Counter-diabatic driving using Floquet engineering

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1 CD driving

$$H_0 = -J \sum_j (c^\dagger_{j+1} c_j + \text{h.c}) + \sum_j V_j(\lambda) c^\dagger_j c_j \tag{1}$$

For this problem, approximate gauge potential is chosen to be $A^*_\lambda = i \sum_j \alpha_j (c^\dagger_{j+1} c_j - h.c)$.

On minimizing action, we get

$$-3J^2(\alpha_{j+1} + \alpha_{j-1}) + (6J^2 + (V_{j+1} - V_j)^2)\alpha_j = -J\partial_\lambda (V_{j+1} - V_j)$$

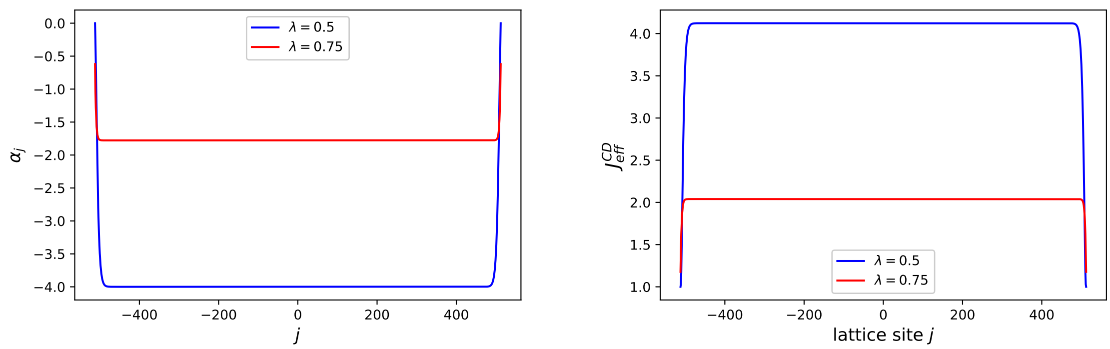

Figure 1: a) α_j for linear potential with vanishing boundary condition b) Effective hopping strength

$$H_{CD} = H_0 + \dot{\lambda} A_\lambda = \sum_j J^{CD}_j (c^\dagger_{i+1} c_i + h.c) + \sum_j U_j c^\dagger_i c_i$$

where

$$J^{CD}_j = J\sqrt{1 + (\dot{\lambda}\alpha_j/J)^2} \quad U_j = V_j(\lambda) - \sum_i^j \frac{J}{J^2 + (\dot{\lambda}\alpha_i/J)^2} (\ddot{\lambda}\alpha_j + \dot{\lambda}^2 \partial_\lambda \alpha_j)$$

Over here, I am going to work with $\dot{\lambda} = 1$ and $L = 1024$.

2 Floquet driving

$$H = H_0 + H_1 = J \sum_j (c^\dagger_{j+1} c_j + \text{h.c}) + \cos(\omega t) \sum_j A_j c^\dagger_j c_j$$

We would go to the rotating frame $|\psi_{rot}\rangle = V^\dagger|\psi_{lab}\rangle$ where $V = \exp(-i\sin(\omega t)/\omega \sum_j A_j c_j^\dagger c_j)$.

$$
\begin{aligned}
H_{rot} &= V^\dagger H V - iV^\dagger \dot{V} \\
&= V^\dagger H_0 V + \cos(\omega t)\sum_j A_j c_j^\dagger c_j + i^2\cos(\omega t)\sum_j A_j c_j^\dagger c_j \\
&= V^\dagger H_0 V = V^\dagger c_{j+1}^\dagger V V^\dagger c_j V + \text{h.c}
\end{aligned}
$$

Using $[n_j, c_j] = -c_j$ and $[n_j, c_j^\dagger] = c_j^\dagger$

$$
H_{rot} = J\sum_j (g^{j,j+1} c_{j+1}^\dagger c_j + \text{h.c}) \quad \text{where} \quad g^{j,j+1} = \exp\left(i\sin(\omega t)\frac{A_{j+1}-A_j}{\omega}\right)
$$

$$
\begin{aligned}
H_F^{(0)} &= \frac{1}{T}\sum_j \int_{t_0}^{T+t_0} (c_{j+1}^\dagger c_j \exp\left(i\sin(\omega t)\frac{A_{j+1}-A_j}{\omega}\right) dt + \text{h.c}) \\
&= \sum_j J_j^F (c_{j+1}^\dagger c_j + \text{h.c}) \quad \text{where } J_j^F = J^F \mathcal{J}_0\left(\frac{A_{j+1}-A_j}{\omega}\right)
\end{aligned}
$$

3 Linear potential

We choose $V(j,\lambda) = j\lambda$.

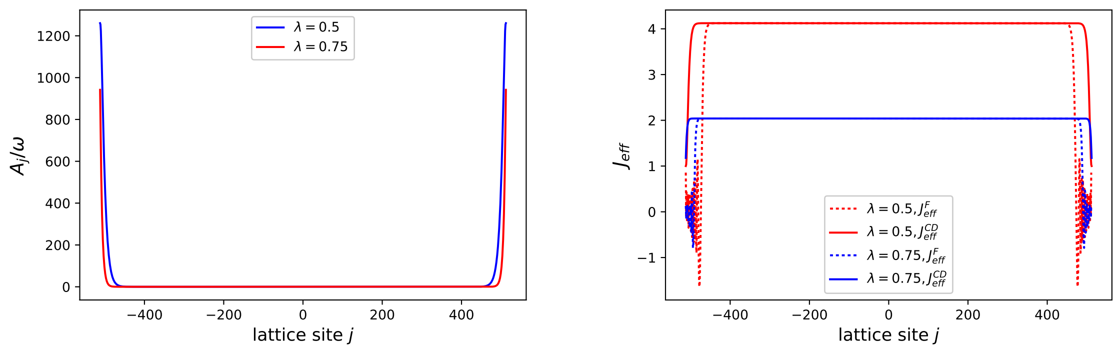

Figure 2: a) Driving field amplitude A_j b) Comparison of effective hopping strength from floquet and CD driving

4 Eckart potential

4.1 Inserting potential

$V(\lambda, j) = \dfrac{\lambda(t)}{\cosh^2 j/\xi}$ where ξ is the localization length.

4.2 Moving potential

$V(\lambda, j) = \dfrac{V_0}{\cosh^2[(j-\lambda)/\xi]}$ where ξ is the localization length. We will use $V_0 = 2J$. And $\partial_\lambda V = \dfrac{2V_0\sinh[(j-\lambda)/\xi]}{\xi\cosh^3[(j-\lambda)/\xi]} = \dfrac{2V_0\tanh[(j-\lambda)/\xi]}{\xi\cosh^2[(j-\lambda)/\xi]}$.

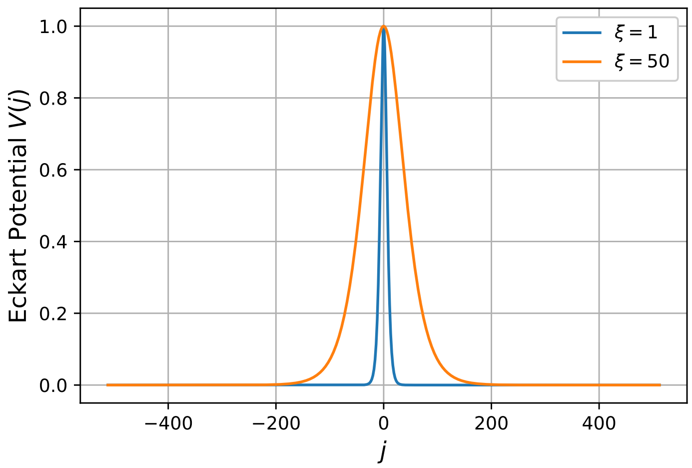

Figure 3: Eckart potential with $\lambda = 1$

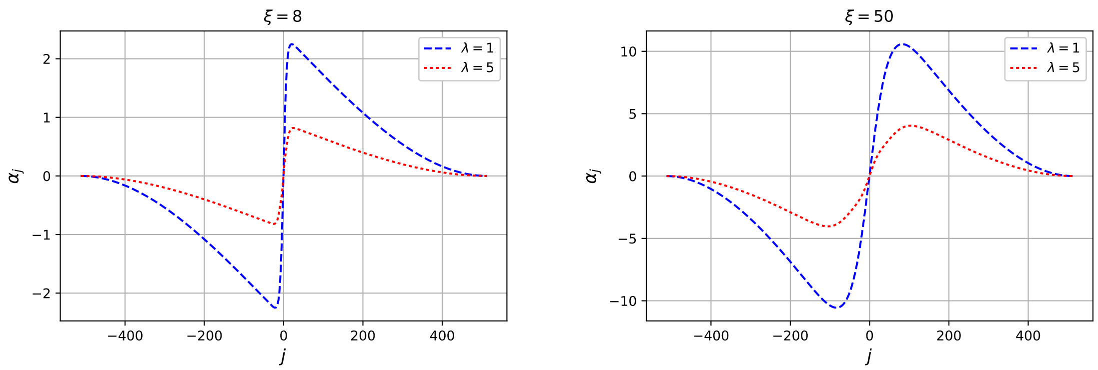

Figure 4: α_j for Eckart potential with vanishing boundary condition with a) $\xi = 8$ b) $\xi = 50$

I still don't know why my numerical simulation is not consistent with Dries's calculation.

A Magnus expansion

For a Hamiltonian which is periodic in time, it's unitary operator over a full driving cycle is given by:

$$U(T + t_0, t_0) = \mathcal{T}_t \exp(-\frac{i}{\hbar}\int_{t_0}^{T} dt H(t)) = \exp(-\frac{i}{\hbar}H_F[t_0]T) \tag{2}$$

$H_F[t_0] = \sum_n H_F^{(n)}[t_0]$ where

$$H_F^{(0)} = \frac{1}{T}\int_{t_0}^{T+t_0} H(t)dt$$

$$H_F^{(1)} = \frac{1}{2!Ti\hbar}\int_{t_0}^{T+t_0} dt_1 \int_{t_0}^{t_1} dt_2 [H(t_1), H(t_2)]$$

Hence,

$$|\psi(T)\rangle = U|\psi(0)\rangle$$

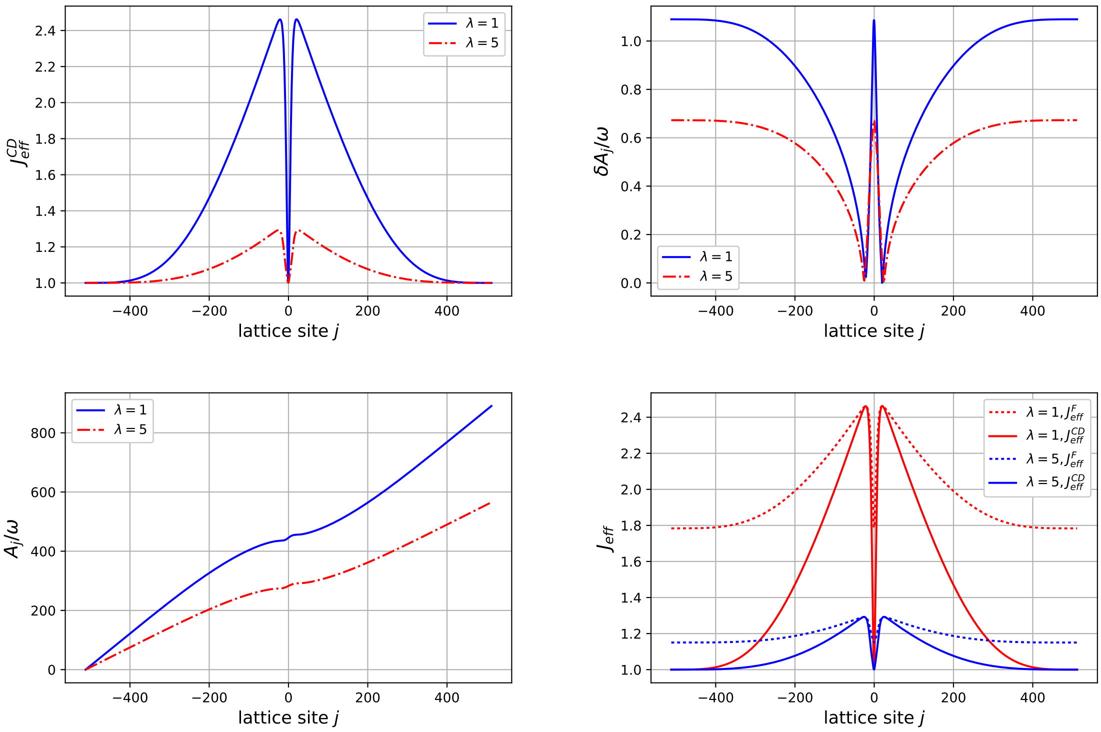

Figure 5: a) Effective hopping strength b) $(A_{j+1} - A_j)/\omega$ c)Driving field's amplitude A_j/ω d) Comparison of effective hopping strength from floquet and CD driving

$$= \exp(-\frac{i}{\hbar} H_F T)|\psi(0)\rangle$$
$$= \lim_{\omega\to\infty} \exp(-\frac{i}{\hbar} H_F^{(0)} T)|\psi(0)\rangle$$

B Numerics of a single body problem

Consider the Hamiltonian operator $\mathbf{H}$ on lattice

$$\mathbf{H} = \sum_n V_n |n\rangle\langle n| + \sum_n (u_{n,n+1}|n\rangle\langle n+1| + u^*_{n,n+1}|n+1\rangle\langle n|) \tag{3}$$

In units of $\hbar = 1$, time-evolution is given by

$$\mathbf{H}|\Psi\rangle = i\frac{d}{dt}|\Psi\rangle \tag{4}$$

We choose $|\Psi\rangle = \sum_n \psi_n |n\rangle$, where ψ_n is the probability amplitude for the quantum particle on n-th lattice site. Hence, we find time-evolution of ψ_n is given by:

$$i\frac{d\psi_n}{dt} = u_{n,n+1}\psi_{n+1} + u^*_{n-1,n}\psi_{n-1} + V_n\psi_n \tag{5}$$

With this, we have converted the problem of solving SE into a problem of solving an ODE.

For us, $u_{j,j+1} = \exp\left(i\sin(\omega t)\frac{A_{j+1} - A_j}{\omega}\right)$ as we are interested in studying the dynamics of this Hamiltonian:

$$H = J\sum_{j=0}^{L-1}(u^{j,j+1}c^\dagger_{j+1}c_j + \text{h.c})$$

where periodic boundary condition is assumed. Let's suppose $A_j = j$ where j goes from 0 to $L-2$ and $A_{j=L-1} = 0$ so that $A_{j+1} - A_j = 1$ for all values of $j = \{0, L-1\}$ [1]. For a lattice-size of $L = 51$, I did numerical simulation with initial condition as $|\psi(t=0)\rangle = \delta_{i,(L-1)/2}$.

$$|\psi(t=T)_{num}\rangle = U|\psi(0)\rangle = \exp(-\frac{i}{\hbar}HT)|\psi(0)\rangle \qquad |\psi_F(T)\rangle = U_F|\psi(0)\rangle \simeq \exp(-\frac{i}{\hbar}H_F^{(0)}T)|\psi(0)\rangle$$

where $H_F^{(0)} = \sum_{j=0}^{L-1} J_j^F(c_{j+1}^\dagger c_j + \text{h.c})$ with $J_j^F = J^F \mathcal{J}_0\left(\frac{A_{j+1}-A_j}{\omega}\right)$

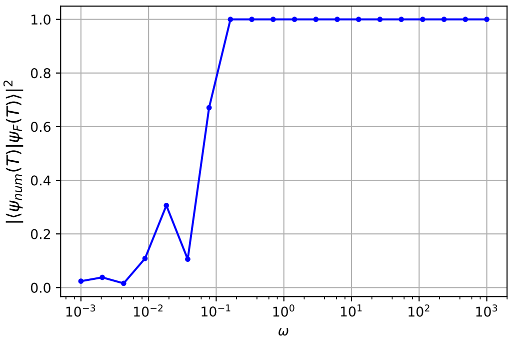

Figure 6: $\psi_{num}(T)$ is the wavefunction obtained after solving numerically and $\psi_F(T)$ is the wevfunction-obtained using zeroth term of Magnus expansion

C Bessel's function of first kind

Integral representation of Bessel's function of first kind $\mathcal{J}_n(x)$ is given by:

$$\mathcal{J}_n(x) = \frac{1}{2\pi}\int_{-\pi}^{\pi} d\tau e^{i(n\tau - x\sin\tau)} = \frac{1}{T}\int_{-T/2}^{T/2} d\tau e^{i(n\omega\tau - x\sin\omega\tau)} \tag{6}$$

For $x \ll 1$, $\mathcal{J}_0(x) = 1 - \frac{x^2}{2}$

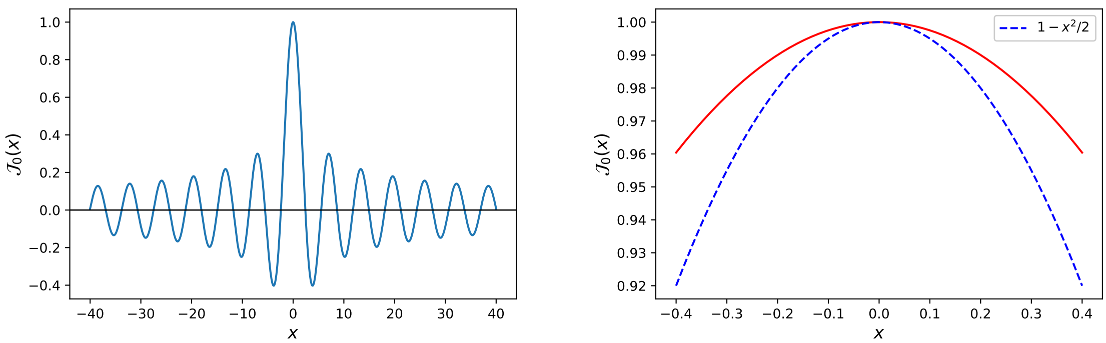

Figure 7: Bessel's function

[1] We should be careful with the boundary terms. For $j = 0, L-1$, $A_1 - A_0 = 1$. But $A_L - A_{L-1} = A_0 - (L-1) = 1 - L$