Chapter 3

2D Transformations

3.1 Transformation

In Computer graphics, Transformation is a process of modifying and re-positioning the existing graphics.

- 2D Transformations take place in a two dimensional plane.
- Transformations are helpful in changing the position, size, orientation, shape etc of the object.

3.1.1 Transformation Techniques :

In computer graphics, various transformation techniques are-

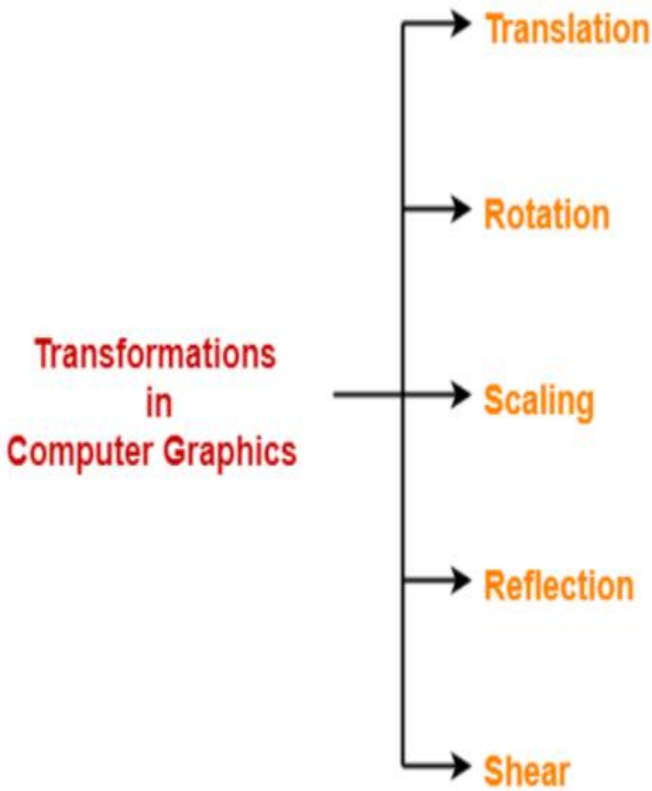

Figure: 3.1 Types of Transformation

3.2 2D Translation in Computer Graphics

In Computer graphics, 2D Translation is a process of moving an object from one position to another in a two dimensional plane.

Consider a point object O has to be moved from one position to another in a 2D plane.

Let-

- Initial coordinates of the object O = (X_{old}, Y_{old})
- New coordinates of the object O after translation = (X_{new}, Y_{new})

- Given a Translation vector (T_x, T_y)-
- T_x defines the distance the X_{old} coordinate has to be moved.
- T_y defines the distance the Y_{old} coordinate has to be moved.

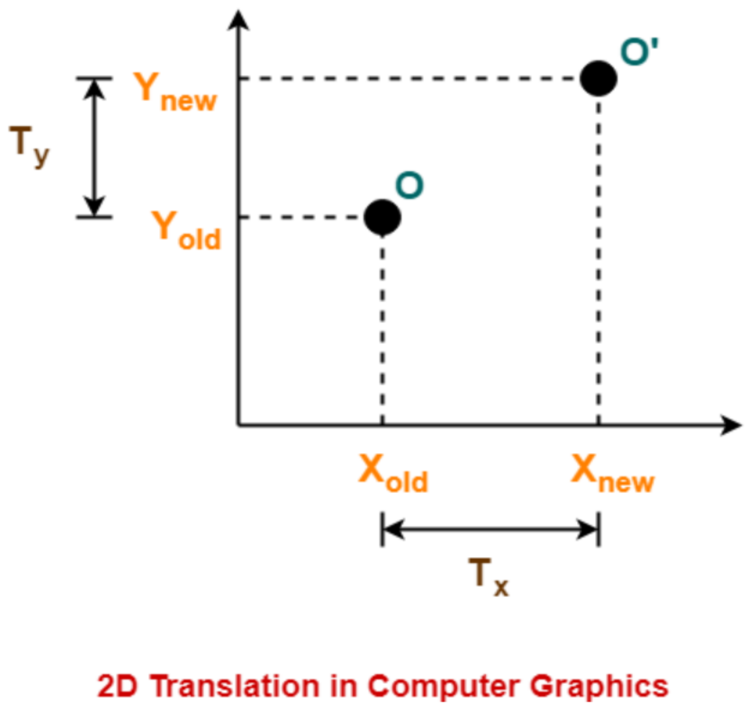

Figure 3.2 2D Translation in Computer Graphics

This translation is achieved by adding the translation coordinates to the old coordinates of the object as-

- $X_{new} = X_{old} + T_x$ (This denotes translation towards X axis)
- $Y_{new} = Y_{old} + T_y$ (This denotes translation towards Y axis)

In Matrix form, the above translation equations may be represented as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} X_{old} \\ Y_{old} \end{bmatrix} + \begin{bmatrix} T_x \\ T_y \end{bmatrix}$$

Translation Matrix

➢ PRACTICE PROBLEMS BASED ON 2D TRANSLATION IN COMPUTER GRAPHICS-

Given a circle C with radius 10 and center coordinates (1, 4). Apply the translation with distance 5 towards X axis and 1 towards Y axis. Obtain the new coordinates of C without changing its radius.

Given-

- Old center coordinates of C = (X_{old}, Y_{old}) = (1, 4)
- Translation vector = (T_x, T_y) = (5, 1)

Let the new center coordinates of C = (X_{new}, Y_{new}).

Applying the translation equations, we have-

- $X_{new} = X_{old} + T_x = 1 + 5 = 6$
- $Y_{new} = Y_{old} + T_y = 4 + 1 = 5$

Thus, New center coordinates of C = (6, 5).

Alternatively,

In matrix form, the new center coordinates of C after translation may be obtained as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} X_{old} \\ Y_{old} \end{bmatrix} + \begin{bmatrix} T_x \\ T_y \end{bmatrix}$$

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \end{bmatrix} + \begin{bmatrix} 5 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} 6 \\ 5 \end{bmatrix}$$

Thus, New center coordinates of C = (6, 5).

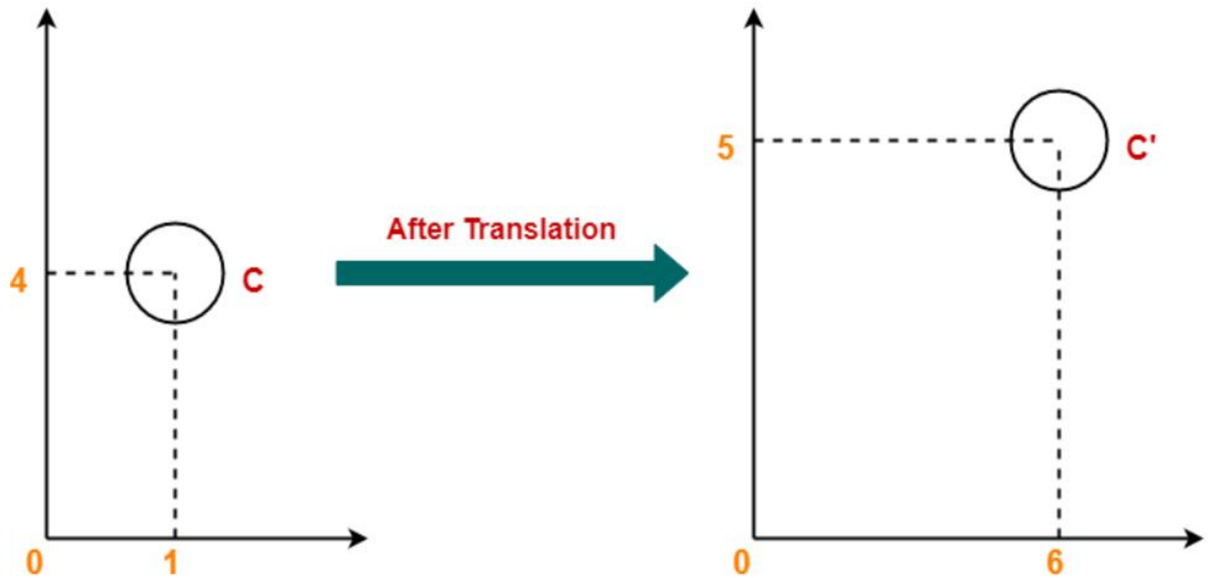

Figure 3.3 Example of Translation

3.3 2D Scaling in Computer Graphics

In computer graphics, scaling is a process of modifying or altering the size of objects.

- Scaling may be used to increase or reduce the size of object.
- Scaling subjects the coordinate points of the original object to change.
- Scaling factor determines whether the object size is to be increased or reduced.
- If scaling factor > 1, then the object size is increased.
- If scaling factor < 1, then the object size is reduced.

Consider a point object O has to be scaled in a 2D plane.

Let-

Initial coordinates of the object O = (Xold, Yold)

Scaling factor for X-axis = Sx

Scaling factor for Y-axis = Sy

New coordinates of the object O after scaling = (Xnew, Ynew)

This scaling is achieved by using the following scaling equations-

Xnew = Xold x Sx

Ynew = Yold x Sy

In Matrix form, the above scaling equations may be represented as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \end{bmatrix}$$

Scaling Matrix

➢ PRACTICE PROBLEMS BASED ON 2D SCALING IN COMPUTER GRAPHICS

Given a square object with coordinate points A(0, 3), B(3, 3), C(3, 0), D(0, 0). Apply the scaling parameter 2 towards X axis and 3 towards Y axis and obtain the new coordinates of the object.

Solution-

Given-

- Old corner coordinates of the square = A (0, 3), B(3, 3), C(3, 0), D(0, 0)
- Scaling factor along X axis = 2
- Scaling factor along Y axis = 3

For Coordinates A(0, 3)

Let the new coordinates of corner A after scaling = (X_{new}, Y_{new}).

Applying the scaling equations, we have-

- $X_{new} = X_{old} \times S_x = 0 \times 2 = 0$
- $Y_{new} = Y_{old} \times S_y = 3 \times 3 = 9$

Thus, New coordinates of corner A after scaling = (0, 9).

For Coordinates B(3, 3)

Let the new coordinates of corner B after scaling = (X_{new}, Y_{new}).

Applying the scaling equations, we have-

- $X_{new} = X_{old} \times S_x = 3 \times 2 = 6$
- $Y_{new} = Y_{old} \times S_y = 3 \times 3 = 9$

Thus, New coordinates of corner B after scaling = (6, 9).

For Coordinates C(3, 0)

Let the new coordinates of corner C after scaling = (X_{new}, Y_{new}).

Applying the scaling equations, we have-

- $X_{new} = X_{old} \times S_x = 3 \times 2 = 6$
- $Y_{new} = Y_{old} \times S_y = 0 \times 3 = 0$

Thus, New coordinates of corner C after scaling = (6, 0).

For Coordinates D(0, 0)

Let the new coordinates of corner D after scaling = (X_{new}, Y_{new}).

Applying the scaling equations, we have-

- $X_{new} = X_{old} \times S_x = 0 \times 2 = 0$
- $Y_{new} = Y_{old} \times S_y = 0 \times 3 = 0$

Thus, New coordinates of corner D after scaling = (0, 0).

Thus, New coordinates of the square after scaling = A (0, 9), B(6, 9), C(6, 0), D(0, 0).

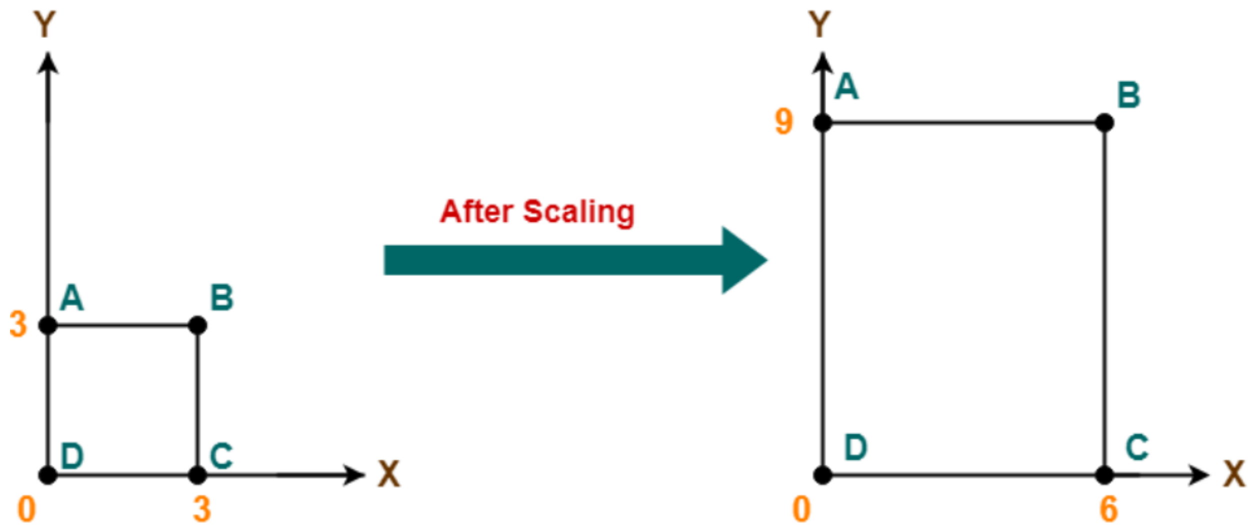

Figure 3.4 Example of Scaling

3.4 2D Rotation in Computer Graphics-

In Computer graphics, 2D Rotation is a process of rotating an object with respect to an angle in a two dimensional plane.

Consider a point object O has to be rotated from one angle to another in a 2D plane.

Let-

- Initial coordinates of the object O = (X_{old}, Y_{old})
- Initial angle of the object O with respect to origin = Φ
- Rotation angle = θ
- New coordinates of the object O after rotation = (X_{new}, Y_{new})

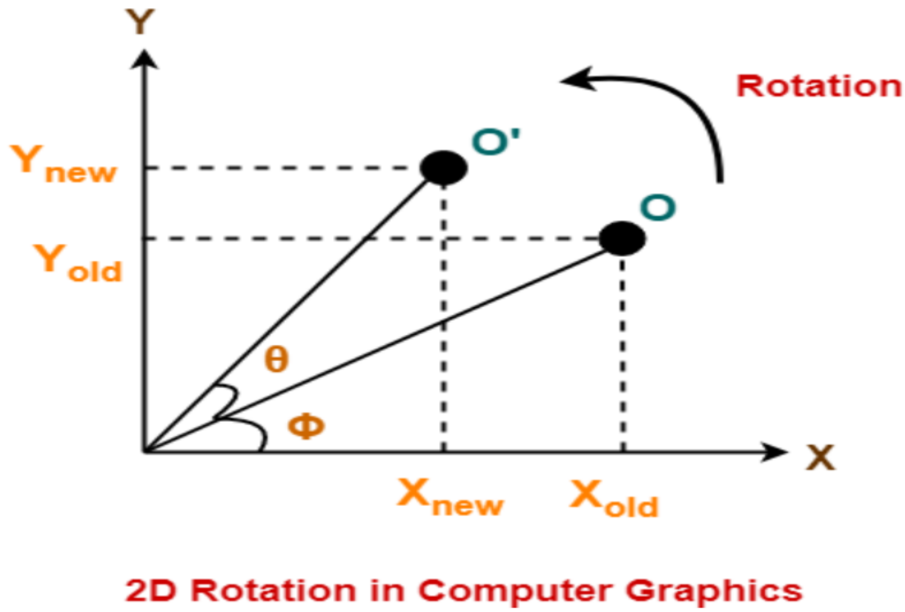

Figure 3.5 Example of Translation

This rotation is achieved by using the following rotation equations-

- $X_{new} = X_{old} \times \cos\theta - Y_{old} \times \sin\theta$
- $Y_{new} = X_{old} \times \sin\theta + Y_{old} \times \cos\theta$

In Matrix form, the above rotation equations may be represented as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \end{bmatrix}$$

Rotation Matrix

➢ PRACTICE PROBLEMS BASED ON 2D ROTATION IN COMPUTER GRAPHICS

Given a line segment with starting point as (0, 0) and ending point as (4, 4). Apply 30 degree rotation anticlockwise direction on the line segment and find out the new coordinates of the line.

Solution-

We rotate a straight line by its end points with the same angle. Then, we re-draw a line between the new end points.

Given-

- Old ending coordinates of the line = (X_{old}, Y_{old}) = (4, 4)
- Rotation angle = $\theta = 30^{o}$

Let new ending coordinates of the line after rotation = (X_{new}, Y_{new}).

Applying the rotation equations, we have-

$X_{new} = X_{old} \times \cos\theta - Y_{old} \times \sin\theta$

$= 4 \times \cos 30^o - 4 \times \sin 30^o$

$= 4 \times (\sqrt{3} / 2) - 4 \times (1 / 2)$

$= 2\sqrt{3} - 2$

$= 2(\sqrt{3} - 1)$

$= 2(1.73 - 1)$

$= 1.46$

$Y_{new} = X_{old} \times \sin\theta + Y_{old} \times \cos\theta$

$= 4 \times \sin 30^o + 4 \times \cos 30^o$

$= 4 \times (1 / 2) + 4 \times (\sqrt{3} / 2)$

$= 2 + 2\sqrt{3}$

$= 2(1 + \sqrt{3})$

$= 2(1 + 1.73)$

$= 5.46$

Thus, New ending coordinates of the line after rotation = (1.46, 5.46).

Alternatively,

In matrix form, the new ending coordinates of the line after rotation may be obtained as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \end{bmatrix}$$

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} \cos30 & -\sin30 \\ \sin30 & \cos30 \end{bmatrix} \times \begin{bmatrix} 4 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} 4 \times \cos30 - 4 \times \sin30 \\ 4 \times \sin30 + 4 \times \cos30 \end{bmatrix}$$

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} 4 \times \cos30 - 4 \times \sin30 \\ 4 \times \sin30 + 4 \times \cos30 \end{bmatrix}$$

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} 1.46 \\ 5.46 \end{bmatrix}$$

Thus, New ending coordinates of the line after rotation = (1.46, 5.46).

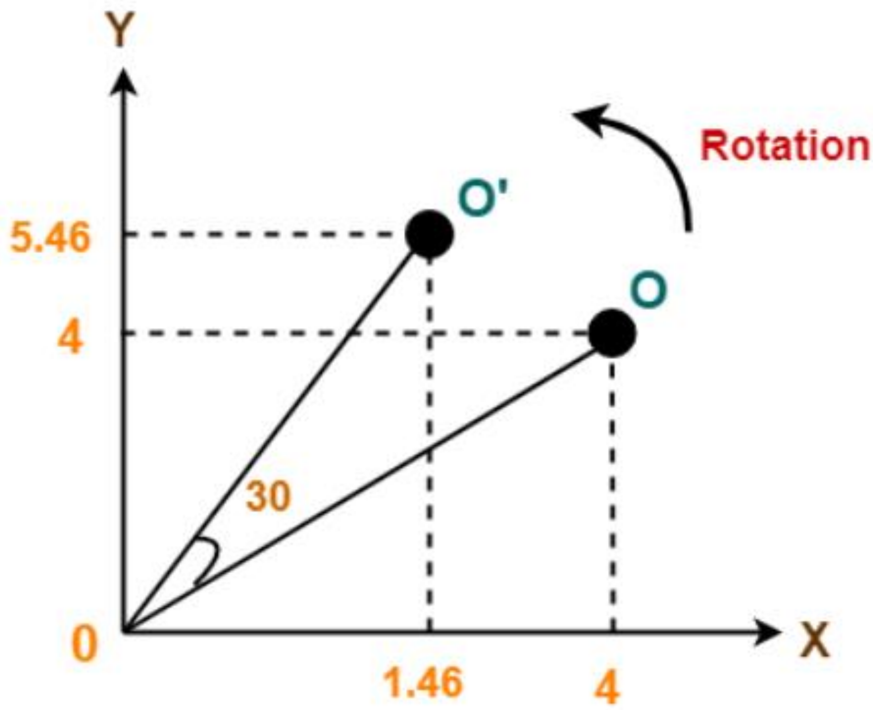

Figure 3.6 Example of Translation

3.5 2D Reflection in Computer Graphics

- Reflection is a kind of rotation where the angle of rotation is 180 degree.
- The reflected object is always formed on the other side of mirror.
- The size of reflected object is same as the size of original object.

Consider a point object O has to be reflected in a 2D plane.

Let-

- Initial coordinates of the object O = (X_{old}, Y_{old})
- New coordinates of the reflected object O after reflection = (X_{new}, Y_{new})

Reflection On X-Axis:

This reflection is achieved by using the following reflection equations-

- $X_{new} = X_{old}$
- $Y_{new} = -Y_{old}$

In Matrix form, the above reflection equations may be represented as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \end{bmatrix}$$

Reflection Matrix

(Reflection Along X Axis)

Reflection On Y-Axis:

This reflection is achieved by using the following reflection equations-

- $X_{new} = -X_{old}$
- $Y_{new} = Y_{old}$

In Matrix form, the above reflection equations may be represented as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \end{bmatrix}$$

Reflection Matrix

(Reflection Along Y Axis)

❖ PRACTICE PROBLEMS BASED ON 2D REFLECTION IN COMPUTER GRAPHICS-

Given a triangle with coordinate points A(3, 4), B(6, 4), C(5, 6). Apply the reflection on the X axis and obtain the new coordinates of the object.

Solution-

Given-

- Old corner coordinates of the triangle = A (3, 4), B(6, 4), C(5, 6)
- Reflection has to be taken on the X axis

For Coordinates A(3, 4)

Let the new coordinates of corner A after reflection = (X_{new}, Y_{new}).

Applying the reflection equations, we have-

- $X_{new} = X_{old} = 3$
- $Y_{new} = -Y_{old} = -4$

Thus, New coordinates of corner A after reflection = (3, -4).

For Coordinates B(6, 4)

Let the new coordinates of corner B after reflection = (X_{new}, Y_{new}).

Applying the reflection equations, we have-

- $X_{new} = X_{old} = 6$
- $Y_{new} = -Y_{old} = -4$

Thus, New coordinates of corner B after reflection = (6, -4).

For Coordinates C(5, 6)

Let the new coordinates of corner C after reflection = (X_{new}, Y_{new}).

Applying the reflection equations, we have-

- $X_{new} = X_{old} = 5$
- $Y_{new} = -Y_{old} = -6$

Thus, New coordinates of corner C after reflection = (5, -6).

Thus, New coordinates of the triangle after reflection = A (3, -4), B(6, -4), C(5, -6).

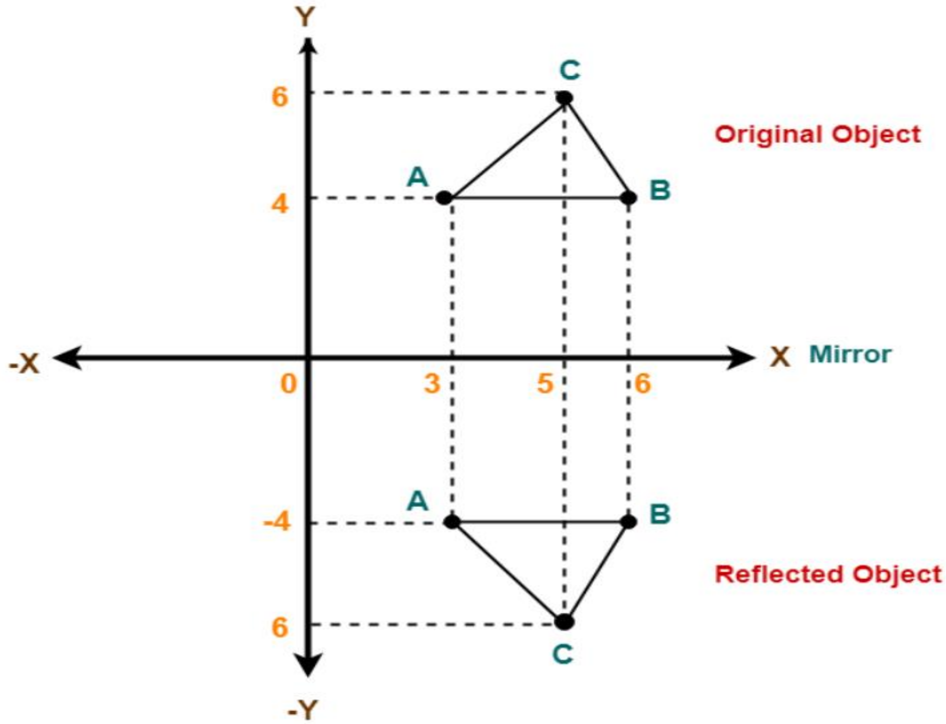

Figure 3.7 Example of Reflection

3.6 2D Shearing in Computer Graphics

In Computer graphics,

2D Shearing is an ideal technique to change the shape of an existing object in a two dimensional plane.

In a two dimensional plane, the object size can be changed along X direction as well as Y direction.

So, there are two versions of shearing-

1. Shearing in X direction
2. Shearing in Y direction

Consider a point object O has to be sheared in a 2D plane.

Let-

- Initial coordinates of the object O = (X_{old}, Y_{old})
- Shearing parameter towards X direction = Sh_x
- Shearing parameter towards Y direction = Sh_y
- New coordinates of the object O after shearing = (X_{new}, Y_{new})

Shearing in X Axis-

Shearing in X axis is achieved by using the following shearing equations-

- $X_{new} = X_{old} + Sh_x \times Y_{old}$
- $Y_{new} = Y_{old}$

In Matrix form, the above shearing equations may be represented as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} 1 & Sh_x \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \end{bmatrix}$$

Shearing Matrix

(In X axis)

Shearing in Y Axis-

Shearing in Y axis is achieved by using the following shearing equations-

- $X_{new} = X_{old}$
- $Y_{new} = Y_{old} + Sh_y \times X_{old}$

In Matrix form, the above shearing equations may be represented as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ Sh_y & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \end{bmatrix}$$

Shearing Matrix

(In Y axis)

❖ PRACTICE PROBLEMS BASED ON 2D SHEARING IN COMPUTER GRAPHICS

Given a triangle with points (1, 1), (0, 0) and (1, 0). Apply shear parameter 2 on X axis and 2 on Y axis and find out the new coordinates of the object.

Solution-

Given-

- Old corner coordinates of the triangle = A (1, 1), B(0, 0), C(1, 0)
- Shearing parameter towards X direction (Sh_x) = 2
- Shearing parameter towards Y direction (Sh_y) = 2

Shearing in X Axis-

For Coordinates A(1, 1)

Let the new coordinates of corner A after shearing = (X_{new}, Y_{new}).

Applying the shearing equations, we have-

- $X_{new} = X_{old} + Sh_x \times Y_{old} = 1 + 2 \times 1 = 3$
- $Y_{new} = Y_{old} = 1$

Thus, New coordinates of corner A after shearing = (3, 1).

For Coordinates B(0, 0)

Let the new coordinates of corner B after shearing = (X_{new}, Y_{new}).

Applying the shearing equations, we have-

- $X_{new} = X_{old} + Sh_x \times Y_{old} = 0 + 2 \times 0 = 0$
- $Y_{new} = Y_{old} = 0$

Thus, New coordinates of corner B after shearing = (0, 0).

For Coordinates C(1, 0)

Let the new coordinates of corner C after shearing = (X_{new}, Y_{new}).

Applying the shearing equations, we have-

- $X_{new} = X_{old} + Sh_x \times Y_{old} = 1 + 2 \times 0 = 1$
- $Y_{new} = Y_{old} = 0$

Thus, New coordinates of corner C after shearing = (1, 0).

Thus, New coordinates of the triangle after shearing in X axis = A (3, 1), B(0, 0), C(1, 0).

Shearing in Y Axis-

For Coordinates A(1, 1)

Let the new coordinates of corner A after shearing = (X_{new}, Y_{new}).

Applying the shearing equations, we have-

- $X_{new} = X_{old} = 1$
- $Y_{new} = Y_{old} + Sh_y \times X_{old} = 1 + 2 \times 1 = 3$

Thus, New coordinates of corner A after shearing = (1, 3).

For Coordinates B(0, 0)

Let the new coordinates of corner B after shearing = (X_{new}, Y_{new}).

Applying the shearing equations, we have-

- $X_{new} = X_{old} = 0$
- $Y_{new} = Y_{old} + Sh_y \times X_{old} = 0 + 2 \times 0 = 0$

Thus, New coordinates of corner B after shearing = (0, 0).

For Coordinates C(1, 0)

Let the new coordinates of corner C after shearing = (X_{new}, Y_{new}).

Applying the shearing equations, we have-

- $X_{new} = X_{old} = 1$
- $Y_{new} = Y_{old} + Sh_y \times X_{old} = 0 + 2 \times 1 = 2$

Thus, New coordinates of corner C after shearing = (1, 2).

Thus, New coordinates of the triangle after shearing in Y axis = A (1, 3), B(0, 0), C(1, 2).

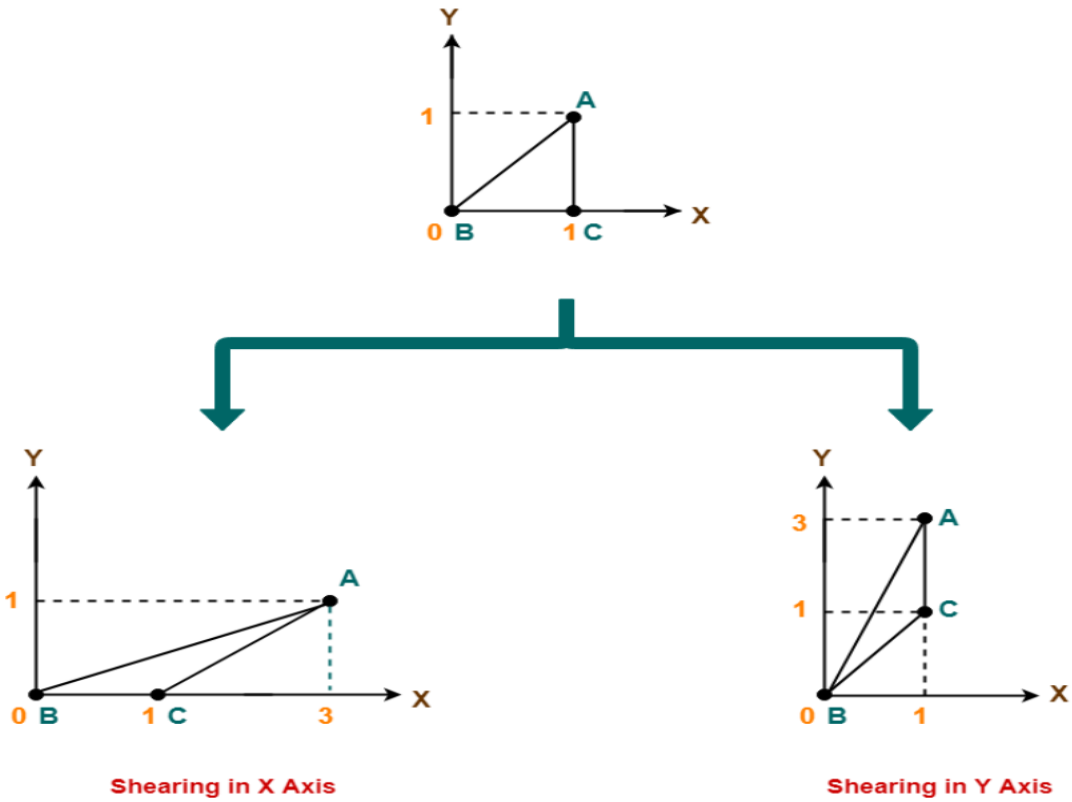

Figure 3.8 Example of Shearing

3D Transformation

3.5 Introduction

Manipulation, viewing, and construction of 3-Dimensional graphic images requires the use of three-dimensional geometric and coordinate transformation. These transformations are formed by composing the basic transformations of translation, scaling and rotation. Each of these transformations can be represented as a matrix transformation. This permits more complex transformations to be built up by use of matrix multiplication or concatenation.

3.6 Geometric Transformation

With respect to some three-dimensional coordinate system, an object *obj* is considered as a set of points.

$Obj = \{P(x, y, z)\}$

If the object is moved to a new position, we can regard it as a new object Obj', all of whose coordinate points P'(x'', y', z') can be obtained from the original coordinate point P (x, y, z) of Obj through the application of a geometric transformation.

3.7 Translation

An object is displaced a given distance and direction from its original position. The direction and displacement from its original position.

The new coordinates of a translated point can be calculated by using the transformation

$$T_{\mathbf{v}}: \begin{cases} x' = x + a \\ y' = y + b \\ z' = z + c \end{cases}$$

(see Fig. 6-1). In order to represent this transformation as a matrix transformation, we need to use homogeneous coordinates (App. 2). The required homogeneous matrix transformation can then be expressed as

$$\begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

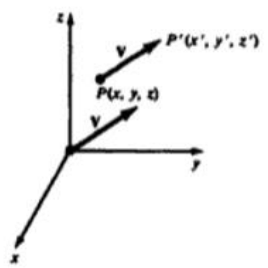

Fig. 6-1

3.8 Scaling

The process of scaling changes the dimensions of an object. The scale factor s determines whether the scaling is a magnification, $s > 1$, or a reduction, $s < 1$.

Scaling with respect to the origin, where the origin remains fixed, is effected by the transformation

$$S_{s_x, s_y, s_z}: \begin{cases} x' = s_x \cdot x \\ y' = s_y \cdot y \\ z' = s_z \cdot z \end{cases}$$

In matrix form this is

$$S_{s_x, s_y, s_z} = \begin{pmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & s_z \end{pmatrix}$$

3.9 Rotation

Rotation in three dimensions is considerably more complex then rotation in two dimensions. In two dimensions, a rotation is prescribed by an angle of rotation θ and a center of rotation P. Three-dimensional rotations require the prescriptions of an angle of rotation and an axis of rotation. The canonical rotations are defined when one of the positive x, y, z coordinate axes is chosen as the axis of rotation. Then the construction of the rotation transformation proceeds just like that of a rotation in two dimensions about the origin.

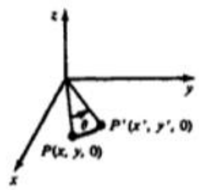

Fig. 6-2

Rotation about the z Axis

From Chap. 4 we know that

$$R_{\theta,\mathbf{K}}:\begin{cases} x' = x\cos\theta - y\sin\theta \\ y' = x\sin\theta + y\cos\theta \\ z' = z \end{cases}$$

Rotation about the y Axis

An analogous derivation leads to

$$R_{\theta,\mathbf{J}}:\begin{cases} x' = x\cos\theta + z\sin\theta \\ y' = y \\ z' = -x\sin\theta + z\cos\theta \end{cases}$$

Rotation about the x Axis

Similarly:

$$R_{\theta,\mathbf{I}}:\begin{cases} x' = x \\ y' = y\cos\theta - z\sin\theta \\ z' = y\sin\theta + z\cos\theta \end{cases}$$

Note that the direction of a positive angle of rotation is chosen in accordance to the right-hand rule with the respect to the axis of rotation.

Reference

1. Chapter 3: Schaum's Outline of Theory & Problems of Computer Graphics
2. https://www.gatevidyalay.com/2d-transformation-in-computer-graphics-translation-examples/
3. https://www.gatevidyalay.com/scaling-in-computer-graphics-definition-examples/
4. https://www.gatevidyalay.com/2d-rotation-in-computer-graphics-definition-examples/
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6. https://www.gatevidyalay.com/2d-shearing-in-computer-graphics-definition-examples/