Quantum control of NV center using counter-diabatic driving

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1 Introduction

The ground state of the NV center is a spin triplet with $|0\rangle, |-1\rangle, |1\rangle$ spin sub-levels. They are defined in S_z basis , where $\hat{z}$ direction is along the NV center axis. The Hamiltonian for the ground state of the NV center can be written as [1]:

$$H_{NV} = \hbar\Delta S_z^2 + g\mu_B \vec{S}.\vec{B}_{ext} \tag{1}$$

where $\Delta = 2\pi \times 2.87$ GHz is zero-field splitting, $g \approx 2$ is the g-factor of electron in the NV center and μ_B is Bohr magneton. If there is no external magnetic field, then $|-1\rangle$ and $|1\rangle$ levels are degenerate, and $\hbar^3\Delta$ is the energy difference between $|0\rangle$ and $|\pm 1\rangle$ energy levels.

Let's choose magnetic field to be in x-direction. Then we have:

$$\begin{aligned} H_{NV} &= \hbar\Delta S_z^2 + g\mu_B S_x B \\ &= \Lambda S_z^2 + \lambda S_x \end{aligned}$$

where $\Lambda = \hbar\Delta$ and $\lambda = g\mu_B B$. Magnetic field is going to be our control parameter in this problem. Using spin algebra, we obtain Hamiltonian in the basis ($|-1\rangle$, $|0\rangle$, $|1\rangle$):

$$H = \begin{bmatrix} \beta & \alpha & 0 \\ \alpha & 0 & \alpha \\ 0 & \alpha & \beta \end{bmatrix} \tag{2}$$

where $\alpha = \hbar\lambda/\sqrt{2}$ and $\beta = \hbar^2\Lambda$.

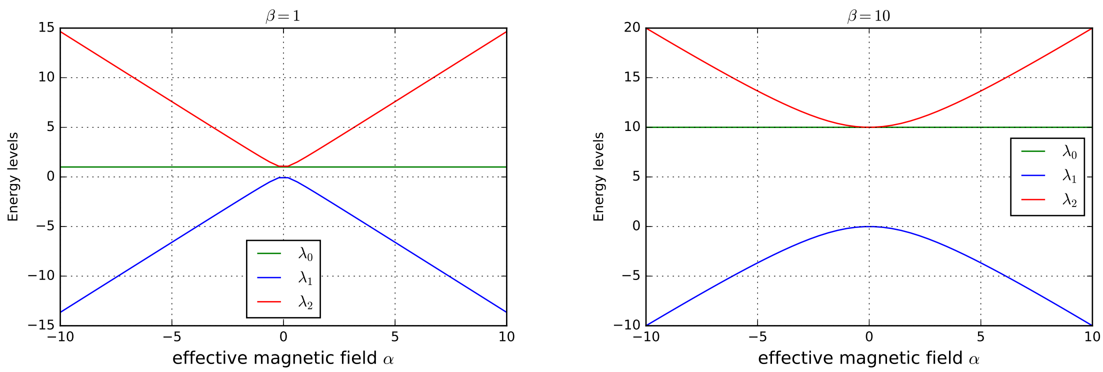

Figure 1: Avoided level crossing as a function of effective magnetic field

Energy eigenvalues are given by:

$$\lambda_0 = \beta, \quad \lambda_1 = (\beta - \sqrt{\beta^2 + 8\alpha^2})/2, \quad \lambda_2 = (\beta + \sqrt{\beta^2 + 8\alpha^2})/2$$

We should remember that $\alpha \propto B$. Hence, it makes sense that when $\alpha = 0$, there is a two -fold degeneracy and zero field energy gap is given by $\beta = \hbar^3\Delta$. Now let's have a look at eigenvectors:

$$\nu_0 = (-1, 0, 1), \quad \nu_1 = (1, -(\beta + \sqrt{\beta^2 + 8\alpha^2})/2\alpha, 1), \quad \nu_2 = (1, -(\beta - \sqrt{\beta^2 + 8\alpha^2})/2\alpha, 1)$$

Now let's compute gauge potential:

Let's find out A_λ for this Hamiltonian for which we need to compute different odd-powered commutator $[H, \partial_\lambda H]$, where $\partial_\lambda H = S_x$. Here we begin:

$$\begin{aligned} C^{(1)} = [H, S_x] &= \Lambda[S_z^2, S_x] \\ &= S_z[S_z, S_x] + [S_z, S_x]S_z \\ &= 2i\hbar(S_zS_y + S_yS_z) \\ &= 2i\hbar([S_z, S_y] + 2S_yS_z) \\ &= 2i\hbar(-i\hbar S_x + 2S_yS_z) \end{aligned}$$

$$\begin{aligned} C^{(2)} = [H, C^{(1)}] &= -2\hbar^2[H, S_x] + 2[H, S_yS_z] \\ &= -2\hbar^2 C^{(1)} + 2S_y[H, S_z] + 2[H, S_y]S_z \\ &= -2\hbar^2 C^{(1)} + 2\lambda S_y[S_x, S_z] + T \\ &= -2\hbar^2 C^{(1)} - 2i\hbar\lambda S_y^2 + T \end{aligned}$$

$$\begin{aligned} T = 2[H, S_y]S_z &= \Lambda[S_z^2, S_y]S_z + \lambda[S_x, S_y]S_z \\ &= \Lambda S_z[S_z, S_y]S_z + \Lambda[S_z, S_y]S_z^2 + 2i\hbar\lambda S_z^2 \\ &= -2i\Lambda(S_zS_xS_z + S_xS_z^2) + 2i\hbar\lambda S_z^2 \\ &= -2i\Lambda([S_z, S_x]S_z + 2S_xS_z^2) + 2i\hbar\lambda S_z^2 \\ &= -4i\Lambda(iS_yS_z + S_xS_z^2) + 2i\hbar\lambda S_z^2 \end{aligned}$$

Hence, we get:

$$C^{(2)} = [H, C^{(1)}] = -2\hbar^2 C^{(1)} - 2i\hbar\lambda(S_y^2 - S_z^2) - 4i\Lambda(iS_yS_z + S_xS_z^2)$$

Further,

$$\begin{aligned} C^{(3)} = [H, C^{(2)}] &= [H, -2\hbar^2 C^{(1)} - 2i\hbar\lambda(S_y^2 - S_z^2) - 4i\Lambda(iS_yS_z + S_xS_z^2)] \\ &= -2\hbar^2 C^{(2)} - 2i\hbar\lambda[H, (S_y^2 - S_z^2)] - 4i\Lambda[H, (iS_yS_z + S_xS_z^2)] \end{aligned}$$

A Spin Algebra

$$S^2|s, m\rangle = \hbar^2 s(s+1)|s, m \pm 1\rangle \quad S_z|s, m\rangle = \hbar m|s, m\rangle \tag{3}$$

$$S_\pm|s, m\rangle = \hbar\sqrt{s(s+1) - m(m \pm 1)}|s, m \pm 1\rangle$$

where $S_+ = S_x + iS_y$ and $S_- = S_x - iS_y$. Hence, we get $S_x = (S_+ + S_-)/2$ and $S_y = (S_+ - S_-)/2i$

References

[1] Shonali Dhingra and Brian D'Urso. Nitrogen vacancy centers in diamond as angle-squared sensors. *Journal of Physics: Condensed Matter*, 29(18):185501, 2017.