Locality of variational gauge fields and slow operators

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THE PROBLEM

Let's separate the entire system in terms of our system of interest S and the environment E, such that we can write the Hamiltonian as $H = H_S + H_E + H_{SE}$. Here we'd like to consider the question, what is the best gauge potential if we only change the Hamiltonian of the system H_S and can only do counter-diabatic driving on the system itself. This has two direct applications: (i) Open systems that are coupled to some bath described by H_E and (ii) closed systems where we only have local control. Here we are primiraly concerned with the latter; specifically we want to see if this is a natural way to remove the small denominator problem.

Variational gauge potentials

To be specific let's consider system-environment interactions of the general form

$$H_{SE} = \sum_i \gamma_i S_i \otimes E_i, \tag{1}$$

where S denotes an operator with support on the system and E an operator with support on the environment only. Moreover, without loss of generality we will consider the operators to be traceless, i.e. $\mathrm{Tr}S_i = \mathrm{Tr}E_i = 0$. Given, we can only access the system we are interested in the most general gauge potential of the form

$$A = A_S \otimes \mathbb{1}_E \tag{2}$$

Using same notation as usual, this variational ansatz results in

$$G = (\partial_\lambda H_S + i[A_S, H_S]) \otimes \mathbb{1}_E + \sum_i \gamma_i i[A_S, S_i] \otimes E_i. \tag{3}$$

Further assuming that the E_i are orthonormal $\mathrm{Tr}E_iE_j = \delta_{ij}$, simply yields

$$S = \mathrm{Tr}G^2 = \underbrace{\mathrm{Tr}_S \left(\partial_\lambda H_S + i[A_S, H_S]\right)^2}_{S_S} + \sum_j \gamma_j^2 \underbrace{\mathrm{Tr}_S \left(i[A_S, S_j]\right)^2}_{\text{coupling}}. \tag{4}$$

We notice that the full action consistes of a sum of two terms. The bare action over the system + a coupling correction coming from the environment. Formally taking the functional derivative with respect to A_S yields the linear equation

$$[\partial_\lambda H_S + i[A_S, H_S], H_S] + \sum_j \gamma_j^2 i[[A_S, S_j], S_j] = 0. \tag{5}$$

Consequently if we define the eigenvalue problem

$$[[O_\nu, H_S], H_S] + \sum_j \gamma_j^2 [[O_\nu, S_j], S_j] = \omega_\nu^2 O_\nu, \tag{6}$$

the solution becomes

$$A = i \sum_\nu \frac{\mathrm{Tr}\left([\partial_\lambda H_S, H_S] O_\nu\right)}{\omega_\nu^2} O_\nu, \tag{7}$$

where sum should run over all non-zero eigenvalues! Note that whenever $\gamma_j = 0$ we recover the exact result since $O_\nu = |n\rangle\langle m|$ and $\omega_\nu = \epsilon_m - \epsilon_n$. For any non-trivial system/environment coupling we actually expect that there is

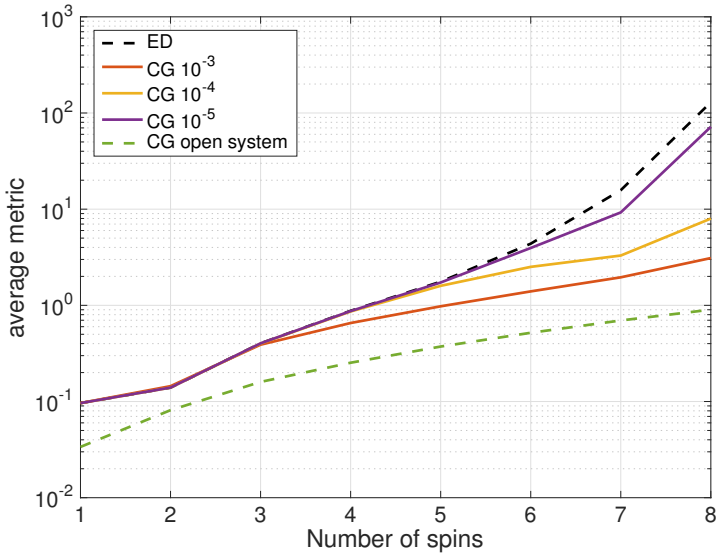

Figure 1. Figure shows the average metric, i.e. $2^{-L}\mathrm{Tr}A^\dagger A$, for a system of L spins and a gauge potential up to L string operator. Here we can compute the exact result by ED (black dashed line) and we used conjugate-gradient minimization as well. The results agree and we see systematic convergence if we reduce the maximum allowed error in the conjugate gradient method.

only a single zero eigenvalue solution to equation (6), namely the identity $\mathbb{1}_S$. Since the numerator of expression (7) vanishes for the identity we can simply shift the spectrum up by ϵ and sum over all states. This could be usefull numerically. Furthermore, the eigenvalue equation (6) recently appeared in a work on slow operators [1] in 1D local quantum spin chains. The takeaway message of the work is that the smallest non-zero eigenvalue ω_1 only scales as a powerlaw with subsystem size L^z. This is in stark contrast to the scaling of the denominator for a closed system, which is exponential in system size. The latter is what causes the gauge potential and all related quantities such as mass and metric tensor to diverge exponentially. I believe the above result indicates that this unphysical divergence is removed by considering only local counter-diabatic drives.

Conjugate gradient method

Results

All numerics is on the following model:

$$H = \sum_i \sigma_i^z \sigma_{i+1}^z 0.9\sigma_i^z + 0.8\sigma_i^x \tag{8}$$

[1] Hyungwon Kim, Mari Carmen Banuls, J. Ignacio Cirac, Matthew B. Hastings, and David A. Huse, Phys. Rev. E 92, 012128, (2015)

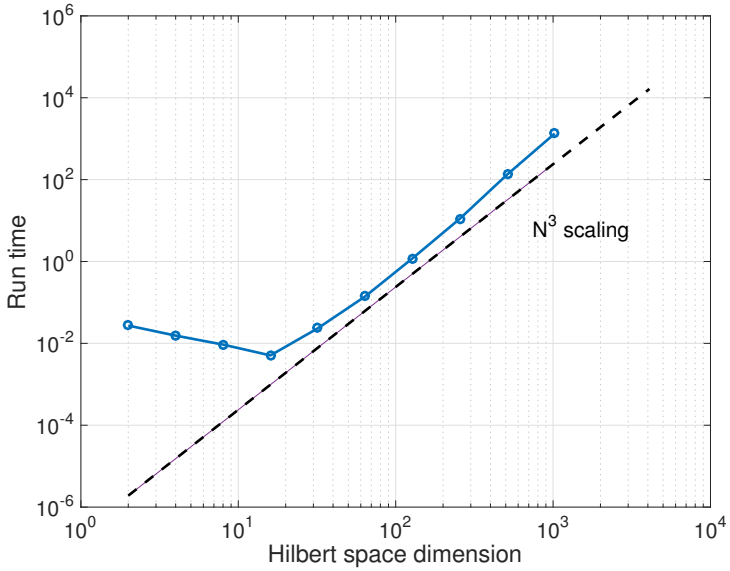

Figure 2. We use the conjugate gradient method to find the gauge potential for a subsystem of size L. The figure shows scaling of the runtime (in second on my laptop) as a function of Hilbert space dimension. The black dashed line corresponds to N^3 scaling, which is the scaling for matrix multiplication. Numerically we also see that the number of conjugate gradient steps seems to go like L^3 so we anticipate that the actual scaling is $(N\log(N))^3$. A subsystem of size 10 took 22 min on my laptop so it would take about 24h to do $L = 12$. Current algorithm doesn't precondition the system nor does it try to make A sparse. I think we can gain some prefactors here.

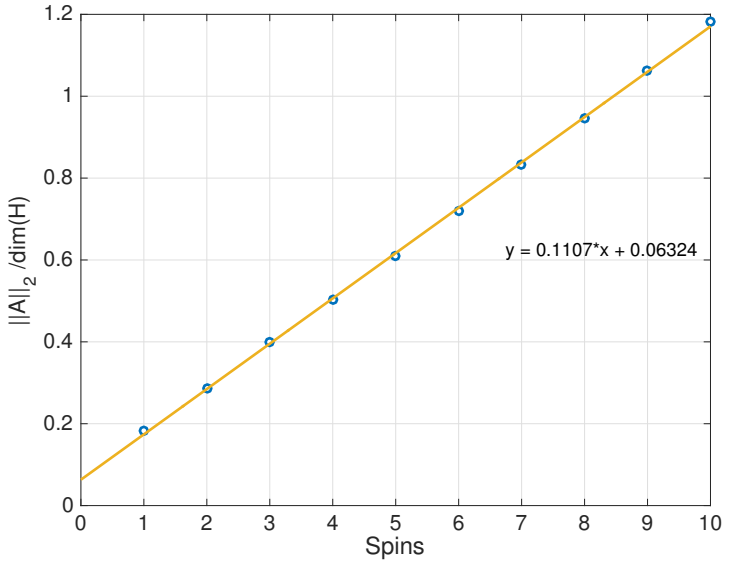

Figure 3. Figure shows $\sqrt{2^{-L}\mathrm{Tr}A^\dagger A}$ vs subsystem size L. Yellow line shows a linear fit. Seem to get nice linear scaling.