Counter-diabatic driving using Floquet engineering

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1 Introduction

$H = H_0 + H_1$

$$H = J\sum_j (c^\dagger_{j+1}c_j + h.c) + \cos(\omega t)\sum_j A_j c^\dagger_j c_j \tag{1}$$

$V = \exp(-i\sin(\omega t)/\omega \sum_j A_j c^\dagger_j c_j)$

$$\begin{aligned}
H_{rot} &= V^\dagger H V - iV^\dagger \dot{V} \\
&= V^\dagger H_0 V + \cos(\omega t)\sum_j A_j c^\dagger_j c_j + i^2\cos(\omega t)\sum_j A_j c^\dagger_j c_j \\
&= V^\dagger H_0 V \\
&= V^\dagger c^\dagger_{j+1} V V^\dagger c_j V
\end{aligned}$$

Using $[n_j, c_j] = -c_j$ and $[n_j, c^\dagger_j] = c^\dagger_j$

$$H_{rot} = J\sum_j (g^{j,j+1} c^\dagger_{j+1}c_j + \text{h.c})$$

where $g^{j,j+1} = \exp\left(i\sin(\omega t)\dfrac{A_{j+1}-A_j}{\omega}\right)$

$$\begin{aligned}
H_F^{(0)} &= \frac{1}{T}\int_{t_0}^{T+t_0} (c^\dagger_{j+1}c_j \exp\left(i\sin(\omega t)\frac{A_{j+1}-A_j}{\omega}\right) dt + \text{h.c}) \\
&= J_{eff}(c^\dagger_{j+1}c_j + \text{h.c})
\end{aligned}$$

where $J_{eff} = J\mathcal{J}_0\left(\dfrac{A_{j+1}-A_j}{\omega}\right)$

Few points: Bessel function is always less than 1 while CD hopping J is always greater than 1. Furthermore, driving frequency would be high so that for all values of $A_{j+1} - A_j$, we would have $\dfrac{A_{j+1}-A_j}{\omega} \ll 1$ so that we are close to origin and away from zeros of Bessel's functions.

2 Magnus expansion

For a Hamiltonian which is periodic in time, it's unitary operator over a full driving cycle is given by:

$$U(T+t_0, t_0) = \mathcal{T}_t \exp(-\frac{i}{\hbar}\int_{t_0}^{T} dt H(t)) = \exp(-\frac{i}{\hbar}H_F[t_0]T) \tag{2}$$

$H_F[t_0] = \sum_n H_F^{(n)}[t_0]$ where

$$H_F^{(0)} = \frac{1}{T}\int_{t_0}^{T+t_0} H(t)dt$$

$$H_F^{(1)} = \frac{1}{2!Ti\hbar}\int_{t_0}^{T+t_0} dt_1 \int_{t_0}^{t_1} dt_2 [H(t_1), H(t_2)]$$

3 Bessel's function of first kind

Integral representation of Bessel's function of first kind $\mathcal{J}_n(x)$ is given by:

$$\mathcal{J}_n(x) = \frac{1}{2\pi}\int_{-\pi}^{\pi} d\tau e^{i(n\tau - x\sin\tau)} = \frac{1}{T}\int_{-T}^{T} d\tau e^{i(n\omega\tau - x\sin\omega\tau)} \tag{3}$$

For $x \ll 1$, $\mathcal{J}_0(x) = 1 - \frac{x^2}{2}$