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PUBLISHED ONLINE: 1 JUNE 2015 | DOI: 10.1038/NPHYS3344
nature physics

Nonlocal adiabatic response of a localized system to local manipulations

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We examine the response of a system localized by disorder to a time-dependent local perturbation that varies smoothly with a characteristic timescale τ. We find that such a perturbation induces a nonlocal response, involving a rearrangement of conserved quantities over a length scale $\sim \ln \tau$. This effect lies beyond linear response, is absent in undisordered insulators and highlights the remarkable subtlety of localized phases. The effect is common to both single-particle and many-body localized phases. Our results have implications for numerous fields, including topological quantum computation in quantum Hall systems, quantum control in disordered environments, and time-dependent localized systems. For example, they indicate that attempts to braid quasiparticles in quantum Hall systems or Majorana nanowires will not succeed if the manipulations are performed asymptotically slowly, and thus using such platforms for topological quantum computation will require considerable engineering. They also establish that disorder-localized insulators suffer from a statistical orthogonality catastrophe.

The study of localization in isolated disordered systems has a rich history dating back to the seminal work of Anderson[1]. Whereas it is well known that strong enough disorder exponentially localizes single-particle wavefunctions, the fate of interacting many-body systems in disordered landscapes remains a long-standing problem[2]. Recent progress[3–5] on the phenomenon of many-body localization (MBL) has led to an intense revival of interest in this subject[6–9]—for a review, see ref. 10. MBL phases have a rich complex of properties including vanishing long wavelength conductivities at finite temperatures[3] and an extensive number of local conserved quantities[11,12] leading to a breakdown of ergodicity. Spectral functions of local operators that show a 'mobility' gap at all temperatures[13,14]. Strikingly, highly excited MBL eigenstates can exhibit localization protected order (LPQO)—both Landau symmetry-breaking and topological order—in dimensions and at energy densities normally forbidden by the Peierls–Mermin–Wagner theorem[15–22].

MBL systems present the tantalizing possibility of using localization to protect high-temperature quantum computation. Localized systems might serve as protected quantum memories because they undergo only slow (logarithmic) dephasing, and even this can be removed by spin echo procedures[13,19,23,24]. Prima facie, one expects to be able to locally manipulate degrees of freedom in such systems without affecting distant q-bits, a property with various quantum control applications. Further, the property of LPQO above raises the interesting possibility of performing topological quantum computation at finite temperatures in the MBL regime by braiding excitations in topologically ordered MBL eigenstates. Although there is no energy gap at finite temperatures, the 'mobility gap' could serve to protect adiabatic braiding instead.

All these applications require local manipulation of quantum degrees of freedom. Motivated by these considerations, we study the adiabatic response of localized phases to local perturbations using a combination of analytic arguments and numerical exact diagonalization. In particular, we study the adiabatic time evolution of a system governed by the time-dependent Hamiltonian

$$H(t) = H_{\mathrm{L}} + V(t/\tau)$$

where H_{L} is a localized Hamiltonian and V is a time-dependent local perturbation which acts only on a small compact subregion in real space, and which is zero in the distant past and future ($t \to \pm\infty$). Finally, τ sets the timescale on which the perturbation changes. In this work, adiabatic time evolution will be understood to mean

$$|\psi(t)\rangle = \lim_{L\to\infty} \lim_{\tau\to\infty} U(t/\tau)|\psi(0)\rangle$$

where $U(t)$ is the unitary time-evolution operator defined by $H(t)$, and the limit $\tau \to \infty$ is taken before the thermodynamic limit $L \to \infty$. We also discuss the opposite order of limits.

A naive understanding of localization suggests that the influence of the perturbation $V(t)$ should be spatially confined to within a localization length ξ of the region in space where V acts. The recent discovery[25] of logarithmic dephasing and entanglement spreading in interacting, localized systems revises this understanding, but nonetheless leaves in place the intuition that conserved charges, such as number and energy, should not move over distances greater than the localization length. We will show that this understanding needs to be further revised.

Our main results are as follows: first, a local perturbation remarkably induces a highly nonlocal adiabatic charge response in distant parts of the system. For an infinitely slow perturbation, $\tau \to \infty$, there is a 'zone of disturbance' where charge rearrangement occurs over a length scale that diverges linearly with system size. For finite τ, charge transfer takes place over length scales $\sim \log(\tau)$. See Fig. 1. This effect is distinct from the logarithmic entanglement growth, as the charge spreading occurs even in the non-interacting problem where there is no entanglement spreading. Second, this effect cannot be captured by linear-response theory, and revises our understanding of susceptibility and transport in localized phases. Our results also modify our understanding of MBL in time-dependent systems[26,27]. And third, there is a statistical Anderson orthogonality catastrophe[28] for both ground and highly excited states in strongly localized systems, contrary to established wisdom for ground states[29]. Importantly, our work places strong constraints on possibilities for quantum control and topological quantum computation in disordered systems, as we will discuss below. We note that

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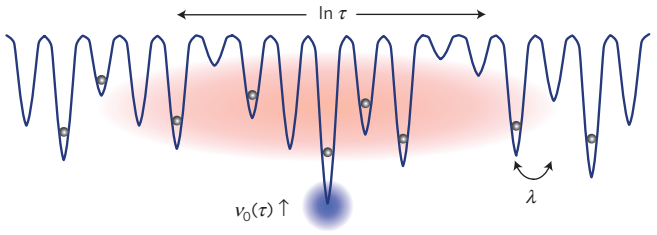

Figure 1 | Schematic illustration of our protocol. A local time-dependent potential $v_0(\tau)$ leads to a highly nonlocal adiabatic charge response in disordered insulators, causing a 'zone of disturbance' with radius $\sim \ln \tau$.

there are parallels to our discussion of local manipulations of disordered systems in the field of optics[30,31]. We emphasize that although our work is motivated by many-body localization, our results apply also to Anderson localized systems, such as systems on a quantum Hall plateau.

Anderson insulator

We start with a disordered single-particle (SP) Anderson insulator in 1D with a time-dependent local potential where most of our results can be described in a transparent setting. Generalization to higher dimensions is straightforward. Many-body (MB) eigenstates are constructed by simply filling the SP levels. To characterize the nonlocal charge response of MB eigenstates, define the adiabatic change in the charge density as:

$$\delta\rho_{\rm ad}(x) = \sum_{\alpha\,{\rm occ}} |\psi_\alpha(x, t=\infty)|^2 - |\psi_\alpha(x, t=-\infty)|^2 \qquad (1)$$

where $\psi_\alpha(x,t)$ is the α-lowest instantaneous SP eigenstate of $H(t)$, and the sum is over occupied SP states in a given MB eigenstate. This expression can be applied generally to many-body states of both non-interacting fermions and hard-core bosons where all SP states can be occupied by at most one particle. Figure 2a shows $\delta\rho^{\rm ad}$ for the MB ground state and an excited state (drawn randomly from the infinite temperature Gibbs ensemble) in a given disorder realization of the Anderson insulator; both show a long-distance rearrangement of charge. We emphasize that this transfer is mediated by the action of a strictly local potential in an insulator! More precisely, define

$$r_{\rm ZD}^2 = \frac{\int_{-L/2}^{L/2} {\rm d}x\, x^2\, \delta\rho_{\rm ad}^2}{\int_{-L/2}^{L/2} {\rm d}x\, \delta\rho_{\rm ad}^2} \qquad (2)$$

where L is the system size and $r_{\rm ZD}$ quantifies the radius of the zone of disturbance over which charge rearrangement takes place. It would be natural to expect $r_{\rm ZD}$ to scale as the localization length ξ. Instead, we find that the disorder-averaged radius diverges linearly with system size, $\sqrt{\overline{r_{\rm ZD}^2}} \sim L$, that is, the zone of disturbance grows without bound in the adiabatic limit. Figure 2b shows the disorder-averaged scaling of $\overline{r_{\rm ZD}^2}$ with system size for both the MB ground state and $T=\infty$ excited states.

To understand these results, let us turn to the specific fermionic Hamiltonian in which our computations were performed:

$$\begin{aligned} H(t) &= H_{\rm hopping} + V_{\rm loc}(t/\tau) \\ &= \sum_{i=-L/2+1}^{L/2} -\lambda(c_{i-1}^\dagger c_i + c_i^\dagger c_{i-1}) + v_i c_i^\dagger c_i + v_0(t/\tau) c_0^\dagger c_0 \end{aligned} \qquad (3)$$

where $c/c^\dagger$ are fermionic annihilation/creation operators, λ is the nearest-neighbour hopping strength, the on-site potentials v_i are drawn uniformly from $[-W, W]$ and $v_0(t)$, the potential on site 0 is changed with time. We now focus on the strong-disorder limit, $\lambda/W \ll 1$, where the localization length $\xi \sim 1$ is on the scale of a lattice constant and an especially simple picture emerges. Denote the eigenstate with localization centre at site r as $|\psi\rangle_r$. As $v_0(t)$ is varied in time, the eigenenergy of $|\psi\rangle_0$ is affected most strongly. To leading order, as $v_0(t)$ sweeps the range from $-W$ to W, $|\psi\rangle_0(t)$ comes into resonance with each of the other eigenstates, giving rise to a set of avoided crossings with gaps that scale as $\lambda \exp(-R\ln(W/\lambda))$, the effective coupling between $|\psi\rangle_0$ and $|\psi\rangle_{\pm R}$. Thus, the smallest gaps (due to a resonance between $|\psi\rangle_0$ and $|\psi\rangle_{O(N)}$) scale exponentially with system size ($\sim\lambda^N$) even though the system is non-interacting, a fact previously discussed by Altshuler *et al.*[32] in the context of the global adiabatic optimization approach to quantum computation. Figure 3 shows the evolution of the spectrum for a given disorder realization in a ten-site chain with $W=1$. We note that such resonances are also present in an unperturbed Anderson insulator with exponentially small probability; however, the local drive ensures that they occur with probability one.

The many-body ground state of fixed number, say $m=N/2$, is constructed by filling the m lowest single-particle states. Thus, if the system evolves adiabatically, a purely local perturbation on site 0 leads to a transfer of charge a distance R_m away, where $|\psi\rangle_{R_m}$ is the mth lowest eigenstate in the distant future! See Fig. 3. The value of R_m differs between disorder realizations, but can take any value from 1 to $N/2$ with equal probability in a system with uniform disorder strength. Thus, the disorder-averaged response to the local perturbation has a very wide spatial distribution, and shows no decay on scales longer than $\xi_{\rm loc}$. When v_0 sweeps only a finite fraction f of the bandwidth ($\sim W$), distant charge transfer in the ground state happens with probability f, occurring only if an occupied state is swept through an avoided crossing with an unoccupied state. The disorder-averaged response still shows no decay. For highly excited MB states, the adiabatic response leads to a multi-particle charge rearrangement in a diverging zone of disturbance, as shown in Fig. 2b.

Having characterized the spatial spread of the adiabatic response, we now turn to the ramp time τ needed for adiabatic time evolution. In particular, we want to know whether τ is set by the exponentially small avoided-crossing gaps or by an $O(1)$ mobility gap[13].

For the nth SP eigenstate of $H(-\infty)$ to remain the nth instantaneous eigenstate of $H(t)$, the adiabaticity condition

$$a_{mn}(t) = \hbar \frac{\langle \psi_m(t) \left| \frac{\partial V(t/\tau)}{\partial t} \right| \psi_n(t) \rangle}{(E_m(t) - E_n(t))^2} \ll 1$$

must be satisfied at all times for all $m \neq n$, where the eigenstates are defined by $H(t)\psi_\alpha(t) = E_\alpha(t)\psi_\alpha(t)$. For a local $V(t/\tau)$, one might expect the numerator of a_{mn} to be significant only when the $\psi_{m,n}$ are centred within a few localization lengths of each other and the potential; however, states within a localization volume in space are separated in energy by the mobility gap, giving a large denominator. Thus, naively $a_{mn} \ll 1$ so long as $\hbar/\tau$ is smaller than the mobility gap.

This reasoning fails at the avoided crossings in our locally perturbed system. At an avoided crossing at time t between eigenstates $|\psi(t)\rangle_0$ and $|\psi(t)\rangle_R$ the energy gap is exponentially small in R, whereas the instantaneous eigenstates look like the symmetric and antisymmetric combinations: $|\psi_{m,n}(t)\rangle \sim |\psi(t)\rangle_0 \pm |\psi(t)\rangle_R(t)$. As $V(t)$ is also localized near site 0, the numerator of $a_{mn}(t)$ receives a substantial contribution from the diagonal piece ${}_0\langle\psi(t)|\dot{V}|\psi(t)\rangle_0$. Thus, the system remains adiabatic only if

$$\tau_{\rm ad} \gg \frac{\hbar W^{2(R-1)} \partial_t V}{\lambda^{2R}}$$

that is, the mobility gap does not protect adiabaticity. Thus, in a system of size L, the drive is adiabatic for all levels only if the ramp is exponentially slow in the system size, even for a single-particle Anderson insulator.

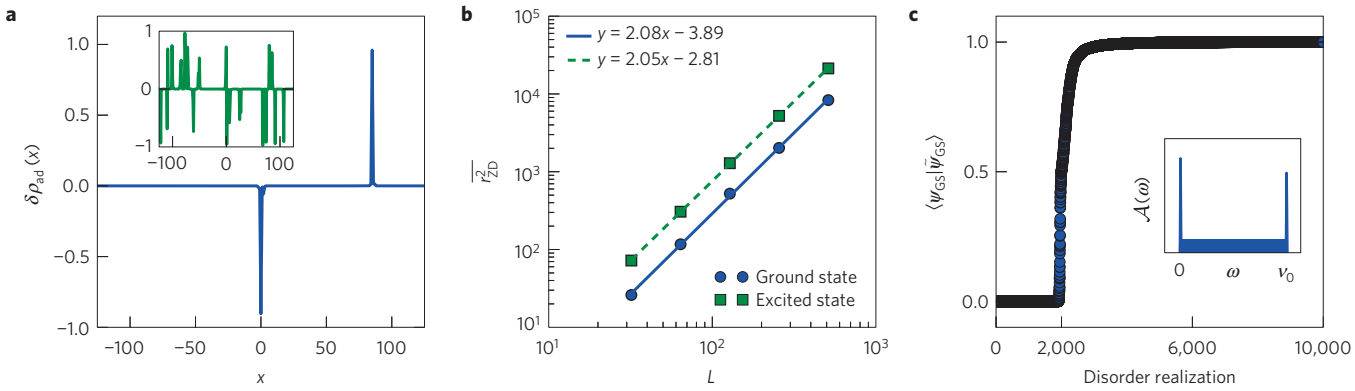

Figure 2 | Charge transfer and the orthogonality catastrophe in an Anderson insulator. a, Adiabatic change in the ground-state charge density (equation (1)) in a given disorder realization in a 250-site Anderson model (equation (3)) subject to a repulsive time-dependent potential on site 0 with $\lambda/W=0.1$. Charge is expelled from site 0 and transferred to a distant location (near site 75). Inset: same for an excited state in the middle of the spectrum (colloquially, a $T=\infty$ excited state) showing a multi-particle rearrangement over a large 'zone of disturbance'. **b**, Scaling of the radius of zone of disturbance (equation (2)) with system size for the ground state (blue circles) and $T=\infty$ excited states (green squares) averaged over 10^4 disorder realizations, showing a linear scaling $r_{ZD}\sim L$ in both cases. **c**, Overlaps of the MB ground states in the presence ($|\tilde{\psi}_{GS}\rangle$) and absence ($|\psi_{GS}\rangle$) of a local potential of strength $v_0=0.4$ at site 0 sorted over 10^4 disorder realizations with $f=v_0/2W=0.2$. The sorted values show a statistical orthogonality catastrophe with probability $f=0.2$. Inset: sketch of the disorder-averaged spectral function (equation (4)) for Anderson insulators. The spectrum is pure point, and the diffuse, non-zero strength between $(0, v_0)$ is a signature of the long-distance charge rearrangement.

The preceding discussion also implies that with a finite ramp time τ, the system is only able to adiabatically avoid level crossings with gaps $>\hbar/\tau$. As the charge transfer is a consequence of avoided crossings and because the gaps decay exponentially with distance $W\exp(-R/\xi)$ (in the strong localization regime), with a finite ramp time τ, charge transfer occurs over a characteristic length scale

$$r_{ZD}\sim\xi\ln\left(\frac{\tau W}{\hbar}\right)\sim\ln(\tau)$$

This logarithmic transfer of charge is our key result. We predict a similar logarithmic spreading in the weak localization regime, on distances larger than the localization length.

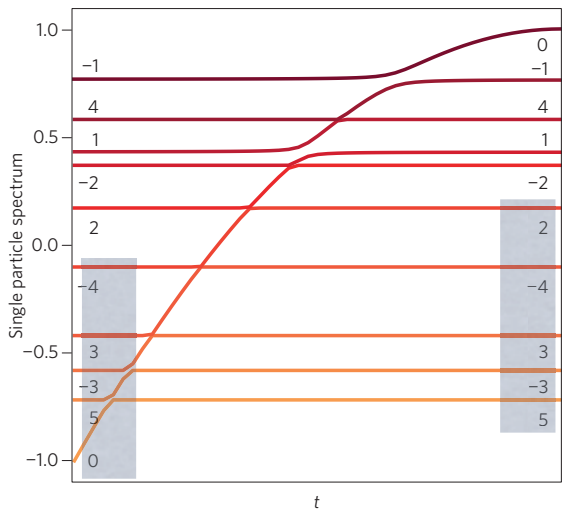

Figure 3 | Single-particle spectrum of a ten-site Anderson insulator, equation (3), in a given disorder realization as a function of time. The numbers denote the localization centres of the corresponding eigenstates. The changing potential on site 0 brings $|\psi\rangle_0(t)$ into resonance with the other eigenstates and leads to a set of avoided crossings. The MB ground states at half filling (shaded levels occupied) in the distant past and future are related through the transfer of charge from site 0 to site $R_m=2$.

Orthogonality catastrophe. This nonlocal charge response implies a statistical Anderson orthogonality catastrophe (OC) in the Anderson insulator. Anderson's original work had shown that the many-body ground states of a clean (metallic) system of fermions in the presence and absence of a local impurity potential were orthogonal in the thermodynamic limit, even for arbitrarily weak (but finite) potentials. In the strongly disordered system under study, adding an on-site potential on site 0 of strength $v_0=2Wf$ with $f<1$ leads to a distant charge transfer and hence an orthogonal new ground state with probability f. Figure 2c shows the ground state overlaps with and without a potential, clearly showing an orthogonality with probability f (roughly when the starting potential on site 0 lies within v_0 of the Fermi energy). For highly excited states, we have a catastrophe with probability one. Previous work[29] on the OC in ground states of strongly disordered systems captured only the non-orthogonal overlaps that occur with probability $1-f$ to incorrectly conclude that strongly disordered insulators do not suffer from the OC.

The OC has important consequences for several dynamical phenomena in metals. Famously, it predicts an X-ray edge singularity[33], where the low-energy X-ray absorption spectrum in a metal has the singular form $\mathcal{A}(\omega)\sim w^{-1+2\eta}$ and η is derived from the Anderson OC. The primary spectral function characterizing local quantum quenches (such as a change in the potential) takes the form:

$$\mathcal{A}(\omega)=\sum_n|\langle n|\mathrm{GS}\rangle|^2\delta(\omega-E_n+E_{GS}+\omega_0)\qquad(4)$$

where $|\mathrm{GS}\rangle$ is the ground state of the system before the quench, and $|n\rangle$, E_n are the eigenstates and eigenvalues of the final Hamiltonian. For the Anderson insulator in the $\lambda/W\ll1$ limit, $A(\omega)$ looks pure point, and is characterized by delta function peaks located between $\omega\in(0,v_0)$ for different disorder realizations with a catastrophe. In disorder realizations where there is no catastrophe and no long-distance charge transfer (probability $1-f$), $\mathcal{A}(\omega)$ has a peak at either $\omega=0$ or $\omega=v_0$. The inset in Fig. 2c shows a representative sketch of the disorder-averaged spectral function—the non-zero weight between $(0,v_0)$ distinguishes the Anderson insulator response from that of ordinary band insulators and is a signature of long-distance charge rearrangement.

Failure of linear response. Before leaving the Anderson insulator it is instructive to compare our description of the adiabatic response to a local perturbation with the more standard account of such a perturbation in linear-response (LR) theory in the $\omega \to 0$ limit. This response is governed by the density susceptibility, which has been calculated using LR, for example, in the classic work by Vollhardt and Wölfle[34] and the LR answer is local on the scale of the localization length. As the standard computations are approximations carried out with disorder-averaged Green's functions and done at fixed chemical potential, it is useful to revisit this question more carefully. For a single disorder realization the density susceptibility is given by a Kubo formula which involves matrix elements of the perturbation between the exact unperturbed eigenstates. We can estimate the density susceptibility in LR by a generalization of the arguments used in the derivation of the celebrated Mott formula for the a.c. conductivity of an Anderson insulator, $\sigma \sim \omega^2 \ln^{d+1} \omega$ (ref. 35) at small ω. Essentially, we translate those arguments from the global dissipative response for the conductivity to the local reactive response for the charge susceptibility. Accordingly, we estimate that the charge rearrangements to distance $r \gg \xi$ away from our chosen site are dominated by unperturbed states in which the site is resonant with another at that distance. As such a state is present with probability exponentially small in r/ξ, the disordered-averaged linear response will indeed decay exponentially on the scale of ξ. This has been verified by numerical computations, as shown in Fig. 4a. The same figure also shows the exact response averaged over the same disorder realizations and it is clear that it is exponentially larger for $r \gg \xi$. This contrast vividly illustrates our central observation that although the states with long-range resonances that contribute to the Mott/LR result are exponentially rare, even a local drive, surprisingly, creates resonances with high probability. For concreteness, we note that the linear-response result in Fig. 4 is obtained by evaluating equation (1) with the single-particle eigenstates perturbed to first order in the local potential, whereas the exact result uses the exact eigenstates in the presence of the potential.

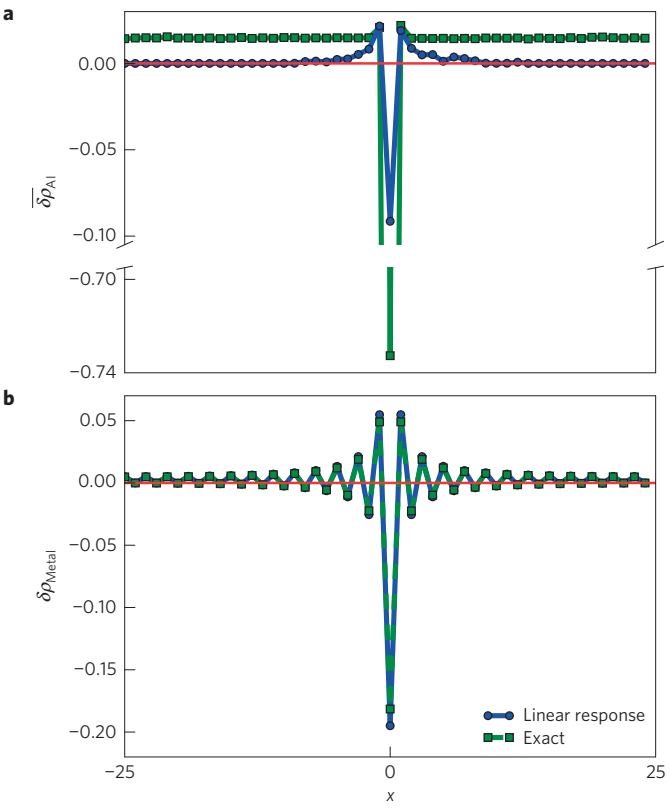

Figure 4 | Exact (green, squares) and linear-response (blue, circle) answers for the ground-state charge density difference (equation (1)), $\delta\rho(r)$ for a metal and an Anderson insulator. a,**b**, The system size is $L = 50$ and a perturbing repulsive potential of strength $v_0 = 0.4$ has been added to the centre of the system. **a**, In an Anderson insulator with $\lambda/W = 0.1$ averaged over 10^5 disorder realizations, whereas $\delta\rho_{\mathrm{LR}}(r)$ rapidly decays away from the location of the potential, the exact $\delta\rho(r)$ shows a uniform response everywhere (with amplitude scaling as $1/L$). **b**, In a metal ($\lambda = 1, W = 0$), the linear-response charge response closely captures the exact answer.

Contrast with clean insulators and with metals

At this point it instructive to contrast the behaviour we have found for the Anderson insulator with that of undisordered insulators (band and Mott) and that of metals. In an undisordered insulator the particles are again localized with a length scale that can be read off from correlation functions and which will scale inversely with the gap. Now the response to an adiabatically prepared local potential is indeed localized with this localization length, there is no orthogonality catastrophe, and the adiabatic response is accurately captured by linear-response/perturbation theory. The case of metals—ballistic and diffusive—is intermediate. In a metal, charge can flow to infinity and thus the adiabatic charge transfer is not restricted to the vicinity of the applied perturbation, there is—famously—an orthogonality catastrophe with a scaling with system size that is modified in the diffusive case, and the adiabatic response is accurately captured by linear-response/perturbation theory, as illustrated in 4b. As a function of the timescale we can be more specific. In both ballistic and diffusive metals we will obtain a power-law spreading of charge $R_\tau \sim \tau^\sigma$ with $\sigma = 1, 1/2$ respectively. Indeed, charge can continue to flow even long after the Hamiltonian stops changing ($t \gg \tau$), allowing the effects of a local perturbation to propagate out to infinity. This is in contrast to both undisordered insulators and Anderson localized systems, where charge transfer occurs only when the Hamiltonian is changing, and thus the influence of the perturbation is restricted to a finite region of space (with linear size τ^0 or $\ln \tau$ respectively). Moreover, the smallest gaps in non-interacting metals scale only polynomially with the system and thus an adiabatic response can be achieved by much faster ramp rates τ as compared to localized systems. Finally, it is interesting to note that in clean insulators and metals the adiabatic limit considered in this paper yields the same charge response as the opposite limit in which $L \to \infty$ before $\tau \to \infty$. In the Anderson insulator the latter limit fails to exist in the mathematical sense that the charge response does not converge to a fixed answer as τ is increased in an infinite system but instead continues to evolve erratically as increasingly small gaps become relevant.

Generalization to MBL

We now generalize our analysis to fully MBL interacting localized systems. Our principal results for the Anderson insulator carry over. MBL systems also exhibit a zone of disturbance that grows as $\ln(\tau)$, a statistical orthogonality catastrophe for the ground state and a certain orthogonality catastrophe for highly excited states, and a failure of linear response to agree with this behaviour. There are three new features that come into play. First, we can consider systems that lack a parent single-particle description as they lack a conserved number and the nonlocal response now involves a rearrangement in the energy density alone. Second, the rearrangement process for highly excited states now exhibits a range of length scales from the ubiquitous $\ln(\tau)$ to the shorter, but still divergent, $(\ln \tau)^{1/(d+1)}$ at which much more comprehensive changes take place in the structure of the ground state. Third, the termination of the perturbation is now followed by the entanglement spreading

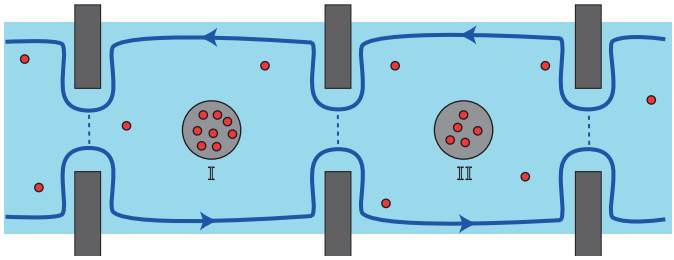

Figure 5 | Schematic illustration of a proposal for topological quantum computation outlined in ref. 48. Regions I and III contain the intended non-Abelian quasiparticles whose joint state is measured by interferometric tunnelling experiments of quasiparticles across the constrictions. In realistic experiments, there will be unintended quasiparticles in the shaded region outside of I and III which will rearrange over long distances in response to the changing potential on the constriction, thereby spoiling the braiding experiment.

discussed in ref. 25. We note that the entanglement spreading is the dominant effect in the recent work on quantum revivals[24] which considers sudden quenches and thus works in the opposite limit from the one considered here. These results can be derived within the 'l-bits' formalism introduced in refs 11,12 and the reader is referred to the Supplementary Methods for more details.

Discussion and ramifications

We conclude by discussing the implications of our work for experiments, other aspects of the physics of localized systems and for quantum engineering. Starting with experiments, it would be gratifying to directly observe the zone of disturbance created in response to a local perturbation and it seems to us that the cold atomic systems which have exhibited Anderson[36–39] and, apparently, MBL (ref. 40) are the best places to look.

Alternatively, experiments could look for the predicted form of the X-ray absorption spectrum sketched in Fig. 2c. Apart from solids hosting disordered electron gases[41], cold atomic gases[42] are again plausible systems to observe this effect.

The tuned resonance behaviour that underlies our chief results can, in principle, be produced in other ways—for example, by sandwiching a localized system between two conducting leads and tuning the chemical potential in the conducting regions. Indeed, this is the well-known setting of the Lifshitz–Azbel[43–45] resonances in Anderson localized systems and a natural extension is to look for generalizations of these to MBL systems (work in progress). Quasiperiodic systems, with[46] and without[47] interactions, are known to exhibit localized states and are natural for studying our results in a setting without disorder (work in progress).

Another problem for which our results have consequences is that of Floquet localization in MBL systems. It has been argued[26,27] that MBL systems subject to a periodic local driving do not absorb energy indefinitely. In particular, the eigenstates of the Floquet operator for such systems are expected to be MBL. Here our results predict that slow, low-frequency local drives (or a slow perturbation of the amplitude of a fast drive) will give rise to a diverging 'zone of disturbance' in the Floquet eigenstates and lead to a transfer of energy deep into the system.

We now turn to the implications of our work for quantum control, engineering and computation, where it might often be necessary to perform local manipulations in disordered environments while leaving distant regions untouched. At the broadest level, our results imply that such control will be problematic if we attempt to carry out such manipulations arbitrarily gently/slowly, as one might wish to for a theoretical analysis of devices. We emphasize that such adiabaticity is implicit in thinking of ideal control by means of gates, for example, or even of small excitation currents which imply slow changes of various potentials.

These results for quantum control have important consequences for topological quantum computation in experimentally realizable samples, including the Anderson localized ground states that underlie quantum Hall plateaux. For concreteness, let us analyse a well-known proposal to use quantum Hall systems as platforms for topological quantum computation by creating and braiding localized excitations. This problem, together with some other related proposals, is discussed in more detail in the Supplementary Methods. In the very simplest setting shown in Fig. 5, taken from ref. 48, a qubit is created from two localized non-Abelian quasiparticles located in regions I and III. Their boundary is defined by two constrictions used interferometrically to detect the joint state of the quasiparticles particles, and hence of the qubit. For our purposes it is enough to focus on the third constriction—separating I and III—which is turned on and off to tunnel a quasiparticle between the edges and thus flip the qubit. This constriction will arise from an electrostatic potential with a dipolar shadow leaking into regions I and III. In a completely ideal device with no localized quasiparticles apart from the ones created by the experimenter, this time-dependent potential in regions I and III will have no effect as long as it is not too big in magnitude. However, for most realistic devices, and all the ones that exhibit a quantum Hall plateaux before patterning, there will be additional localized quasiparticles which will be subjected to this potential and can then rearrange in response. If these localized 'background' quasiparticles move across the braiding path as a result of the nonlocal response, the resulting braid will be different from the intended braid, causing the computational step to fail especially if the gate is operated arbitrarily slowly. Thus, there is no safe asymptotic limit, and braiding in these devices will require considerable engineering of various timescales. Similar problems will bedevil attempts to perform topological quantum computation in Majorana nanowire networks[49,50], in the presence of localized gapless Majorana excitations and quasiparticles, which will probably also be present in realistic samples, as we discuss in the Supplementary Methods.

Finally, the discussion above unfortunately has a negative implication for MBL physics. As outlined in ref. 15, highly excited MBL eigenstates with localized quasiparticles can still support topological order. This observation has led to the recent, novel possibility of performing high-temperature topological quantum computation using MBL systems. Although still nascent, this exciting idea has attracted a lot of attention. The discussion in the preceding paragraph directly generalizes to braiding in MBL systems because the localized background quasiparticles can again respond to the slow braid and spoil the computation; thus our work places strong limits on the implementations of this idea and will inform future experiments and engineering in this field. Further, our results here imply that because one cannot define an adiabatic process with a well-defined Berry phase, quasiparticle statistics are, as such, ill-defined in the localized setting. Instead the topological information in the excited MBL states must be reconstructed from other data, as we will discuss elsewhere (work in progress).

Altogether, our results place natural limits on the manipulation of local degrees of freedom in localized phases and help further elucidate the remarkably subtle nature of localization.

Received 8 October 2014; accepted 22 April 2015;
published online 1 June 2015

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Acknowledgements

We acknowledge useful conversations with B. L. Altshuler, P. W. Anderson, J. E. Avron, R. Bhatt, A. Elgart, M. S. Rudner and, especially, J. Chalker. We thank J. Chalker and D. Huse for comments on a draft. This work was supported by NSF Grant Numbers DMR 1006608, 1311781 and PHY-1005429 and the John Templeton Foundation (V.K. and S.L.S.) and by a PCTS fellowship (R.N.).

Author contributions

All authors contributed extensively to the work presented in the paper and the writing of the manuscript.

Additional information

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Competing financial interests

The authors declare no competing financial interests.