

Assisted Finite-Rate Adiabatic Passage Across a Quantum Critical Point: Exact Solution for the Quantum Ising Model

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The dynamics of a quantum phase transition is inextricably woven with the formation of excitations, as a result of critical slowing down in the neighborhood of the critical point. We design a transitionless quantum driving through a quantum critical point, allowing one to access the ground state of the broken-symmetry phase by a finite-rate quench of the control parameter. The method is illustrated in the one-dimensional quantum Ising model in a transverse field. Driving through the critical point is assisted by an auxiliary Hamiltonian, for which the interplay between the range of the interaction and the modes where excitations are suppressed is elucidated.

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The complexity involved in describing a generic many-body quantum system prompted Feynman to suggest the use of a highly controllable quantum system as a simulator of another generally complicated quantum system of interest [1]. From this perspective, the quantum systems of interest are those with a large amount of entanglement that are hardly tractable in classical computers [2]. Quantum simulation has become an exciting field of research, which is being developed experimentally, exploring a variety of platforms including ultracold atoms, trapped ions, photonic quantum systems, and superconducting circuits, among others. Simulation of many-body interacting systems is particularly advanced in implementations with trapped ions [3], where the building blocks of a digital quantum simulator for both closed [4] and open [5] quantum systems have been demonstrated. Moreover, while early experimental efforts have been limited to a somewhat low number of qubits, the simulation of a few hundreds of spins with variable-range spin-spin Ising-type interactions has recently been reported [6].

In a continuous quantum phase transition, divergence of length and time scales across a quantum critical point (QCP) leads inevitably to nonadiabatic dynamics. When a parameter λ of the Hamiltonian is changed across its critical value λ_c , the energy gap between the ground state and the first excited state vanishes, and adiabaticity breaks down. The Kibble-Zurek mechanism (KZM), originally developed for classical continuous phase transitions [7,8], predicts that the resulting density of excitations obeys a power-law scaling with the quench rate. The power-law exponent is expressed using the critical exponents at equilibrium and the dimensionality of the system [9,10]. As a result, quantum quenches are useful to characterize universal features of a system and to shed some light on its dynamics out of equilibrium.

The inevitable formation of excitation is however undesirable for a wide range of applications, such as the preparation of novel quantum phases in quantum simulation, and

adiabatic quantum computation. Suppressing excitations is also of interest to a variety of operations in the laboratory, like entangling strings of atoms [11]. This has motivated studies including the use of the energy gap arising from the finite size of the system [12], optimal nonlinear passage across a QCP [13,14], inhomogeneous quenches [15–17], and optimal quantum control strategies [18]. All these approaches can be regarded as strategies to exploit or engineer a spectral gap. Nevertheless, there is a need for new methods to ensure adiabaticity [2]. In this Letter, we shall exploit recent advances in the simulation of coherent k -body interactions [5,19] and transitionless quantum driving [20,21] to explore an alternative to quantum adiabatic protocols, and assist a fully adiabatic finite-rate passage across a QCP.

Shortcut to the adiabatic driving of a two-level system.—Demirplak and Rice [20] and Berry [21] have shown the possibility of implementing a transitionless quantum driving in multilevel systems. Let us consider the Landau-Zener (LZ) transition, the simplest model supporting the KZM [22], described by a Hamiltonian:

$$H_0 = \begin{pmatrix} \lambda(t) & \Delta \\ \Delta & -\lambda(t) \end{pmatrix} = \lambda(t)\sigma^z + \Delta\sigma^x, \quad (1)$$

where $\sigma^{x,y,z}$ are the usual Pauli matrices. The instantaneous eigenbasis reads

$$|1(\lambda)\rangle = \sin\theta|1(-\infty)\rangle - \cos\theta|2(-\infty)\rangle,$$

$$|2(\lambda)\rangle = \cos\theta|1(-\infty)\rangle + \sin\theta|2(-\infty)\rangle,$$

where the angle θ obeys the relations

$$\cos 2\theta = \frac{\lambda}{\sqrt{\lambda^2 + \Delta^2}}, \quad \sin 2\theta = \frac{\Delta}{\sqrt{\lambda^2 + \Delta^2}},$$

and the energy gap is $E_2(t) - E_1(t) = 2\sqrt{\Delta^2 + \lambda^2}$. Following Refs. [20,21], it is found that the Hamiltonian

that drives the exact evolution of the system along the adiabatic solution associated with the instantaneous eigenbasis $\{|n(\lambda)\rangle\}$ of H_0 in Eq. (1) is given by $H = H_0 + H_1$ with [23]

$$H_1 = i\lambda'(t) \sum_n [|\partial_\lambda n\rangle\langle n| - \langle n|\partial_\lambda n\rangle|n\rangle\langle n|]. \quad (2)$$

For the LZ crossing, upon explicit calculation, one finds

$$H_1 = -\lambda'(t) \frac{1}{2} \frac{\Delta}{\Delta^2 + \lambda(t)^2} \sigma^y. \quad (3)$$

The adiabatic solution of H_0 , in which the instantaneous eigenstates exclusively pick up a phase along the evolution, that is, the sum of the dynamical and Berry phases, becomes the exact solution of the time-dependent Schrödinger equation associated with $H = H_0 + H_1$, no matter how fast the transition is crossed, i.e., how large the rate $\lambda'(t)$ is. This approach has recently been verified in the laboratory with an effective two-state model arising in a Bose-Einstein condensate in the presence of an optical lattice [24].

Models.—We now turn our attention to the family of d -dimensional free-fermion Hamiltonians,

$$\mathcal{H}_0 = \sum_{\mathbf{k}} \psi_{\mathbf{k}}^\dagger [\vec{a}_{\mathbf{k}}(\lambda(t)) \cdot \vec{\sigma}_{\mathbf{k}}] \psi_{\mathbf{k}}, \quad (4)$$

where $\vec{\sigma}_{\mathbf{k}} \equiv (\sigma_{\mathbf{k}}^x, \sigma_{\mathbf{k}}^y, \sigma_{\mathbf{k}}^z)$ denote the Pauli matrices acting on the \mathbf{k} -mode and $\psi_{\mathbf{k}}^\dagger = (c_{\mathbf{k},1}^\dagger, c_{\mathbf{k},2}^\dagger)$ are fermionic operators. The function $\vec{a}_{\mathbf{k}}(\lambda) \equiv (a_{\mathbf{k}}^x(\lambda), a_{\mathbf{k}}^y(\lambda), a_{\mathbf{k}}^z(\lambda))$ is specific for the model, and the sum goes over the independent \mathbf{k} -modes. Such a Hamiltonian represents a variety of systems with QCP, including in particular the Ising and the XY models in $d = 1$ [25], as well as the Kitaev model in $d = 2$ [26] and $d = 1$ [27], and as such has been the subject of a recent series of works on defect production induced by a quantum quench [9].

We shall use this to illustrate and investigate the possibility of driving an adiabatic passage across a QCP. Let us consider the instantaneous eigenstates of \mathcal{H}_0 with eigenenergies associated with the \mathbf{k} -mode $\varepsilon_{\mathbf{k},\pm} = \pm |\vec{a}_{\mathbf{k}}(\lambda)| = \pm \sqrt{a_{\mathbf{k}}^x(\lambda)^2 + a_{\mathbf{k}}^y(\lambda)^2 + a_{\mathbf{k}}^z(\lambda)^2}$. We generalize Eq. (3) to find the modified Hamiltonian $\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_1$, where

$$\mathcal{H}_1 = \lambda'(t) \sum_{\mathbf{k}} \frac{1}{2\varepsilon_{\mathbf{k}}^2} \psi_{\mathbf{k}}^\dagger [(\vec{a}_{\mathbf{k}}(\lambda) \times \partial_\lambda \vec{a}_{\mathbf{k}}(\lambda)) \cdot \vec{\sigma}_{\mathbf{k}}] \psi_{\mathbf{k}}$$

induces the adiabatic crossing of the QCP by driving the dynamics exactly along the instantaneous eigenmodes of \mathcal{H}_0 .

Without further knowledge of the explicit form of the matrix elements of \mathcal{H}_0 , its form in real space cannot be determined. Next, we turn our attention to a specific model.

The quantum Ising model in a transverse field.—Consider a chain of N spins described by the one-dimensional quantum Ising model in a transverse magnetic field g ,

$$\mathcal{H}_0 = - \sum_{n=1}^N (\sigma_n^x \sigma_{n+1}^x + g \sigma_n^z), \quad (5)$$

a paradigmatic model to study quantum phase transitions [25] of relevance to current experimental efforts in quantum simulation [28]. We assume periodic boundary conditions $\sigma_{N+1} = \sigma_1$ and, for simplicity, even N . This model exhibits a quantum phase transition at $g_c = \pm 1$ between the paramagnetic phase ($|g| > 1$) and ferromagnetic phase ($|g| < 1$).

The Jordan-Wigner transformation $\sigma_n^z = 1 - 2c_n^\dagger c_n$, $(\sigma_n^x + i\sigma_n^y) = 2c_n \prod_{l<n} (1 - 2c_l^\dagger c_l)$, where c_n are fermionic annihilation operators, allows us to rewrite the Hamiltonian (5) as a free-fermion model. Below, we will limit our discussion to the plus-one parity subspace of the Hilbert space, including the ground state; note that \mathcal{H}_0 commutes with the parity operator $P = \prod_{n=1}^N \sigma_n^z$. In fermionic representation,

$$\mathcal{H}_0 = \sum_{n=1}^N [(c_n - c_n^\dagger)(c_{n+1}^\dagger + c_{n+1}) - g(c_n c_n^\dagger - c_n^\dagger c_n)],$$

with antiperiodic boundary conditions $c_{N+1} = -c_1$. Using the Fourier transform $c_n = e^{-i\pi/4} \sum_{\mathbf{k}} c_{\mathbf{k}} e^{ikn} / \sqrt{N}$ with momenta consistent with the boundary conditions $\mathbf{k} \in k^+ = (\pm\pi/N, \pm 3\pi/N, \dots, \pm(N-1)\pi/N)$, we can conveniently rewrite this as

$$\mathcal{H}_0 = 2 \sum_{\mathbf{k}>0} \psi_{\mathbf{k}}^\dagger [\sigma_{\mathbf{k}}^z (g - \cos k) + \sigma_{\mathbf{k}}^x \sin k] \psi_{\mathbf{k}},$$

where the operator $\psi_{\mathbf{k}}^\dagger \equiv (c_{\mathbf{k}}^\dagger, c_{-\mathbf{k}})$. In this form, it becomes apparent that the Ising model can be decomposed into a series of independent LZ transitions, as was first realized in Ref. [29]. Now we can directly use the results of the previous section and write the supplementary Hamiltonian required for adiabatic driving across the QCP,

$$\mathcal{H}_1 = -g'(t) \sum_{\mathbf{k}>0} \frac{1}{2} \frac{\sin k}{g^2 + 1 - 2g \cos k} \psi_{\mathbf{k}}^\dagger \sigma_{\mathbf{k}}^y \psi_{\mathbf{k}}.$$

This expression is expected to be highly nonlocal in real space, and can be written as

$$\mathcal{H}_1 = -g'(t) \left[\sum_{m=1}^{N/2-1} h_m(g) \mathcal{H}_1^{[m]} + \frac{1}{2} h_{N/2}(g) \mathcal{H}_1^{[N/2]} \right]. \quad (6)$$

The Hamiltonian $\mathcal{H}_1^{[m]}$ includes an interaction over a range m . Above, we have a factor of $\frac{1}{2}$ for $m = N/2$ (this is the largest distance in the system with periodic boundary condition) because for an even N there is only one spin over the distance $N/2$, while there are two different spins over smaller distances. Every $\mathcal{H}_1^{[m]}$ is independent of g [all dependence on g is included in coefficients $h_m(g)$] and reads

$$\mathcal{H}_1^{[m]} = 2i \sum_{n=1}^N (c_n c_{n+m} + c_n^\dagger c_{n+m}^\dagger).$$

The coefficients $h_m(g)$ are given by the Fourier transform

$$h_m(g) = \frac{1}{N} \sum_k f(k) \sin(mk)$$

of the function

$$f(k) = \frac{1}{4} \frac{\sin k}{g^2 + 1 - 2g \cos k}. \quad (7)$$

In the limit of a large N , we can approximate $h_m \simeq \int_0^\pi f(k) \times \sin(mk) dk / \pi$, with the result

$$h_m = \frac{1}{8} \begin{cases} g^{m-1} & \text{for } |g| < 1, \\ g^{-m-1} & \text{for } |g| > 1. \end{cases}$$

Mapping back to spins, the supplementary Hamiltonians read

$$\begin{aligned} \mathcal{H}_1^{[m]} = & \sum_{n=1}^N (\sigma_n^x \sigma_{n+1}^z \cdots \sigma_{n+m-1}^z \sigma_{n+m}^y \\ & + \sigma_n^y \sigma_{n+1}^z \cdots \sigma_{n+m-1}^z \sigma_{n+m}^x). \end{aligned} \quad (8)$$

Some comments are in order. First, since we have represented the class of Hamiltonians in Eq. (4)—and in particular the Ising model Eq. (5)—as independent LZ crossings, the supplementary Hamiltonian \mathcal{H}_1 allows us to adiabatically drive any eigenstate of the model under consideration [30]. This means that one can further tailor the Hamiltonian \mathcal{H}_1 for the purpose of driving exclusively a given subset of states, e.g., the ground states. Further considerations along this line of reasoning are beyond the scope of this Letter [31].

Second, the coefficients $h_m(g)$ can be neatly written as $|h_m(g)| = e^{-(m \pm 1)/\xi(g)}/8$, where $\xi(g) = |\ln(|g|)|^{-1}$ is the correlation length in the Ising model [32]. It follows then that $h_m(g) \sim \mathcal{O}(1)$ for distances m up to the correlation length and go to zero exponentially fast for longer interaction ranges. At the critical point, this means that \mathcal{H}_1 is acting along the whole chain.

Finally, Hamiltonians of the $\mathcal{H}_1^{[m]}$ form can be implemented in trapped-ion quantum simulators using stroboscopic techniques [19,33], already demonstrated in the laboratory [5].

Finite range interactions, filtering, and the KZM.—We next consider a linear quench of the coupling $g(t) = g_c - \nu t$ through the QCP at $g_c = 1$, bringing the system from the paramagnetic to the ferromagnetic phase. The evolution induced by the Hamiltonian \mathcal{H}_1 [see Eq. (6)] is adiabatic in the instantaneous eigenbasis of the Ising model Eq. (5) [30]. However, the range of the interaction, for example, at the critical point, spans over the whole chain. As a result, from a practical point of view, one might be interested in assisting the crossing of the QCP with an approximation to \mathcal{H}_1 that involves interactions of only the restricted range. In this section, we examine the simplest approximation, namely, a direct truncation of Eq. (6) that limits the range of interaction to M sites,

$$\tilde{\mathcal{H}}_1(M) = \nu \sum_{m=1}^M s_m h_m(g) \mathcal{H}_1^{[m]}, \quad (9)$$

where s_m will be a filter function.

We start by examining the limit of a fast transition, which for now means $\nu \gg 1$ (this will be made more precise later). We consider the initial ground state in the paramagnetic phase and evolve it using (i) only the supplementary Hamiltonian $\mathcal{H} = \tilde{\mathcal{H}}_1(M)$ and then (ii) both the supplementary and the Ising Hamiltonians $\mathcal{H} = \mathcal{H}_0 + \tilde{\mathcal{H}}_1(M)$. For both cases, we numerically solve the time-dependent Bogoliubov–de Gennes equations that describe the evolution of the system, as explained in Ref. [29]. Figure 1 shows the probability of excitation in the k -mode, p_k .

First, we have verified that p_k does not depend on the quench rate ν for $\nu \gg 1$, and that they coincide in both cases (i) and (ii) in that limit. That is, the presence of \mathcal{H}_0 results only in phase difference and does not affect how well the approximated supplementary Hamiltonian $\tilde{\mathcal{H}}_1$ is able to drive the transition.

Second, the cutoff M in Eq. (9) implies approximating the function $f(k)$ in Eq. (7) by its truncated Fourier series. Since $f(k)$ is divergent and discontinuous at $g = g_c = 1$ and $k = 0$, $\tilde{\mathcal{H}}_1(M)$ suffers from the so-called Gibbs phenomenon, this is, the problem of recovering point values of a nonperiodic or discontinuous function from its Fourier coefficients [34]. In Fig. 1(a), we present the results for the truncation $\tilde{\mathcal{H}}_1(M)$ with $s_m = 1$ (Dirichlet kernel). The Gibbs phenomenon is seen here in the appearance of side lobes at large k . This can be prevented by using a Fourier

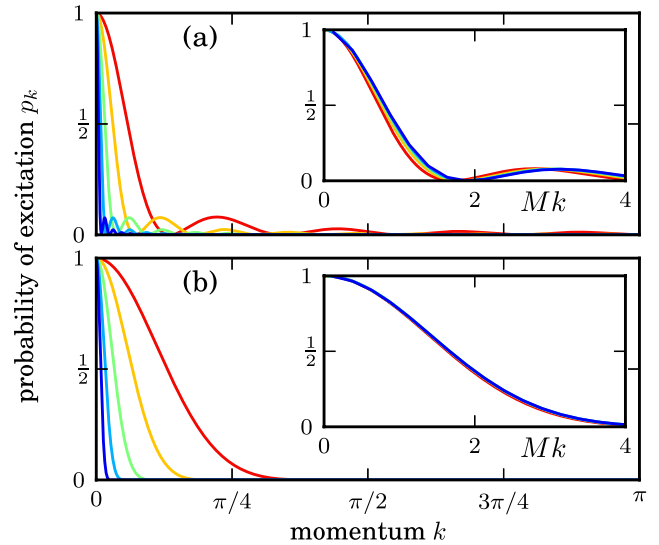


FIG. 1 (color online). Excitation probability p_k as a function of the wave vector k , following a shortcut to adiabaticity in the 1d quantum Ising model. (a) The crossing of the critical point is assisted by a truncated Hamiltonian $\tilde{\mathcal{H}}_1(M)$ ($s_m = 1$), (b) and a modified truncation where the expansion coefficients are modulated by a raised cosine Fourier filter. The range of the interaction increases from right to left with cutoff $M = 4, 8, 16, 32, 64$ ($N = 1600$). Above, the system evolves from $g_i = 10$ to $g_f = 0$ across QCP at $g_c = 1$ at quench rate $\nu = 50$ using $\mathcal{H} = \mathcal{H}_0 + \tilde{\mathcal{H}}_1(M)$. The insets show the scaling of p_k as a function of $k/k_M \sim Mk$.

space filter s_m that modifies the expansion coefficients [34]. In Fig 1(b), we use the raised cosine filter $s_m = \frac{1}{2} \times [1 + \cos(m\pi/M)]$. This improves the convergence away from the discontinuity, making the decay of p_k with k (almost) monotonic, and suppresses the sidelobes observed in its absence, at the expense of broadening p_k . However, it remains impossible to recover $f(k)$ close to its discontinuity, so the modes with $k \approx 0$ are still excited.

As an upshot, in the limit of fast quenches, the effect of approximating $\mathcal{H}_1(M)$ by $\tilde{\mathcal{H}}_1(M)$ depends only on M , and to recover a fully adiabatic dynamics, we need $M = N/2$. Based on the above considerations and the relation of h_m to the correlation function in the Ising model, we draw the conjecture that an approximation of the form in Eq. (9) induces an adiabatic dynamics of the modes with $k \gg k_M \sim M^{-1}$. This is corroborated in the insets in Fig. 1, where we rescale k for different values of the cutoff M , and the corresponding excitation probabilities p_k collapse onto each other.

We consider as well the mean number of excitations, $n_{\text{ex}} = \frac{1}{\pi} \int_0^\pi p_k dk$, as a function of the quench rate ν and cutoff M . The results are presented in Fig. 2. During a fast transition and for a given cutoff M , we are able to adiabatically drive modes with $k \gg k_M \sim M^{-1}$. This means that the mean number of defects saturates at $n_{\text{ex}} \sim M^{-d}$. This limit can be seen at the right-hand side of Fig. 2.

Next, we focus on slower transitions that are induced by the Ising Hamiltonian and approximated supplementary

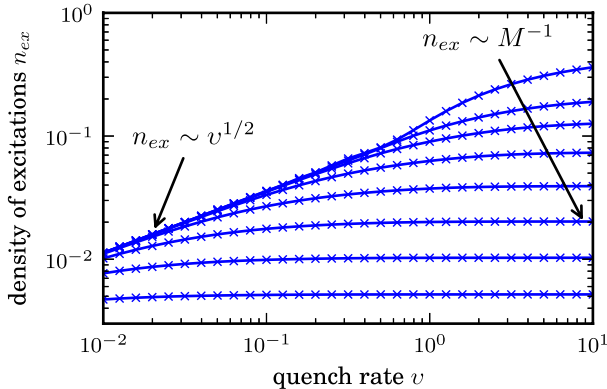


FIG. 2 (color online). Suppression of the total number of excitations n_{ex} as a function of the quench rate ν of the transverse field in a 1d Ising chain following a quench through the QCP ($g = 1$). The dynamics is assisted by a truncated auxiliary Hamiltonian $\tilde{\mathcal{H}}_1(M)$, with a cutoff $M = 0, 1, 2, 4, 8, 16, 32, 64$ from top to bottom. The numerics for \mathcal{H}_0 ($M = 0$) is the reference case where the KZM dictates a power-law scaling of n_{ex} for $\nu \ll 1$. This scaling is recovered at slow quench rates in a passage through the QCP assisted by $\mathcal{H}_1(M)$, while at faster rates there is an efficient suppression of excitations. As the range of the interactions is increased, the dynamics in all modes is driven through the instantaneous eigenbasis of \mathcal{H}_0 , a complete suppression of excitations is achieved. In all simulations $N = 1600$, and we evolve from $g_i = 10$ to $g_f = 0$, and no filtering is applied ($s_m = 1$).

Hamiltonian $\mathcal{H} = \mathcal{H}_0 + \tilde{\mathcal{H}}_1(M)$. In the limiting case of $M = 0$, the system is driven only by the Ising Hamiltonian, and the nonadiabatic dynamics is correctly described by KZM [29,35,36]. We do not consider here the limit where the quench across QCP is adiabatic due to the gap resulting from the finite size of the system, which is reached for extremely slow quenches with $\nu \ll N^{-2}$ [12,29].

For $M = 0$, i.e., for $\mathcal{H} = \mathcal{H}_0$, there appears a characteristic value of momenta described by KZM: $k_{\text{KZ}} \sim \nu^{\nu/(1+\nu)} = \nu^{1/2}$ for the Ising model [37] (to be precise, we expect the power-law behavior for $\nu \ll 1$, when the system goes out of equilibrium close to the QCP). The modes with $k \gg k_{\text{KZ}}$ are expected to cross QCP adiabatically and $n_{\text{ex}} = \nu^{d\nu/(1+d\nu)} = \nu^{1/2}$. We recover this limit in the left-hand side of Fig. 2 when M is small enough compared to ν^{-1} . A crossover between the two quench rate limits occurs for intermediate values of M and ν . The one that dominates for a given set of parameters depends on whether k_{KZ} is smaller or greater than k_M .

Relation to fidelity susceptibility.—Finally, it is interesting to draw a connection with the so-called fidelity susceptibility, which puts some constraints on \mathcal{H}_1 . Fidelity susceptibility, $\chi_F(\lambda)$, can be defined in the leading-order expansion of the overlap of the ground states calculated for slightly different values of external parameter λ . For a finite system in the limit $\delta \rightarrow 0$ [38], we can Taylor expand the overlap in δ [39] as follows:

$$|\langle \lambda | \lambda + \delta \rangle|^2 \approx 1 - \delta^2 \chi_F(\lambda),$$

where $\chi_f(\lambda)$ for a nondegenerated ground state reads [30,40]

$$\chi_f(\lambda) = \sum_{n \neq 0} \frac{|\langle 0(\lambda) | \partial_\lambda \mathcal{H}_0 | n(\lambda) \rangle|^2}{|E_n - E_0|^2},$$

$\{|n(\lambda)\rangle\}$ and E_n are instantaneous eigenstates and eigenenergies of $\mathcal{H}_0(\lambda)$, respectively, and $|0(\lambda)\rangle$ is the ground state.

In addition, the supplementary Hamiltonian \mathcal{H}_1 (2), which would be able to drive the evolution along the instantaneous ground state, must satisfy

$$\langle 0(\lambda) | \mathcal{H}_1 | n(\lambda) \rangle = i\lambda'(t) \frac{\langle 0(\lambda) | \partial_\lambda \mathcal{H}_0 | n(\lambda) \rangle}{E_n - E_0}, \quad (10)$$

for $n \neq 0$. Thus, we can verify that the mean variance of \mathcal{H} in the instantaneous ground state is [23]

$$\Delta \mathcal{H}^2 = \langle 0(\lambda) | \mathcal{H}_1^2 | 0(\lambda) \rangle = |\lambda'(t)|^2 \chi_F(\lambda). \quad (11)$$

Fidelity susceptibility for translationally invariant systems is expected to typically scale at the critical point as $\chi_F(\lambda_c) \sim N^{2/d\nu}$ [40–42], and away from the critical point as $\chi_F(\lambda) \sim N|\lambda - \lambda_c|^{d\nu-2}$. It is divergent in the vicinity of the QCP as long as fidelity susceptibility is dominated by low-lying excitations [40].

In conclusion, for a broad family of many-body systems exhibiting a quantum phase transition, we have presented a method to assist the adiabatic crossing of the critical point

at a finite rate by supplementing the system with a finite-range time-dependent interaction. Our proposal is suited to access the ground state manifold in quantum simulators. The nonlocal terms of the $\mathcal{H}_1^{[m]}$ type in the auxiliary Hamiltonian can be implemented using the stroboscopic techniques recently demonstrated in the laboratory (see Refs. [5,19,33]). We have focused on the finite-rate adiabatic crossing of a quantum phase transition, where suppressing excitations is particularly challenging due to the critical slowing down in the proximity of the critical point. Nonetheless, the method can be applied as well to the preparation of many-body states as an alternative to optimal control techniques [43] or in combination with them.

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 - [30] To be precise, we are confined here in the subspace with plus-one parity, which, for example in the ferromagnetic phase, singles out one ground state that does not break the (Z_2) parity symmetry.
 - [31] For example, if we limit ourselves to driving adiabatically only the ground state of the system, the supplementary Hamiltonian \mathcal{H}_1 can be modified by any Hamiltonian of the form $(1 - |0(g)\rangle\langle 0(g)|)\mathcal{H}_2(1 - |0(g)\rangle\langle 0(g)|)$.
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