

UNIT 4

SCATTERING MATRIX AND MICROWAVE MEASUREMENTS



Syllabus

SCATTERING MATRIX : Scattering Matrix Properties, Directional Couplers – 2 Hole, Bethe Hole, [S] matrix of Magic Tee and Circulator.

MICROWAVE MEASUREMENTS : Description of Microwave Bench – Different Blocks and their Features, Errors and Precautions, Measurement of Attenuation, Frequency, Standing Wave Measurements, measurement of Low and High VSWR, Cavity Q, Impedance Measurements.

LEARNING OBJECTIVES

- ☛ Concept of scattering matrix and its properties.
- ☛ Basic idea and types of directional couplers.
- ☛ Construction and working of two hole directional coupler and Bethe hole type directional coupler.
- ☛ S-matrix of various waveguide components such as directional coupler, E-plane tee, H-plane tee, magic tee, gyrator, isolator and circulator.
- ☛ Description of microwave bench setup with explanation of its various blocks and its features.
- ☛ Basic idea about errors in microwave measurements and precautions to avoid them.
- ☛ Measurement of various parameters like attenuation, frequency, low & high VSWR, cavity Q and impedance.

INTRODUCTION

In the previous unit, we studied about the construction and working of commonly used waveguide components such as directional couplers, waveguide tees (like E-plane tee, H-plane tee and magic tee) and ferrite devices (i.e., gyrator, isolator, and circulator). In this unit, we discuss the mathematical analysis of these components using Scattering parameters, also known as S-parameters, are the most common network parameters used to describe the performance of the microwave devices or circuits or networks in the frequency domain.

In the field of microwave engineering, we often come across the need of measuring different values while using in different applications such as power, attenuation, phase shift, VSWR, impedance and Q factor etc. for the effective usage. In this unit, we will discuss about the different measurement techniques.

PART-A SHORT QUESTIONS WITH SOLUTIONS

Q1. Define S-matrix and mention the use of scattering parameters in microwaves.

Ans: State the significance of S-parameters at high frequencies.
(Refer Only Significance of S-matrix or S-parameters)

(or)

Define scattering matrix. Nov/Dec.-18, (R13), Q1(i)

(Refer Only S-matrix)

(or)

Explain the significance of scattering matrix. Nov/Dec.-17, (R13), Q1(i)

Why the s-parameters are used in microwaves? Nov/Dec.-16, (R13), Q1(i)

(Refer Only Significance of S-matrix or S-parameters)

Ans: S-matrix

The S-matrix (or) a scattering matrix is a matrix which is used to represent all inputs which are applied to the ports of a given network in a matrix form. The elements in this matrix are known as 'scattering coefficients' (or) scattering parameters'. Significance of S-matrix or S-parameters

S-parameters can be used at any frequencies, but at high frequencies such as RF and microwave frequencies powers associated with travelling waves must be calculated. S-parameters at high frequencies make these calculations simple and accurate. S-parameters are used at high frequencies to describe the behaviour of complex networks.

Q2. Why Z and Y parameters are not measured at microwave frequencies? Dec.-19, (R16), Q4(i)

(or)
Explain why h, Y and Z parameters cannot be measured at microwave frequencies.

Ans: Z, Y and h-parameters are used in the analysis of low frequency circuits. Because at low frequencies, the total voltages and total currents are measured in the network analysis. But at microwave frequencies, travelling waves come into the analysis. For this, the microwave powers associated with travelling waves must be calculated. Due to this reason s-parameters are used at microwave frequencies instead of Z, Y and h-parameters.

Q3. What are the properties of S-matrix?
Ans: The properties of S-matrix include,

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UNIT-4 (Scattering Matrix and Microwave Measurements)

Q7. A 20 dB coupler has a directivity of 30 dB. Calculate the value of isolation.

Ans: Given that,

For a coupler,

Coupling factor, $C = 20 \text{ dB}$

Directivity, $D = 30 \text{ dB}$

Isolation, $I = ?$

The isolation can be defined as,

Isolation, $I = \text{Coupling factor (C)} + \text{Directivity (D)}$

On substituting corresponding values in the above equation,

$$I = 20 + 30 = 50 \text{ dB}$$

\therefore Isolation, $I = 50 \text{ dB}$

Q8. List out the different components used in a microwave bench.

Ans: The different components used in a microwave bench are,

1. Reflex klystron or gun oscillator as a source

2. Isolator

3. Attenuator

4. Wave meter

5. Standardizing wave detectors

6. Slotted waveguide

7. Tunable probe vernier scale

8. Rack and pinion arrangement

9. Waveguide detector

Q9. What type of slot is used in microwave bench?

Ans:

The slotted line is one of the basic instruments used in radio frequency test and measurement at microwave frequencies. It consists of a precision transmission line with a movable insulated probe inserted into a longitudinal slot cut into the line. The type of slot used in microwave bench is transmission line (waveguide) type.

Q10. Why Isolator is used in microwave measurements?

(or)

What is the need for an isolator in MW measurements and where it is placed? Nov/Dec.-17, (R13), Q4(i)

Ans:

Isolator is used in several microwave circuits such as microwave bench set up because it provides a very small amount of attenuation when transmitted from port (1) to port (2) and maximum amount of attenuation, when transmitted from port (2) to port (1). This feature is used in matching a source with a variable load.

It is placed between variable attenuator and frequency meter.

Q11. How are microwave measurements different from low frequency measurements?

Ans:

The differences between low frequency and microwave measurements are mentioned in table.

Low Frequency Measurement

1. In order to measure the power, initially the voltage and current must be calculated.

2. In case of power measurement, both input and output powers must be known.

3. Circuits are lumped elements.

4. Exact values must be known.

Microwave Measurement

1. Power is measured directly without calculating the current and the voltage.

2. In case of power measurement, Ratio of powers is enough.

3. Circuits are distributed.

4. Exact values are not necessary.

Q12. What are the possible errors in high frequency measurements?

Ans:

1. Error due to hysteresis and nonlinearity of the modulator characteristics.
2. Mismatch error in the channel.
3. Error due to microwave frequency instability.
4. Error may be caused by vibrations of equipment and other external disturbances.
5. These measurements desire the manual interaction which, ultimately leads to an error.

Q13. What are the characteristics of detectors used in microwave measurements?

Ans:

The characteristics of detectors used in microwave measurements are,

1. In order to detect low frequency square wave modulated microwave signal, a non-reciprocal detector diode must be mounted in the transmission line.
2. To detect the modulated signal, a crystal detector is connected at one end of the waveguide.
3. In order to avoid amplitude and phase variations and to get perfect detection, a tunable stub is used to match the detector to the microwave transmission system and that tunable stub may be,

- (i) Tunable waveguide detector
- (ii) Tunable probe detector
- (iii) Tunable coaxial detector.

4. Schottky detector can be used in which the output is proportional to the input power.

Q14. What is Q of a cavity resonator?

Ans:

The quantity which is a measure of the frequency selectivity of a resonant or non-resonant circuit is known as quality factor 'Q'.

Mathematically,

$$Q = 2\pi \times \frac{\text{Maximum energy stored}}{\text{Energy dissipated per cycle}}$$

i.e.,
$$Q = \frac{\omega W}{P}$$

Where,

$$W = 2\pi \times \frac{\text{Maximum energy stored}}{\text{Energy dissipated per cycle}}$$

P – Average power loss.

Q15. What is a VSWR meter and how will you determine the VSWR?

Ans:

A VSWR meter is a high gain, high Q, low noise amplifier which is tuned normally to a fixed frequency modulated microwave signal.

VSWR can be determined by direct method (slotted line) and double minimum method.

Q16. What are the different possible errors that will effect VSWR measurements?

(or) April/May-14, (R13), Q10

1. Account for the different types of errors associated with the measurement of VSWR using a slotted line setup.
2. The different types of errors associated with the measurement of VSWR using a slotted line setup are,

1. Linearity error.
2. There is no accuracy in obtained results because of the limited dynamic range of detectors.
3. This process desires the manual interaction which, ultimately leads to an error.

WARNING: X-RAY AND MICROWAVE EQUIPMENT IS CRIMINAL/ OFF AND ONE FOUND GUILTY IS UNABLE TO FACE LEGAL PROCEEDINGS.**PART-B ESSAY QUESTIONS WITH SOLUTIONS**

Ans:

Q17. What is a scattering matrix? Discuss the importance of S-parameters. List the properties of S-matrix.

Ans:

S-Matrix

The S-matrix (or) scattering matrix is a square matrix gives all possible relations between incident and reflected wave power of a microwave junction. The elements in this matrix are known as 'scattering coefficients' (or) 'scattering parameters'.

It is denoted as,

$$\begin{bmatrix} [b] \\ \vdots \\ b_n \end{bmatrix} = \begin{bmatrix} s_{11} & s_{12} & \dots & s_{1n} \\ s_{21} & s_{22} & \dots & s_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ s_{n1} & s_{n2} & \dots & s_{nn} \end{bmatrix} \begin{bmatrix} [a] \\ \vdots \\ a_n \end{bmatrix}$$

Column matrix

Scattering column matrix $[s]$ of order $n \times n$

to incident wave (or)

to reflected wave

$n \times n$

input powers.

(or) output powers.

The scattering parameters are fixed properties of the linear circuits, which describe how the energy couples between a pair of ports or transmission lines connected to the circuit.

Formally, S-parameters can be defined for linear electronic components. They are algebraically related to the impedance, admittance and also to transmission parameters.

Properties

1. Scattering matrix is always a square matrix i.e., the order of S-matrix is $n \times n$.
2. The diagonal elements of an ideal N-port network are zero, if all the ports matched perfectly.
3. A reciprocal microwave device holds symmetric property. i.e.,

$$S_i = S_{i'} \quad i \neq i'$$

In matrix notation, it can be expressed as,

$$[S] = [S]^T$$

For a lossless network, S-matrix exhibits unitary property. i.e.,

$$[S][S]^* = [I]$$

If the ports of a network (2-port) shifted by a phase of ϕ_1 and ϕ_2 then S-matrix of device after shift are evaluated as,

$$[S'] = \begin{bmatrix} e^{-j\phi_1} & 0 \\ 0 & e^{-j\phi_2} \end{bmatrix} [S] \begin{bmatrix} e^{-j\phi_1} & 0 \\ 0 & e^{-j\phi_2} \end{bmatrix}$$

This is known as phase shift property and it is applicable for a microwave junction having number of ports.

Significance

The S-matrix is a square matrix, it gives the power relationships between the various input and output ports of a microwave junction.

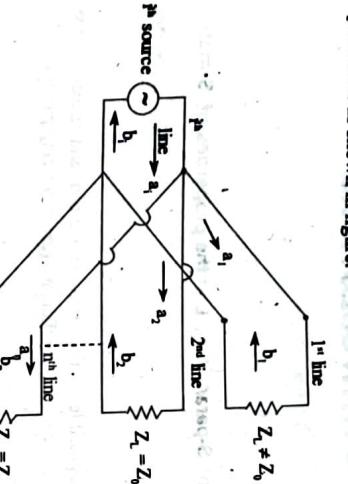


Figure: n-Port Microwave Device

' a_i ' is the incident power wave at the junction from i^{th} source, and b_i is the reflected power wave from i^{th} line (of load). Then, a_i and b_i (when the 1^{st} line is terminated to $Z_L \neq Z_0$ remaining all terminated to Z_0 characteristic impedance) are related as, $b_i = S_{11}a_i$ where, $i = 1, 2, 3, \dots, n$.

The value of b_i also depends on the a_2, a_3, \dots values (when all line loads are terminated by $Z_L \neq Z_0$). It is expressed as,

$$b_i = S_{11}a_i + S_{22}a_2 + \dots + S_{nn}a_n$$

Thus, for different values of i , n different equations are obtained as follows,

$$\begin{aligned} b_1 &= S_{11}a_1 + S_{12}a_2 + \dots + S_{1n}a_n \\ b_2 &= S_{21}a_1 + S_{22}a_2 + \dots + S_{2n}a_n \\ &\vdots \\ b_n &= S_{n1}a_1 + S_{n2}a_2 + \dots + S_{nn}a_n \end{aligned} \quad \dots (1)$$

All these equations are represented in matrix form as,

$$b_i = S_{ii}a_i + S_{i2}a_2 + \dots + S_{in}a_n$$

$$\begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & \dots & S_{1n} \\ S_{21} & S_{22} & \dots & S_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ S_{n1} & S_{n2} & \dots & S_{nn} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix} \quad \dots (2)$$

i.e., $[b] = [S][a]$

Here, ' $[S]$ ' is the scattering matrix and each of its element is called as scattering parameter that gives the power relations between the input and output ports.

The diagonal elements $S_{11}, S_{22}, S_{33}, \dots, S_{nn}$ are called reflection coefficients of that respective line. Equation (2) also interprets the contribution to the outward travelling wave. Thus, with the help of scattering matrix the behavioral characteristics of the device can be determined.

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Ans:

Dec.-19 (R16), Q1(b)

Unitary property states that the sum of products of each term in S -matrix of a lossless network with its conjugate is unity, i.e., $\sum_{n=1}^N S_{ni} S_{ni}^* = 1$

Where, N - Number of ports.

For a lossless network, the sum of power incident at input ports is equal to the total power reflected from output ports. It can be expressed as,

$$\sum_{n=1}^N |b_n|^2 = \sum_{n=1}^N |a_n|^2 \quad \dots (1)$$

Where,

a_n - Incident wave power

b_n - Reflected wave power.

Since, $[b] = [S][a]$

$$b_n = \sum_{i=1}^N S_{ni} a_i$$

On substituting the value of b_n in equation (1),

$$\sum_{n=1}^N \sum_{i=1}^N S_{ni} a_i^2 = \sum_{n=1}^N |a_n|^2 \quad \dots (2)$$

Assuming, only the 1^{st} port is excited and all remaining ports are terminated with matched loads. Then equation (2) becomes,

$$\sum_{n=1}^N |S_{11}a_1 + S_{21}a_2 + \dots + S_{n1}a_n|^2 = \sum_{n=1}^N |a_n|^2 \quad \dots (3)$$

Here, the incident power a_1 is zero for all ' n ' except for $n = i$. Which implies,

$$\sum_{n=1}^N |a_n|^2 = |a_i|^2$$

From equation (3),

$$\sum_{n=1}^N |S_{ni}|^2 |a_i|^2 = |a_i|^2$$

The structure of coupler determines the degree of coupling between port (1) and port (4) and between port (2) and port (3) respectively. When power is obtained at port (1) and port (4) respectively but no power is obtained at port (2) and port (3) because no coupling exists between port obtained from port (3) because no coupling exists between port (1) and port (3). Similarly, when the power is fed at port (2), the resulting power is obtained at port (1) and port (3) but, yields no power at port (4).

Applications of directional coupler includes,

1. Power monitoring
2. Source leveling
3. Isolation of signal sources
4. Swept transmission and reflection measurements.

UNIT-4 (Scattering Matrix and Microwave Measurements)

Q18. State and derive the unitary property of S matrix.

Dec.-19 (R16), Q1(b)

If the ports i and j are excited and remaining ports terminated. Then,

$$\sum_{n=1}^N S_{ni} S_{nj}^* = 0 ; \forall i \neq j$$

The above expression can be written in matrix representation as,

$$[S]^T [S]^T = [U]$$

Where,

$[U]$ - Unit matrix (or) identity matrix.

Q19. Explain the functional features of Directional coupler.

(or)

Discuss about directional coupler with neat diagram. (or)

Ans:

Directional couplers are the microwave devices that are specially designed for power measurements. These can be built by connecting two or more waveguides together.

Figure shows that a directional coupler is a four-port waveguide junction made up of two waveguides namely, main waveguide (1-2) and auxiliary waveguide (3-4). There is free transmission (without reflection) of power between the pair of port (1) and port (2) and no transmission of power between the pair of port (1) and (3) or ports (2) and (4) respectively, when all the ports are terminated with their characteristic impedances.

When an incident power, P_i is fed at the port (1) of two-hole directional coupler, received power P_r is obtained at port (2). The incident power coming out from holes A and B are out of phase by 180° , since the incident power travels a distance of $(\lambda/4 + \lambda/4)$ when it comes back from hole B. So, the incident powers cancel each other and produces a zero power ($P_r = 0$) at port (3). At hole B, both the incident powers are in phase so, they add up and produce a forward coupled power, P_f at port (4). The magnitude of the power coming out of port 2 depends upon the dimension of the two holes.

The degree of coupling is determined by the size and location of the holes in the waveguide walls. It is very difficult to produce high degree directivity over a band of frequencies.

Coupling Coefficient

The factor which is a measure of the ratio of power levels in main and auxiliary waveguides of a two-hole directional coupler (shown in figure) is known as coupling factor.

Mathematically,

$$\text{Coupling factor} = \frac{P_f}{P_r}$$

Coupling factor (DB) = $10 \log_{10} \left(\frac{P_f}{P_r} \right)$

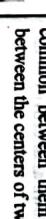
Ans:

Two-Hole Directional Coupler (Model Paper, Q1(a) | Nov-Dec-16, (R15), Q5(a))

A two-hole directional coupler with a travelling wave propagation consists of two waveguides, namely, main waveguide and auxiliary waveguide with two tiny holes common between them as shown in figure. The spacing between the centers of two tiny holes must be,

$$L = (2n+1) \frac{\lambda_x}{4} \quad \dots (1)$$

Where,



λ_x - Guide wavelength
Main waveguide
Port (1) Main
Port (2) Auxilliary
Port (3) Device
Port (4) Waveguide
Cancelled
Added
 $L = (2n+1) \lambda_x / 4$

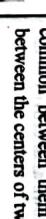
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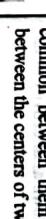
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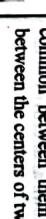
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$$L = (2n+1) \frac{\lambda_x}{4} \quad \dots (1)$$

Where,



λ_x - Guide wavelength
Main waveguide
Port (1) Main
Port (2) Auxilliary
Port (3) Device
Port (4) Waveguide
Cancelled
Added
 $L = (2n+1) \lambda_x / 4$

Where,

$$P_i - \text{Power input to port (1)}$$

$$P_o - \text{Power output from port (4)}$$

Coupling coefficient of an ideal two-hole directional coupler is in the range of 3 to 20 dB.

Directive

The factor which is a measure of how well the forward travelling wave in the main waveguide couples only to a specific port of the auxiliary waveguide, is known as directivity of the directional coupler shown in figure.

Mathematically,

$$\text{Directivity, } D = \frac{P_o}{P_i} \text{ and.}$$

$$\text{Directivity, } D(\text{dB}) = 10 \log_{10} \left(\frac{P_o}{P_i} \right)$$

Where,
 P_o - Power output from port (3)

Directive can also be written as,

$$D = 10 \log \left(\frac{P_o}{P_i} \right) + 10 \log \left(\frac{P_o}{P_p} \right)$$

$$\Rightarrow D = 10 \log \left(\frac{P_o}{P_i} \right) - 10 \log \left(\frac{P_p}{P_i} \right)$$

= Isolation (dB) - coupling factor (dB)

Since, Isolation (dB) = $10 \log \left(\frac{P_p}{P_i} \right)$

\therefore Directivity (dB) = Isolation (dB) - Coupling factor (dB)

For an ideal directional coupler, directivity must be infinity. The typical value of the directivity is only 30 to 35 dB.

Insertion Loss

The amount of signal attenuation in the main guide is called 'insertion loss'. It is given by the ratio of power incident to the power transmitted in the main arm.

$$\text{i.e., } I = \frac{\text{Power in port 4}}{\text{Power in Port 1}} \text{ (dB)}$$

The insertion loss of an ideal two-hole directional coupler is infinite.

Q21. Explain the principle of operation of Bethe hole directional coupler.

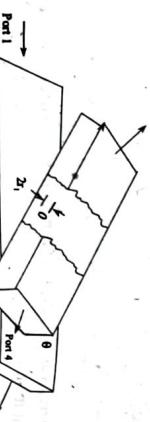
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Ans:

Bethe-hole coupler or waveguide coupler contains two coupled waveguides arranged through a single hole as shown in figure.

Bethe-hole Coupler

Bethe-hole coupler or waveguide coupler contains two coupled waveguides arranged through a single hole as shown in figure.

**Figure: Bethe-Hole Directional Coupler**

A hole is located either at the centre of common broad wall of inclined waveguides at an angle θ , or at an offset position a' of two parallel waveguides.

The following considerations should be taken when the aperture is smaller than the wavelength.

(i) It works as a electric dipole, which is perpendicular to the aperture, and having dipole moment directly proportional to the normal component of the electric field in the main guide.

(ii) It also works as a magnetic dipole in the plane of aperture, having dipole moment which is directly proportional to the tangential component of the exciting magnetic field at the aperture.

(iii) Due to radiations of electric and magnetic dipole, coupling is available to the auxiliary guide.

(iv) Magnetic dipole radiates asymmetrically in longitudinal directions, while electric dipole radiates equally and longitudinally in both the directions.

(v) The port 4 and port 3 powers can be controlled by adjusting the distance ' d' or by varying the angle θ between the two waveguides.

Centre Hole: Generally, $P_o \neq$ for centre coupling hole. Therefore, the coupling and directivity are defined as,

$$C = -20 \log \left[\frac{4}{3} \frac{\beta_0^3}{ab} \cos \theta + \frac{\lambda_0^2}{2\lambda_0^2} \right] \text{ dB}$$

Hence, there is no coupling between the ports (1), (3) and the ports (2), (4).

i.e., $S_{13} = S_{31} = 0$ and $S_{24} = S_{42} = 0$

\therefore Equation (1) can be written as,

$$C = -20 \log \left[\frac{4}{3} \frac{\beta_0^3}{ab} \cos \theta + S_{11} \right] \text{ dB}$$

Where,

r_0 - Radius of hole

$a \times b$ - Cross-sectional waveguide dimensions

$$\beta = \frac{2\pi}{\lambda_0} \sqrt{1 - \left(\frac{\lambda_0}{2a} \right)^2}$$

$$\lambda_0 = \frac{2\pi}{\beta} \sqrt{1 - \left(\frac{\lambda_0}{2a} \right)^2}$$

UNIT-4 (Scattering Matrix and Microwave Measurements)

Here, the offset value 'd' is given by,

$$\sin \left(\frac{\pi d}{a} \right) = \frac{\lambda_0}{\sqrt{2a}}$$

While, Coupling 'C' = $20 \log[1 + x^2/x]$

Where,

$$x = \left(\frac{16\pi r_0^3}{3ab\lambda_0} \right) \sin^2 \left(\frac{\pi d}{a} \right)$$

Q22. What are the characteristics of two hole direction coupler and derive the S-matrix of it.

(or)
Ans: Derive the S matrix of directional coupler and define all the parameters?

Refer Only S-matrix

Dec-19 (R15, Q10)

Characteristics of Two Hole Direction Coupler

For answer refer Unit-4, Q20. (Refer Only Coupling Coefficient and Directivity).

Derivation of S-matrix

The directional coupler contains four ports. So a 4×4 matrix is required to represent [S] matrix,

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix} \quad \dots (1)$$

i.e., $[S] =$

The four ports of a directional coupler are perfectly matched to the junction.

$\therefore S_{11} = S_{22} = S_{33} = S_{44} = 0$

From the symmetrical property, $S_{ij} = S_{ji}$

By using the above property, $S_{12} = S_{21}, S_{13} = S_{31}, S_{14} = S_{41}, S_{23} = S_{32}, S_{24} = S_{42}, S_{34} = S_{43}$

For ideal 3 dB directional coupler, the back power (P_b) is zero, i.e., $P_b = 0$

Hence, there is no coupling between the ports (1), (3) and the ports (2), (4).

i.e., $S_{13} = S_{31} = 0$ and $S_{24} = S_{42} = 0$

\therefore Equation (1) can be written as,

$$S_{12} = -S_{21} \quad [\because S_{12} = S_{21}]$$

$$S_{14} = -S_{41} \quad [\because S_{14} = S_{41}]$$

$$S_{23} = -S_{32} \quad [\because S_{23} = S_{32}]$$

$$S_{24} = -S_{42} \quad [\because S_{24} = S_{42}]$$

$$S_{34} = -S_{43} \quad [\because S_{34} = S_{43}]$$

$$S_{13} = -S_{31} \quad [\because S_{13} = S_{31}]$$

$$S_{21} = -S_{12} \quad [\because S_{21} = S_{12}]$$

$$S_{32} = -S_{23} \quad [\because S_{32} = S_{23}]$$

$$S_{43} = -S_{34} \quad [\because S_{43} = S_{34}]$$

$$S_{14} = -S_{41} \quad [\because S_{14} = S_{41}]$$

$$S_{24} = -S_{42} \quad [\because S_{24} = S_{42}]$$

$$S_{34} = -S_{43} \quad [\because S_{34} = S_{43}]$$

Using the relation $[S][S]^* = I$,

$$\begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{21} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{34} \\ S_{41} & 0 & S_{34} & 0 \end{bmatrix} \begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{21} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{34} \\ S_{41} & 0 & S_{34} & 0 \end{bmatrix} = \begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{21} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{34} \\ S_{41} & 0 & S_{34} & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{21} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{34} \\ S_{41} & 0 & S_{34} & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{21} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{34} \\ S_{41} & 0 & S_{34} & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{21} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{34} \\ S_{41} & 0 & S_{34} & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{21} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{34} \\ S_{41} & 0 & S_{34} & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{21} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{34} \\ S_{41} & 0 & S_{34} & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{21} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{34} \\ S_{41} & 0 & S_{34} & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{21} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{34} \\ S_{41} & 0 & S_{34} & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{21} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{34} \\ S_{41} & 0 & S_{34} & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{21} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{34} \\ S_{41} & 0 & S_{34} & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{21} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{34} \\ S_{41} & 0 & S_{34} & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{21} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{34} \\ S_{41} & 0 & S_{34} & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{21} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{34} \\ S_{41} & 0 & S_{34} & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{21} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{34} \\ S_{41} & 0 & S_{34} & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{21} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{34} \\ S_{41} & 0 & S_{34} & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{21} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{34} \\ S_{41} & 0 & S_{34} & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{21} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{34} \\ S_{41} & 0 & S_{34} & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{21} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{34} \\ S_{41} & 0 & S_{34} & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{21} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{34} \\ S_{41} & 0 & S_{34} & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{21} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{34} \\ S_{41} & 0 & S_{34} & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{21} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{34} \\ S_{41} & 0 & S_{34} & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{21} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{34} \\ S_{41} & 0 & S_{34} & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{21} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{34} \\ S_{41} & 0 & S_{34} & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{21} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{34} \\ S_{41} & 0 & S_{34} & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{21} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{34} \\ S_{41} & 0 & S_{34} & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{21} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{34} \\ S_{41} & 0 & S_{34} & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{21} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{34} \\ S_{41} & 0 & S_{34} & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{21} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{34} \\ S_{41} & 0 & S_{34} & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{21} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{34} \\ S_{41} & 0 & S_{34} & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{21} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{34} \\ S_{41} & 0 & S_{34} & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{21} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{34} \\ S_{41} & 0 & S_{34} & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{21} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{34} \\ S_{41} & 0 & S_{34} & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{21} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{34} \\ S_{41} & 0 & S_{34} & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{21} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{34} \\ S_{41} & 0 & S_{34} & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{21} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{34} \\ S_{41} & 0 & S_{34} & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{21} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{34} \\ S_{$$

- Q23.** A directional coupler is having coupling factor = 10 dB and directivity = 40 dB. Determine the power coupled in forward and reverse direction when input power is 10 W assuming the coupler is lossless.

Ans: Given that,

Given that,

For a directional coupler,

Input power, $P_f = 10 \text{ W}$

Coupling factor, $c = 10 \text{ dB}$

Directivity = 40 dB

Coupled port power, $P_b = ?$

Isolated port power, $P_i = ?$

The power through coupled and isolated ports are calculated by considering the expression of coupling factor and directivity.

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Given that,

Given that,

For a directional coupler,

Input power, $P_f = 10 \text{ W}$

Coupling factor, $c = 10 \text{ dB}$

Directivity = 40 dB

Coupled port power, $P_b = ?$

Isolated port power, $P_i = ?$

The power through coupled and isolated ports are calculated by considering the expression of coupling factor and directivity.

Ans: Given that,

Given that,

For a directional coupler,

Input power, $P_f = 10 \text{ W}$

Coupling factor, $c = 10 \text{ dB}$

Directivity = 40 dB

Coupled port power, $P_b = ?$

Isolated port power, $P_i = ?$

The power through coupled and isolated ports are calculated by considering the expression of coupling factor and directivity.

Given that,

Given that,

For a directional coupler,

Input power, $P_f = 10 \text{ W}$

Coupling factor, $c = 10 \text{ dB}$

Directivity = 40 dB

Coupled port power, $P_b = ?$

Isolated port power, $P_i = ?$

The power through coupled and isolated ports are calculated by considering the expression of coupling factor and directivity.

Given that,

Given that,

For a directional coupler,

Input power, $P_f = 10 \text{ W}$

Coupling factor, $c = 10 \text{ dB}$

Directivity = 40 dB

Coupled port power, $P_b = ?$

Isolated port power, $P_i = ?$

The power through coupled and isolated ports are calculated by considering the expression of coupling factor and directivity.

Given that,

Given that,

For a directional coupler,

Coupling coefficient, $C = 10 \text{ dB}$

Directivity, $D = 40 \text{ dB}$

Insertion loss, $I = 1 \text{ dB}$

Input power, $P_f = 10 \text{ mW}$

- Q25.** For a directional coupler, the incident power is 550 mW. Calculate the power in the main and auxiliary arm. The coupling factor is 30 dB.

Ans: Given that,

- Q26.** Incident power to a directional coupler is 80 watts. The directional coupler has coupling factor of 20 dB, directivity of 30 dB and insertion loss of 0.5 dB. Find the output power at,
- Main arm
 - Coupled
 - Isolated ports.

Ans: Given that,

- Q27.** For a directional coupler, the incident power is 550 mW. Calculate the power in the main and auxiliary arm. The coupling factor is 30 dB.

Ans: Given that,

4.1.3 [S] Matrix of Magic Tee and Circulator

Q27. Explain the S-matrix representation of a multiport microwave network and its significance.

Nov-Dec-16, (R15), Q4(b)

Derive the S-parameters for a two port microwave junction.

Ans:

The two-port microwave junction is as shown in figure

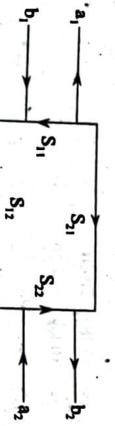


Figure (1)

The S-matrix for a two-port microwave junction is,

$$[S] = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$

$$\therefore \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \quad [\because [b] = [S][a]]$$

In a 2-port microwave junction, a total of four S-parameters are present, S_{11} , S_{12} , S_{21} and S_{22} .

S_{11} corresponds to reflection coefficient at port (1), when the input is applied at port (1), with matched terminated port (2) i.e., $a_2 = 0$.

S_{12} corresponds to transmission coefficient at port (2), when the input is applied at port (1).

S_{21} corresponds to reflection coefficient at port (1), when the input is applied at port (2).

S_{22} corresponds to reflection coefficient at port (2), when the input is applied at port (2), with matched terminated port (1), i.e., $a_1 = 0$.

Significance

For answer refer Unit-4, Q17, Topic: Significance.

Q28. Build the S-matrix of E-plane Tee junction.

Nov-Dec-12, (R09), Q4(c)

E-plane tee has three-ports. Therefore, S-matrix is a 3×3 matrix

$$\text{i.e., } [S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$

The output obtained at ports (1) and (2) is 180° out of phase, i.e., $S_{23} = -S_{13}$

And $S_{33} = 0$, if port (3) (i.e., input port) is perfectly matched to the junction.

WARNING: Xerox/Photocopying of this book is a CRIMINAL act. Anybody found guilty is LIABLE to face LEGAL proceedings.

By using symmetric property,

$$\begin{aligned} S_{12} &= S_{21} \\ S_{23} &= S_{32} \\ S_{13} &= S_{31} \\ \text{i.e., } [S] &= \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} \end{aligned}$$

By using unitary property,

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} \begin{bmatrix} S_{11}^* & S_{12}^* & S_{13}^* \\ S_{21}^* & S_{22}^* & S_{23}^* \\ S_{31}^* & S_{32}^* & S_{33}^* \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

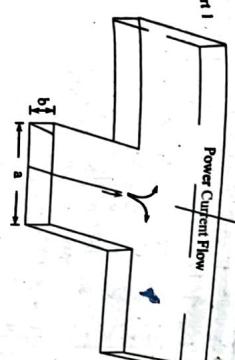
From the above matrix,

$$\begin{aligned} |S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 &= 1 \\ |S_{21}|^2 + |S_{22}|^2 + |S_{23}|^2 &= 1 \\ |S_{31}|^2 + |S_{32}|^2 + |S_{33}|^2 &= 1 \end{aligned} \quad \dots (1)$$

$$\begin{aligned} |S_{11}|^2 + |S_{21}|^2 + |S_{31}|^2 &= 1 \\ |S_{12}|^2 + |S_{22}|^2 + |S_{32}|^2 &= 1 \\ |S_{13}|^2 + |S_{23}|^2 + |S_{33}|^2 &= 1 \end{aligned} \quad \dots (2)$$

$$\begin{aligned} |S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 &= 1 \\ |S_{21}|^2 + |S_{22}|^2 + |S_{23}|^2 &= 1 \\ |S_{31}|^2 + |S_{32}|^2 + |S_{33}|^2 &= 1 \end{aligned} \quad \dots (3)$$

$$\begin{aligned} |S_{11}|^2 + |S_{21}|^2 + |S_{31}|^2 &= 1 \\ |S_{12}|^2 + |S_{22}|^2 + |S_{32}|^2 &= 1 \\ |S_{13}|^2 + |S_{23}|^2 + |S_{33}|^2 &= 1 \end{aligned} \quad \dots (4)$$



Figure

If the two waves are incident into port 1 and port 2, the addition of these two waves will be out through port 3.

The required S-matrix can be obtained by the following steps.

1. As H-plane has 3-ports, S-matrix is a 3×3 matrix.

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} \quad \dots (5)$$

Where,

$$S_i = \text{Scattering coefficient}$$

$$i = \text{Output port}$$

2. For the H-plane to be symmetric, scattering coefficients of S-matrix must satisfy the following condition.

$$S_{13} = S_{23} \quad \dots (6)$$

3. From the symmetric property,

$$S_{12} = S_{21}, S_{23} = S_{32}, S_{13} = S_{31}$$

4. $S_{33} = 0$, because the port-3 is terminated in matched load.

By applying all the above properties, [S] matrix can be written as,

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} \quad \dots (7)$$

5. From the unitary property,

$$\text{i.e., } [S][S^*] = 1$$

From the above matrix we get,

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} \begin{bmatrix} S_{11}^* & S_{12}^* & S_{13}^* \\ S_{21}^* & S_{22}^* & S_{23}^* \\ S_{31}^* & S_{32}^* & S_{33}^* \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

From the above matrix we get,

$$\begin{aligned} |S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 &= 1 \\ |S_{21}|^2 + |S_{22}|^2 + |S_{23}|^2 &= 1 \\ |S_{31}|^2 + |S_{32}|^2 + |S_{33}|^2 &= 1 \end{aligned} \quad \dots (1)$$

$$\begin{aligned} |S_{11}|^2 + |S_{21}|^2 + |S_{31}|^2 &= 1 \\ |S_{12}|^2 + |S_{22}|^2 + |S_{32}|^2 &= 1 \\ |S_{13}|^2 + |S_{23}|^2 + |S_{33}|^2 &= 1 \end{aligned} \quad \dots (2)$$

$$\begin{aligned} |S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 &= 1 \\ |S_{21}|^2 + |S_{22}|^2 + |S_{23}|^2 &= 1 \\ |S_{31}|^2 + |S_{32}|^2 + |S_{33}|^2 &= 1 \end{aligned} \quad \dots (3)$$

$$\begin{aligned} & \text{By solving equation (1) and (2), we get,} \\ & |S_{11}|^2 + |S_{21}|^2 + |S_{31}|^2 = 1 \\ & |S_{12}|^2 + |S_{22}|^2 + |S_{32}|^2 = 1 \\ & |S_{13}|^2 + |S_{23}|^2 + |S_{33}|^2 = 1 \\ & \therefore |S_{11}|^2 - |S_{22}|^2 = 0 \quad \dots (5) \end{aligned}$$

$$\begin{aligned} & \text{From equation (3),} \\ & |S_{11}|^2 + |S_{31}|^2 = 1 \\ & |S_{21}|^2 + |S_{31}|^2 = 1 \\ & |S_{11}|^2 = \frac{1}{2} \quad \dots (6) \end{aligned}$$

$$\begin{aligned} & \text{From equation (4) we get,} \\ & S_{13}(S_{11}^* + S_{21}^*) = 0 \\ & S_{13}(S_{11}^* - S_{21}^*) = 0 \\ & S_{13}^* \neq 0 \\ & \text{Hence, } S_{11}^* - S_{21}^* = 0 \\ & S_{11} = -S_{21} \quad \dots (7) \end{aligned}$$

$$\begin{aligned} & \text{By substituting equations (6) and (7) in equation (1) we get,} \\ & |S_{11}|^2 + |S_{21}|^2 + \left| \frac{1}{2} \right|^2 = 1 \\ & |S_{11}|^2 + |S_{21}|^2 + \frac{1}{4} = 1 \end{aligned}$$

$$\begin{aligned} & 2|S_{11}|^2 = 1 - \frac{1}{4} \\ & 2|S_{11}|^2 = \frac{3}{4} \\ & |S_{11}|^2 = \frac{3}{8} \quad \dots (8) \end{aligned}$$

$$\begin{aligned} & \text{From equations (5), (6) and (7) we get,} \\ & S_{12} = -\frac{1}{2} \\ & S_{22} = -\frac{1}{2} \\ & S_{33} = 0 \end{aligned}$$

$$\begin{aligned} & \text{By substituting all corresponding values in [S] matrix we get,} \\ & [S] = \begin{bmatrix} 1/2 & -1/2 & 1/\sqrt{2} \\ -1/2 & 1/2 & 1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \end{bmatrix} \quad \dots (9) \end{aligned}$$

Q30. Construct and explain about Magic Tee and calculate its S-parameters. Oct/Nov-20, (R13), Q1(a)

Ans: (or)
Find the S-matrix of a magic Tee.

What is magic tee? Derive the S-matrix for magic tee.

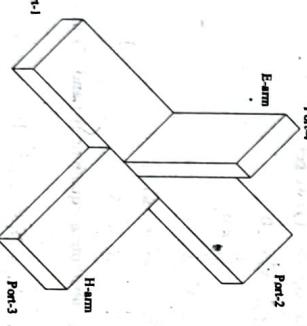
Magic tee is also known as hybrid tee or E-H plane tee.

It is used to obtain completely matched three port tee junction. Magic tee can be used to measure the impedance, as a duplexer and as a mixer.

S-matrix of Magic Tee

By using the properties of E-H plane tee, we can obtain the S-matrix of a magic tee.

Figure shows the E-H plane magic tee.



It has 4 ports, hence $[S] 4 \times 4$ matrix.

$$\text{i.e., } S = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix}$$

Port-3 has H-plane tee section. Hence,

$S_{23} = S_{13}$

Similarly port-4 has E-plane tee section,

$S_{34} = -S_{14}$

From figure, $S_{14} = S_{43} = 0$ [:: ports (3) and (4) are isolated]

By using the symmetrical property,

$$\text{i.e., } S_{ij} = S_{ji}$$

$$\Rightarrow S_{12} = S_{21}$$

$$\Rightarrow S_{13} = S_{31}$$

$$\Rightarrow S_{23} = S_{32}$$

$$\Rightarrow S_{34} = S_{43}$$

$$\Rightarrow S_{24} = S_{42}$$
 and

$$\Rightarrow S_{14} = S_{14}$$

UNIT-4 (Scattering Matrix and Microwave Measurements)

Q31. Explain the characteristics of a 3-port circulator Isolator? (or)

Ans: The most commonly used circulator is port-3 or port-4 circulator. But, a basic circulator can have any number of ports. Characteristic of circulator is that each arm is connected to its next arm only. That means, a signal transmitted from port (1) reaches port (2) and then port (3), (4).

S-matrix of a 3-port Circulator

According to Carine theorem, a 3-port circulator is a lossless, perfectly matched and non-reciprocal 3-port micro wave junction.

Q32. Find the S matrix of isolator. (or)

Derive the scattering matrix of an isolator.

The above matrix is called the S-matrix of a magic Tee.

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If the two ports are perfectly matched, then $S_{11} = S_{22} = 0$

From the property of isolator $S_{12} = 0$

Then, the S-matrix of an ideal isolator becomes as,

$$[S] = \begin{bmatrix} 0 & 0 \\ S_{21} & 0 \end{bmatrix}$$

$$[S_{21}]^2 = 1$$

Therefore, $S_{21} = 1$

Then, the S-matrix of an ideal isolator is becomes as,

$$S = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

Q33. A reciprocal two-port microwave device has a VSWR of 1.5 and an insertion loss of 2 dB. Find the magnitude of S-parameters of the device.

Ans: Given that,

For a reciprocal two port microwave device,

$$\text{VSWR} = 1.5$$

$$\text{Insertion loss, } I = 2 \text{ dB}$$

For reciprocal two port microwave device, S-parameters are related as,

$$S_{11} = S_{22} \text{ and}$$

$$S_{12} = S_{21}$$

The expression for insertion loss of microwave device in terms of S-parameters is given as,

$$I(\text{dB}) = -20 \log |S_{21}|$$

$$\Rightarrow |S_{21}| = 10^{-1(\text{dB}/20)}$$

$$\Rightarrow S_{21} = 10^{-2/20}$$

$$\Rightarrow S_{21} = 10^{-1/10} = 0.794$$

$$\therefore S_{21} = 0.79$$

The expression of S_{11} in terms of VSWR is given as,

$$S_{11} = S_{22} = \frac{\text{VSWR}-1}{\text{VSWR}+1}$$

$$\therefore S_{11} = \frac{1.5-1}{1.5+1} = 0.5$$

$$= \frac{1}{3} = 0.2$$

$$\boxed{S_{11} = 0.2}$$

S-matrix of given two port microwave device is given as,

$$S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} = \begin{bmatrix} 0.2 & 0.79 \\ 0.79 & 0.2 \end{bmatrix}$$

Isolation = $-20 \log |S_{12}|$

$$25 = -20 \log |S_{12}|$$

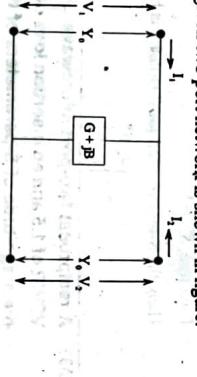
$$S_{12} = 10^{-2.5}$$

$$S_{12} = 0.0562 \text{ and}$$

$$S_{11} = S_{22} = 0$$

Figure

The given two port network is shown in figure.



Figure

From figure,

$$Y_0 = \frac{1}{G + jB}$$

The ABCD-parameters for the network shown in figure are obtained as,

$$S_{11} = \frac{V_1}{V_0} = 1$$

$$S_{12} = \frac{V_2}{V_0} = 0$$

$$S_{21} = \frac{V_1}{V_2} = 0$$

$$S_{22} = \frac{V_0}{V_2} = 0$$

Since $V_1 = V_2$

$$A = 1 \text{ and } B = 0$$

By solving the above equation for specific conditions,,

$$C = Y_0 = D = 1$$

Therefore,

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ Y_0 & 1 \end{bmatrix}$$

From the above parameters, scattering matrix can be obtained as,

$$S_{11} = \frac{\bar{A} + \bar{B} - \bar{C} - \bar{D}}{\bar{A} + \bar{B} + \bar{C} + \bar{D}}$$

$$S_{22} = \frac{-\bar{A} + \bar{B} - \bar{C} + \bar{D}}{\bar{A} + \bar{B} + \bar{C} + \bar{D}}$$

$$S_{12} = \frac{2(\bar{A}D - \bar{B}\bar{C})}{\bar{A} + \bar{B} + \bar{C} + \bar{D}}$$

$$S_{21} = \frac{2}{\bar{A} + \bar{B} + \bar{C} + \bar{D}}$$

Where,

$$\bar{A} = \frac{1}{Y_0 Z_{01}} \quad ; \quad \bar{B} = \frac{B}{Y_0 Z_{02}}$$

$$\bar{C} = C \sqrt{Z_{01} Z_{02}} \quad ; \quad \bar{D} = D \sqrt{\frac{Z_{01}}{Z_{02}}}$$

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In this case,

$$Z_{01} = \text{Characteristic impedance of input}$$

$$Z_{02} = \text{Characteristic impedance of output}$$

$$\bar{A} = A, \bar{B} = Y_0 B, \bar{C} = \frac{1}{Y_0} C, \bar{D} = D$$

Then,

$$S_{11} = \frac{A + Y_0 B + \frac{C}{Y_0} + D}{A + Y_0 B + \frac{C}{Y_0} + D}$$

$$= \frac{1+0-1+1}{1+0+1+1} = \frac{-1}{3} \quad \left(\because A = 1, B = 0 \right)$$

$$S_{22} = \frac{-A + Y_0 B - \frac{C}{Y_0} + D}{A + Y_0 B + \frac{C}{Y_0} + D} = \frac{-1+0-1+1}{1+0+1+1} = \frac{-1}{3}$$

$$S_{12} = \frac{2(A - B Y_0 \times \frac{C}{Y_0})}{A + Y_0 B + \frac{C}{Y_0} + D} = \frac{2(1-0)}{1+0+1+1} = \frac{2}{3}$$

$$S_{21} = \frac{2}{A + Y_0 B + \frac{C}{Y_0} + D} = \frac{2}{1+0+1+1} = \frac{2}{3}$$

\therefore Scattering coefficients, $[S] = \begin{bmatrix} 0 & 0.0562 \\ 0.0562 & 0 \end{bmatrix}$

Q36. A 20 mW signal is fed to one of the collinear port1 of a lossless H-plane Tee. Calculate the power delivered through each port when other ports are match terminated.

Ans:

Given that,

For a lossless H-plane Tee,

Power fed to the collinear port 1 = 20 mW

The other two ports are terminated with matched loads,

Power delivered to port-1 = ?

Power delivered to port-2 = ?

Power delivered to port-3 = ?

S-matrix for H-plane Tee is given as,

$$S = \begin{bmatrix} \frac{Y_1}{Y_0} & -\frac{Y_2}{Y_0} & \frac{Y_3}{Y_0} \\ -\frac{Y_2}{Y_0} & \frac{Y_1}{Y_0} & \frac{Y_3}{Y_0} \\ \frac{Y_3}{Y_0} & \frac{Y_3}{Y_0} & 0 \end{bmatrix}$$

But from the given data,

\Rightarrow Port-1 is fed with some power i.e., $\alpha_1 = 20 \text{ mW}$

\Rightarrow Port-2 and port-3 are terminated i.e., $\alpha_2 = \alpha_3 = 0$

Then, the power at ports are given as,

$$\Rightarrow \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} = \begin{bmatrix} \frac{Y_1}{Y_0} & -\frac{Y_2}{Y_0} & \frac{Y_3}{Y_0} \\ -\frac{Y_2}{Y_0} & \frac{Y_1}{Y_0} & \frac{Y_3}{Y_0} \\ \frac{Y_3}{Y_0} & \frac{Y_3}{Y_0} & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$\Rightarrow b_1 = \frac{1}{2} \alpha_1 - \frac{1}{2} \alpha_2 + \frac{\alpha_3}{\sqrt{2}}$

$\Rightarrow b_2 = -\frac{1}{2} \alpha_1 + \frac{1}{2} \alpha_2 + \frac{\alpha_3}{\sqrt{2}}$

$\Rightarrow b_3 = \frac{\alpha_1 + \alpha_2}{\sqrt{2}}$

$\Rightarrow b_3 = \frac{\alpha_1 + \alpha_2}{\sqrt{2}}$

Consider $b_1 = \frac{1}{2} \alpha_1 - \frac{1}{2} \alpha_2 + \frac{\alpha_3}{\sqrt{2}}$ then the power at port-1

is obtained as,
Insertion loss = $-20 \log |S_{11}|$
Scattering coefficients = ?
The expression for insertion loss of a matched isolator is given by,

0.5 = $-20 \log |S_{11}|$
 $\therefore S_{11} = 10^{-0.5}$

$\therefore S_{11} = 0.944$

tained as,

$$P_3 = \left(\frac{1}{\sqrt{2}} \right)^2 |\alpha_1| = \frac{1}{2} \times 20 = 10 \text{ mW}$$

\therefore Power at port-2 = 5 mW

Consider, $b_3 = \frac{\alpha_1}{\sqrt{2}} + \frac{\alpha_2}{\sqrt{2}}$ then the power at port-3 is ob-

tained as,
Consider, $b_3 = \frac{\alpha_1}{\sqrt{2}} + \frac{\alpha_2}{\sqrt{2}}$ then the power at port-3 is ob-

$$P_3 = \left(\frac{1}{\sqrt{2}} \right)^2 |\alpha_1| = \frac{1}{2} \times 20 = 10 \text{ mW}$$

\therefore Power at port-3 = 10 mW

Q37. For the given scattering parameters for a two-port network calculate the equivalent impedance parameters if the characteristics impedance is 50Ω .

Ans: Given that,

For a two-port network,

Scattering parameters, $S_{11} = 0.4 + j0.7$

$S_{12} = S_{21} = j0.6$

$S_{22} = 0.3 - j0.8$

Given that,

For a two-port network,

Scattering parameters, $S_{11} = 0.4 + j0.7$

$S_{12} = S_{21} = j0.6$

$S_{22} = 0.3 - j0.8$

Characteristic Impedance, $Z_0 = 50 \Omega$

Equivalent Impedance parameters, $Z_{11}, Z_{12}, Z_{21}, Z_{22} = ?$

Impedance parameters in terms of S parameters are obtained as follows,

$$Z_{11} = Z_0 \left| \frac{(1 + S_{11})(1 - S_{22}) + S_{12}S_{21}}{(1 - S_{11})(1 - S_{22}) - S_{12}S_{21}} \right|$$

$$= 50 \left| \frac{(1 + 0.4 + j0.7)(1 - 0.3 + j0.8) + (j0.6)(j0.6)}{(1 - 0.4 - j0.7)(1 - 0.3 + j0.8) - (j0.6)(j0.6)} \right|$$

$$= 50 \left| \frac{(1 + 4 + 0.7j)(0.7 + 0.8j) - 0.36}{(0.6 - 0.7j)(0.7 + 0.8j) + 0.36} \right|$$

$$= 50 \left| \frac{1.14 - 0.01j}{1.14 + 1.20j} \right|$$

$$= 1.75 + 60j$$

Consider $b_1 = \frac{1}{2} \alpha_1 - \frac{1}{2} \alpha_2 + \frac{\alpha_3}{\sqrt{2}}$ then the power at port-1

is obtained as,
Consider $b_1 = \frac{1}{2} \alpha_1 - \frac{1}{2} \alpha_2 + \frac{\alpha_3}{\sqrt{2}}$ then the power at port-1

$P_1 = \left(\frac{1}{2} \right)^2 |\alpha_1|$ [if $\alpha_2 = \alpha_3 = 0$]

\therefore Power at port-1 = 5 mW

The S-matrix for 3-port circulator is given by,

$$Z_{12} = Z_0 \left[\frac{2S_{12}}{(1-S_{11})(1-S_{22}) - S_{12}S_{21}} \right]$$

$$= 50 \left[\frac{2}{(1-0.4-j0.7)(1-0.3+j0.8) - (j0.6)(j0.6)} \right]$$

$$= 50 \left[\frac{1.2j}{1.34-0.01j} \right]$$

$$Z_{21} = Z_0 \left[\frac{2S_{21}}{(1-S_{11})(1-S_{22}) - S_{12}S_{21}} \right]$$

$$= 50 \left[\frac{2}{(1-0.4-j0.7)(1-0.3+j0.8) - (j0.6)(j0.6)} \right]$$

$$= 50 \times 2 \times j \frac{0.6}{1.34-0.01j} = -0.33 + 44.77j$$

$$Z_{21} = -0.33 + 44.77j$$

Similarly,
 $|S_{21}| = |S_{13}| = |S_{32}| = 0.944$

And Isolation = $-20 \log |S_{12}|$

$$20 = -20 \log |S_{12}|$$

$$S_{12} = 10^{-1}$$

$$= 50 \left[\frac{(1-0.4-j0.7)(1+0.3-j0.8) + (j0.6)(j0.6)}{(1-0.4-j0.7)(1-0.3+j0.8) - (j0.6)(j0.6)} \right]$$

$$= 50 \left[\frac{(0.6-0.7j)(1.3-0.8j) - 0.36}{(1.34-0.01j)} \right]$$

$$= 50 \left[\frac{0.22-1.39j-0.36}{1.34-0.01j} \right]$$

$$= 50 \left[\frac{-0.14-1.39j}{1.34-0.01j} \right]$$

$$= 50(-0.09-1.038j)$$

$$Z_{22} = -4.5 - 51.9j$$

$$\therefore Z_{22} = -4.5 - 51.9j$$

$$\Rightarrow Z = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$$

$$= \begin{bmatrix} 1.75 + 60j & -0.33 + 44.77j \\ -0.33 + 44.77j & -4.5 - 51.9j \end{bmatrix}$$

∴ Equivalent impedance parameters,
 $Z = \begin{bmatrix} 1.75 + 60j & -0.33 + 44.77j \\ -0.33 + 44.77j & -4.5 - 51.9j \end{bmatrix}$

Q38. Obtain the scattering matrix of a 3-port circulator.

Given insertion loss of 0.5 dB, Isolation of 20 dB and VSWR of 2.

Ans:
 Given that,
 For a 3-port circulator,
 Insertion loss, $J = 0.5$ dB
 Isolation = 20 dB
 VSWR = 2
 S matrix, $|S| = ?$

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} \quad \dots (1)$$

Since,

$$\text{Insertion loss} = -20 \log_{10} |S_{12}|$$

$$\Rightarrow 0.5 = -20 \log_{10} |S_{12}|$$

$$-0.025 = \log |S_{12}|$$

$$S_{12} = 10^{-0.025}$$

$$\therefore S_{12} = 0.944$$

Similarly,
 $|S_{21}| = |S_{13}| = |S_{32}| = 0.944$

And Isolation = $-20 \log |S_{12}|$

$$20 = -20 \log |S_{12}|$$

$$S_{12} = 10^{-1}$$

$$\therefore S_{12} = 0.1$$

Similarly,
 $|S_{11}| = |S_{33}| = |S_{22}| = 0.1$

$$P = \frac{S-1}{S+1} = \frac{2-1}{2+1}$$

$$\therefore P = 0.334$$

$$S_{11} = S_{22} = S_{33} = 0.334$$

On substituting corresponding values in equation (1), the scattering matrix as,

$$[S] = \begin{bmatrix} 0.334 & 0.1 & 0.944 \\ 0.944 & 0.334 & 0.1 \\ 0.1 & 0.944 & 0.334 \end{bmatrix}$$

Ans:

4.2 MICROWAVE MEASUREMENTS

4.2.1 Description of Microwave Bench

Different Blocks and Their Features, Errors and Precautions

Q39. Describe the blocks of microwave bench and their features.

(Model Paper, Q9(a) | Nov/Dec.-16, (R13), Q11(a))

(or)

Draw the block schematic of a typical microwave bench and explain the functionality of each component.

Ans:

Given that,

For a 3-port circulator,

Insertion loss, $J = 0.5$ dB

Isolation = 20 dB

VSWR = 2

S matrix, $|S| = ?$

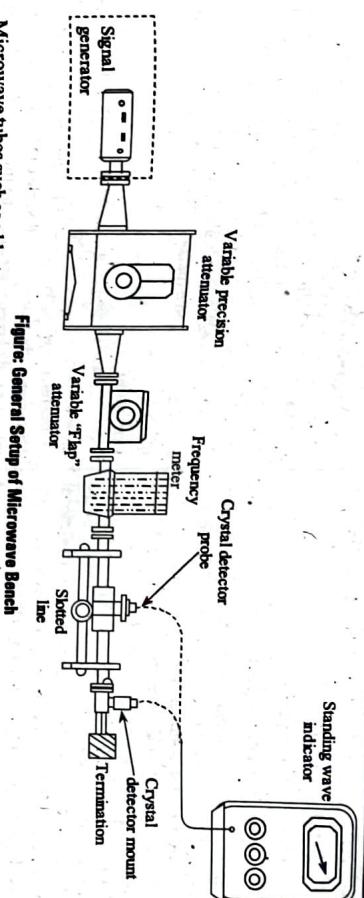
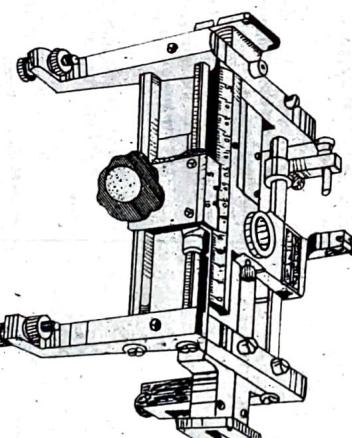


Figure: General Setup of Microwave Bench

Microwave tubes such as a klystron oscillator or a backward wave oscillator or a Gunn diode oscillator is used as a microwave signal generator. They generate low microwave power (about mW).

The output from the source is either a square wave or a continuous wave (CW) modulated at 1 kHz. The sweep oscillator can also be used as a source generator to generate periodic signals. The variable precision attenuator provides the attenuation ranging from 0 to 50 dB upon insertion loss. A variable flap attenuator is employed to check its readings with that of precision attenuator. The frequency wavemeter or simply a wavemeter with a movable cylindrical cavity gives the frequency calibration when it is tuned to resource condition. It is connected to a slotted line carriage which consists of a crystal detector. The modulated signals are detected by using crystal detector inserted in the E probe. The SWR indicator is a tunable voltmeter that reads the value of SWR in decibels and various types of terminators are used to terminate the end of the waveguide as indicated in the figure.

Precautions required in Microwave Measurements

Two basic requirements in microwave measurements using experimental setup are mentioned below.

(i) Due to improper matched terminations, reflections or standing waves occur in waveguide and the load admittance changes resulting variations in output power and resonant frequency of microwave source generator. These errors can be minimized by providing excess attenuation between the source frequency and the remaining experimental setup.

(ii) Deeper insertion of a coupling probe causes reflections in the wave guide and results change in the position of standing waves and hence SWR value. To obtain the desired output at the detector, the attenuation must not be so high. The reflection and wave discontinuities can be avoided by inserting the probe loosely and by maintaining full amplifier gain.

Q40. With a neat diagram, explain the construction of a slotted line.

The main function of slotted section is to detect the standing wave pattern present inside the waveguide. Figure shows the typical construction of slotted line.

Figure

Where,

 P_i - Input power
 P_o - Output power.

The slotted line mainly consists of three parts. They are,

1. Slotted section of a transmission line.
2. A travelling probe carriage.
3. A facility for attaching detecting instruments.

The broad face of the waveguide contains a slot at center which is parallel to the axis of the waveguide. The slot doesn't radiate any power for the dominant mode travelling inside the waveguide. The relative field strength of the standing wave pattern inside the waveguide is sensed or detected by a small probe, which is inserted through the slot. This probe is placed on a carriage plate, which moves on the top surface of the waveguide. The output from the crystal detector is proportional to the square of the input voltage due to the connection between probe and detector. An output proportional to the standing wave pattern inside the waveguide is obtained when the position of the probe is moved along the waveguide slot.

The square root of the ratio of maximum output to the minimum output gives VSWR, as the crystal is a square law device. A centimeter scale with vernier reading of 0.1 mm (minimum) is used for noting the positions. A low frequency modulating signal can be obtained by the slotted line carriage with a tunable detector on the oscilloscope. The loose coupling between the probe and the inner conductor of the line is used for measurement purpose. The characteristic impedances of both main line and slotted line are the same.

Q41. What is VSWR meter? How is it different from voltmeter? Explain its features.

Ans:

VSWR meter is a tunable voltmeter with high gain, high Q, low noise voltage amplifier operating at a fixed frequency of 1 kHz.

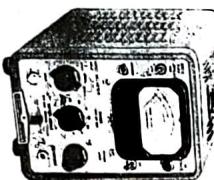


Figure illustrates microwave VSWR meter.

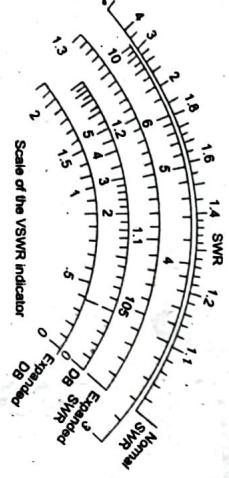


Figure: Microwave VSWR meter

The meter consists of three scales which indicates normal SWR, expanded SWR and dB scales. Besides these, a voltage scale with maximum reading of 10 mV is included. Usually, the total gain (about 125dB) is calibrated into 10 steps. The normal scale ranges from 1 to 4. The scale which is below normal scale reads the VSWR between 3 and 10. The calibration of expanded SWR scale is between 1 and 1.3 and is used for reading VSWR of value less than 1.3 with higher precision. dB scale is placed below along with an expanded dB (0 to 2dB) for measurement of VSWR in dB.

Operation

VSWR is defined as the ratio of maximum voltage to the minimum voltage. The input to the VSWR meter or tunable voltmeter is the signal detected by the microwave detector. Maximum deflection on VSWR scale can be achieved by tuning by using pad of probe carriage. The full scale deflection (FSD) indicates a VSWR of 1. Assuming that FSD corresponds to 10 mV with a VSWR of 1 and probe is tuned to obtain a deflection of 5 mV, then VSWR is measured as $\frac{10 \text{ mV}}{5 \text{ mV}} = 2$

The maximum VSWR that can be measured by using VSWR meter is 10. For $\text{VSWR} > 10$, slotted line technique is employed.

Applications of VSWR Meter

VSWR meter is used to measure the standing wave ratio in high frequency communication systems. Some of them include,

1. Wireless Communication Systems
2. Cellular Communications
3. High frequency aerial communications (HF/VHF/UHF)
4. Winmax Communications.

4.2.2 Measurement of Attenuation, Frequency, Standing Wave Measurements - Measurement of Low and High VSWR, Cavity Q

Q42. Explain the procedure for measuring attenuation with neat diagram.

Nov/Dec-17, (R13), Q11(b)

(or)
Explain how the microwave attenuation can be measured.

Nov-15, (R09), Q8(b)

(or)
How to measure an attenuation of a given microwave signal?

Nov-13, (R09), Q8(a)

Explain the RF substitution method of measurement of attenuation.

The ratio of input power to the output power is known as attenuation. Generally, it is expressed in decibels (dB).

$$\text{Attenuation (dB)} = 10 \log \frac{P_i}{P_o}$$

The commonly used method for the measurement of attenuation is RF substitution method. This method is particularly suitable for the networks with large attenuation and low input powers because the attenuation is measured at a single power position. Thus, the results obtained are accurate compared to the power ratio method. The output power of a microwave signal is estimated by placing a network whose attenuation has to be measured as shown in figure (1).

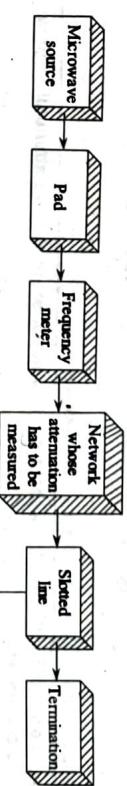


Figure (1): Experimental Setup to Measure Attenuation Before Substitution

Using the experimental setup shown in figure (1), power of the signal is obtained. In this case, the attenuation of the network is measured by directly reading value from precision attenuator.

Q43. Explain the power ratio method of measurement of attenuation.

Ans:

Power ratio method is the most simplest way of measuring attenuation. The experimental setup for the measurement of attenuation is as shown below.

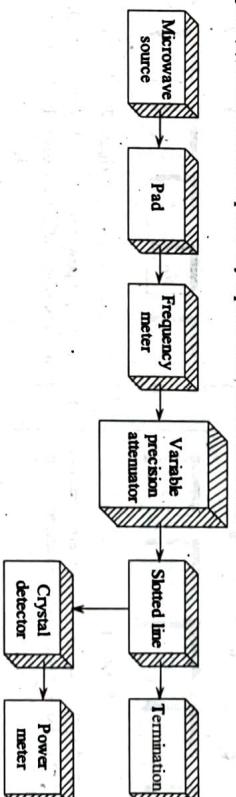


Figure : Experimental Setup for the Measurement of Attenuation Using Power Ratio Method

The measurement of attenuation is carried out in two steps.

1. The output power of a microwave signal is measured as P_1 using a slotted line carriage without attenuation device.
2. The output power of a microwave signal is obtained as P_2 using a slotted line carriage with an attenuation device between frequency meter and slotted line.

Consider, P_1 and P_2 are the output powers of a microwave signal measured using slotted line carriage without and with an attenuation device between frequency meter and slotted line respectively.

The ratio of power measured in each setup, i.e., P_1/P_2 , gives the attenuation in decibels.

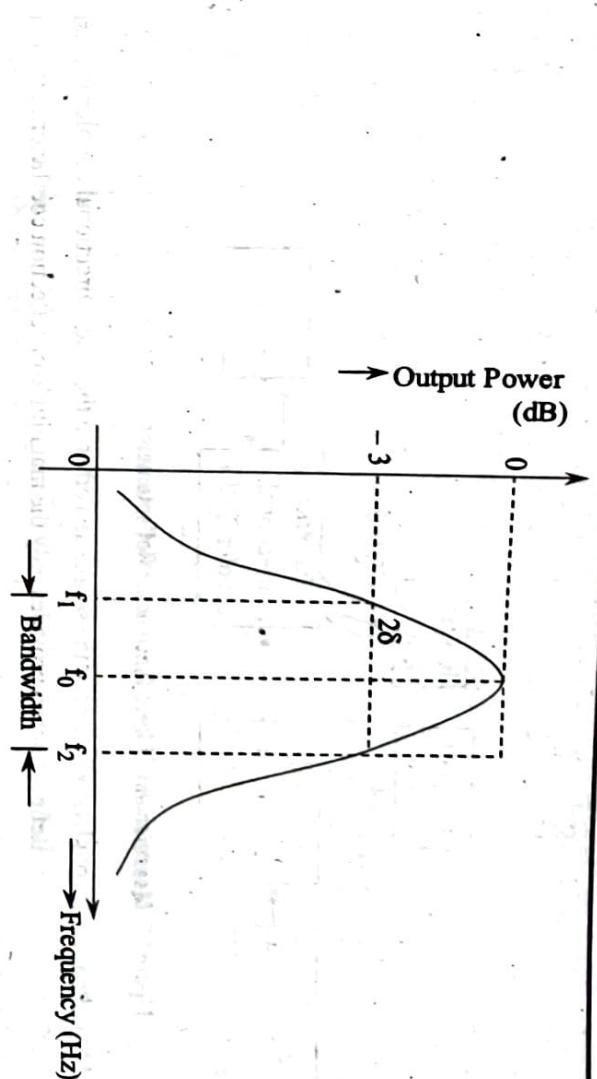


Figure (2)

By neglecting the coupling between microwave source and cavity and that of detector and cavity, $Q_L = Q_0$.

Disadvantage

In the case of very high Q systems, the accuracy of this method is poor due to narrow band of operation.

4.2.3 Impedance Measurements

Q55. Describe the measurement of impedance using slotted line, reflectometer and Smith chart.

Ans:

Measurement of Impedance using Slotted Line

Microwave test bench holds good for measurement for almost any microwave parameter is as shown in figure (1).

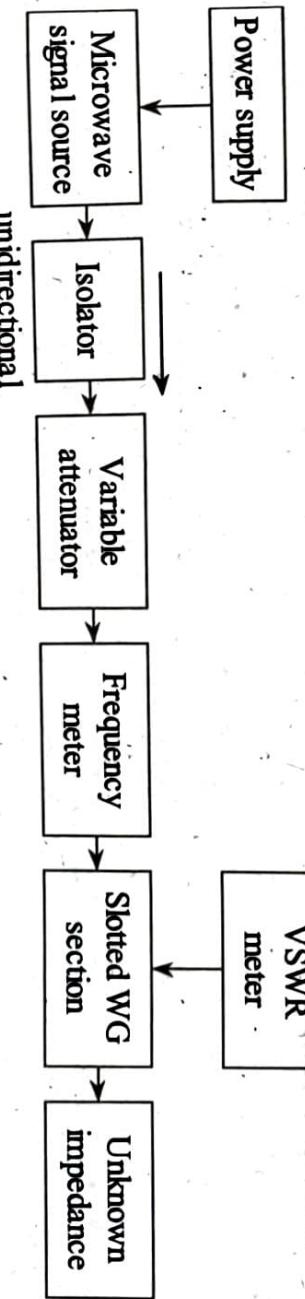


Figure (1)

An unknown impedance is connected to the load, to measure VSWR, wavelength and position of voltage at first minima (x_1). The unknown impedance and load at terminated output are then replaced by short circuit. The position of voltage at first minima (x_2) is now measured. The shift in minimum is measured as, $\frac{(x_2 - x_1)}{\lambda}$. On Smith chart a circle is plotted with center VSWR and the load as radius. The shift in minima on the circumference of the Smith chart is marked. The shift in minima point on the Smith chart is then corrected to the centre of the VSWR circle (i.e., one) by drawing the line. The point at which this line cuts the circle gives the value of unknown impedance as $R + jX$ or $R - jX$.

Measurement of Impedance using Reflectometer

Reflectometer is a microwave measurement device used to measure the impedance at microwave frequencies. The schematic diagram of measuring impedance using reflectometer is as shown in figure (2).

Frequently Asked & Important Questions

Q1. Define S-matrix and mention the use of scattering parameters in microwaves.

Ans: Refer Q1. (April/May-18, (R13), Q1(i) | Nov/Dec.-18, (R13), Q1(i) | Nov/Dec.-17, (R13), Q1(i) | Nov/Dec.-16, (R13), Q1(i))

Q2. Why Isolator is used in microwave measurements?

Ans: Refer Q10.

Q3. Explain the functional features of Directional coupler.

Ans: Refer Q19.

Q4. What are the characteristics of two hole direction coupler and derive the S-matrix of it.

Ans: Refer Q22.

Q5. Incident power to a directional coupler is 80 watts. The directional coupler has coupling factor of 20 dB, directivity of 30 dB and insertion loss of 0.5 dB. Find the output power at,

- (i) Main arm
- (ii) Coupled
- (iii) Isolated ports.

Ans: Refer Q26.

Q6. Derive the S-matrix for H-plane Tee.

Ans: Refer Q29.

Q7. Construct and explain about Magic Tee and calculate its S-parameters.

Ans: Refer Q30.

(Oct/Nov.-20, (R16), Q8(a) | April/May-18, (R13), Q10(a) | Dec.-14, (R09), Q4(a))

Q8. Describe the blocks of microwave bench and their features.

Ans: Refer Q39.

(Nov/Dec.-16, (R13), Q11(a) | Dec.-14, (R09), Q8(e))

Q9. Explain the procedure for measuring attenuation with neat diagram.

Ans: Refer Q42.

(Nov/Dec.-17, (R13), Q11(b) | Nov.-15, (R09), Q8(b) | Nov.-13, (R09), Q8(a))

Q10. How to find low and high VSWR of a given load at microwave frequencies? Explain.

(Dec.-19, (R16), Q11 | May/June -19, (R15), Q11 | April/May-18, (R13), Q10(b) | Nov/Dec.-17, (R13), Q10(b) | Nov.-15, (R09), Q8(a) | Nov.-13, (R09), Q8(b) | Nov/Dec.-17, (R13), Q11(a))

Q11. Give the measurement procedure for measuring Q factor of resonant cavity.

Ans: Refer Q54.

REPEATED	4
REPEATED TIMES	

REPEATED	2
REPEATED TIMES	

REPEATED	3
REPEATED TIMES	

REPEATED	2
REPEATED TIMES	

REPEATED	3
REPEATED TIMES	

Q12. What is a scattering matrix? Discuss the importance of S-parameters. List the properties of S-matrix.

Ans: Refer Q17.

Important Question

Q13. Explain the working of a two-hole directional coupler with a neat diagram and derive the expression for the coupling and directivity of a two-hole directional coupler.

Ans: Refer Q20.

Important Question

Q14. Explain the S-matrix representation of a multiport microwave network and its significance.

Ans: Refer Q27.

Important Question

Q15. Build the S-matrix of E-plane Tee junction.

Ans: Refer Q28.

Important Question

Q16. Find the S matrix of isolator.

Ans: Refer Q32.

Important Question

Q17. Find the S matrix for a matched isolator having an insertion loss of 0.5 dB and isolation of 25 dB.

Ans: Refer Q35.

Important Question

Q18. Describe how the frequency of a given microwave source can be measured using two different methods.

Ans: Refer Q46.

Important Question

Q19. Draw the block diagram of microwave bench setup and explain the slotted line method of frequency measurement.

Ans: Refer Q47.

Important Question

Q20. Describe the measurement of impedance using slotted line, reflectometer and Smith chart.

Ans: Refer Q55.

Important Question

Exercise Questions

1. In an H-plane Tee junction, 20 mW power is applied to port ③ that is perfectly matched to the junction. Calculate the power delivered to the load $60\ \Omega$ and $75\ \Omega$ connected to ports ① and ②. [Ans: 9.92 mW, 9.6 mW]

2. An isolator has an insertion loss of 0.5 dB and an isolation of 30 dB. Determine the scattering matrix of the isolator, if the isolated ports are perfectly matched to the junction. [Ans: $[S] = \begin{bmatrix} 0 & 10^{-1.5} \\ 10^{-0.5} & 0 \end{bmatrix}$]

3. Double minimum method is used to determine the VSWR value on a waveguide. If the separation between two nulls is 3.5 cm and that between twice minimum power points is 2.5 mm. Determine the value of VSWR. [Ans: 8.9126]

4. In a phase shift measurement setup, without the waveguide component (whose phase is to be measured) the guide wavelengths measured 7.2 cm and the reference null was at 10.5 cm. With the component, the reference null got shifted to 9.3 cm. Determine the phase shift of the component. [Ans: 60°]