

# Counter-diabatic driving using Floquet engineering

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January 31, 2018

## 1 CD driving

$$H_0 = -J \sum_j (c_{j+1}^\dagger c_j + \text{h.c.}) + \sum_j V_j(\lambda) c_j^\dagger c_j \quad (1)$$

For this problem, approximate gauge potential is chosen to be  $A_\lambda^* = i \sum_j \alpha_j (c_{j+1}^\dagger c_j - \text{h.c.})$ .

On minimizing action, we get

$$-3J^2(\alpha_{j+1} + \alpha_{j-1}) + (6J^2 + (V_{j+1} - V_j)^2)\alpha_j = -J\partial_\lambda(V_{j+1} - V_j)$$

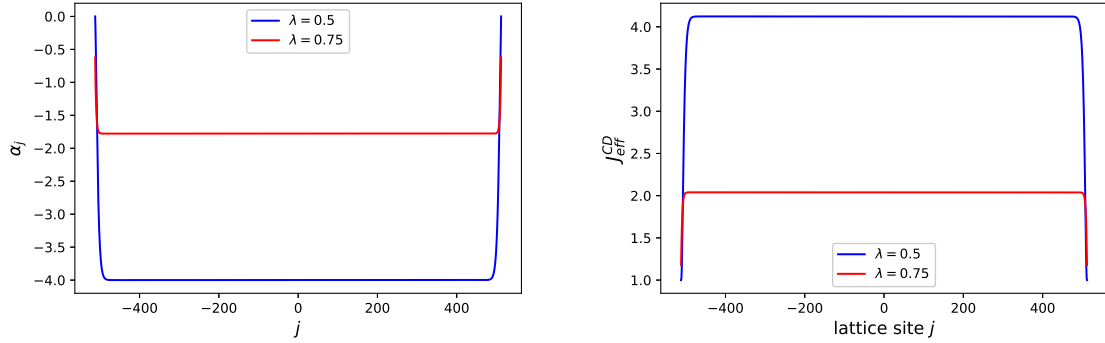


Figure 1: a)  $\alpha_j$  for linear potential with vanishing boundary condition b) Effective hopping strength

$$H_{CD} = H_0 + \dot{\lambda} A_\lambda = \sum_j J_j^{CD} (c_{i+1}^\dagger c_i + \text{h.c.}) + \sum_j U_j c_i^\dagger c_i$$

where

$$J_j^{CD} = J \sqrt{1 + (\dot{\lambda} \alpha_j / J)^2} \quad U_j = V_j(\lambda) - \sum_i^j \frac{J}{J^2 + (\dot{\lambda} \alpha_i / J)^2} (\ddot{\lambda} \alpha_j + \dot{\lambda}^2 \partial_\lambda \alpha_j)$$

Over here, I am going to work with  $\dot{\lambda} = 1$  and  $L = 1024$ .

## 2 Floquet driving

$$H = H_0 + H_1 = J \sum_j (c_{j+1}^\dagger c_j + \text{h.c.}) + \cos(\omega t) \sum_j A_j c_j^\dagger c_j$$

We would go to the rotating frame  $|\psi_{rot}\rangle = V^\dagger |\psi_{lab}\rangle$  where  $V = \exp(-i \sin(\omega t)/\omega \sum_j A_j c_j^\dagger c_j)$ .

$$\begin{aligned} H_{rot} &= V^\dagger H V - i V^\dagger \dot{V} \\ &= V^\dagger H_0 V + \cos(\omega t) \sum_j A_j c_j^\dagger c_j + i^2 \cos(\omega t) \sum_j A_j c_j^\dagger c_j \\ &= V^\dagger H_0 V = V^\dagger c_{j+1}^\dagger V V^\dagger c_j V + \text{h.c} \end{aligned}$$

Using  $[n_j, c_j] = -c_j$  and  $[n_j, c_j^\dagger] = c_j^\dagger$

$$H_{rot} = J \sum_j (g^{j,j+1} c_{j+1}^\dagger c_j + \text{h.c}) \quad \text{where} \quad g^{j,j+1} = \exp\left(i \sin(\omega t) \frac{A_{j+1} - A_j}{\omega}\right)$$

$$\begin{aligned} H_F^{(0)} &= \frac{1}{T} \sum_j \int_{t_0}^{T+t_0} (c_{j+1}^\dagger c_j \exp\left(i \sin(\omega t) \frac{A_{j+1} - A_j}{\omega}\right) dt + \text{h.c}) \\ &= \sum_j J_j^F (c_{j+1}^\dagger c_j + \text{h.c}) \quad \text{where} \quad J_j^F = J^F \mathcal{J}_0\left(\frac{A_{j+1} - A_j}{\omega}\right) \end{aligned}$$

### 3 Linear potential

We choose  $V(j, \lambda) = j\lambda$ .

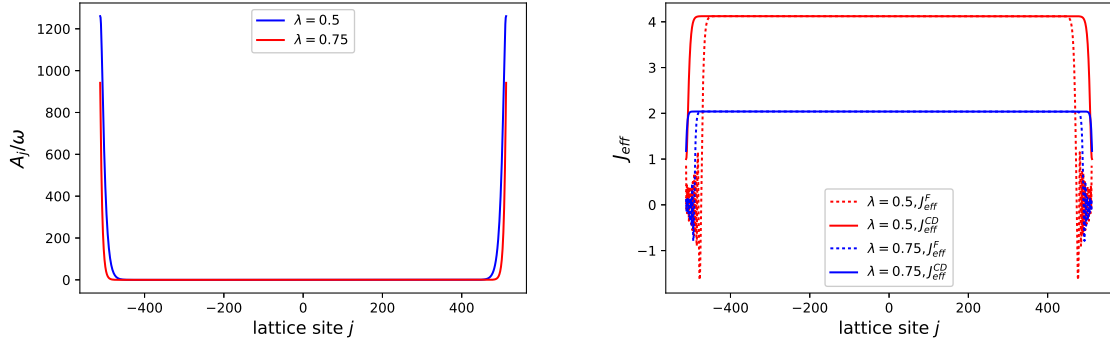


Figure 2: a) Driving field amplitude  $A_j$  b) Comparison of effective hopping strength from floquet and CD driving

### 4 Eckart potential

#### 4.1 Inserting potential

$V(\lambda, j) = \frac{\lambda(t)}{\cosh^2 j/\xi}$  where  $\xi$  is the localization length.

### A Magnus expansion

For a Hamiltonian which is periodic in time, it's unitary operator over a full driving cycle is given by:

$$U(T + t_0, t_0) = \mathcal{T}_t \exp\left(-\frac{i}{\hbar} \int_{t_0}^T dt H(t)\right) = \exp\left(-\frac{i}{\hbar} H_F[t_0]T\right) \quad (2)$$

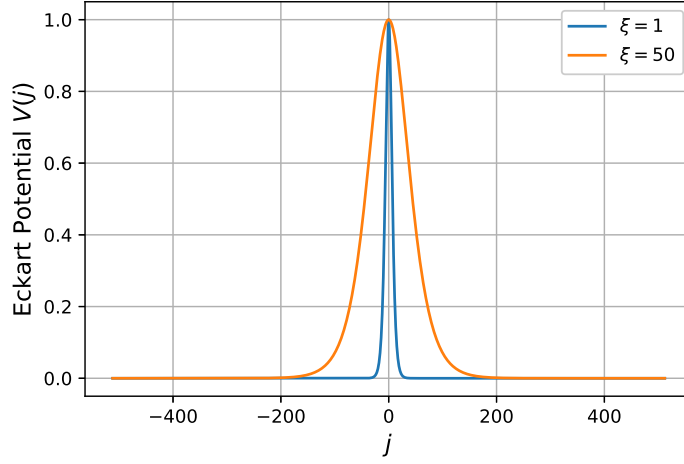


Figure 3: Eckart potential with  $\lambda = 1$

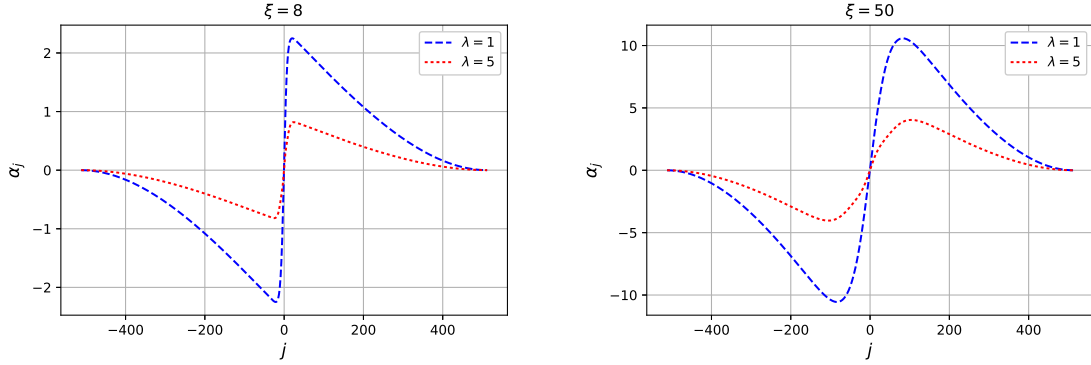


Figure 4:  $\alpha_j$  for Eckart potential with vanishing boundary condition with a)  $\xi = 8$  b)  $\xi = 50$

$H_F[t_0] = \sum_n H_F^{(n)}[t_0]$  where

$$H_F^{(0)} = \frac{1}{T} \int_{t_0}^{T+t_0} H(t) dt$$

$$H_F^{(1)} = \frac{1}{2!T i \hbar} \int_{t_0}^{T+t_0} dt_1 \int_{t_0}^{t_1} dt_2 [H(t_1), H(t_2)]$$

## B Bessel's function of first kind

Integral representation of Bessel's function of first kind  $\mathcal{J}_n(x)$  is given by:

$$\mathcal{J}_n(x) = \frac{1}{2\pi} \int_{-\pi}^{\pi} d\tau e^{i(n\tau - x \sin \tau)} = \frac{1}{T} \int_{-T/2}^{T/2} d\tau e^{i(n\omega\tau - x \sin \omega\tau)} \quad (3)$$

For  $x \ll 1$ ,  $\mathcal{J}_0(x) = 1 - \frac{x^2}{2}$

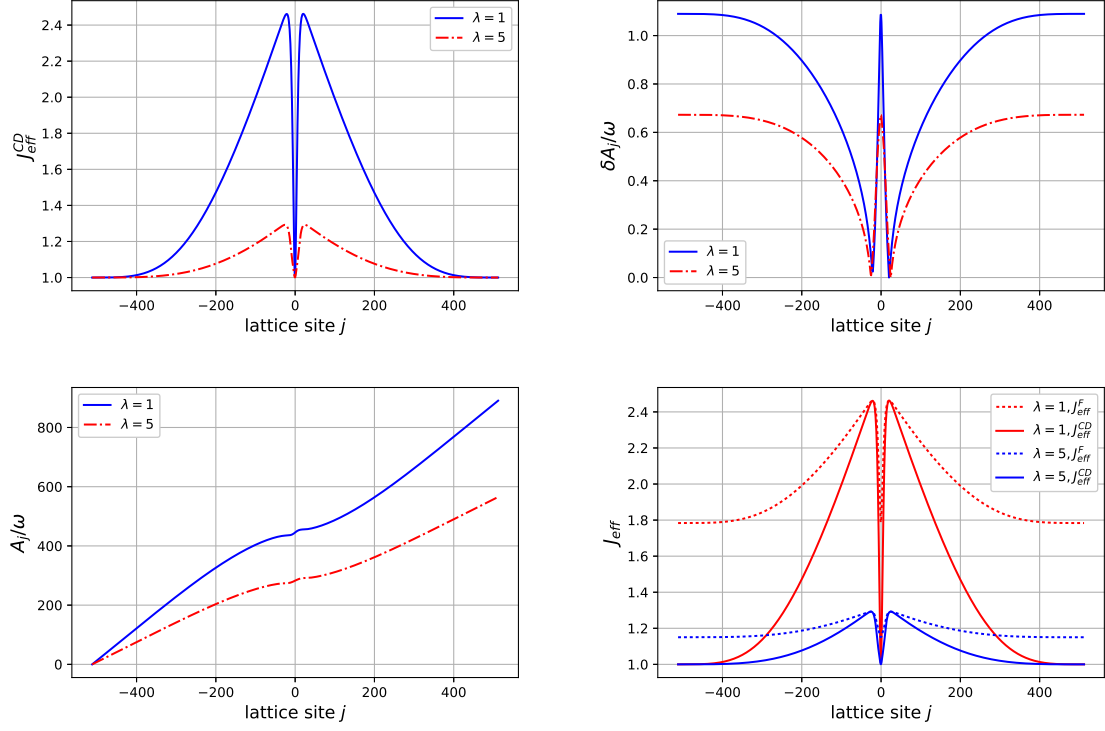


Figure 5: a) Effective hopping strength b)  $(A_{j+1} - A_j)/\omega$  c) Driving field's amplitude  $A_j/\omega$  d) Comparison of effective hopping strength from floquet and CD driving

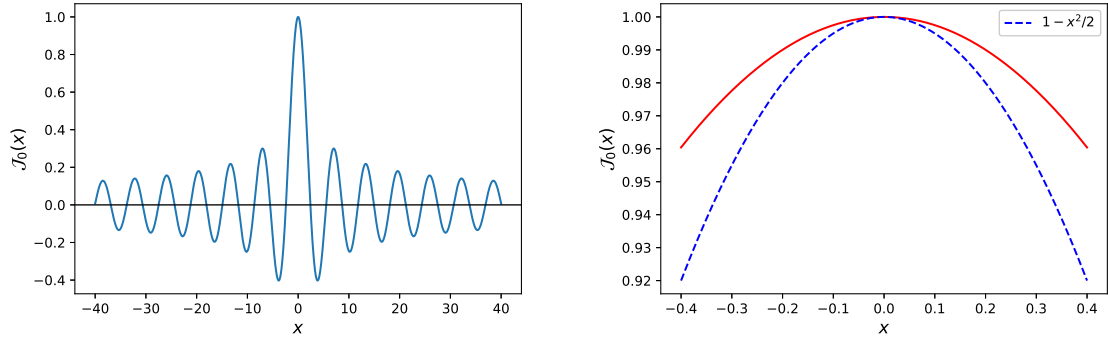


Figure 6: Bessel's function