UNIT-1 (PART-A)
BASICS OF RADAR

BASICS OF RADAR

- Introduction
- Maximum unambiguous Range
- Radar Block diagram and operation
- Simple form of the radar equation
- Radar frequencies and Applications
- Prediction of range performance
- Minimum Detectable signal
- Receiver Noise
- Modified Radar Range equation
- Illustrative problems

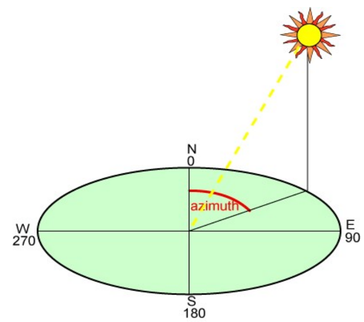
RADAR is an Acronym for
RAdio Detection And Ranging

Radar is a detection system that uses radio waves to determine the range, angle, or velocity of objects. It can be used to detect aircraft, ships, spacecraft, guided missiles,motor vehicles, weather formations, and terrain.

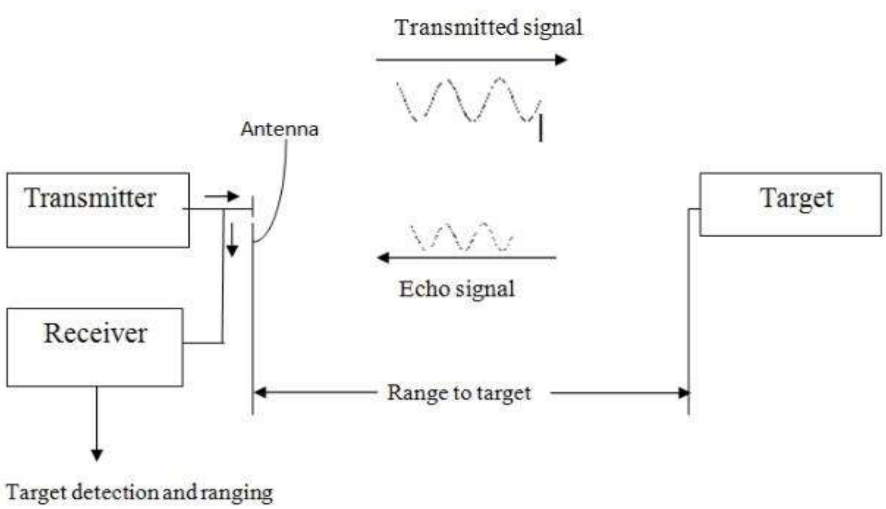
A radar system consists of a transmitter producing electromagnetic waves in the radio or microwaves domain, a transmitting antenna, a receiving antenna (often the same antenna is used for transmitting and receiving) and a receiver and processor to determine properties of the object(s).

FUNCTIONS OF RADAR

- Detects the presence of target
- Gives the range of the target from the Radar station
- Gives the azimuth angle and elevation angle of the target
- Gives the radial velocity of target.



PRINCIPLE OF WORKING



Two basic radar systems exist

1. Monostatic
2. Bistatic

Measurement of Range

PRF: The no of radar pulses transmitted per second is known as pulse repetition frequency or pulse repetition rate. F_p

PRT: The time from beginning of first pulse to the beginning of the next is called pulse repetition time. T_p

$$F_p = \frac{1}{T_p}$$

T_R --Time taken by EM pulse to travel to target and come back to same antenna

R- range of target

C –Velocity of EM waves = 3×10^8 Meters/sec

Rest Time or Receiver Time : The time between two successive transmitted pulse is called as Rest Time or Receiver Time

Radar range determination

- The most common radar waveform is a train of narrow, rectangular-shape pulses modulating a sine wave carrier.
- The distance, or range, to the target is determined by measuring the time T_R taken by the pulse to travel to the target and return.
- Electromagnetic energy in free space travels with the speed of light **c (3×10^8 m/s)** therefore range **R** is given by : $\mathbf{R = cT_R / 2}$
- The range **R** in kilometers or nautical miles, and T_R in microseconds, the above relation becomes: $\mathbf{R(km) = 0.15 \times T_R\ (\mu S)}$ **or** $\mathbf{R(nmi) = 0.081 \times T_R\ (\mu S)}$
- Each microsecond of round-trip travel time corresponds to a distance of **0.081** nautical mile, **0.093** statute mile, **150** meters, **164** yards, or **492** feet.
- (1 mile = 0.8689 nautical mile or 1.6 km
  1 nautical mile = 1.15078 miles or 1.8412 km)
- It takes 12.35 µs for radar signal to travel a nautical mile and back

Maximum unambiguous range

- Once signal is transmitted into space by a radar, sufficient time must elapse to allow all echo signals to return to the radar before the next pulse is transmitted .
- The rate which pulses may be transmitted T_P is determined by the longest range at which the target is expected.
- If the time between the pulses T_P is too short, the echo signal from target may arrive after the next pulse transmitted and it leads to incorrect or ambiguous measurement of the range.
- The echoes that arrive after the transmission of next pulse are called **second time around echoes or second return echoes**.
- The maximum range from which a transmitted radar pulse can be reflected and received before the next pulse is transmitted.

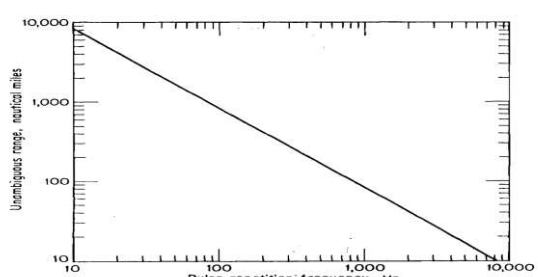
  or

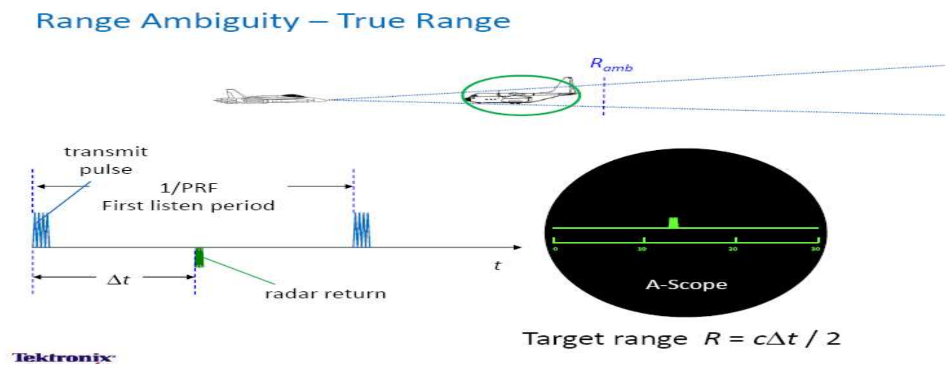
  the range beyond which targets appear as second time around echoes is called the **maximum unambiguous range**.

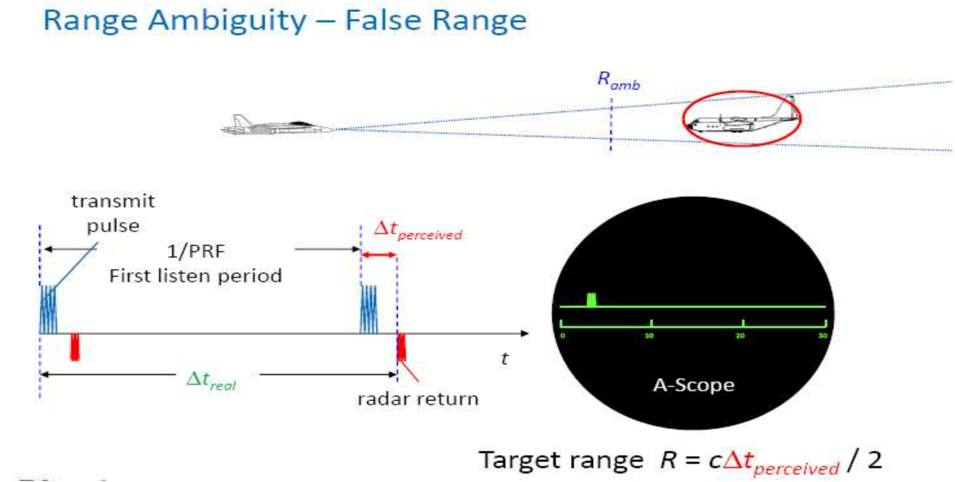
- R_{max} is the farthest target range that can be detected by a Radar without ambiguity and is also called Maximum Unambiguous Range of the Radar. Since PRF $f_P = 1/T_P$ It is also given by :

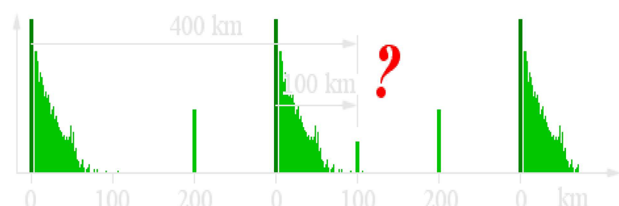
  $$R_{un} \text{ or } R_{max} = CT_P/2 = C/2f_P$$

- If the range of target is more than the Maximum Unambiguous Range, multiple time around echoes occur and range computed would be erroneous.
- The relation between PRF and Maximum Unambiguous Range is linear and shown in the next slide.









The first transmitted pulse, after being reflected from the target in 200 km, is received by the radar before the second pulse is transmitted. There will be no ambiguity here as the reflected pulse can be easily identified as a reflection of the first pulse. But in same Figure, we notice that the reflection of a target of the first pulse is received after the second pulse has been transmitted (in range of 400 km). This causes some confusion since the radar, without any additional information, cannot determine whether the received signal is a reflection of the first pulse or of the second pulse. This leads to an ambiguity in determining the range, this received echo signal be mistaken as a short-range echo of the next cycle.

Therefore maximum unambiguous range R_{max} is the maximum range for which $t < T$.

$R_{max} = c_0 \cdot (T - \tau)/2$ where R_{max} = Unambiguous Range in [m]

c_0 = Speed of light [$3 \cdot 10^8$ m/s]

T = Pulse Repetition Time [sec]

τ = length of the transmitted pulse

The simple form of the radar equation

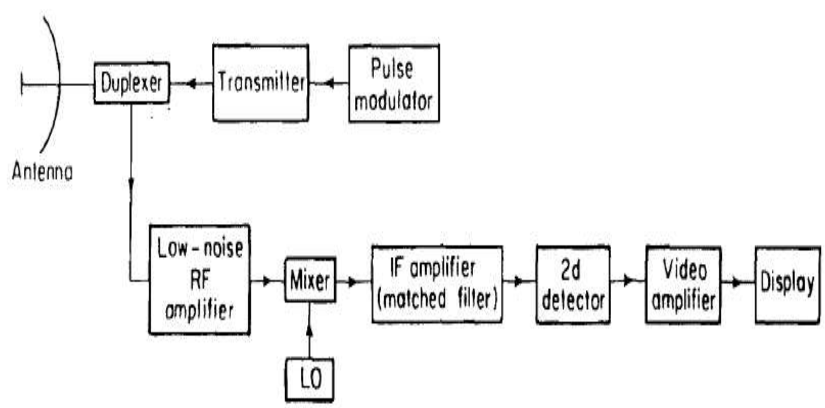
- The radar equation Relates the range of a Radar to the characteristics of the transmitter, receiver, antenna, target, and environment.
- It is useful not only for determining the maximum range, but it can serve for understanding the factors affecting radar performance.

Limitations:

- Does not adequately describe the performance of practical radar.
- Many important factors that affect range are not explicitly included.
- In practice, the observed maximum radar ranges are usually much smaller than what would be predicted by the above equations, sometimes by as much as a factor of two.

There are many reasons for the failure of the simple radar equation to correlate with actual performance and these will be explained subsequently in the modified Radar range equation .

Radar block diagram



There are two sections of radar

1. Transmitter section
2. Receiver section

Radar shown in the block diagram is called monostatic Radar since same antenna is used for transmission and reception.

Transmitter section

- **Transmitter** : the transmitter may be a power amplifier such as klystron, travelling wave tube or transistor amplifier. This will generates the Electrical energy at R.F.(Radio Frequency).
- **Pulse modulator :** The power amplifier (Such as Klystron, TWT) produces a high power signal, may be in terms of megawatts. Pulse modulator shown in the block is used as a switch, which will turn on and off the power amplifier.
- **Wave form generator**: A low power signal is produced by the waveform generator which is given as an input to the power amplifier.
- **Duplexer:** The duplexer allows a single antenna to be used on a time shared basis for both transmitting and receiving. The duplexer is generally a gaseous device that produces a short circuit at the input to the receiver when the transmitter is operating, so that high power flows to the antenna and not to the receiver. On the reception, the duplexer directs echo signal to the receiver and not to the transmitter. Solid state ferrite circulators and receiver protector devices can also be part of the duplexer

Receiver section:

- **Low noise RF amplifier:** The receiver is almost always a super heterodyne. LNA is used immediately after the antenna. This reduces the Noise Figures and produces the RF pulse proportional to the transmitted signal.
- **Mixer and local oscillator**: It converts the RF signal to an intermediated frequency where it is amplified by the IF amplifier. The IF frequency might be 30 or 60 MHz.
- **IF amplifier**:

  i) It amplifies the IF pulse.

  ii) IF amplifier is designed as a matched filter which maximizes the output peak signal to mean noise ratio.

  iii) The matched filter maximizes the detectability of weak echo signals and attenuates unwanted signals.

  iv) The signal bandwidth of super heterodyne receiver is determined by the bandwidth of its IF stage.

  v) For example when pulse width is of the order of 1µs the IF bandwidth would be about 1MHz.

- **Second Detector**: the IF amplifier followed by a crystal diode which is called the second detector or demodulator. Its purpose is to assist in extracting the echo signal modulation from the carrier. It is called as 2ndDetector since it is the second diode used in the chain. The first diode is used in the mixer. Output of the 2ndDetector is the Video Pulse.
- **Video amplifier**: It is designed to provide the sufficient amplification to rise the level of the input signal to a magnitude where it can be diplay (CRT or Digital computer).
- **Threshold decision**: The output of video amplifier is given to the threshold detector where it is decided whether the received signal is from a target or just because of the presence of noise.
- **Display**: The Display is generally a CRT (Cathode Ray Tube)

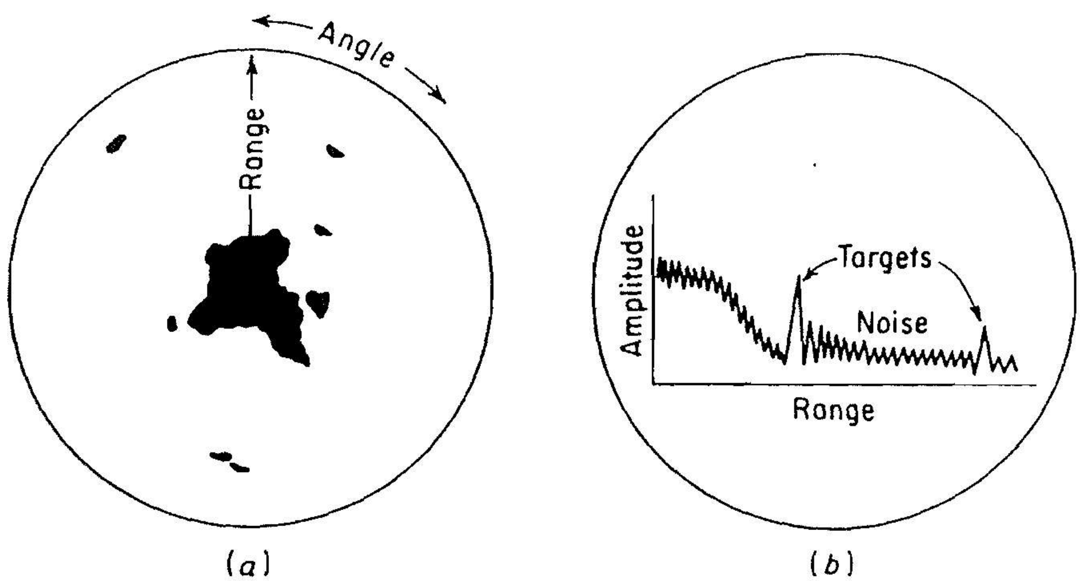
  (a) ‘A’ scope (b) PPI

  i) ’A’ scope provided Range and Echo power.

  ii) PPI measures Range and bearing (azimuth angles)

  iii) In addition there are other displays like ‘B’ scope, ‘ D ‘ scope etc.

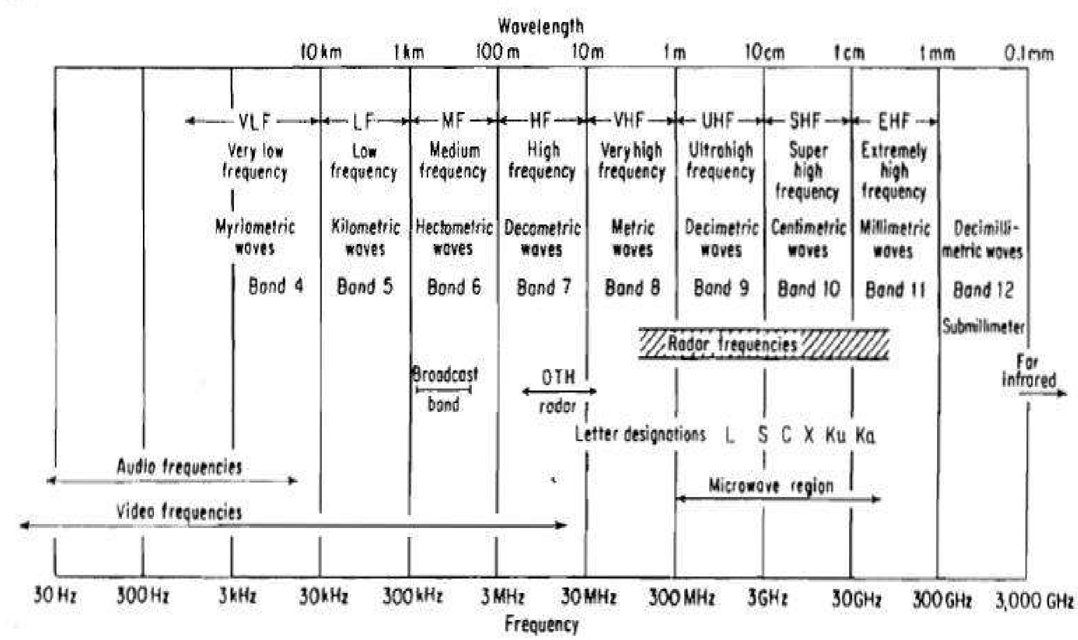
(a) PPI presentation displaying Range vs. Angle (intensity modulation)
(b) A-scope presentation displaying Amplitude vs. Range (deflection modulation)



RADAR FREQUENCIES

- RF spectrum is very scarce and as such Radars are allotted only a certain frequency bands for their operation by International Telecom Union ITU
- During 2ndworld war, to keep the secrecy, certain code words were used. The same designations are continued even today
- Lema Band (L) 1GHZ-2GHZ, Sierra band(S) 2GHZ-4GHZ, Charlie Band (C) 4GHZ-8GHZ, Xera Band (X) 8GHZ-12GHZ
- ITU(International Telecommunication Union) allocated a portion of these bands for Radar

ELECTROMAGNETIC SPECTRUM



Standard radar-frequency letter-band nomenclature

Band designation	Nominal frequency range	Specific radiolocation (radar) bands based on ITU assignments for region 2
HF	3–30 MHz	
VHF	30–300 MHz	138–144 MHz 216–225
UHF	300–1000 MHz	420–450 MHz 890–942
L	1000–2000 MHz	1215–1400 MHz
S	2000–4000 MHz	2300–2500 MHz 2700–3700
C	4000–8000 MHz	5250–5925 MHz
X	8000–12,000 MHz	8500–10,680 MHz
K_u	12.0–18 GHz	13.4–14.0 GHz 15.7–17.7
K	18–27 GHz	24.05–24.25 GHz
K_a	27–40 GHz	33.4–36.0 GHz
mm	40–300 GHz	

FIELDS OF APPLICATION

- MILITARY
- REMOTE SENSING
- AIR TRAFFIC CONTROL
- LAW ENFORCEMENT AND HIGHWAY
- SECURITY
- AIRCRAFT SAFETY AND NAVIGATION
- SHIP SAFETY
- SPACE
- MISCELLANEOUS APPLICATIONS

MILITARY:

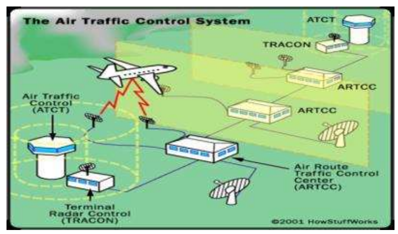
- Important part of air defence system, operation of offensive missiles & other weapons.
- Target detection, target tracking & weapon control .
- Also
- used in area, ground & air surveillance.

AIR TRAFFIC CONTROL

- Used to safely control air traffic in the vicinity of the airports and enroute.
- Ground vehicular traffic & aircraft taxing.
- Mapping of regions of rain in the vicinity of airports & weather.

LAW ENFORCEMENT & HIGHWAY SAFETY:

- Radar speed meters are used by police for enforcing speed limits.
- It is used for warning of pending collision, actuating air bag or warning of obstruction or people behind a vehicle or in the side blind zone



REMOTE SENSING

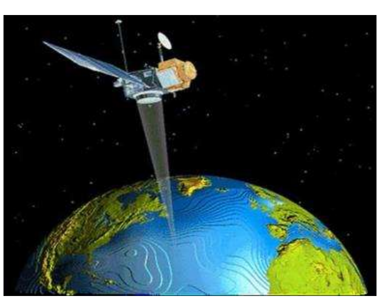
- Weather observation-t.V.Reporting
- Planetary observation
- Below ground probing
- Mapping of sea ice

AIRCRAFT SAFETY & NAVIGATION

- Low flying military aircrafts rely on terrain avoidance & terrain following radars to avoid collision with high terrain & obstrucions

SHIP SAFETY

- Radar is found on ships & boats for collision avoidance & to observe navigation buoys, when the visibility is poor.
- Shore based radars are used for surveillance of harbours & river traffic.



SPACE

- Space vehicles have used radar for clocking & for landing on the moon.
- Used for planetary exploration.
- Ground based radars are used for detection & tracking of satellites & other space objects.
- Used for radio astronomy.

OTHER APPLICATIONS

- It is used for in industry for the non contact measurement of speed & distance.
- Used for oil & gas exploration.
- Used to study movements of insects & birds.

PREDICTION OF RANGE PERFORMANCE

The simple form of the radar equation derived earlier expresses the maximum radar range R_{max} in terms of radar and target parameters:

$$R_{max} = [(Pt . G. A_e. \sigma)/ (4\pi)^2. S_{min}]^{1/4}$$

where P_t = transmitted power, watts

G = antenna gain

A_e = antenna effective aperture, m^2

σ = radar cross section, m^2

S_{min} = minimum detectable signal, watts

All the parameters are to some extent under the control of the radar designer, except for the target cross section **σ.**

The radar equation states that if long ranges are desired,

1. The transmitted power must be large,
2. The radiated energy must be concentrated into a narrow beam (high transmitting antenna gain),
3. The received echo energy must be collected with a large antenna aperture (also synonymous with high gain) and
4. The receiver must be sensitive to weak signals.

In practice, however, the simple radar equation does not predict the range performance of actual radars. The predicted values of radar range are usually optimistic. In some cases the actual range might be only half of that is predicted.

The failure of the simple form of radar equation is due to

1. The statistical nature of the minimum detectable signal determined by receiver noise.
2. Fluctuations and uncertainty in radar cross-section.
3. The losses throughout the radar system.
4. Propagation effects caused by the earth's surface and atmosphere.

Because of statistical nature of receiver noise and target cross section, the maximum radar range is described probabilistically rather than single number.

Therefore the radar range equation includes

1. Probability that radar will detect a target at a particular range(pd).
2. Probability of making a false detection when no target is present(pfa).

From the above facts it can be concluded that the range of radar is a function of probability of detection(pd) and probability of false alarm(pfa). The prediction of radar range is not accurate as there is uncertainty in various parameters. Still radar range equation is an important tool for i) Assessing the performance of radar ii) Generating technical requirements and Determining system tradeoffs for designing new radar systems.

MINIMUM DETECTABLE SIGNAL

- The ability of a radar receiver to detect a weak echo signal is limited by the noise present in the frequency spectrum.
- The weakest signal that the receiver can detect is called the ***minimum detectable signal.*** It is difficult to define what is ***minimum detectable signal*** (MDS) because of its statistical nature and the criterion for deciding whether a target is present or not may not be too well defined.
- Detection is normally based on establishing a threshold level at the output of the receiver (as shown by the dotted line). Whenever Rx output signal which is a mixture of echo and noise crosses this threshold then it is detected as a target. This is called ***threshold detection.***
- Consider the output of a typical radar receiver as a function of time as shown in the figure below which typically represents one sweep of the video output displayed on an A-scope.

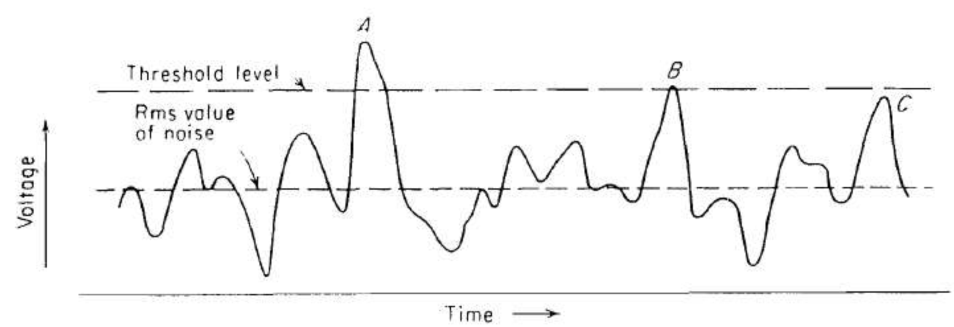


Fig : Typical envelope of a radar receiver output as a function of time. *A,* B, and *C* are three targets representing signal plus noise. A and B are valid detections, but C is a missed detection

1. If the threshold level were set properly, the signal would not generally exceed the threshold if noise alone were present, but would exceed it if a strong signal were present along with the noise.
2. If the threshold level is set too low, noise might exceed it and be mistaken for a target. This is called a false alarm.

3. If the threshold level were set too high, noise might not be large enough to cause false alarms, but weak target echoes might not exceed the threshold and would not be detected. This is called missed detection.
4. Here points A,B and C represents signal plus noise.
5. The signal at **A** is large which has a much larger amplitude than the noise. Hence target detection is possible without any difficulty and ambiguity.
6. Next consider the two signals at **B** and **C**, representing target echoes of equal amplitude. The noise voltage accompanying the signal at **B** is large enough so that the combination of signal plus noise exceeds the threshold and target detection is still possible. Thus the presence of noise will sometimes enhance the detection of weak signals.
7. But ,for the target **C ,** the noise is not as large and the resultant signal plus noise does not cross the threshold and hence target is not detected.

- **Threshold Level setting:** Weak signals such as C would not be lost if the threshold level were lower. But too low threshold causes false alarms. If the threshold is set too low, false target indications are obtained, but if it is set too high, targets might be missed. The selection of the proper threshold level is necessary to avoid the mistakes of

  1.Failing to recognize a signal that is present (missed detection) or

  2.Falsely indicating the presence of a signal when it does not exist (false alarm)

  The signal-to-noise ratio is a better measure of a radar's detection performance that the minimum detectable signal.

RECEIVER NOISE

- Noise is an unwanted EM energy which interferes with the ability of the receiver to detect the wanted signal thus limiting the receiver sensitivity.
- It may originate within the receiver itself or it may enter via the receiving antenna along with the desired signal.
- If the radar were to operate in a perfectly noise free environment so that no external noise accompany the target signal.
- If the receiver itself were so perfect that it didn't generate any excess noise, there would be still be noise generated by the thermal motion of the conduction electrons in the ohmic portion of the receiver i/p stages. This is called **Thermal noise or jhonson noise**.

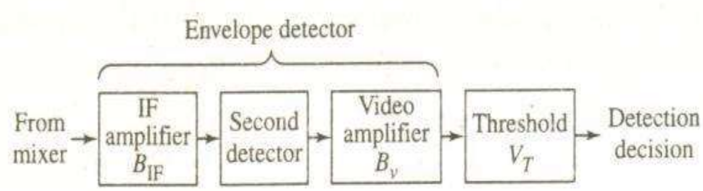
RADAR EQUATION

- SNR
- Envelop Detector
- False Alarm time and Probability
- Integration of Radar Pulses
- Radar Cross Section of Targets (simple targets: sphere and cone sphere)
- Transmitter Power
- PRF and Range Ambiguities
- System Losses (qualitative treatment)

SNR

- Signal to noise ratio is very important as far as radar is concerned. Because presence of target or not have small difference.
- Statistical noise theory will be applied to obtain S/N at the o/p of the IF amplifier necessary to achieve a specified prob of detection and prob of false alarm.

- **Envelope Detector**:



• Consider an IF amplifier with bandwidth B_{IF} followed by a second detector and a video amplifier with bandwidth B_v

- The second detector and video amplifier are assumed to form an envelope detector, that is one which rejects the carrier freq but passes the modulation envelop.
- To extract the modulation envelope, the video bandwidth must be wide enough to pass the low freq components generated by the second detector but no so wide as to pass the high frequency components at or near the IF.
- The video bandwidth B_v must be greater than $B_{IF}/2$ in order to pass all video modulation.

Radar Cross Section of Targets

- A radar cross section is defined as the ratio of its effective isotropic scattered power to the incident power density.

$$\sigma = \frac{\text{Power reflected towards source/Unit Angle}}{\text{Incident Power density}/4\pi}$$

$$= \lim_{R \to \infty} 4\pi R^2 \left| \frac{E_r}{E_i} \right|^2$$

where $\mathbf{R}$ = distance between radar and target
$\mathbf{E_r}$ = strength of reflected field at radar
$\mathbf{E_i}$ = strength of incident field at target

- The radar cross section depends on the characteristic dimensions of the object compared to the radar wavelength.

RCS of Simple Targets:

Sphere: A perfectly conducting sphere acts a isotropic radiator i.e. Incident radiation scattered in all directions.

The radar cross section of the sphere is characterized into three regions

1. Rayleigh region $\frac{2\pi a}{\lambda} << 1$:When the wavelength is large compared to the object's dimensions is said to be Rayleigh region.
2. Optical region $\frac{2\pi a}{\lambda} >> 1$:When the wavelength is small compared to the object's dimensions is said to be Optical region.
3. Resonance region $\frac{2\pi a}{\lambda} = 1$ In between the Rayleigh and Optical regions is the Resonance region where the radar wavelength is comparable to the objects dimensions.

For many objects the radar cross section is larger in the resonance region than in the other two regions.

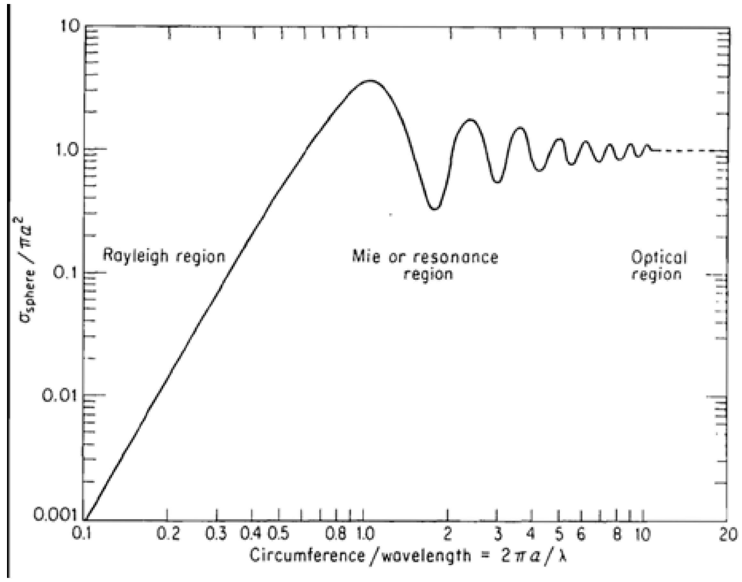
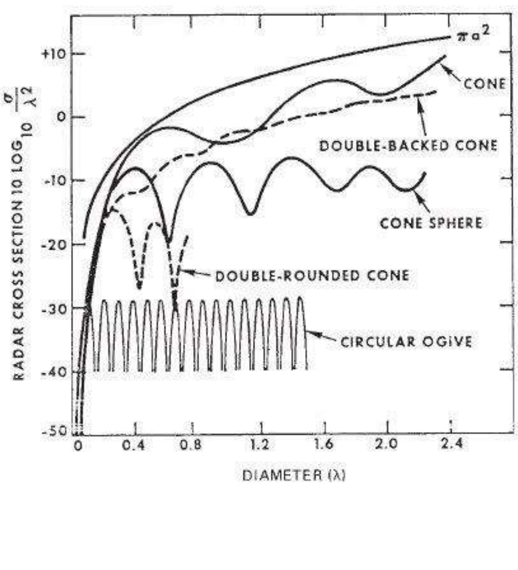
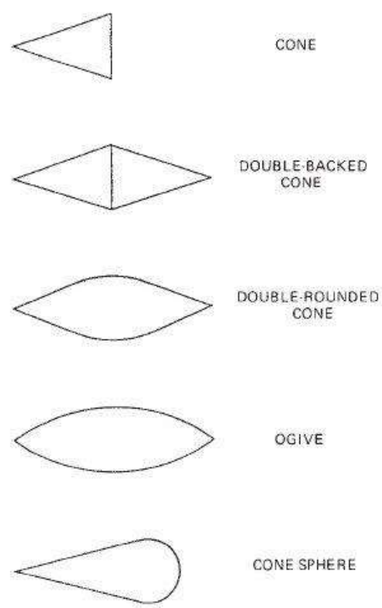


Fig: Radar cross section of a sphere as a function of circumference ($2\pi a$) measured in wavelength

- Cone sphere: It is a cone whose base is capped with a sphere. A large cross section occurs when a radar views the cone perpendicular to its surface.



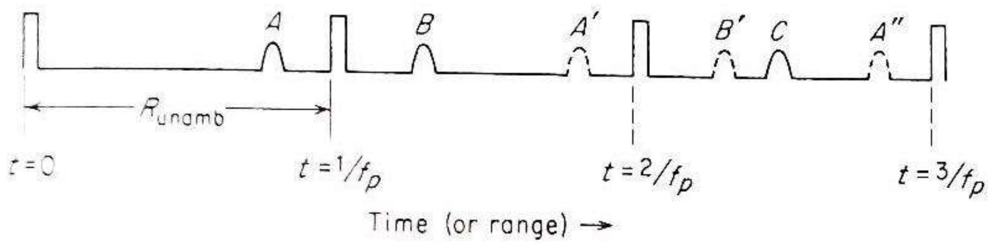


PRF and Range Ambiguities

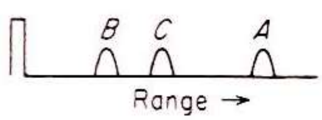
- The pulse repetition frequency (prf) is determined primarily by the maximum range at which targets are expected.

$$f_p = \frac{c}{2R_{un}}$$

- Echo signals that arrive at a time later than the pulse repetition period are known as second time around echoes or multiple time around echoes. These echoes may cause error and confusion. Also it can mask unambiguous target echoes at shorter ranges.
- Pulse Doppler radars have usually problem of range ambiguities because of prf.
- Consider the three targets located at three different positions A,B and C
- Target A is located within the maximum unambiguous range Runamb [= C.TP /2] of the radar, target B is at a distance greater than R*unamb* but less than 2R*unamb* and the target C is greater than 2R*unamb* but less than 3R*unamb*

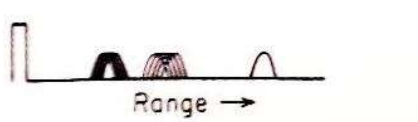


- The appearance of the three targets on an A-scope is shown below.



The ambiguous echoes B and C looks very similar to unambiguous range echo A. Out of these three echoes only the range of A is correct ,for B and C are not correct.

- The ambiguous range echoes are recognized by changing the prf of the radar. When the prf is changed the unambiguous echo remains at its true range. Ambiguous range echoes appear at different apparent ranges for each prf shown in below fig



- Let if prf f_{r1} has unambiguous range $R_{unambiguous1}$ and the range corresponds to it is R_1 then the true range is given by

$$R_{true} = R_1$$

or $$R_{true} = R_1 + R_{unambiguous1}$$

or $$R_{true} = R_1 + 2R_{unambiguous2}$$

- Let if prf f_{r2} has unambiguous range $R_{unambiguous2}$ and the range corresponds to it is R_2 then the true range is given by

$$R_{true} = R_2$$

or $$R_{true} = R_2 + R_{unambiguous2}$$

or $$R_{true} = R_2 + 2R_{unambiguous2}$$

The correct range is same for two prfs. Thus two or more prfs can be used to correct range ambiguity with increased accuracy and avoiding false values.

System Losses

The losses within the radar system is called system losses. The losses in a radar system reduce the signal-to-noise ratio at the receiver output.

1. **Microwave plumbing losses** : There is always loss in the transmission line that connects the antenna to the transmitter and receiver. In addition there can be loss in the various microwave components, such as ***duplexer, receiver protector, rotary joints, directional couplers, transmission line connectors, bends in the transmission lines and the mismatch at the antenna***.

   a) **Transmission line losses:** Generally same transmission line used for both transmission and reception , the loss to be inserted in the radar eq is twice the one way loss. At lower radar frequencies, the transmission line introduces little loss. At higher radar frequencies attenuation may not be small and may have to be taken in account. In practical the transmitter and receiver should be placed close to the antenna to keep the transmission line loss small.

   b) **Duplexer loss:** the loss due to a gas duplexer that protects the receiver from the high power of the transmitter is generally different on transmission and reception. It also depends on the type of duplexer used.

- In an S-band (3000 MHz) radar, for example, the plumbing losses might be as follows:

100 ft of RG-113/U A1 waveguide transmission line (two-way)	:	1.0 dB
Duplexer loss	:	2.0 dB
Loss due to poor connections (estimate)	:	0.3 dB
Rotary-joint loss	:	0.8 dB
Other RF devices	:	0.4dB
Total plumbing loss	:	4.5 dB

2. Antenna losses:

a) Beam shape loss: In radar equation antenna gain is assumed as constant at its maximum value but in practice as a search antenna scans across a target, it does not offer its peak gain to all echo pulses. When the system integrates several echo pulses maximum antenna gain occurs when the peak of antenna beam is in direction of target.

- The beam shape loss is computed by,

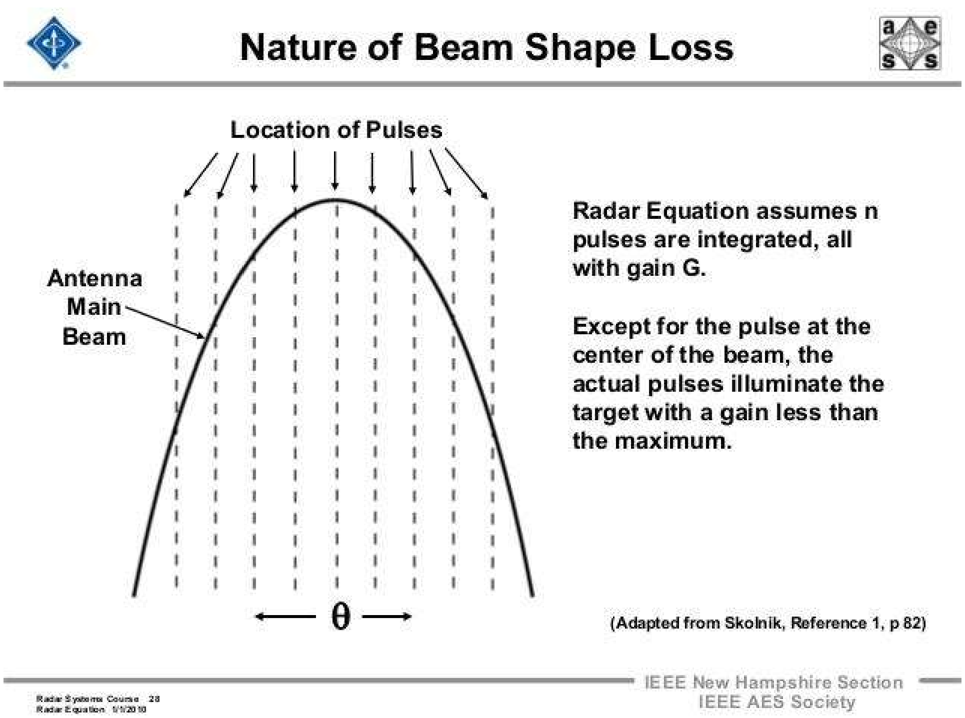
$$\text{Beam shape loss} = \frac{n}{1+2\sum_{k=1}^{\left(\frac{n-1}{2}\right)} e^{-\left[\frac{5.55k^2}{(n_B-1)^2}\right]}} \quad \ldots (2.8.1)$$

Where,

n is total number of pulses integrated.

n_B is number of pulses received within one-way half power beamwidth (θ_B).

θ_B is half power beamwidth.



b) **Scanning loss**:

When a radar antenna scans rapidly compared to round trip time of the echo signal, the antenna gain may not be same for transmission and while receiving of echoes. This results in the direction of additional loss called the Scanning loss.

The scanning loss is most significant in long range scanning radars, such as space surveillance and ballistic missile defense radars.

c) Radome:

The loss introduced by radome is decided by its type and operating frequency. A commonly used ground based metal space frame radome offers a loss of 1.2dB for two way transmission.

d) Phased array losses:

Some phased array radars have additional transmission line losses due to the distribution network that connects the receiver and transmitter to multiple elements of array. These losses reduces antenna power gain.

3. Signal Processing Losses:

For detecting targets in clutters and in extraction information from the radar echo signals is very important and lossless signal processing is necessary. Various losses accounted during signal processing are

No.	Process / Components	Loss
1.	Non matched filter	0.5 to 1.0 dB
2.	Constant False-Alarm Rate (CFAR)	> 2.0 dB
3.	Automatic integrator	1.5 to 2.0 dB
4.	Limiting loss	1 dB
5.	Straddling loss	1.0 to 2.0 dB
6.	Sampling loss	2.0 dB
7.	Threshold level loss	Upto 1 dB

2.8.4 Collapsing Loss

- When additional noise samples are integrated with signal + noise pulses, this added noise causes degradation called *collapsing loss*.
- The collapsing loss is given by L_c

$$L_{c(m,n)} = \frac{L_{i(m+n)}}{L_{i(n)}}$$

where, $L_{i(m+n)}$ is integration loss for m + n pulses.

$L_{i(n)}$ is integration loss for n pulses.

m is noise pulses.

n is signal - to - noise pulses.

2.8.5 Propagation Effects

- The propagation effects of radar wave has significant impact on losses. Major effects of propagation on radar performance are under mentioned.
  1. Reflections from earth's surface 2. Refraction
  3. Propagation in atmospheric ducts 4. Attenuation in clear atmosphere.
- The propagation effects are not computed under system loss but under propagation factor.

The simple form of the Radar Equation

If the transmitter power 'P_t' is radiated by an isotropic antenna (one that radiates uniformly in all directions), the power density (watt/Area) at a distance R from the radar is equal to the radiated power divided by the surface area $4\pi R^2$ of an imaginary sphere of radius 'R'.

∴ power density at Range R from an isotropic antenna $= \dfrac{P_t}{4\pi R^2}$

The power density at the target from a directive antenna with a transmitting Gain 'G' is then

power density at Range R from a directive antenna $= \dfrac{P_t G}{4\pi R^2}$

The Radar cross section of the target determines the power density returned to the radar for a particular power density incident on the target. It is denoted by σ

Reradiated power density back at the radar $= \dfrac{P_t G}{4\pi R^2} \cdot \dfrac{\sigma}{4\pi R^2}$

The radar antenna captures a portion of the echo energy incident on it. If the effective area of the receiving antenna is 'Ae' and the power Pr received by radar is

$$P_r = \frac{P_t G}{4\pi R^2} \cdot \frac{\sigma}{4\pi R^2} \cdot A_e$$

$$= \frac{P_t G \sigma A_e}{(4\pi R^2)^2}$$

The maximum Radar range (Rmax) is the distance beyond which the target can't be detected. It occurs when the received echo signal power Pr just equals the minimum detectable signal Smin

$$(R_{max})^4 = \frac{P_t G \sigma A_e}{(4\pi)^2 S_{min}}$$

$$R_{max} = \left(\frac{P_t G \sigma A_e}{(4\pi)^2 S_{min}}\right)^{1/4}$$

This eqn is called fundamental radar eqn.

$$* \quad G = \frac{4\pi A_e}{\lambda^2} \qquad A_e = \frac{G\lambda^2}{4\pi}$$

$$\therefore R_{max} = \left(\frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 S_{min}}\right)^{1/4}$$

→ In a Radar system it is observed that the echo received is after 9.15 μsec. Calculate the distance of the stationary object from the radar.

Sol

$$T_R = 9.15\,\mu\,sec$$

$$R = \frac{C T_R}{2}$$

$$= \frac{3\times10^8\ m/sec \times 9.15\times10^{-6}\ sec}{2}$$

$$= 1.372\ Km$$

→ A radar system transmits pulses of duration 2μs and repetition rate of 1KHz. Find the maximum and minimum range of radar.

Sol

$$PRF\ f_r = 1\,KHz$$

$$PW = 2\mu sec.$$

The maximum Range of radar is

$$R_{max} = \frac{c}{2f_r} = \frac{3\times10^8\ m/s}{2\times1\times10^3} = 1.5\times10^5\ m.$$

$$R_{min} = \frac{c\cdot PW}{2} = \frac{3\times10^8\times2\times10^{-6}}{2}$$

$$= 300\ meters.$$

→ what is the duty cycle of radar with a pulse width of 4 μsec and a pulse repetition time of 8 msec?

Sol

$$Duty\ cycle = \frac{PW}{PRT} = \frac{4\times10^{-6}}{8\times10^{-3}} = 0.0005.$$

→ A Radar is to have a maximum of 300km. what is the maximum allowble Pulse repetition freq for unambiguous reception?

sol

The maximum unambiguous range is given by

$$R_{max} = \frac{c}{2f_r}$$

$$f_r = \frac{c}{2R_{max}}$$

$$= \frac{3\times10^8}{2\times300\times10^3}$$

$= 500$ pulse per sec

$= 0.5\,KHz$

→ calculate the maximum range of radar which operates at a freq of 10GHz, Peak pulse power of 600Kw. If the antenna effective area is $5m^2$ and the area of target is $20m^2$, minimum receivable power is 10^{-13} watt.

sol $P_t = 600\,Kw$ $\quad \sigma = 20m^2$ $\quad A_e = 5m^2$ $\quad$ (or S_{min}) $P_{min} = 10^{-13}\,W$

$f = 10\times10^9\,Hz$

$$\lambda = \frac{c}{f} = \frac{3\times10^8}{10\times10^9} = 0.03\,m$$

$$G_t = \frac{4\pi A_e}{\lambda^2} = 69.813\times10^3$$

$$R_{max} = \left(\frac{P_t\, G\, A_e\, \sigma}{(4\pi)^2\, P_{min}}\right)^{1/4}$$

$= 717\,Km$

→ A marine radar operating at 10GHz has a max. range of 50 Km with an antenn gain of 4000. If the transmitter has a Power of 250 KW and and minimum detectable signal of 10^{-11} W. Determine the cross-sec of the target the radar can sight.

sol

$f = 10 \times 10^9$ Hz $R_{max} = 50$ Km

$P_t = 250$ KW $G_t = 4000$

P_{min} (or) $S_{min} = 10^{-11}$ W

$\lambda = \frac{c}{f} = 0.03$ m

$$\therefore R_{max} = \left[\frac{P_t G_t^2 \lambda^2 \sigma}{(4\pi)^3 S_{min}}\right]^{1/4}$$

$$\sigma = \frac{R_{max}^4 (4\pi)^3 S_{min}}{P_t \cdot G_t^2 \lambda^2}$$

$$= 34.45 \text{ m}^2$$

→ An S-band radar transmitting at 3GHz radiates 200 KW. Determine the signal power density at ranges 100 nautical miles if the effective area of the radar antenna is 9 m².

sol

$f = 3$GHz $= 3 \times 10^9$ Hz

$P_t = 200$KW $= 200 \times 10^3$ watts

$R = 100$ nautical miles

1 nautical mile $= 1.8412 \times 10^3$ m

$\therefore R = 1.8412 \times 10^5$ meter

$$A_e = 9\,m^2 \qquad \lambda = \frac{c}{f} = \frac{3\times10^8}{3\times10^9} = 0.1\,m$$

$$G_t = \frac{4\pi A_e}{\lambda^2} = \frac{4\pi\times 9}{(0.1)^2} = 11.3\times10^3$$

Power density by directive antenna is

$$P = \frac{P_t G_t}{4\pi R^2}$$

$$= 5.248\ mW/m^2$$

-> A Radar operating at 3GHz radiating power of 200 KW. Calculate the power of the reflected signal at the radar with a $20m^2$ target at 300 nautical miles. Take $A_e = 9m^2$

sol

$$P_r = \frac{P_t G_t A_e \sigma}{(4\pi)^2 R^4} \qquad G_t = \frac{4\pi A_e}{\lambda^2}$$

$$R = 300\times1.8412\times10^3\ m$$

$$= 27.034\times10^{-15}\ \text{watts}$$

-> Find the maximum range of a radar, the transmitted power is 250KW, cross sectional area of the target is $12.5\,m^2$, minimum power received is 10^{-3} watt. receiver antenna gain is 2000 and operating wave length 16cm.

sol

$$R_{max} = \left(\frac{P_t G_t^2 \lambda^2 \sigma}{(4\pi)^3 S_{min}}\right)^{1/4}$$

$G_r = G_t$
Common antenna is used for transmission & reception
$\therefore G_r = G_t$

$$= 200.39\ Km$$

→ A pulsed radar operating at 10GHz has an antenna with a gain of 28dB and a transmitting power of 2KW. If it is defined to detect a target with cross section of 12 m^2 and the minimum detectable signal Smin = −90dBm. what is the maximum range of radar.

Sol

$f = 10\times10^9$ Hz $\quad \lambda = \frac{c}{f} = 0.03m$

$G_t = 28dB = 630.95 \quad P_t = 2\times10^3 W$

$\sigma = 12\ m^2 \quad P_{min}$ (or) $S_{min} = -90\ dBm$

$= 10^{-12}$ watt

$$\therefore R_{max} = \left(\frac{P_t G_t \cancel{A_e} \lambda^2 \sigma}{(4\pi)^3 S_{min}}\right)^{1/4}$$

$= 1619 = 1.619$ km.

Limitation of Radar Range eq^n

$$R_{max} = \left(\frac{P_t G A_e \sigma}{(4\pi)^2 S_{min}}\right)^{1/4} \text{ — (1)} \qquad G = \frac{4\pi A_e}{\lambda^2}$$

$$R_{max} = \left(\frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 S_{min}}\right)^{1/4} \text{ — (2)} \qquad A_e = \frac{G\lambda^2}{4\pi}$$

$$R_{max} = \left(\frac{P_t A_e^2 \sigma}{4\pi \lambda^2 S_{min}}\right)^{1/4} \text{ — (3)}$$

These three forms of the radar eq^{n}s are basically the same but there are diff in interpretation.

for example in eq ②

$$R_{max} \propto \lambda^{1/2}$$

& in eq ③

$$R_{max} \propto \lambda^{-1/2}$$ which is just opposite.

The correct interpretation depends on whether the antenna gain is held constant with change in wavelength or freq as implied by eq ②

(or) the effective area is held constant for eq ③

Receiver Noise

Noise is unwanted electromagnetic energy which interferes with the ability of the receiver to detect the wanted signal thus limiting the receiver sensitivity. It may originate within the receiver itself or it may enter via the receiving antenna along with the desired signal.

If the radar were to operate in a perfectly noise free environment so that no external noise accompany the target signal, & if the receiver itself were so perfect that it did not generate any excess noise, there would be still be noise generated by the thermal motion of the conduction electrons in the ohmic portion of the receiver i/p statges. This is called thermal noise or Johnson noise.

The available thermal noise power generated by a receiver of B.W B_n (hertz) at a temparature T (°K) is

$$\text{thermal noise power} = KTB_n$$

K = Boltzmann's constant $= 1.38 \times 10^{-23}$ J/deg.

~~The~~ For the superhetrodyne receiver mostly used in radar, the receiver B.W is approximately that of the intermediate freq B.W. (IF amplifier)

$$B_n = \frac{\int_{-\infty}^{\infty} |H(f)|^2 \, df}{|H(f_0)|^2}$$

$H(f)$ = freq response funⁿ of the IF amplifier

f_0 = freq of maximum response.

The Band width B_n is called the noise band width is the band width of the equivalent rectangular filter whose noise power o/p is same as the filter with freq response funⁿ $H(f)$.

But noise band width is not theoretically same as the 3-dB (or) half power band width. The 3-dB Bw is defined by the separation b/w the points of the freq response funⁿ $H(f)$ where the response is reduced 0.707 (3dB in power) from its maximum value.

Actually 3dB - bandwidth is used in many cases as an approximation to the noise B.W.

The measure of the noise out of a real receiver to that from the ideal receiver with only thermal noise is called the noise figure and is defined as

$$F_n = \frac{N_{out}}{K T_o B_n G_a} = \frac{\text{Noise out of practical receiver}}{\text{Noise out of ideal receiver at std temp } T_o}$$

G_a = available gain

T_o = 290 K std temp

The available gain G_a is the ratio of the signal S_{out} to the signal in S_{in}, with both o/p and i/p matched to deliver maximum o/p power.

The i/p Noise N_{in}, in an ideal receiver is $K T_o B_n$

$$\therefore \text{Noise figure } F_n = \frac{N_{out}}{\frac{S_{out}}{S_{in}} N_{in}}$$

$$= \frac{S_{in}/N_{in}}{S_{out}/N_{out}}$$

$\therefore$ i/p signal $\quad S_{in} = \dfrac{K T_o B_n F_n S_o}{N_o}$

If the minimum detectable signal S_{min} is that value of S_{in}, which corresponds to the min detectable signal to noise at the o/p of the IF $(S_{out}/N_{out})_{min}$.

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$$S_{min} = KT_0 B_n F_n \left(\frac{S_{out}}{N_{out}}\right)_{min}$$

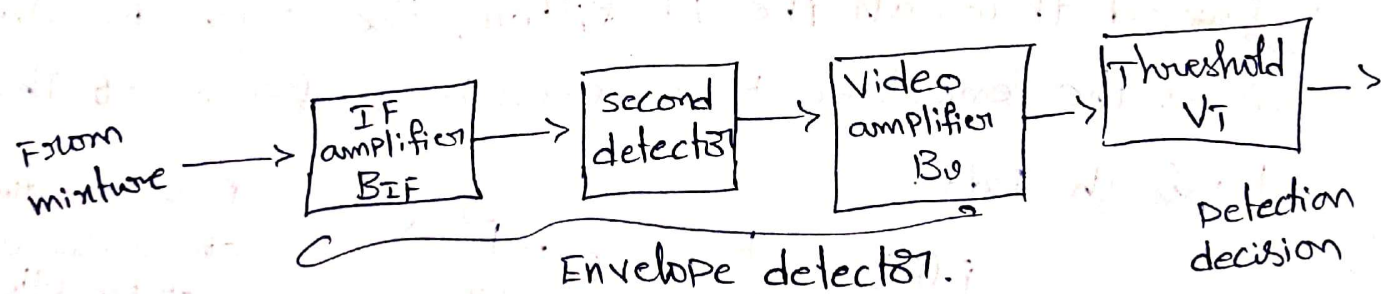
∴ the modified Radar eqn is

$$R^4_{max} = \frac{P_t G A_e \sigma}{(4\pi)^2 K T_0 B_n F_n (S_0/N_0)_{min}}$$

signal to noise Ratio

signal to noise ratio is very important as far as, radar is concerned. Because presence of target or not have small diff. statistical noise theory will be applied to obtain S/N at the o/p of the IF amplifier necessary to achieve a specified prob of detection without exceeding a specified prob of false alarm.

The Details of system that is considered



Conside

-> IF amplifier with Band width B_{IF} followed by a second detector and a video amplifier with Band width B_v.

-> the second detector and video amplifier are assumed to form an envelope detector, that is one which rejects the carrier freq. but passes the modulation envelope.

→ To extract the modulation envelope, the video BW must be wide enough to pass the low freq components generated by the second detector but no so wide as to pass the high freq components at or near the IF.

→ The video bandwidth B_v must be greater than $B_{IF}/2$ in order to pass all video modulation.

Probability of false alarm

The receiver noise at the I/P to the IF filter is described by the gaussian probability density fun with mean value zero.

$$P(v) = \frac{1}{\sqrt{2\pi \psi_0}} \exp \frac{-v^2}{2\psi_0}$$

$$P(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-a_x)^2}{2\sigma^2}}$$

S.O. Rice has shown that when the gaussian noise is passed through the IF filter, the prob density fun of the envelope R is given by a form of the Rayleigh pdf.

Rayleigh $P(x) = \frac{2}{b}(x-a) e^{-(x-a)^2/b}$

$$P(R) = \frac{R}{\psi_0} \exp\left(\frac{-R^2}{2\psi_0}\right)$$

R → amplitude of envelope of the filter

The prob that the envelope of the noise voltage will exceed the voltage threshold V_T

$$P(V_T < R < \infty) = \int_{V_T}^{\infty} \frac{R}{\psi_0} \exp\left(\frac{-R^2}{2\psi_0}\right) dR$$

$$= \exp\left(\frac{-V_T^2}{2\psi_0}\right) = P_{fa}$$

whenever the voltage envelope exceeds the threshold V_T, a target is considered to have been detected, since the prob of a false alarm is the prob that noise will cross the threshold.

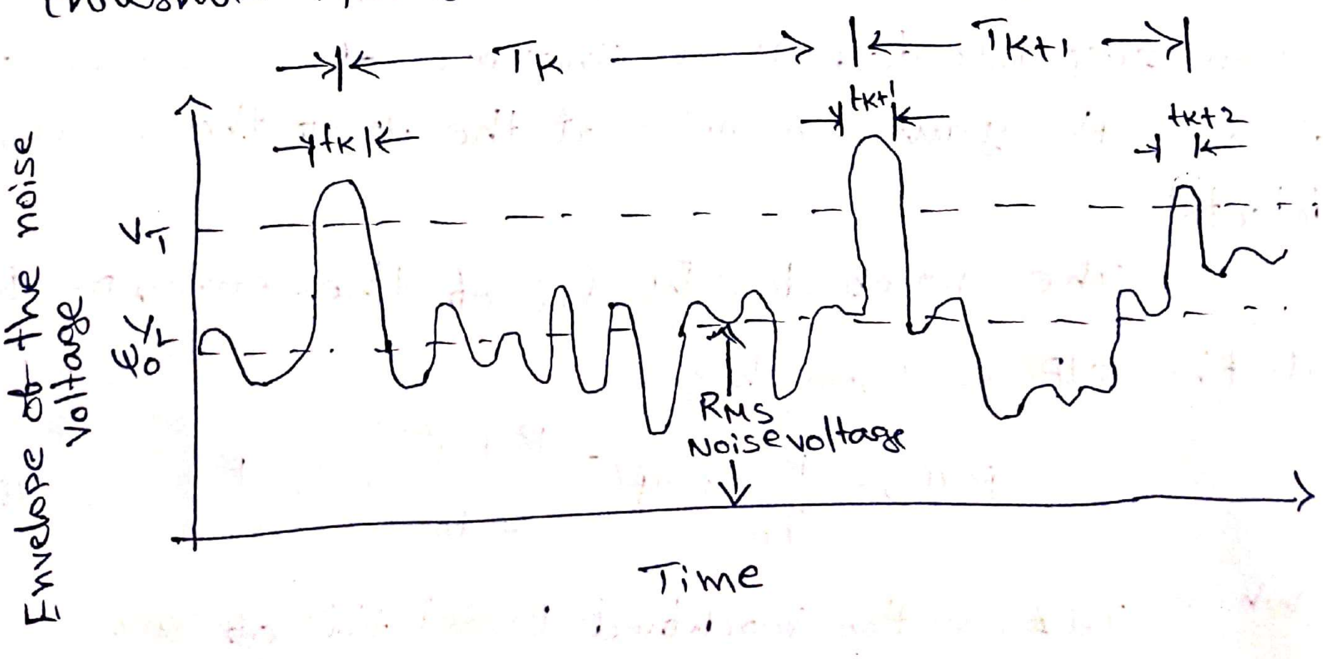
∴ prob of false alarm

$$P_{fa} = \exp\left(\frac{-V_T^2}{2\psi_0}\right)$$

The time b/w false alarms T_{fa} is also a better measure of the effect of Noise on the radar performance. The average time b/w crossings of the threshold by noise alone is defined as the false alarm time T_{fa}.

$$T_{fa} = \lim_{N \to \infty} \frac{1}{N} \sum_{K=1}^{N} T_K$$

T_K is the time b/w crossings of the threshold V_T by the noise envelope.



The false alarm probability may also be defined as the ratio of the duration of time the envelope is actually above the threshold to the total time it could have been above the threshold.

i.e; $$P_{fa} = \frac{\sum_{K=1}^{N} t_K}{\sum_{K=1}^{N} T_K} = \frac{(t_K)_{av}}{(T_K)_{av}} = \frac{1}{T_{fa} B}$$

The avg duration of a threshold crossing by noise $(t_K)_{avg}$ is approximately the reciprocal of the IF Bandwidth B. The avg of T_K is the false alarm time T_{fa}.

$$T_{fa} = \frac{1}{B} \exp\left(\frac{V_T^2}{2\psi_0}\right)$$

prob of detection

so far we have discussed only the noise i/p at the radar receiver. Now consider an echo signal represented as a sine wave of amplitude A along with gaussian noise at the i/p of the envelope detector.

The prob density fun of the envelope R at the o/p is given by

$$P_s(R) = \frac{R}{\psi_0} \exp\left(-\frac{R^2+A^2}{2\psi_0}\right) I_0\left(\frac{RA}{\psi_0}\right) \quad \text{—(1)}$$

where $I_0(Z)$ is the modified Bessel fun of zero order and argument Z.

For large z, an asymptotic expansion for $I_0(Z)$ is

$$I_0(Z) = \frac{e^Z}{\sqrt{2\pi Z}}\left(1+\frac{1}{8Z}+\cdots\right)$$

when the signal is absent $A=0$ eq ① reduced to

then $$P(R) = \frac{R}{\psi_0}\exp\left(-\frac{R^2}{2\psi_0}\right)$$

The prob of detecting the signal is the prob that the envelope R will exceed the thresh-old V_T.

∴ The prob of detection is

$$P_d = \int_{V_T}^{\infty} P_s(R)\,dR$$

$$= \int_{V_T}^{\infty} \frac{R}{\psi_0} \exp\left[\frac{-(R^2+A^2)}{2\psi_0}\right] I_0\left(\frac{RA}{\psi_0}\right) dR$$

In radar systems analysis it is more convenient to use signal to noise power ratio S/N than $A^2/2\psi_0$

$$\frac{A^2}{\psi_0^{1/2}} = \frac{\text{signal amplitude}}{\text{rms noise voltage}} = \frac{\sqrt{2}\,(\text{rms signal voltage})}{\text{r.m.s noise voltage}}$$

$$= \left(2\,\frac{\text{signal power}}{\text{noise power}}\right)^{1/2} = \left(\frac{2S}{N}\right)^{1/2}$$

Integration of Radar Pulses

The no. of pulses returned after hitting target is given by

$$n = \frac{\theta_B f_P}{\theta_S} = \frac{\theta_B f_P}{6 w_r}$$

θ_B = antenna Beam width deg

f_P = pulse repetition freq Hz

θ_S = antenna scanning rate deg/sec

w_r = revolutions per minute (rpm) if 360° rotating antenna.

The process of summing all the radar echos available from a target is called integration.

If the integration of pulses is performed before second detector in radar receiver is known as predetection integration or coherent integration. after second detector is called post detection integration or non coherent integration.

→ If n pulses of sam S/N were perfectly integrated by an ideal lossless predetection integrator, the integrated S/N will be exactly n times that of a single pulse

$$\therefore \left(\frac{S}{N}\right)_n = \frac{\left(\frac{S}{N}\right)_1}{n}$$

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→ If n pulses of same S/N are integrated by post detection integrator, the integrated S/N will be less than n time of single pulse.

This loss in integration efficiency is caused by the non linear action of the second dector, which converts some of the signal energy to noise energy in the rectification process.

The integration efficiency for post-detection integration is given by

$$E_i(n) = \frac{(S/N)_1}{n(S/N)_n} \quad — ①$$

The improvement in S/N ratio when n pulses are integrated is called the "integration improvement factor" $I_i(n) = n E_i(n)$.

Marcum defined an integration loss in dB as $L_i(n) = 10 \log[1/E_i(n)]$.

The radar eqn with n pulses integrated can be written as

$$R_{max}^4 = \frac{P_t G A_e \sigma}{(4\pi)^2 K T_0 B_n F_n (S/N)_n}$$

from eq ①

$$R_{max}^4 = \frac{P_t G A_e \sigma\, n\, E_i(n)}{(4\pi)^2 K T_0 B_n F_n (S/N)_1}$$

Radar cross-section of targets

A radar cross section is defined as the ratio of its effective isotropic scattered power to the incident power density.

$$\sigma = \frac{\text{power reflected towards source / unit solid angle}}{\text{incident power density} / 4\pi}$$

$$= \lim_{R \to \infty} 4\pi R^2 \left|\frac{E_r}{E_i}\right|^2$$

R is the range of the target

E_r is the strength of reflected field at radar

E_i is the " incident at target

→ The radar cross-section depends on the characteristic dimensions of the object comp-ared to the radar wavelength.

(i) when the wavelength is large compared to the object's dimensions, scattering is said to be in Rayleigh region

(ii) when the wavelength is small compared to the the object's dimensions is called the optical region.

(iii) In b/w the Rayleigh and optical regions is the resonance region where the radar wavelength is comparable to the objects dimensions

For many objects the radar cross section is larger in the resonance region than in the other two regions.

Transmitter Power

$$R_{max} = \left(\frac{P_t G A_e \sigma}{(4\pi)^2 S_{min}} \right)^{1/4} \quad -①$$

the power P_t in the eq ① is also called peak power of the pulse. The avg power P_{avg} of a radar is also an important measure of radar performance than the peak power. It is defined as the average transmitter power over the duration of the total transmission.

If the transmitter waveform is a train of rectangular pulses of width τ and constant pulse repetition period $T_p = 1/f_p$ then the avg power is related to peak power by

$$P_{av} = \frac{P_t \tau}{T_p} = P_t \tau f_p$$

$$\text{The radar duty cycle} = \frac{P_{avg}}{P_t} = \tau f_p$$

$$\therefore R^4_{max} = \frac{P_{av} G A_e \sigma n E_i(n)}{(4\pi)^2 K T_0 F_n (B_n \tau) f_p (S/N)_1}$$

If the transmitted waveform is not rectangular pulse, sometimes it is more convenient to express radar eq^n in terms of energy

The energy per pulse

$$E_p = P_t \tau = \frac{P_{av}}{f_p}$$

$$\therefore R_{max}^4 = \frac{E_p G A_e \sigma \, n \, E_i(n)}{(4\pi)^2 K T_0 F_n (B_n \tau)(S/N)_1}$$

$$= \frac{E_T G A_e \sigma \, E_i(n)}{(4\pi)^2 K T_0 F_n (B_n \tau)(S/N)_1}$$

where E_T is the total energy of the 'n' pulses which equals nE_p i.e; $E_T = nE_p$

The bandwidth and the pulse width are grouped together since the product of two is approximately unity in a well designed radar.