Counter-diabatic driving using Floquet engineering

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1 CD driving

$$H_0 = -J\sum_j (c_{j+1}^\dagger c_j + \text{h.c}) + \sum_j V_j(\lambda) c_j^\dagger c_j \tag{1}$$

For this problem, approximate gauge potential is chosen to be $A_\lambda^* = i\sum_j \alpha_j(c_{j+1}^\dagger c_j - h.c)$.

On minimizing action, we get

$$-3J^2(\alpha_{j+1}+\alpha_{j-1}) + (6J^2 + (V_{j+1}-V_j)^2)\alpha_j = -J\partial_\lambda(V_{j+1}-V_j)$$

$$H_{CD} = H_0 + \dot\lambda A_\lambda = \sum_j J_j(c_{i+1}^\dagger c_i + h.c) + \sum_j U_j c_i^\dagger c_i$$

where

$$J_j = J\sqrt{1+(\dot\lambda\alpha_j/J)^2} \quad U_j = V_j(\lambda) - \sum_i^j \frac{J}{J^2+(\dot\lambda\alpha_i/J)^2}(\ddot\lambda\alpha_j + \dot\lambda^2\partial_\lambda\alpha_j)$$

2 Floquet driving

$$H = H_0 + H_1 = J\sum_j (c_{j+1}^\dagger c_j + \text{h.c}) + \cos(\omega t)\sum_j A_j c_j^\dagger c_j$$

We would go to the rotating frame $|\psi_{rot}\rangle = V^\dagger|\psi_{lab}\rangle$ where $V = \exp(-i\sin(\omega t)/\omega\sum_j A_j c_j^\dagger c_j)$.

$$\begin{aligned}
H_{rot} &= V^\dagger H V - iV^\dagger\dot V \\
&= V^\dagger H_0 V + \cos(\omega t)\sum_j A_j c_j^\dagger c_j + i^2\cos(\omega t)\sum_j A_j c_j^\dagger c_j \\
&= V^\dagger H_0 V = V^\dagger c_{j+1}^\dagger V V^\dagger c_j V + \text{h.c}
\end{aligned}$$

Using $[n_j, c_j] = -c_j$ and $[n_j, c_j^\dagger] = c_j^\dagger$

$$H_{rot} = J\sum_j (g^{j,j+1} c_{j+1}^\dagger c_j + \text{h.c})$$

where $g^{j,j+1} = \exp\left(i\sin(\omega t)\dfrac{A_{j+1}-A_j}{\omega}\right)$

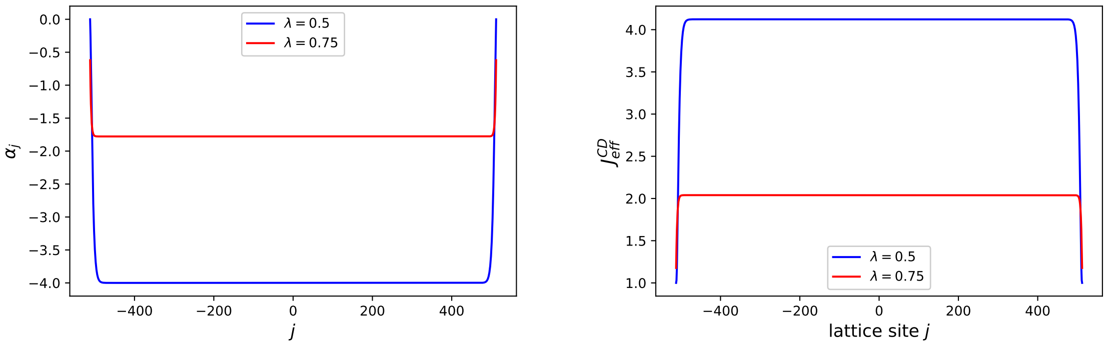

Figure 1: a) α_j for linear potential with vanishing boundary condition b) Effective hopping strength

$$
\begin{aligned}
H_F^{(0)} &= \frac{1}{T}\int_{t_0}^{T+t_0} (c_{j+1}^{\dagger} c_j \exp\left(i \sin(\omega t) \frac{A_{j+1} - A_j}{\omega} \right) dt + \text{h.c}) \\
&= J_{eff}(c_{j+1}^{\dagger} c_j + \text{h.c})
\end{aligned}
$$

where $J_{eff} = J\mathcal{J}_0\left(\frac{A_{j+1}-A_j}{\omega}\right)$

A necessary condition for cold atom experiments to work is that the driving frequency be smaller than the band gap between the lowest two Bloch bands, or otherwise higher bands will get populated.

3 Linear potential

We choose $V(j, \lambda) = j\lambda$.

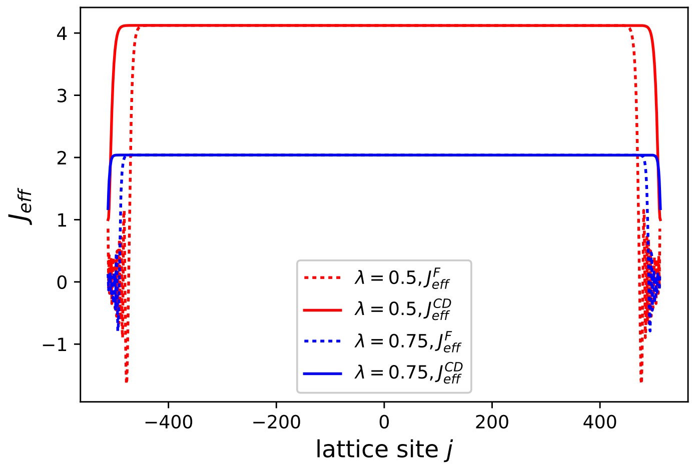

Figure 2: Comparison of effective hopping strength from floquet and CD driving

4 Eckart potential

$V(\lambda, j) = \dfrac{\lambda(t)}{\cosh^2 j/\xi}$. This potential is localized and I expect it to better amenable for present purpose.

A Magnus expansion

For a Hamiltonian which is periodic in time, it's unitary operator over a full driving cycle is given by:

$$U(T+t_0, t_0) = \mathcal{T}_t \exp(-\frac{i}{\hbar}\int_{t_0}^{T} dt H(t)) = \exp(-\frac{i}{\hbar} H_F[t_0]T) \tag{2}$$

$H_F[t_0] = \sum_n H_F^{(n)}[t_0]$ where

$$H_F^{(0)} = \frac{1}{T}\int_{t_0}^{T+t_0} H(t)dt$$

$$H_F^{(1)} = \frac{1}{2!Ti\hbar}\int_{t_0}^{T+t_0} dt_1 \int_{t_0}^{t_1} dt_2 [H(t_1), H(t_2)]$$

B Bessel's function of first kind

Integral representation of Bessel's function of first kind $\mathcal{J}_n(x)$ is given by:

$$\mathcal{J}_n(x) = \frac{1}{2\pi}\int_{-\pi}^{\pi} d\tau e^{i(n\tau - x\sin\tau)} = \frac{1}{T}\int_{-T/2}^{T/2} d\tau e^{i(n\omega\tau - x\sin\omega\tau)} \tag{3}$$

For $x \ll 1$, $\mathcal{J}_0(x) = 1 - \frac{x^2}{2}$

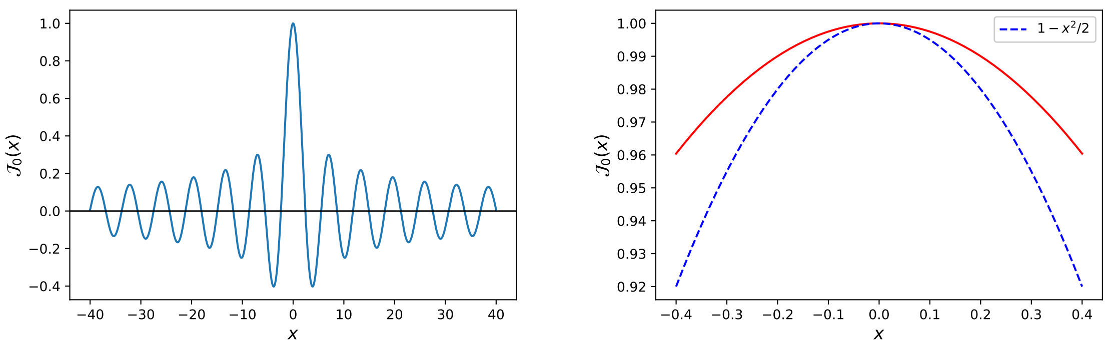

Figure 3: Bessel's function