Week-1 Problem-8

(b) For $f_s = 4000$ Hz

i) $T = \frac{1}{F_s} = 2.5 \times 10^{-4}$

ii) $\Omega_p = \frac{2}{T} \tan\left(\frac{2\pi \times 1000 \times T}{2}\right)$

$$\Omega_p = \frac{2}{2.5 \times 10^{-4}} \tan\left(\frac{2000\pi \times 2.5 \times 10^{-4}}{2}\right)$$

$$\Omega_p = 8000 \text{ rad/s}.$$

iii) $\Omega_s = \frac{2}{T} \cdot \tan\left(\frac{2\pi \times 350 \times 2.5 \times 10^{-4}}{2}\right)$

$$\Omega_s = 2256 \text{ rad/s}.$$

iv) $$N = \frac{\log \sqrt{\frac{10^{(0.1\alpha_p)} - 1}{10^{(0.1\alpha_s)} - 1}}}{\log\left(\frac{\Omega_s}{\Omega_p}\right)}$$

$$N = \frac{0.47}{0.54} = 0.87 \approx 1$$

v) $H(s) = \frac{1}{1+s} = \frac{1}{1 + \frac{8000}{s}} = \frac{s}{s + 8000}$

$$H(s) = \frac{s}{s+8000}$$

vi) $H(z) = H(s) \Big|_{s = \frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$

$$H(z) = \frac{2}{2.5\times10^{-4}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$$

$$\frac{2}{2.5\times10^{-4}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)+8000}$$

$$H(z) = \frac{8000(1-z^{-1})}{8000(1-z^{-1}) + 8000(1+z^{-1})}$$

$$H(z) = \frac{8000(1-z^{-1})}{16000}$$

$$H(z) = \frac{y(n)}{x(n)} = 0.5 - 0.5z^{-1}$$

$$y(n) = 0.5x(n) - 0.5x(n-1)$$

(b) For $f_s = 5000$

i) $T = \frac{1}{f_s} = \frac{1}{5000} = 2 \times 10^{-4}$

iii) $\Omega_p = \frac{2}{2 \times 10^{-4}} \tan\left(\frac{2\pi \cdot 1000 \times 2 \times 10^{-4}}{2}\right)$

$\Omega_p = 7265.4$ rad/sec

iii) $\Omega_s = \frac{2}{2 \times 10^{-4}} \tan\left(\frac{2\pi \times 350 \times 2 \times 10^{-4}}{2}\right)$

$\Omega_s = 2235.26$ rad/sec.

iv) $N = \frac{0.478}{0.511} = 0.935 \approx 1$

v) $H(s) = \frac{1}{1+s}$ for $N = 1$

$s = \frac{\Omega_p}{s} = \frac{1}{1 + \frac{7265}{s}}$

$H(s) = \frac{s}{s + 7265}$

vi) $H(z) = H(s) \Big| s = \frac{2}{T}\left(\frac{1 - z^{-1}}{1 + z^{-1}}\right)$

$H(z) = \frac{\frac{2}{2 \times 10^{-4}}\left(\frac{1 - z^{-1}}{1 + z^{-1}}\right)}{\frac{2}{2 \times 10^{-4}}\left(\frac{1 - z^{-1}}{1 + z^{-1}}\right) + 7265}$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{10^4\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}{10^4\left(\frac{1-z^{-1}}{1+z^{-1}}\right) + 7265}$$

$$= \frac{10^4 - 10^4 z^{-1}}{10^4 - 10^4 z^{-1} + 7265 + 7265 z^{-1}}$$

$$= \frac{10^{-4} - 10^{-4} z^{-1}}{10^{-4} - 2735 z^{-1} + 7265}$$

$$= \frac{10000(1-z^{-1})}{17265\left(1 - \frac{2735}{17265} z^{-1}\right)}$$

$$= \frac{0.579(1-z^{-1})}{1 - 0.158 z^{-1}}$$

$$Y(n) = 0.158 y(n-1) + 0.579 x(n) - 0.579 x(n-1)$$

(c) For $F_s = 25000$

i) $T = 4 \times 10^{-4}$

ii) $\Omega_p = \frac{2}{4 \times 10^{-4}} \tan\left(\frac{2\pi \times 1000 \times 4 \times 10^{-4}}{2}\right)$

$\Omega_p = 15388.41$ rad/s.

iii) $\Omega_s = \frac{2}{4 \times 10^{-4}} \tan\left(\frac{2\pi \times 8500 \times 4 \times 10^{-4}}{2}\right)$

$\Omega_s = 23528.82$ rad/s

iv) $N = \frac{\log + 0.85}{+0.815}$ $N = 1$

v) $H(s) = \frac{8}{8 + 15388}$

vi) $H(z) = \frac{2}{4 \times 10^{-4}} \left(\frac{1 - z^{-1}}{1 + z^{-1}}\right)$

$\frac{2}{4 \times 10^{-4}} \left(\frac{1 - z^{-1}}{1 + z^{-1}}\right) + 15388 \left(1 + z^{-1}\right)$

$= \frac{5000 - 5000 z^{-1}}{20388 + 10388 z^{-1}}$.

$\frac{y(n)}{x(n)} = \frac{0.24 - 0.24 z^{-1}}{1 + 0.509 z^{-1}}$.

$y(n) = 0.24 x(n) - 0.24 x(n-1) - 0.509 y(n-1)$

⑥ $F_s = 7500$ Hz

i) $T = \frac{4}{3} \times 10^{-4}$

ii) $\Omega_p = \frac{2}{1.33 \times 10^{-4}} \tan\left(\frac{2\pi \times 1000 \times 1.33 \times 10^{-4}}{2}\right)$

$\Omega_p = 6678.43$ rad/s.

iii) $\Omega_s = \left(\frac{2}{T}\right) \tan\left(\frac{2\pi \times 350 \times T}{2}\right)$

$\Omega_s = 2211.79$ rad/sec.

iv) $N = \frac{-0.47}{-0.474}$

v) $H(s) = \frac{s}{s + 6678}$

vi) $H(z) = \frac{2 \times 3}{4 \times 10^{-4}} \frac{(1 - z^{-1})}{(1 + z^{-1})}$

$\frac{3}{2 \times 10^{-4}} \cdot \frac{(1 - z^{-1})}{(1 + z^{-1})} + 6678$

$\frac{Y(z)}{X(z)} = \frac{1500 - 1500\, z^{-1}}{21659.61 - 8340.39 z^{-1}}$

$y(n) = 0.69x(n) - 0.69x(n-1) + 0.38y(n-1)$.

⑥ For $F_s = 10000$ Hz

i) $T = 1 \times 10^{-4}$

ii) $\Omega_p = \frac{2}{T} \tan\left(\frac{2\pi \times 1000 \times T}{2}\right)$

$\Omega_p = 6498.39$ rad/s

iii) $\Omega_s = \frac{2}{T} \tan\left(\frac{2\pi \times 3501 \times T}{2}\right)$

$\Omega_s = 22080.2$ rad/s

iv) $N = \frac{-0.47}{-0.46} = 1.03 \approx 1$

v) $H(s) = \frac{s}{s+6498}$

vi) $H(z) = \frac{\frac{2}{10^{-4}}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}{\frac{2}{10^{-4}}\left(\frac{1-z^{-1}}{1+z^{-1}}\right) + 6498}$

$H(z) = \frac{20000(1-z^{-1})}{26498 - 13502 z^{-1}}$

$\frac{y(n)}{x(n)} = \frac{0.754 - 0.754 z^{-1}}{1 - 0.509 z^{-1}}$

$y(n) = 0.754\, x(n) - 0.754\, x(n-1) + 0.509\, y(n-1)$