-

Three types of capacitive sensors.

⇒ Variable capacitance type

⇒ It varying distance between two or more parallel electrodes.

⇒ Variable area between the electrodes

→ Variable dielectric constant materials.

⇒ The movement of the moving electrodes of the type shown in fig.

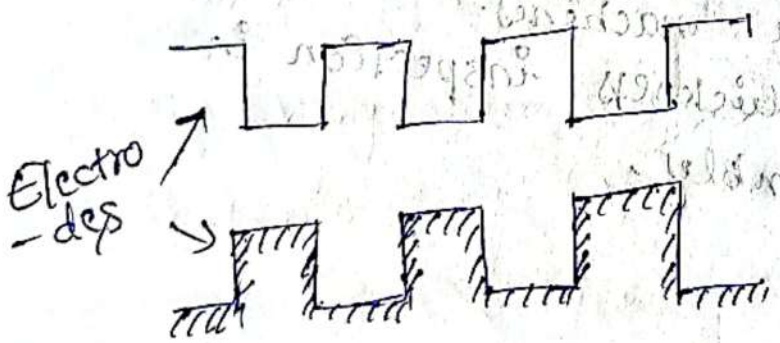


fig :- capacitance type.

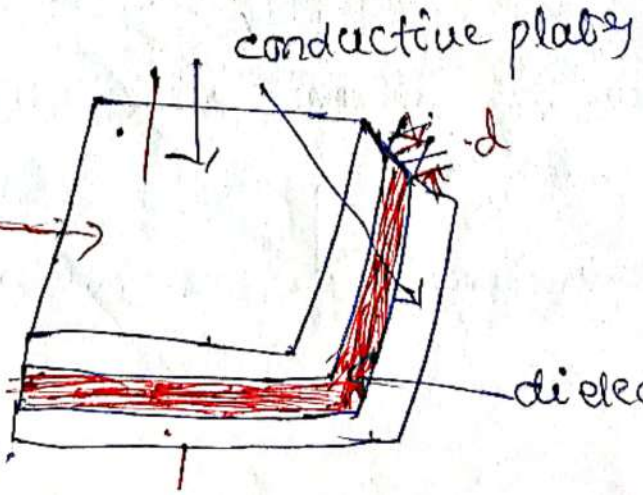
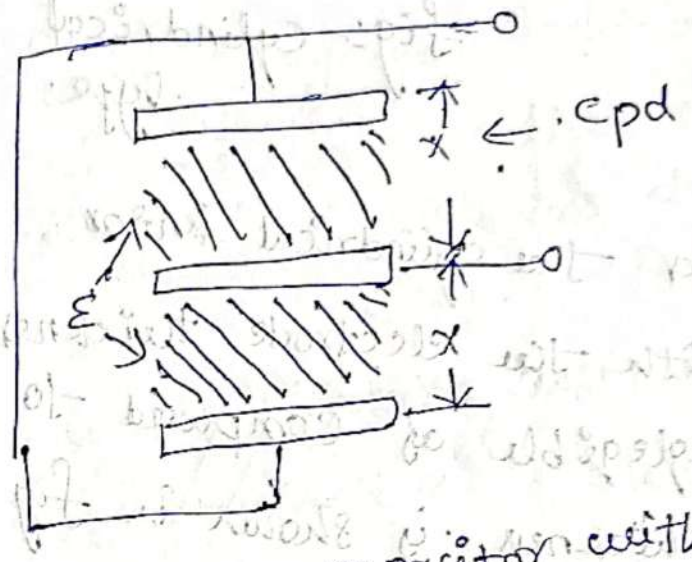


fig :- parallel plate capacitance sensor.

⇒ The parallel plate capacitive sensor is often used in a differential form with three plates as shown in fig.



⇒ For a parallel plate capacitor with dielectric constant or permitivity ε_1.

→ Its relative permitivity and the permitivity of the free space of value 8.85×10^{-12} F/m and plate area A, each seperated by a distance x is from the otter capacitance

$$C_p = \frac{\varepsilon A}{x} \quad \text{—} \quad (1)$$

⇒ Three plate capacitor arrangement is shown in figure.

⇒ The capacitance Cpd is then given as

$$C_{pd} = \frac{2\varepsilon A}{x} \quad \text{--- (2)}$$

fig:- cylindrical type.

⇒ for the cylindrical sensor with the electrode thickness neglegible as compared to dielectric thickness is shown in fig.

the capacitace is

$$C_c = \frac{2\pi\varepsilon l}{\ln\frac{D}{d}} \quad \text{--- (3)}$$

where 'l' is the cylinder length.

⇒ for very thin layer of dielectric meterial, eqn (3) can be approximated to

$$C_{ca} = \frac{\pi\varepsilon l(D+d)}{(D-d)} \quad \text{--- (4)}$$

⇒ In a parallel plate pair the dielectric has a number of layers of dielectric constant with corresponding permitivity ε_i. for thickness x_i, the relation

eq'n (1) can be modified to

$$C_{pi} = \frac{\alpha}{\sum x_i / \varepsilon_i} \quad \text{--- (5)}$$

=> The capacitance is in general associated with a high resistance, called leackage,

=> because the dielectric meterials do not have infinite permittivity.

=> The leackage, by a parallel resistance R_p, represented.

=> particularly at low frequencies of measurment.

=> with increasing frequency, the load resistance R_l contribute to loss factors and the complete equivalent circuit is given by the circuit of ~~fig show~~ fig.

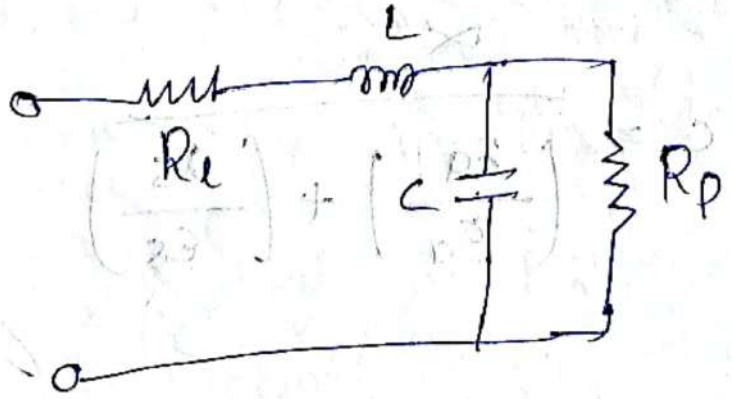


fig:- Equivalent circuit of the capacitance transducer

⇒ the inductance L represents the inductance between the terminals as also the cable inductance.

⇒

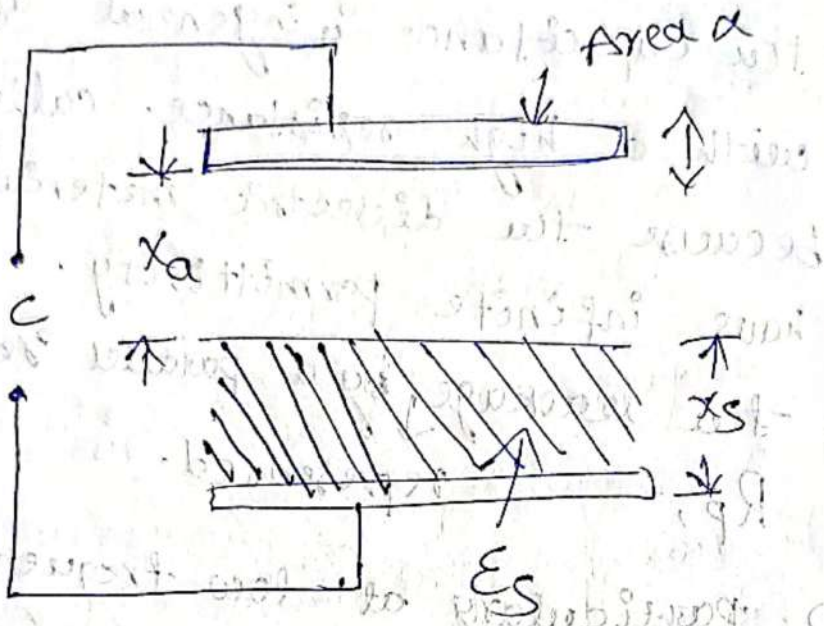


fig:- Parallel plate sensor.

⇒ parallel plate with a solid dielectric of a certain thickness x_s, and an air gap x_a as shown in fig. the capacitance C is given by

$$C = \frac{\alpha}{\left(\frac{x_a}{\varepsilon_a}\right) + \left(\frac{x_s}{\varepsilon_s}\right)}$$

Electrostatic transducer :-

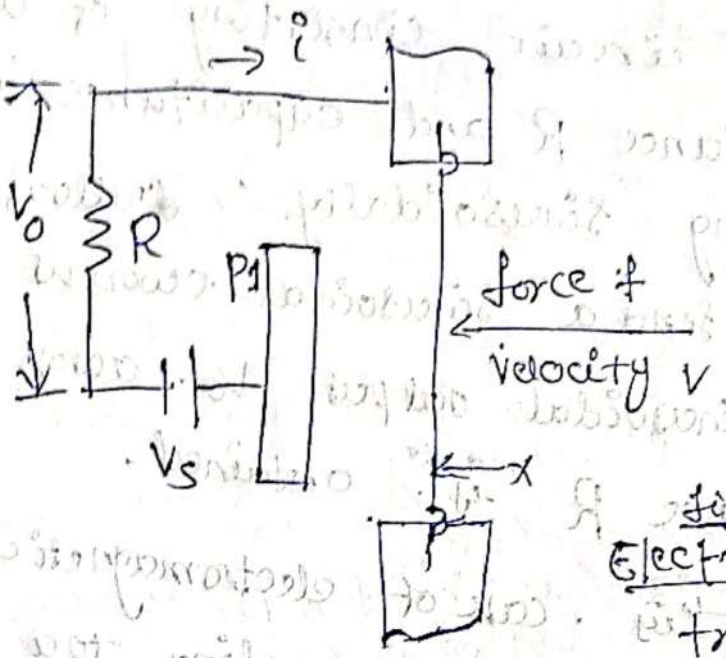


fig :- Electrostatic transducer.

⇒ A transducer is also reffered to as an electrostatic transducer. A typical scheme of such a system is shown in fig,

⇒ A capacitor formed with a flexible diagram which can move due to application of force and a grid plate P1.

⇒ There is bias voltage V_s which is sufficiently large.

⇒ When the system acts as a transducer, the gap x between the plates changes as by some pressure in case of an electrostatic microphone.

⇒ This pressure may be considered sinusoidal for analysis purpose.

⇒ The circuit consisting of a Resistance R and capacitance 'C' varying sinusoidally allows V_s.

⇒ to send a sinusoidal current i

⇒ A sinusoidal output V_o across resistance R is obtained.

⇒ In this case of electromagnetic transducer, V_o corresponding to a force f can be obtained in terms of the parameters V_s, x, R, C, ω, mass m, stiffness k, and damping ζ of the system.

⇒ The dynamic transfer function is given by

$$\frac{V_o(s)}{f(s)} = \frac{s\, x_0 R C_0 V_s}{s^3 x_0^2 m R C_0 + s^2 (m x_0^2 + x_0^2 R C_0 \zeta) + s(x_0^2 \zeta + x_0^2 R C_0 k) + (x_0^2 k + V_s^2 C_0)}$$

where, C_0 and x_0 are the initial values of x and C.

⇒ In case of generating along with bias V_s, a sinusoidal input voltage is also applied so that the diaphragm undergoes electrostatic vibration.

Force / ~~Strell~~ stress sensors using Quartz Resonators :

⇒ Stress is applied to a flexurally vibrating quartz beam through its mountings. The beam has a fundamental mode of flexural resonance frequency ~~is~~ f given by

$$f = f_0 \sqrt{1 + K_1 \left(\frac{S}{Y}\right)\left(\frac{l}{t}\right)^2} \quad \text{— (1)}$$

with f_0 as the frequency in absence of stress and given by

$$f_0 = K_2 \left(\frac{t}{l^2}\right)\left(\frac{Y}{\rho}\right)^{1/2} \quad \text{— (2)}$$

Where = S = Stress.

Y = Young's modulus.

l = beam length

t = beam thickness

p = quartz density, and

k_1 and k_2 are constants.

(1)

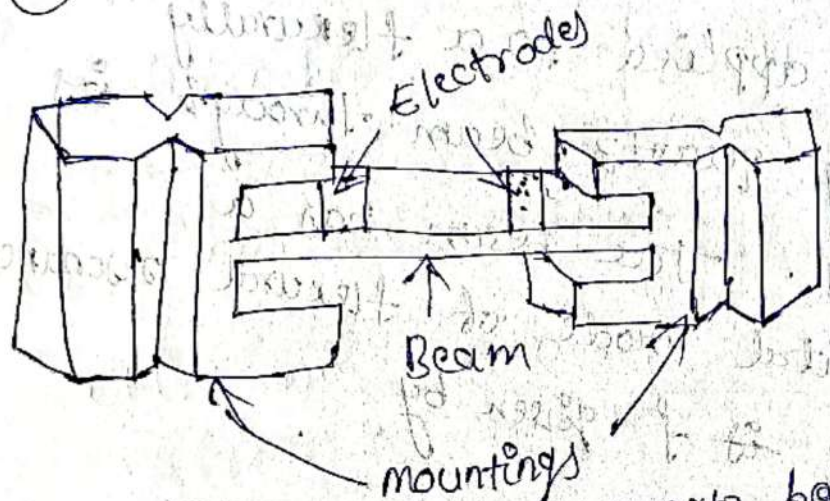


fig:- A quartz beam with semi flexible mounting

(2)

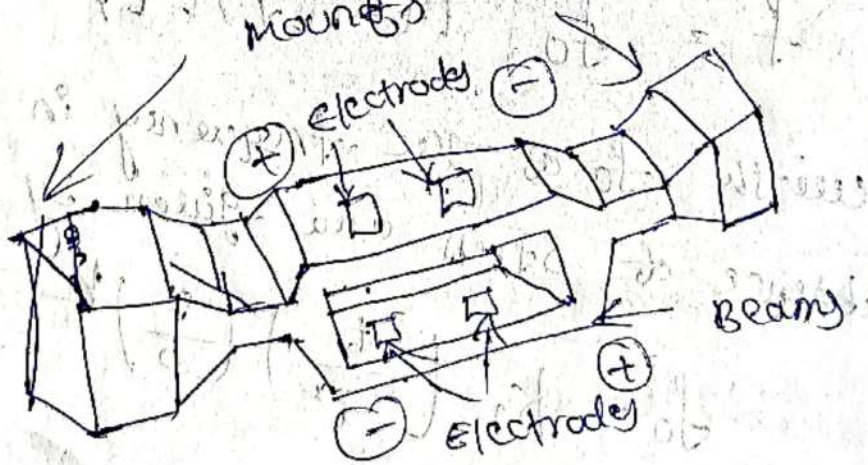


fig:- Double beam design of the cantilever.

=) The beam is generally machined out from a stock of quartz with semi-flexible mounting. for minimizing the mounting misalignments that can effect the vibrating frequency mode. shown in fig (1)

=) The vibrational bending moments that might induce distortion are sought to be cancelled by a double beam design. as shown in fig (2)

=) The vibration in the two beams are opposite to each other

=) The two beams are polarized by oscillator input.

=) Tuning fork type design of the beams has also been developed using photolithographic techniques.

=) Two schemes of which are shown in figure (a) and fig (b).

=) The two beams are polarized by oscillator input in such a way that the beams vibrate in opposite phase.

⇒ vibrates in opposition to the side beams.

⇒ The stress sensitivity of frequency $\Sigma_s^f = (df/ds)/(f/s)$ is obtained from eqn. (1)

$$\Sigma_s^f = \frac{K_1\left(\frac{S}{Y}\right)\left(\frac{l}{t}\right)^2}{1+K_1\left(\frac{S}{Y}\right)\left(\frac{l}{t}\right)^2} \quad - (3)$$

⇒ Indicating non linearity in f-S relationship. Σ_s^f increases with (l/t) but increases of (l/t) reduces mechanical strength.

Ultrasonic sensors :-

⇒ Piezoelectric effect of certain crystalline materials has been successfully utilized in ultrasound production and sensing.

⇒ The property is utilized in generating acoustic or ultrasound wave.

⇒ piezoelectric transducer can generate continuous wave ultrasound

or pulsed ultrasound latter being used in SONAR or other similar systems.

⇒ ultrasonic piezocrystals operate in the range of 0.5-10 MHz.

⇒ They are directly attached to the transmitting medium.

⇒ for continuous wave operation the sensor is energized by a tuned oscillator while for pulsed application relaxation oscillators are used to charge a capacitor which is discharged across the sensor.

Sensitivity and linearity of the sensor :-

⇒ ~~For~~ A small air gap l_g and effective permeability of the core μ_s

⇒ The inductance is given by

calc $$L = \left[\frac{\mu_s}{\left\{1+\left(\frac{l_g}{l}\right)\mu_s\right\}}\right]\left(\frac{n^2 a}{l}\right) \quad \text{--- (1)}$$

n and a are constants, using

$$K_L = 4\pi \times 10^{-7} n^2 a.$$

eq'n (1) can be written as

$$L = \frac{K_L}{\left(l_g + \frac{l}{\mu_s}\right)} \quad \text{--- (2)}$$

Assuming $l >> l_g$, for small increase and decrease in gap l_g and δl_g.

$$\frac{dL}{L} = \frac{dl_g}{\left(l_g \pm dl_g + \frac{1}{\mu_s}\right)}$$

$$= \frac{dl_g / l_g}{1 + \frac{1}{l_g \mu_s}} \cdot \frac{1}{1 + \frac{(dl_g / l_g)}{\{1 + l / l_g \mu_s\}}} \quad \text{--- (3)}$$

and for $(dl_g / l_g) / (1 + l / l_g \mu_s) << 1$

$$\frac{dL}{L} = \frac{dl_g / l_g}{1 + \frac{1}{l_g \mu_s}} \left[1 \pm \frac{dl_g / l_g}{1 + \frac{l}{l_g \mu_s}} + \left(\frac{dl_g / l_g}{1 + \frac{1}{l_g \mu_s}} \right)^2 \mp \cdots \right] \quad \text{--- (4)}$$

If only the first term is accepted for dl_g being very small, ∴ there appears to be a linear variation between L and l_g, and the sensitivity $S_{l_g}^{L}$ = ∴ $(dL/L) / (dl_g / l_g)$ is given as

$$S_{l_g}^{L} = \frac{1}{1 + l / (l_g \mu_s)}$$

Inductive Sensor :-

The inductive transducer utilizes the simple principle that the physical quantity to be measured can be made to vary the inductance of a coil, maintaining a relation between the two.

⇒ This variation of inductance can often be measured by ac bridge circuit.

⇒ If a magnetostrictive core material is used.

⇒ The most common methods of achieving variation in inductance are

⇒ by changing the reluctance of the magnetic path and by coupling two or more elements.

(a) change of mutual inductance.

(b) change of eddy current.

(c) transformer action.

These are shown in figs.

(a) fig. (a), (b) and (c)

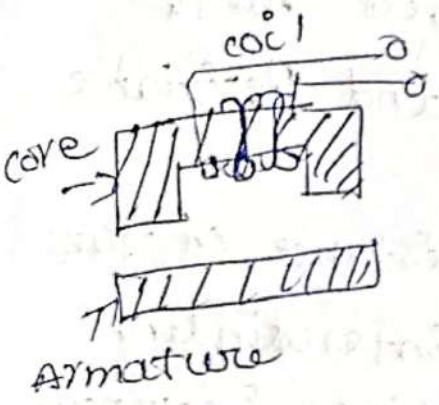


fig (a)

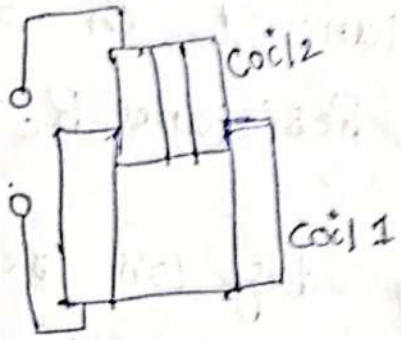


fig (b)

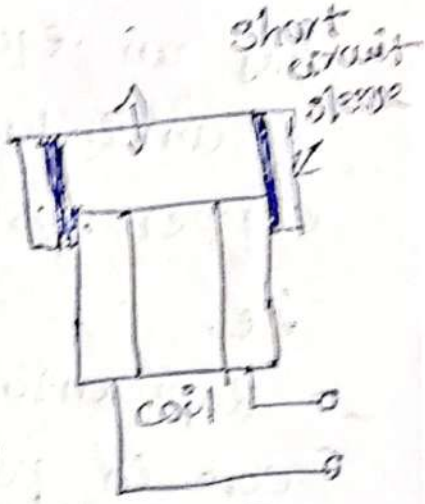


fig (c)

=) There are inductive sensors of the magnetic type which are bilateral in operation with electrical and mechanical input/output relationship and

=) The ~~magnetic~~ magnetostrictive type.

=) A sensor that uses a magnetostrictive core meterial is shown in figure.

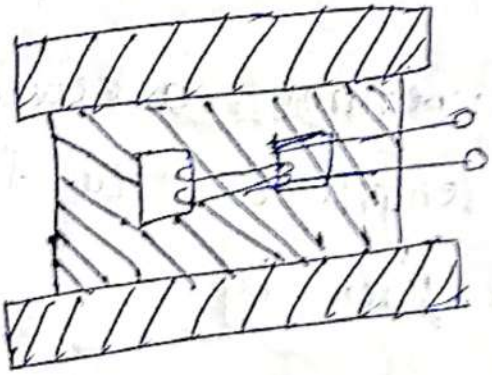


fig:- sensor using a magnetostrictive effect.

⇒ the copper coil on a ferromagnetic core has an equivalent circuit that consists of an inductance L in series with copper loss Resistance R_c and Resistance R_e,

→ Representing eddy loss resistance in the core in parallel with 'L'. interwinding are self capacitance. at high frequency.

⇒ I_s in parallel to the coil resistance R_c and inductance L. the equivalent circuit shown in figure

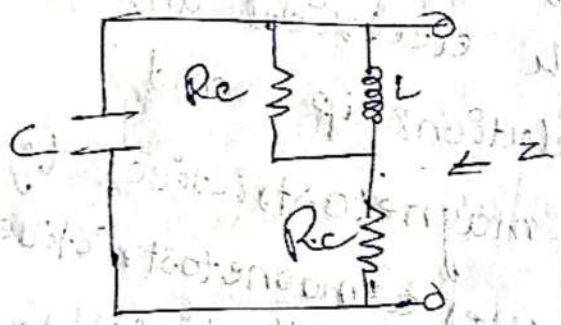


fig:- Equivalent circuit of a ferromagnetic coil.

⇒ If a coil 'n' turns, a current I, and the core length l, the field Strength H is given by

$$H = \frac{nI}{l} \ (A/m) \quad \text{--- (1)}$$

for a core material of permeability μ,

⇒ It is relative permeability and the permeability of the free space or vacuum. ($\mu_0 = 4\pi \times 10^{-7}$ H/m), and core cross section area a,

the inductance L of the coil is the flux linkage per unit current so that

$$L = \frac{n\phi}{I} = n = \frac{Ba}{I} = n\frac{\mu Ha}{I} \quad \text{— (2)}$$

where B is in Tesla or Wb/m² and ϕ is in Wb. using eq. (2).

$$L = \frac{\mu n^2 a}{l} \quad \text{— (3)}$$

⇒ The copper resistance Rc is also calculated if the coil wire diameter d and the copper resistivity ρ are known; so that

$$R_c = \frac{\rho n l_t}{\pi d^2} \quad \text{— (4)}$$

where l_t is the average length per turn of the coil.

=) The coil dissipation factor Dc is usually defined.

$$D_c = \frac{R_c}{\omega L} \quad \text{—— (5)}$$

with decreases with increasing frequency. for reducing eddy loss or core loss is called. the depth of penetration of eddy current, dp is given by.

$$d_p = \sqrt{\frac{\rho_e}{\pi \mu f}} \quad \text{—— (6)}$$

where ρ_e is the resistivity of the core material and $f = \omega/(2\pi)$ is the frequency. the eddy loss resistance is then given by.

$$R_e = \left(\frac{2 d_p \omega L}{t_l}\right)\left[\frac{\cosh\left(\frac{t_l}{d_p}\right) - \cos\left(\frac{t_l}{d_p}\right)}{\sinh\left(\frac{t_l}{d_p}\right) + \sin\left(\frac{t_l}{d_p}\right)}\right] \quad \text{—— (7)}$$

eq'n (6) and (7) for low frequencies.

when $P_t = (t_l / d_p) \leq 2$.

=) this range of frequency eq'n (7). can be simplified using eq'n (8) and (6)

$$R_e = \frac{6\omega L}{(t_l/d_p)^2}$$

$$= \frac{12 \rho_e a n^2}{(l t_l)^2} \quad \text{--- (8)}$$

The eddy loss dissipation factor is defined by

$$D_e = \frac{\omega L}{R_e} \quad \text{--- (9)}$$

and is directly proportional to frequency.

The hysteresis curve is given by

$$A_h = \int B \cdot dH \quad \text{--- (10)}$$

where α_r is the ~~Rely~~ Rayleigh's constant which may be defined by the eqn.

$$\alpha_r = 2 \frac{\left[\frac{\Delta B}{\mu_0} - \mu_i H\right]}{(\Delta H)^2} \quad \text{--- (11)}$$

is permeability $H = 0$, with change from zero value of B and H eqn (12) is written as

$$\alpha_r = \frac{2(B/\mu_0 - \mu_i H)}{H^2} \quad \text{--- (13)}$$

using $P_h = E^2/R_h$, R_h, the equivalent hysteresis-loss resistance, is

$$R_h = \frac{\omega^2 L^2 I^2}{P_h} \quad (14)$$

which is independent of frequency.

A sensor or a transducer involves the movement of an armature.

=) the effective permeability of the core

=) the permeability μ_s, and a relation between L and the gap length l_g.

=) the total path length l, gap length l_g cross sectional area a,

the effective permeability μ.

$$\frac{\left(\frac{l - l_g}{\mu_s}\right) + l_g}{a} = \frac{l}{\mu a} \quad (15)$$

$$\mu = \frac{\mu_s}{\left\{1 + \left(\frac{l_g}{l}\right)(\mu_s - 1)\right\}} \quad (16)$$

Since $\mu_s \gg 1$

$$\mu \approx \frac{\mu_s}{\left[1 + \left(\frac{l_g}{l}\right)\mu_s\right]} \quad (17)$$

Substituting this eq'n (8)

$$L = \frac{\mu' n^2 a}{l}$$

$$L = \left[\frac{\mu_s}{\left[1 + \left(\frac{l_g}{l}\right)\mu_s\right]}\right]\left(\frac{n^2 a}{l}\right) \quad \text{— (18)}$$