CW and Frequency Modulated Radar

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Doppler Effect

- Doppler effect implies that the frequency of a wave when transmitted by the source is not necessarily the same as the frequency of the transmitted wave when picked by the receiver.
- The received frequency depends upon the relative motion between the transmitter and receiver.
- If transmitter and receiver both are moving towards each other the received frequency higher, this is true even one is moving.
- If they are moving apart the received signal frequency decreases and if both are stationary, the frequency remains the same. This change in frequency is known as ***Doppler shift***.
- Doppler shift depends upon the relative velocity between radar and target

- If **R** is the distance from the radar to target, the total number of wavelengths **λ** contained in the two-way path between the radar and the target are ***2R/λ.***
- Each wavelength corresponds to a phase change of **2π** radians. The total phase change in the two way propagation path is then

$$\phi = 2\pi * \frac{2R}{\lambda} = 4\pi R/\lambda$$

- If the target is in motion relative to the radar, R is changing and so will the phase. Differentiating ϕ w.r.t time gives the rate of change of phase, which is the angular frequency

$$w_d = \frac{d\phi}{dt} = \frac{4\pi}{\lambda}\frac{dR}{dt} = \frac{4\pi v_r}{\lambda} = 2\pi f_d$$

- $v_r = \frac{dR}{dt}$ is the radial velocity or rate of change range with time
- $w_d = 2\pi f_d$ is the rate of change of ϕ with time is the angular frequency, where f_d is the Doppler frequency shift.

$$f_d = \frac{2v_r}{\lambda} = \frac{2f v_r}{c}$$

- The relative velocity may be written as $\mathbf{V_r = V.cos\ \theta}$ where $\mathbf{V}$ is the target speed and $\boldsymbol{\theta}$ is angle made by the target trajectory and the line joining radar and target. When $\boldsymbol{\theta=0}$ the Doppler frequency is maximum. The Doppler is zero when the trajectory is perpendicular to the radar line of sight $\mathbf{(\theta = 90^0)}$.

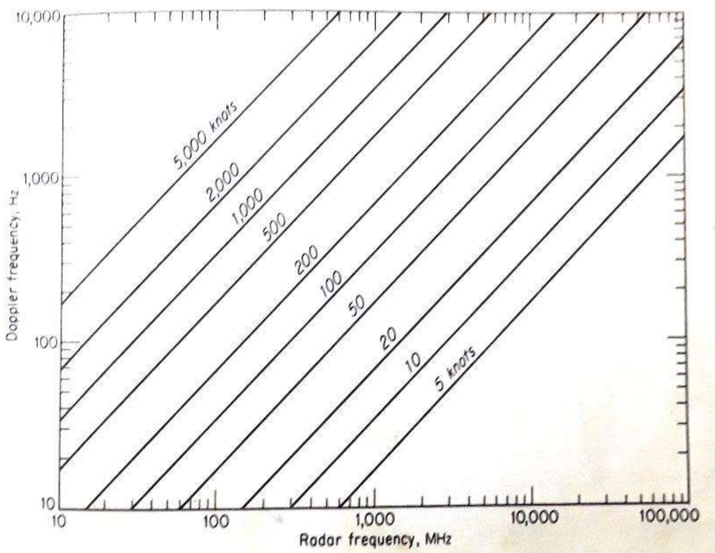


Figure: Doppler frequency f_d as a function of radar frequency and target relative velocity

CW RADAR

- It is possible to detect moving targets by radiating unmodulated Continuous wave energy instead of radiating in the form of pulses. Continuous Wave radars makes use of Doppler effect for target speed measurements.

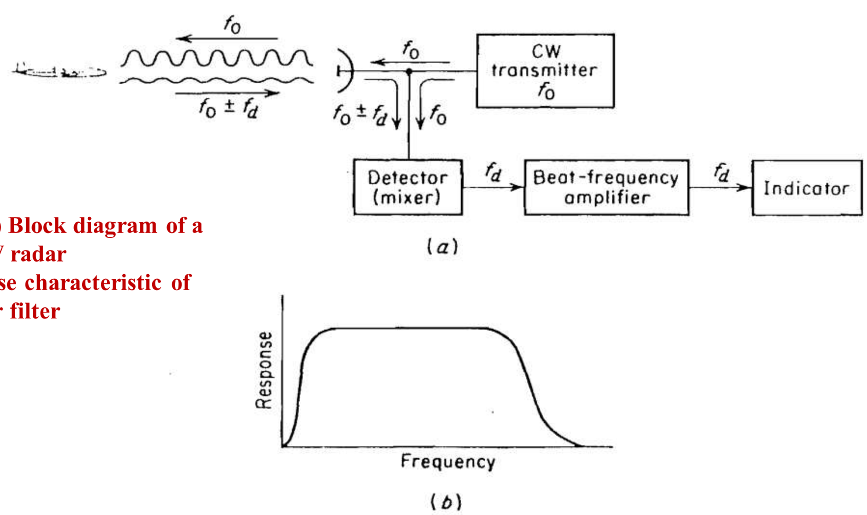


Figure : (a) Block diagram of a Simple CW radar
(b) Response characteristic of the doppler filter

- Consider the simple CW radar shown the block diagram. The transmitter generates a continuous (unmodulated) oscillation of frequency f_0 which is radiated by the antenna.
- A portion of the radiated energy is intercepted by the target and is scattered, some of it in the direction of the radar, where it is collected by the receiving antenna.
- If the target is in motion with a velocity $\mathbf{V_r}$ relative to the radar, the received signal will be shifted in frequency from the transmitted frequency f_0 by an amount $\pm f_d$ as given by the equation $f_d = \frac{2v_r}{\lambda} = \frac{2fv_r}{c}$
- ***The plus sign associated with the Doppler frequency shift applies if the distance between target and radar is decreasing (approaching target) that is, when the received signal frequency is greater than the transmitted signal frequency. The minus sign applies if the distance is increasing (receding target).***
- The received echo signal at a frequency $f_0 \pm f_d$ enters the radar via the antenna and is heterodyned in the detector (mixer) with a portion of the transmitter signal $\mathbf{f_o}$ to produce a Doppler beat note of frequency $\mathbf{f_d}$**.** The sign of $\mathbf{f_d}$ is lost in this process.

- The beat frequency amplifier eliminates the echoes from stationary targets and amplifies the Doppler echo signal.
- The low-frequency cutoff must be high enough to reject the d-c component caused by stationary targets, but yet it must be low enough to pass the smallest Doppler frequency expected. Sometimes both conditions cannot be met simultaneously and a compromise is necessary. The upper cutoff frequency is selected to pass the highest Doppler frequency expected.

Advantages of CW Radar

- CW Doppler radar has no blind speed.
- CW Doppler radar is capable of giving accurate measurements of relative velocities.
- CW Doppler radars are always on, they need low power and are compact in size.
- They can be used for small to large range with high degree of efficiency and accuracy.
- The performance of radar is not affected by stationary object.

Disadvantages of CW Doppler radar

- The maximum range of CW Doppler radar is limited by the power that radar can radiate.
- The target range can not be calculated by CW Doppler radar.
- There is possibility of ambiguous results when number of targets are more.

Applications of CW Radar

- CW Doppler radars are used where only velocity information is of intrest and actual range is not needed. E.g: in LAW and Enforcement radar applications
- Measuring motion of wave on water level.
- Runway monitors.
- Cricket ball speed measurement.

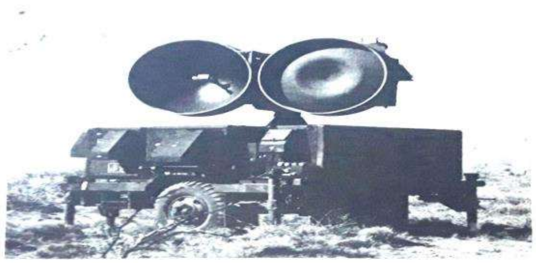
Isolation between transmitter and receiver

- A single antenna serves the purpose of both transmission and reception in the simple CW radar. In principle, a single antenna is sufficient as the necessary isolation is obtained by the separation in frequency (as a result of doppler effect), in practice there is considerable transmitter leakage.
- However, there are two reasons why the amount of transmitter leakage power should be kept at a low value.

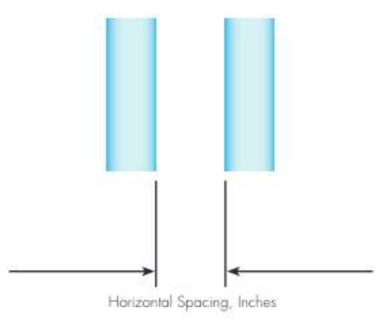
  1. The maximum power the receiver input circuitry can withstand, without being physically damaged or having its sensitivity reduced, is quite low.

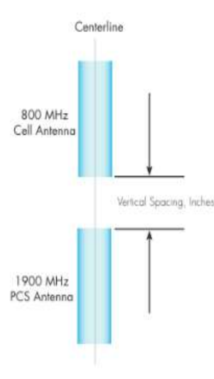
  2. The transmitter noise which enters the receiver from the transmitter reduces receiver sensitivity.
- The amount of isolation required depends on the transmitter power and the accompanying transmitter noise as well as the ruggedness and sensitivity of the receiver.
- ***For example, If the safe value of power which might be applied to a receiver is 10mW and if the transmitter power is 1 kW, the isolation between transmitter and receiver must be at least 50 dB.***

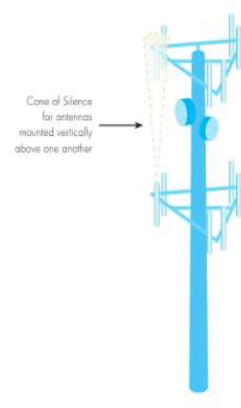
- In long range CW applications, it is the level of the noise accompanying the transmitter leakage signal, rather than the damage this leakage might cause to the receiver circuitry, which determines the amount of isolation required.
- The amount of isolation which can be readily achieved between the arms of practical hybrid junctions such as the magic-T, rat race, or short-slot coupler is of the order of 20 to 30 dB.
- In some instances, when extreme precision is exercised, an isolation of perhaps 60 dB or more might be achieved. But one limitation of the hybrid junction is the 6-dB loss in overall performance.
- The largest isolations are obtained with two antennas one for transmission, the other for reception-physically separated from one another. Isolations of the order of 80 dB or more are possible with high-gain antennas. The more directive the antenna beam and the greater the spacing between antennas, the greater will be the isolation.
- The separate antennas of the AN/MPQ-46 CW tracker-illuminator of the Hawk missile system are shown



- The correct degree of isolation between transmitters and receivers can be implemented by one of two methods:
    1) *Use two antennas, physically separated by a given distance*, or;
    2) *Use the appropriate duplexer with a single-antenna system.*







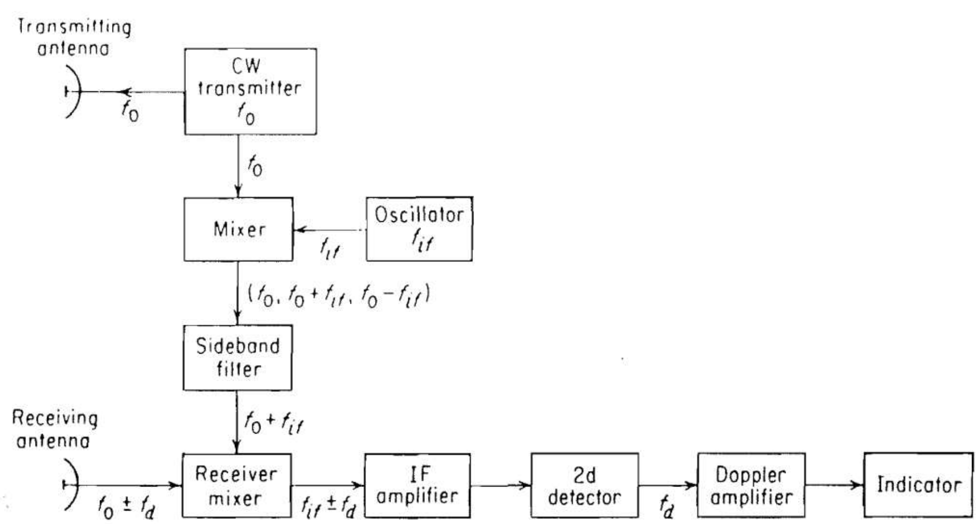
Intermediate-frequency receiver

Limitation of Zero IF receiver:

- Receivers of super heterodyne receiver type are also called homodyne receivers, or super heterodyne receivers with zero IF.
- However, this simpler receiver is not very sensitive because of increased noise at the lower intermediate frequencies caused by flicker effect.
- Flicker-effect noise occurs in semiconductor devices such as diode detectors and cathodes of vacuum tubes. The noise power produced by the flicker effect varies as **$1/f^{\alpha}$ where α is approximately unity.**
- At the lower range of frequencies (audio or video region), where the Doppler frequencies usually are found, the detector of the CW receiver can introduce a considerable amount of flicker noise, resulting in reduced receiver sensitivity.
- For short-range, low-power, applications this decrease in sensitivity might be tolerated
- But for maximum efficiency with CW radar, the reduction in sensitivity caused by the simple Doppler receiver with zero IF cannot be tolerated.

Non zero IF Receiver:

- Flicker effect noise reduces the receiver sensitivity of a CW Radar with zero IF (Simple Doppler radar). In order to increase the sensitivity and efficiency we go for CW Radar with Non-zero IF.



- Figure above shows the block diagram of a CW radar whose receiver operates with a nonzero IF**.** Separate antennas are shown for transmission and reception.
- Instead of the usual local oscillator found in the conventional super heterodyne receiver, the local oscillator (or reference signal) is derived in the receiver from a portion of the transmitted signal mixed with a locally generated signal of frequency equal to that of the receiver IF.
- Since the output of the mixer Consists of two sidebands on either side of the carrier plus higher harmonics, a narrow band filter selects one of the sidebands as the reference signal.
- The improvement in receiver sensitivity with an intermediate-frequency super heterodyne might be as much as 30 Db.

Limitations of CW radar with Non Zero IF

- False targets
- Unable to detect the range of the target

Receiver bandwidth requirements

- Bandwidth *B*, *BW* or *Δf* is the difference between the upper and lower cut-off frequencies of a radar receiver, and is typically measured in hertz.
- In case of a baseband channel or video signal, the bandwidth is equal to its upper cut-off frequency. In a Radar receiver the bandwidth is mostly determined by the IF filter stages.
- IF amplifier should be wide enough to pass the expected range of Doppler frequencies.
- Usually expected range of Doppler frequencies will be much higher than the frequency spectrum occupied by the signal energy . So a wide band amplifier is needed.
- which result in an increase in noise and a lowering of the receiver sensitivity and S/N.
- If the frequency of the Doppler-shifted echo signal are known beforehand, narrowband filter-that is just wide enough to reduce the excess noise without eliminating a significant amount of signal energy might be used.

- If the received waveform were a sine wave of infinite duration, its frequency spectrum would be a delta function as shown in the figure (a) below and the receiver bandwidth would be infinitesimal.

- But a sine wave of infinite duration and an infinitesimal bandwidth cannot occur in nature. The more normal situation is an echo signal which is a sine wave of finite duration.

- The frequency spectrum of a finite-duration sine wave has a shape of the form

$$[\sin\pi(f-f0)\delta]/\pi(f-f0)]$$

where f0 and δ are the frequency and duration of the sine wave, respectively, and f is the frequency variable over which the spectrum is plotted (Fig b).

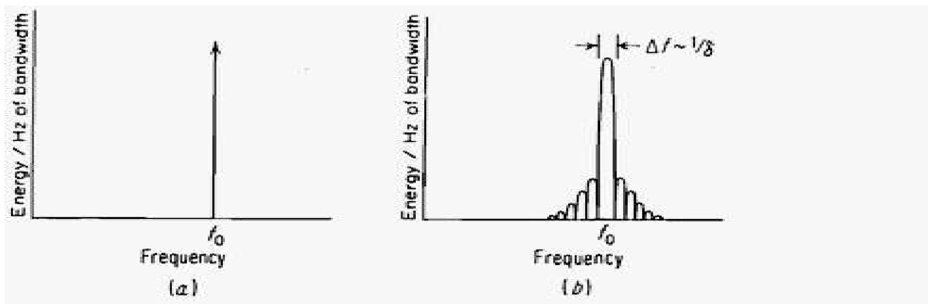


Figure 3.5 Frequency spectrum of CW oscillation of (*a*) infinite duration and (*b*) finite duration.

Filter Banks in CW radar Receiver

- A bank of narrowband filter is required to measure the frequency of echo signals. The filter bank also increases signal to noise ratio of radar receiver.

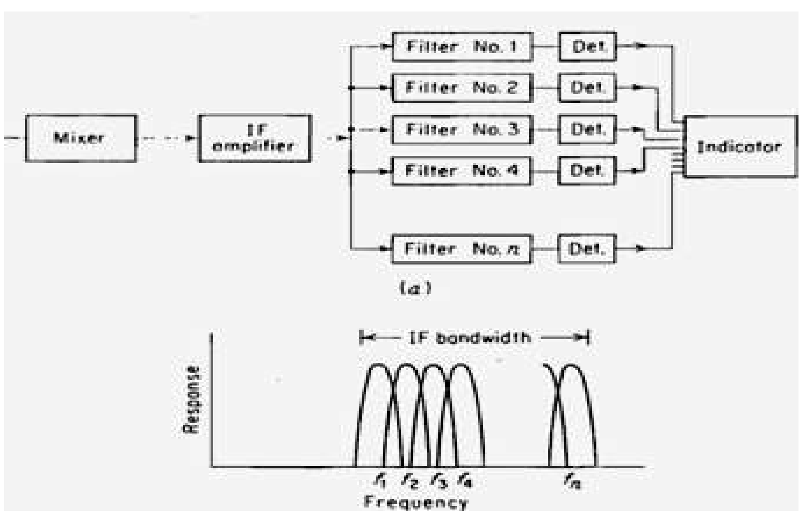


Figure: (a) Block diagram of IF Doppler filter bank (b) frequency-response characteristic of Doppler filter bank.

- BW of each filter is wide enough to accept the signal energy. But not so wide to accept the noise.
- The more the filters used less will be the SNR loss and less chance of missing a target.
- The ability to measure the magnitude of Doppler frequency and improvement in signal to noise ratio is better in IF filter bank than in video filter bank.
- Also the sign of Doppler shift (+ or -) is available which is not present in video filter bank.
- Each filter of filter bank has different bandwidth.

Sign of the radial velocity

- In many applications of CW radar it is of interest to know if the target is approaching or receding. This might be determined with separate filters located on either side of the intermediate frequency.
- If the echo-signal frequency lies below the carrier, then the target is receding; whereas if the echo frequency is greater that the carrier, then the target is approaching.

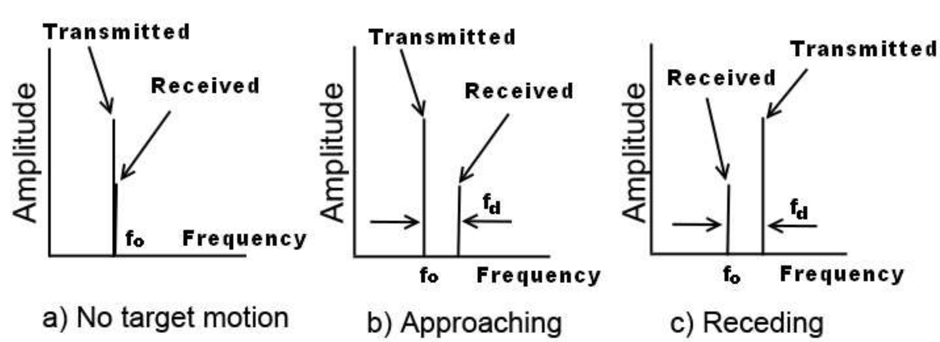


Figure: Spectra of received signals. (a) No Doppler shift, no relative target motion; (b) approaching target; (c) receding target.

- However, the Doppler-frequency spectrum "folds over" in the video because of the action of the detector, and hence the information about whether the doppler shift is positive or negative is lost. But it is possible to determine its sign from a technique borrowed from single-sideband communication.
- If the transmitter signal is given by,

$$E_t = E_o \cos w_o t$$

- The echo signal from the moving target will be,

$$E_r = K_1 E_0 \cos [(w_o + w_d)t + \varphi]$$

where, E_0 = *amplitude of the transmitted signal*

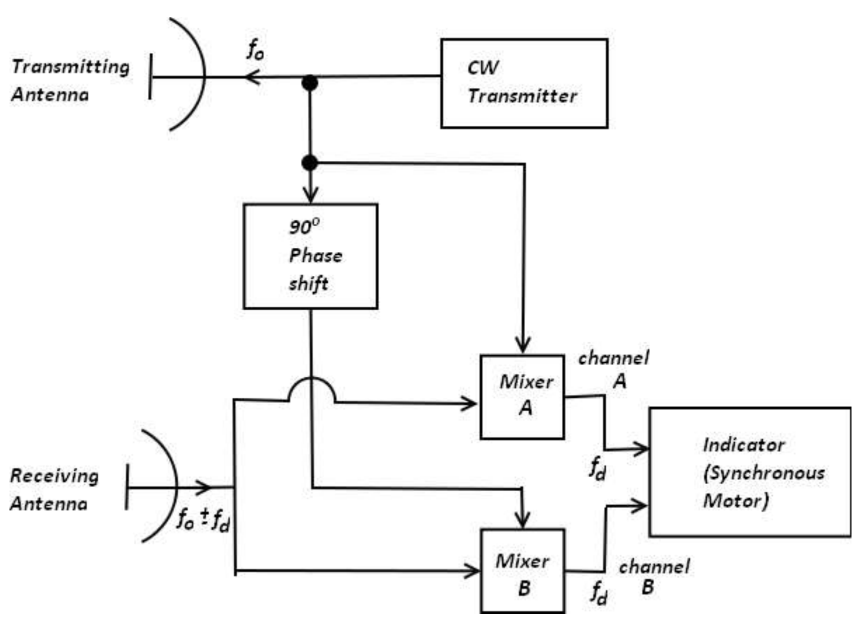
K_1 = *a constant determined from the radar equation*

w_o = *angular frequency of transmitted signal, rad/sec*

w_d = *dopper angular frequency shift, rad/sec*

φ = *a constant phase shift, which depends upon the range of initial detection (i.e., distance between the radar and the target)*

- The sign of the Doppler frequency, and therefore the direction of target motion, may be found by splitting the received signal into two channels as shown



- In channel A the signal is processed as in a simple CW radar. The receiver signal and a portion of the transmitter signal heterodyne in the detector (mixer) to yield a difference signal,

$$E_A = K_2 E_0 cos(\pm w_d t + \varphi)$$

- The channel B has $\pi/2$ phase delay introduced in the reference signal. The output of the channel B mixer is

$$E_B = K_2 E_0 \, cos(\pm w_d t + \varphi + \pi/2)$$

- If the target is approaching (positive doppler),the outputs from the two channels are,

$$E_A = K_2 E_0 \, cos(w_d t + \varphi)$$

$$E_B = K_2 E_0 \, cos(w_d t + \varphi + \pi/2)$$

on the other hand, if the target is receding (negative doppler),

$$E_A(-) = K_2 E_0 \, cos(w_d t - \varphi)$$

$$E_B(-) = K_2 E_0 \, cos(w_d t - \varphi - \pi/2)$$

- The sign of *wd and the direction of the target's motion may be determined* according to whether the output of channel B leads or lags the output of channel A.
- One method of determining the relative phase relationship between the two channels is to apply the outputs to a ***synchronous two-phase motor.*** The direction of motor rotation is an indication of the direction of the target motion.

Applications of CW radar

- Police speed monitor
- Rate-of-climb meter (During aircraft take off)
- Vehicle counting
- As a replacement for "5th wheel speedometer" in vehicle testing
- Antilock braking system
- Collision avoidance
- In railways as speedometer instead of tachometer
- Advance warning system for approaching targets
- Docking speed measurement of large ships
- Intruder alarms
- Measurement of velocity of missiles, baseball etc

FM CW RADAR

- FM CW radar is capable of measuring the relative velocity and the range of the target with the expense of bandwidth.
- The inability of the simple CW radar to measure range is related to the relatively narrow spectrum (bandwidth) of its transmitted waveform.
- By providing timing marks into the Tx signal the time of transmission and the time of return can be calculated. This will increase the bandwidth
- More distinct the timing, more accurate the result will be and more broader will the Tx spectrum
- The spectrum of a CW transmission can be broadened by the application of modulation, either amplitude. frequency, or phase.
- An example of an amplitude modulation is the pulse radar. The narrower the pulse, the more accurate the measurement of range and the broader the transmitted spectrum
- A widely used technique to broaden the spectrum of CW radar is **to frequency-modulate** the carrier. The timing mark is the changing frequency.

Range and Doppler measurement:

- In the frequency-modulated CW radar (abbreviated FM-CW), the transmitter frequency is changed as a function of time.
- Assume that the transmitter frequency increases linearly with time, as shown by the solid line in the next slide.
- If there is a reflecting object at a distance **R**, an echo signal will return after a time **T = 2R/C.** The dashed line in the figure represents the echo signal.
- If the echo signal is heterodyned with a portion of the transmitter signal in a nonlinear element such as a diode, a beat note ***fb will be produced.***
- If there is no Doppler frequency shift, the beat note (difference frequency) is a measure of the target's range and ***fb = fr where fr is the beat frequency due only*** to the target's range.

- If the rate of change of the carrier frequency is **f_0 (dot)** ,then the beat frequency is given by

$$f_r = \dot{f}_0 T = \frac{2R}{c}\dot{f}_0$$

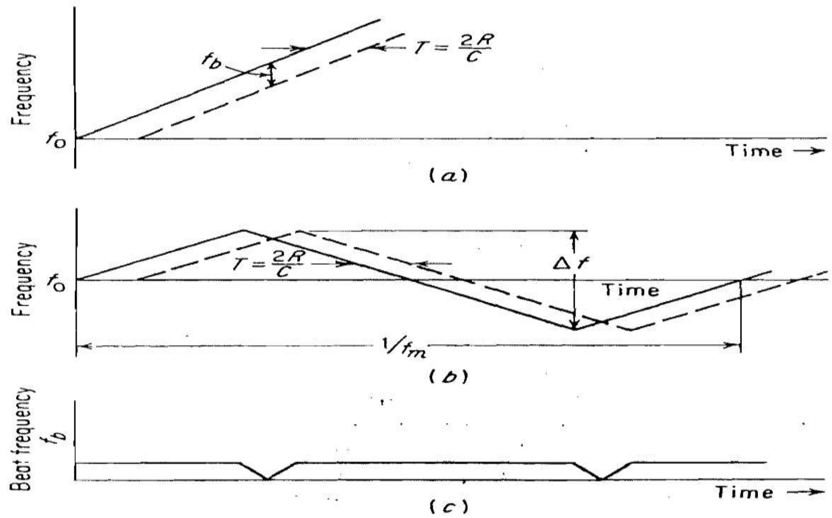


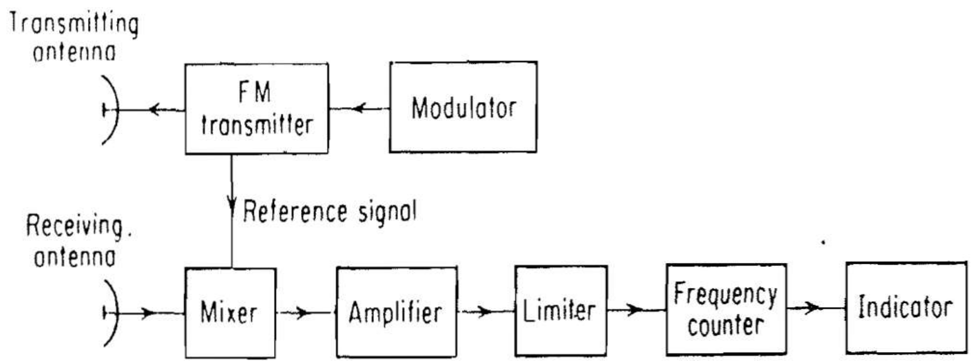
Figure: Frequency-time relation-ships in FM-CW radar. Solid curve represents transmitted signal; dashed curve represents echo. (a) Linear frequency modulation; (b)triangular frequency modulation; (c) beat note of (b).

- In any practical CW radar, the frequency cannot be continually changed in one direction. Periodicity in the modulation is necessary, as in the triangular frequency-modulation waveform shown in the figure (b**)**. The modulation need not necessarily be triangular. It can be saw tooth, sinusoidal, or some other shape.
- The resulting beat frequency as a function of time is shown in figure(c) for triangular modulation.
- If the frequency is modulated at a rate f_m over a range **Δf** the beat frequency is

$$f_r = (2R/C).2f_m.\Delta f = 4Rf_m.\Delta f /C$$

- Thus the measurement of the beat frequency determines the range R.

Block diagram illustrating the principle of the FM-CW radar



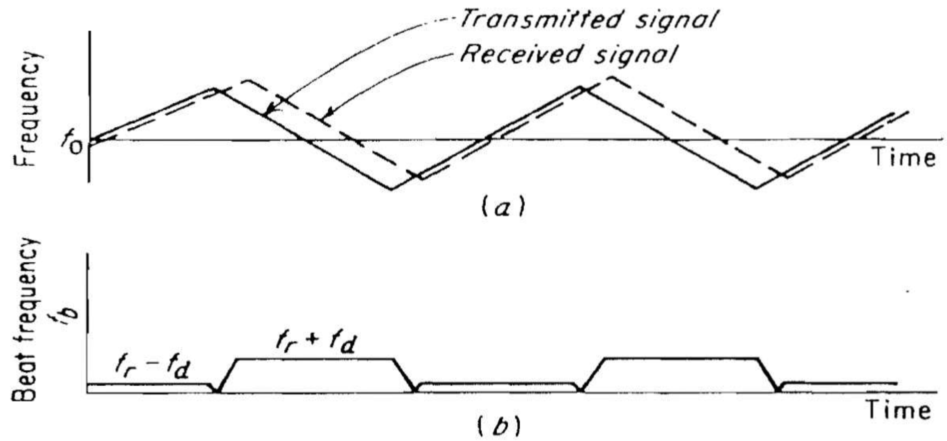
- A portion of the transmitter signal acts as the reference signal required to produce the beat frequency. It is introduced directly into the receiver via a cable or other direct connection.

- Ideally the isolation between transmitting and receiving antennas is made sufficiently large so as to reduce to a negligible level the transmitter leakage signal which arrives at the receiver via the coupling between antennas.
- The beat frequency is amplified and limited to remove any amplitude fluctuations.
- The frequency of the amplitude-limited beat note is measured with a cycle-counting frequency meter calibrated in distance.
- If the target is not stationary Doppler frequency shift will be superimposed on the FM range beat note and an erroneous range measurement results.
- The Doppler frequency shift causes the frequency-time plot of the echo signal to be shifted up or down as shown in the figure (a). On one portion of the frequency-modulation cycle, the beat frequency (Fig. b) is increased by the Doppler shift, while on the other portion, it is decreased.
- If for example, the target is approaching the radar, the beat frequency $\mathbf{f_b(up)}$ produced during the increasing or up will be the difference between the beat frequency due to the range $\mathbf{f_r}$ and the Doppler frequency shift f_d. Similarly, on the decreasing portion, the beat frequency $\mathbf{f_b(down)}$ is the sum of the two

$$f_b(\text{up}) = f_r - f_d$$

$$f_b(\text{down}) = f_r + f_d$$

Figure: Frequency-time relationships in FM-CW radar when the received signal is shifted in frequency by the Doppler effect (a) Transmitted (solid curve) and echo (dashed curve) (b) beat frequency



- The beat frequency due to range f_r can be calculated as

$$\frac{1}{2}[f_b(\text{up}) + f_b(\text{down})] = f_r$$

FM Altimeter

- The FM-CW radar principle is used in the aircraft radio altimeter to measure height above the surface of the earth.
- Relatively short ranges of altimeters permit Low Tx power and low antenna gain.
- Since the relative motion between the aircraft and ground is small, the effect of the Doppler frequency shift may usually be neglected.
- Frequency range: 4.2 to 4.4 GHz (reserved for altimeters)
- Solid state Tx is used here.
- High sensitive super-heterodyne Rx is preferred for better sensitivity and stability

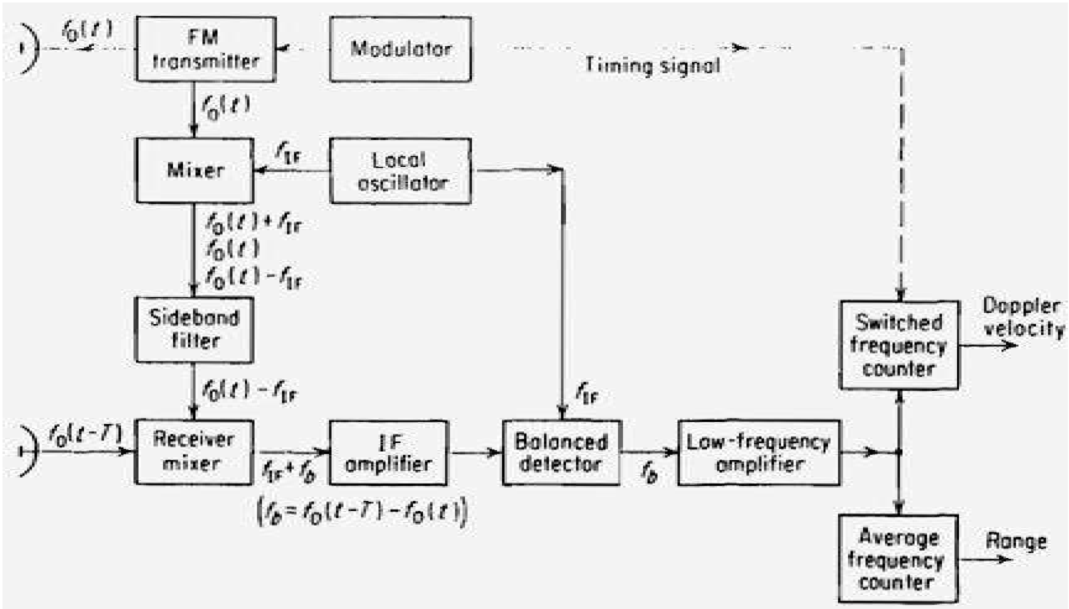


Figure: Block diagram of a FM-CW radar using sideband super heterodyne receiver

- A portion of the frequency-modulated transmitted signal is applied to a mixer along with the oscillator signal.

- The selection of the local-oscillator frequency is a bit different from that in the usual super heterodyne receiver. The local-oscillator frequency $\mathbf{f_{IF}}$ is the same as the intermediate frequency used in the receiver.

- The output of the mixer consists of the varying transmitter frequency $\mathbf{f_o(t)}$ plus two sideband frequencies, one on either side of $\mathbf{f_o(t)}$ and separated from $\mathbf{f_o(t)}$ by the local-oscillator frequency $\mathbf{f_{IF}}$.

- The filter selects the lower sideband, $\mathbf{f_o(t) - f_{IF}}$ and rejects the carrier and the upper sideband. The sideband filter must have sufficient bandwidth to pass the modulation, but not the carrier or other sideband. The filtered sideband serves the function of the local oscillator.

- When an echo signal is present, the output of the receiver mixer is an IF signal of frequency ($\mathbf{f_{IF}}+f_b$) where f_b is composed of the range frequency $\mathbf{f_r}$ and the Doppler velocity frequency $\mathbf{f_d}$.

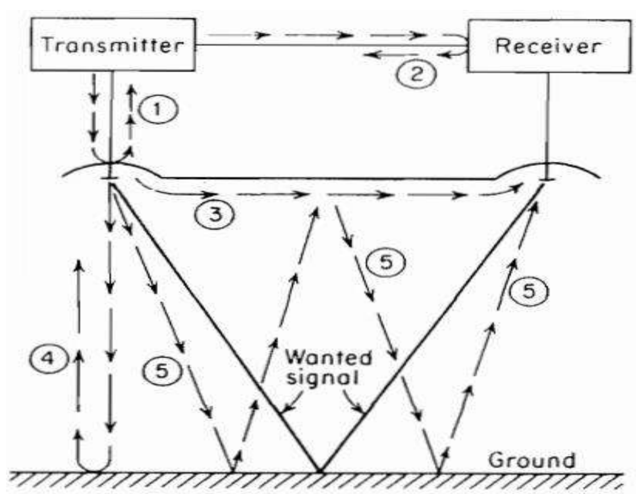
- The IF signal is amplified and applied to the balanced detector along with the local-oscillator signal $\mathbf{f_{IF}}$.

- The output of the detector contains the beat frequency (range frequency and the Doppler velocity frequency), which is amplified to a level where it can actuate the frequency-measuring circuits.

- The output of the low-frequency amplifier is divided into two channels: one feeds an average-frequency counter to determine the range, and the other feeds a switched frequency counter to determine the Doppler velocity (assuming $f_r > f_d$).
- Only the averaging frequency counter need be used in an altimeter application.
- A target at short range will generally result in a strong signal at low frequency, while one at long range will result in a weak signal at high frequency. Therefore the frequency characteristic of the low frequency amplifier in the FM-CW radar may be used to provide attenuation at the low frequencies corresponding to short ranges and large echo signals. Less attenuation is applied to the higher frequencies, where the echo signals are weaker.
- **Unwanted signals in FM altimeter:**

1. The reflection of the transmitted signals at the antenna caused by impedance mismatch.
2. The standing-wave pattern on the cable feeding the reference signal to the receiver, due to poor mixer match.
3. The leakage signal entering the receiver via coupling between transmitter and receiver antennas. This can limit the ultimate receiver sensitivity, especially at high altitudes.

3. The interference due to power being reflected back to the transmitter, causing a change in the impedance seen by the transmitter. This is usually important only at low altitudes. It can be reduced by an attenuator introduced in the transmission line at low altitude or by a directional coupler or an isolator.
4. The double-bounce signal.



Advantages of FM-CW Radar

- Range can be measured by simple broadening of frequency spectrum.
- FM modulation is easy to generate than linear modulation.
- Synchronization is not required as in multiple frequency CW radar.
- For measuring range, single frequency is required.

Disadvantages of FM-CW Radar

- FM CW radar can be used to detect single targets only.
- Accuracy of FM CW radar is less compared to Multiple frequency radar.
- Measurement of range is more difficult when FM signal is non uniform or mixer is not operating in linear region.

Multiple-frequency CW Radar

- Consider a CW radar radiating a single-frequency sine wave of the form sin $2\pi f_o t$
- The signal travels to the target at a range **R** and returns to the radar after a time **T = 2R/c** where **c** is the velocity of propagation.
- The echo signal received at the radar is $\sin[2\pi f_o(t-T)]$.
- If the transmitted and received signals are compared in a phase detector, the output **is** proportional to the phase difference between the two and is given by :

$$\Delta\varphi = 2\pi f_0 T = 4\pi f_o R/c.$$

- The phase difference may therefore be used as a measure of the range, or

$$R = \frac{c\,\Delta\phi}{4\pi f_0} = \frac{\lambda}{4\pi}\Delta\phi$$

- However, the measurement of the phase difference $\Delta\varphi$ is unambiguous only if $\Delta\varphi$ does not exceed 2π radians. Substituting $\Delta\varphi = 2\pi$ into above eqn gives the maximum unambiguous range as $\lambda/2$.

- Unambiguous range may be extended considerably by utilizing two separate CW signals differing slightly in frequency.
- The transmitted waveform is assumed to consist of two continuous sine waves of frequency f_1 *and* f_2 separated by an amount **Δf.**
- For convenience, the amplitudes of all signals are set equal to unity.
- The voltage waveforms of the two components of the transmitted signal v_{1T} and v_{2T} may be written as:

$$v_{1T} = \sin(2\pi f_1 t + \phi_1)$$
$$v_{2T} = \sin(2\pi f_2 t + \phi_2)$$

where **Ø1 and Ø2** are arbitrary (constant) phase angles.

- The echo signal is shifted in frequency by the Doppler effect. The form of the Doppler-shifted signals corresponding to the two frequencies f_1 *and* f_2 are:

$$v_{1R} = \sin\left[2\pi(f_1 \pm f_{d1})t - \frac{4\pi f_1 R_0}{c} + \phi_1\right]$$
$$v_{2R} = \sin\left[2\pi(f_2 \pm f_{d2})t - \frac{4\pi f_2 R_0}{c} + \phi_2\right]$$

Where $\mathbf{R_0}$ = range to target at a particular time $t=t_0$

$\mathbf{f_{d1}}$ = Doppler frequency shift associated with frequency $\mathbf{f_1}$

$\mathbf{f_{d2}}$ = Doppler frequency shift associated with frequency $\mathbf{f_2}$

- Since the two RF frequencies $\mathbf{f_1}$ and $\mathbf{f_2}$ are approximately the same (that is $\mathbf{f_2} = \mathbf{f_1} + \mathbf{\Delta f}$, where $\Delta \mathbf{f} <<< \mathbf{f_1}$) the Doppler frequency shifts $\mathbf{f_{d1}}$ and $\mathbf{f_{d2}}$ can be assumed to be equal to each other.
- Therefore we may write $\mathbf{f_{d1}} = \mathbf{f_{d2}} = \mathbf{f_d}$
- The receiver separates the two components of the echo signal and heterodynes each received signal component with the corresponding transmitted waveform and extracts the two Doppler-frequency components given below:

$$v_{1D} = \sin\left(\pm 2\pi f_d t - \frac{4\pi f_1 R_0}{c}\right)$$
$$v_{2D} = \sin\left(\pm 2\pi f_d t - \frac{4\pi f_2 R_0}{c}\right)$$

- The phase difference between these two components is

$$\Delta\phi = \frac{4\pi(f_2 - f_1)R_0}{c} = \frac{4\pi\, \Delta f R_0}{c}$$

Hence

$$R_0 = \frac{c\, \Delta\phi}{4\pi\, \Delta f}$$

- which is the same as that of R in 37th slide with **Δf** substituted in place of $\mathbf{f_0}$.
- The two-frequency CW technique for measuring range was described as using the Doppler frequency shift.
- A large difference in frequency between the two transmitted signals improves the accuracy of the range measurement since large **Δf** means a proportionately large change in **ΔØ** for a given range.
- However, there is a limit to the value of **Δf** since **ΔØ** cannot be greater than **2π** radians if the range is to remain unambiguous.
- The maximum unambiguous range $\mathbf{R_{unamb}}$ is:

$$R_{\mathrm{unamb}} = \frac{c}{2\, \Delta f}$$

- Therefore **Δf** must be less than $\mathbf{c/2R_{unamb}}$. Note that when **Δf** is replaced by the pulse repetition rate, above eq gives the maximum unambiguous range of a pulse radar.
- The two-frequency CW radar is essentially a single-target radar since only one phase difference can be measured at a time. If more than one target is present, the echo signal becomes complicated and the meaning of the phase measurement becomes doubtful.
- The theoretical accuracy with which range can be measured with the two-frequency CW radar can be found and it can be shown that the theoretical rms range error is

$$\delta R = \frac{c}{4\pi\, \Delta f\,(2E/N_0)^{1/2}}$$

- Where ***E*** = energy contained in received signal and
- ***No*** = noise power per hertz of bandwidth.

The above Equation indicates that the greater the separation **Δf** between the two frequencies, the lesser will be the **rms** error.

- Therefore the frequency difference must not be too large if unambiguous measurements are to be made.
- The selection of **Δf** represents a compromise between the requirements of accuracy and ambiguity.
- Both accurate and unambiguous range measurements can be made by transmitting three or more frequencies instead of just two.
- For example, if the three frequencies f_1,f_2 and f_3 are such that $f_3 - f_1 = k(f_2 – f_1)$ where **k** is a factor of the order of 10 or 20, the pair of frequencies f_3, f_1 (with greater Δf) gives an ambiguous but accurate range measurernent while the pair of frequencies f_2, f_1(with lesser Δf) resolve the ambiguities in the measurement of Range.
- Likewise, if further accuracy is required a fourth frequency can be transmitted and its ambiguities resolved by the less accurate but higher unambiguous measurement obtained from the three frequencies, f1, f2 and f3.
- As more frequencies are added the spectrum and target resolution approach that obtained with a pulse or an **FM-CW**waveform.

- The multiple-frequency CW radar technique has been applied to the accurate measurement of distance in surveying and in missile guidance. The ***Tellurometer is the name given to a*** portable electronic surveying instrument which is based on this principle.
- In addition to its use in surveying, the multiple CW frequency method of measuring range has been applied in

    1. Range-instrumentation radar for the measurement of the distance to a transponder-equipped missile,
    2. The distance to satellites,
    3. In satellite navigation systems based on range measurement
    4. For detecting the presence of an obstacle in the path of a moving automobile by measuring the distance,
    5. The Doppler velocity, and the sign of the Doppler (whether the target is approaching or receding).

- **Example1: Determine the Range and Doppler velocity of an approaching target using a triangular modulation FMCW Radar. Given : Beat frequency fb(up) = 15KHz and fb (down) = 25KHz , modulating frequency : 1MHz, Δf : 1KHz and Operating frequency : 3Ghz**

Solution:

We know fr = ½[fb(up)+ fb (down)] = ½(15+25) = 20 Khz

fd = ½[fb (down) - fb(up)] = ½(25-15) = 5 Khz

The Range R in terms of fr , fm and Δf is given by : R = c fr / 4fm.Δf = (3x10^8)20x10^3 / 4(1x10^6x1x10^3) mtrs = 1500 mtrs = 1.5 Kms