A quick recap of the material covered in lectures

Effective Mass

Effective mass relates the motion of a particle in a crystal to an externally applied force and takes into account the effect of the crystal lattice on the motion of the particle. The effective mass of an electron, m^*, is generally calculated by approximating the band edges by a parabolic equation -

$$\boxed{E - E_c = C_1 k^2} \tag{1}$$

The energy E_c is the energy at the bottom of the band. Since $E > E_c$, the parameter C_1 is a positive quantity. The effective mass, m^*, is the calculated as -

$$\boxed{\frac{1}{\hbar^2}\frac{d^2E}{d^2k} = \frac{2C_1}{\hbar^2} = \frac{1}{m^*}} \tag{2}$$

$Eq.2$ implies that m^* is inversely proportional to the curvature of the energy band. A similar analogy can be applied to the holes in the valance band.

Equilibrium carrier concentrations - Intrinsic semiconductor

For an intrinsic semiconductor, the thermal equilibrium electron and hole concentrations are given by

$$\boxed{n_0 = N_c\ exp\left(\frac{-(E_c - E_F)}{kT}\right)} \quad \boxed{p_0 = N_v\ exp\left(\frac{-(E_F - E_v)}{kT}\right)} \tag{3}$$

The intrinsic carrier concentration is given by -

$$\boxed{n_i^2 = N_c N_v\ exp\left(\frac{-E_g}{kT}\right)} \tag{4}$$

The intrinsic Fermi level w.r.t the midgap position depends on the effective masses of electron and hole, and is given by -

$$\boxed{E_{Fi} - E_{midgap} = \frac{3kT}{4} ln\left(\frac{m_p^*}{m_n^*}\right)} \tag{5}$$

Equilibrium carrier concentrations - Extrinsic semiconductor

For an extrinsic semiconductor, the thermal equilibrium electron and hole concentrations are given by -

$$\boxed{n_0 = n_i\ exp\left(\frac{(E_F - E_{Fi})}{kT}\right)} \quad \boxed{p_0 = n_i\ exp\left(\frac{(E_{Fi} - E_F)}{kT}\right)} \tag{6}$$

Alternatively,

$$\boxed{n_0 = \frac{N_d - N_a}{2} + \sqrt{\left(\frac{N_d - N_a}{2}\right)^2 + n_i^2}} \quad \boxed{p_0 = \frac{N_a - N_d}{2} + \sqrt{\left(\frac{N_a - N_d}{2}\right)^2 + n_i^2}} \tag{7}$$

where N_d and N_a are donor and acceptor concentrations, respectively.

The mass action law is given by -

$$\boxed{n_0 p_0 = n_i^2} \tag{8}$$

The position of Fermi level w.r.t intrinsic Fermi level is -

$$\boxed{E_F - E_{Fi} = kT\ ln\left(\frac{n_0}{n_i}\right)} \quad \boxed{E_{Fi} - E_F = kT\ ln\left(\frac{p_0}{n_i}\right)} \tag{9}$$

Carrier Drift

The drift current density and drift velocity for a semiconductor in an applied electric field, $\mathscr{E}$, is given by-

$$\boxed{J_{drift} = e(\mu_n n + \mu_p p)\mathscr{E} = \sigma\mathscr{E}} \quad \boxed{v_{drift} = \frac{e\tau\mathscr{E}}{m*}} \tag{10}$$

where σ is the conductivity, τ is the mean time between collisions, and μ_n, μ_p are mobilities of electrons and holes respectively.

Solve the following questions. There are 18 questions, for a total of 25 marks.

1. (1 mark) Consider a region of Si, which is a perfect single crystal except for the phosphorous atom shown in the figure 1. Note that the P atom has donated one electron to the lattice. The net charge in region A and B is -

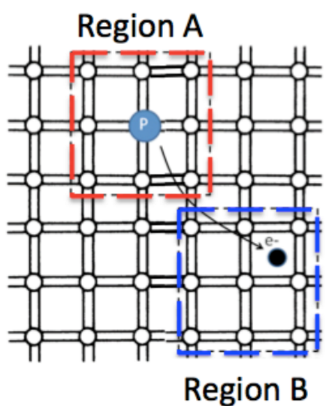


Figure 1: Silicon Lattice

   A. neutral, negative

   B. positive, positive

   C. positive, negative

   D. neutral, neutral

   E. negative, positive

   F. negative, negative

2. (1 mark) As temperature increases from $T = 5\ K$ to $T = 700\ K$, the carrier concentration goes through three regions. In what order does the transition occur?

   A. intrinsic, extrinsic, partially ionized

   B. extrinsic, partially ionized, intrinsic

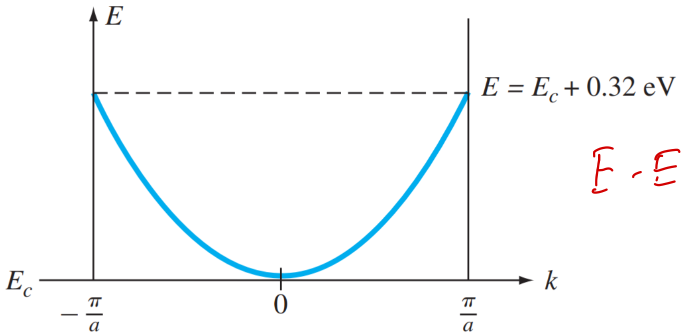
   C. partially ionized, intrinsic, extrinsic

   D. intrinsic, partially ionized, extrinsic

   E. partially ionized, extrinsic, intrinsic

   F. extrinsic, intrinsic, partially ionized

3. (2 marks) A simplified E vs k curve for an electron in the conduction band is shown in figure 2. The value of a is 10 $\mathring{A}$. Determine the relative effective mass $\frac{m^*}{m0}$. (*Hint:* Use the $E - E_c$ value at $k = \frac{\pi}{a}$ to compute the constant C_1.)



$E - E_c = 0.32\,eV$ @ $k = \pi/a$

Figure 2: Effective mass

A. 0.2985

B. 1.175

C. 0.597

D. 0.1175

E. 2.985

F. 2.350

Sq 1 $\Rightarrow$ $E - E_c = C_1 k^2$

$$\frac{1}{\hbar^2}\frac{d^2E}{dk^2} = \frac{2C_1}{\hbar^2} = \frac{1}{m^*}$$

$$0.32 \times 1.6\times10^{-19} = C_1 \times \frac{\pi^2}{(1\times10^{-9})^2}$$

$$C_1 = \frac{0.32 \times 1.6\times10^{-19}\times10^{-18}}{\pi^2}$$

$$= 5.19\times10^{-39}$$

$$\frac{m^*}{m_0} = \frac{\hbar^2}{2C_1 m_0} = \frac{(1.05\times10^{-34})^2}{2\times5.19\times10^{-39}\times9.1\times10^{-31}}$$

$$= 1.17$$

4. (1 mark) Assume that the Fermi level is near the valence band. Which of the following is true?

   A. $n = p = n_i$

   B. $n > p,\ n >> n_i$

   C. $p >> n,\ p > n_i$

   D. $n + p = n_i$

   E. $n^2 + p^2 = n_i^2$

   F. $p = n,\ p >> n_i$

Analyze why other options are wrong

5. (1 mark) Which of the following is the Fermi function?

   A. $f = \dfrac{1}{1 - e^{(E-E_F)/k_BT}}$

   B. $f = \dfrac{1}{1 + e^{(E-E_F)/k_BT}}$

   C. $f = 1 - e^{(E+E_F)/k_BT}$

   D. $f = \dfrac{1}{1 + e^{(E+E_F)/k_BT}}$

   E. $f = \dfrac{1}{1 - e^{(E+E_F)/k_BT}}$

   F. $f = 1 + e^{(E+E_F)/k_BT}$

6. (2 marks) The probability that a quantum state at energy $E = E_c + kT$ is occupied by an electron~~, and its electron concentration (n_0), in GaAs at $T = 300\ K$~~ if the Fermi energy is $0.25\ eV$ below E_c is ______ (take $kT = 0.025\ eV$, $N_{c,GaAs} = 4.7 \times 10^{17}\ cm^{-3}$ and $\frac{m_n^*}{m_0} = 0.067$)

   Also, calculate the electron concentration assuming the material to be GaAs

   A. 1.67×10^{-5}, $n_0 = 1.5 \times 10^{10}\ cm^{-3}$

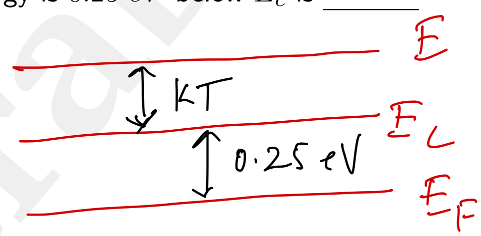
   B. 1.67×10^{-3}, $n_0 = 1.9 \times 10^{15}\ cm^{-3}$

   C. 2.13×10^{-3}, $n_0 = 1.67 \times 10^{15}\ cm^{-3}$

   D. 1.67×10^{-5}, $n_0 = 2.13 \times 10^{13}\ cm^{-3}$

   E. 1.5×10^{-5}, $n_0 = 1.67 \times 10^{10}\ cm^{-3}$

   F. 1.67×10^{-3}, $n_0 = 1.5 \times 10^{15}\ cm^{-3}$



$E = E_F + 0.25 + kT$

$$f(E) = \frac{1}{1 + e^{(E-E_F)/kT}} = \frac{1}{1 + \exp\left((kT+0.25)/kT\right)} = 1.67 \times 10^{-5}$$

$$n = N_c \exp\left(-\frac{(E_c - E_F)}{kT}\right) = 4.7 \times 10^{17} \times \exp\left(\frac{-0.25}{0.025}\right) = 2.13 \times 10^{13}\ cm^{-3}$$

7. (1 mark) Two semiconductor materials have exactly the same properties except material A has a bandgap energy of $0.90\ eV$ and material B has a bandgap energy of $1.10\ eV$. The ratio of intrinsic concentration 'n_{iB}' of material B to that of intrinsic concentration 'n_{iA}' of material A at $T = 400\ K$ is ______

   A. 18.175

   B. 1.8175

   C. 0.305

   D. 0.055

   E. 0.55

   F. 3.05×10^{-3}

$$n_i = \sqrt{N_C N_V}\ \exp \frac{-E_g}{2KT}$$

$$\frac{n_{iB}}{n_{iA}} = \frac{\exp(-1.1/2KT)}{\exp(-0.9/2KT)}$$

$$= \exp(-0.2/2KT)$$

$$= \exp\left(\frac{-0.2}{2 \times 0.0345}\right)$$

$$= 0.055$$

KT at 400K

$$= \frac{1.38 \times 10^{-23} \times 400}{1.602 \times 10^{-19}}$$

$$= 0.0345$$

8. (2 marks) A company X aims to "design" a new semiconductor material. The semiconductor is to be p type and doped with $N_a = 5 \times 10^{15}\ cm^{-3}$ acceptor atoms. Assume complete ionization and $N_d = 0$. The effective density of states functions are $N_c = 1.2\times10^{19}\ cm^{-3}$ and $N_v = 1.8\times10^{19}\ cm^{-3}$ at $T = 300\ K$, and vary as T^2. A special semiconductor device fabricated with this material requires that the hole concentration be no greater than $5.08 \times 10^{15}\ cm^{-3}$ at $T = 350\ K$. The minimum bandgap energy required in this new material is ______ eV (*Hint:* Use the charge neutrality equation to find n_i at $T = 350\ K$.)

   A. 1.25

   B. 0.625

   C. 1.625

   D. 0.325

   E. 1.12

   F. 1.42

We are given the required P_0 and asked to calculate E_g.

We know $n_i = \sqrt{N_c N_v} \exp\left(-\frac{E_g}{2KT}\right)$

We need to calculate n_i, N_c and N_v at 350 K.

$N_{c,350K} = N_{c,300K} \cdot \left(\frac{350}{300}\right)^2$

$= 1.2\times10^{19} \times \left(\frac{350}{300}\right)^2$

$= 1.63\times10^{19}$

$N_{v,350K} = N_{v,300K} \cdot \left(\frac{350}{300}\right)^2$

$= 2.45\times10^{19}$

But what is n_i? We will determine this using charge neutrality relationship

$P_0 + \cancel{N_D}^{\,0} = n_0 + N_A$

$5.08\times10^{15} \quad \frac{n_i^2}{P_0} \quad 5\times10^{15}$

$\Downarrow$

$n_i^2 = 4.06\times10^{29}$

Since $n_i^2 = N_c N_v \exp\left(-\frac{E_g}{KT}\right)$

$4.06\times10^{29} = 1.63\times2.45\times10^{38} \times \exp\left(\frac{-E_g}{KT}\right)$

Solving $E_g = 0.625\ eV$

Remember KT @ 350 K is

$\frac{1.38\times10^{-23} \times 350}{1.6\times10^{-19}} = 0.03\ eV$

9. (2 marks) Silicon at $T = 300\ K$ is doped with Arsenic atoms such that the concentration of electrons is $n_0 = 7 \times 10^{15}\ cm^{-3}$. The position of Fermi level w.r.t E_c and $E_{F,i}$ (intrinsic Fermi level) is ______ (take $n_i = 1.5 \times 10^{10}\ cm^{-3}$, $kT = 0.025\ eV$, $N_c = 2.8 \times 10^{19}\ cm^{-3}$)

N_D

A. $0.326\ eV$ below E_c and $0.207\ eV$ above $E_{F,i}$

B. $0.163\ eV$ below E_c and $0.414\ eV$ above $E_{F,i}$

C. $0.326\ eV$ above E_v and $0.207\ eV$ below $E_{F,i}$

D. $0.207\ eV$ below E_c and $0.326\ eV$ above $E_{F,i}$

E. $0.207\ eV$ above E_v and $0.326\ eV$ below $E_{F,i}$

F. $0.414\ eV$ below E_c and $0.163\ eV$ above $E_{F,i}$

$$n_0 = N_c \exp\left(-\left(\frac{E_c - E_F}{KT}\right)\right)$$

$$E_c - E_F = KT \ln\left(\frac{N_c}{n_0}\right)$$

$$= 0.025 \times \ln\left(\frac{2.8 \times 10^{19}}{7 \times 10^{15}}\right)$$

$$= 0.207\ eV$$

Also, $$n_0 = n_i \exp\left(\frac{E_F - E_i}{KT}\right)$$

$$\therefore E_F - E_i = KT \ln\left(\frac{n_0}{n_i}\right) = 0.025 \ln\left(\frac{7 \times 10^{15}}{1.5 \times 10^{10}}\right)$$

$$= 0.326\ eV$$

10. (1 mark) A silicon sample is $2.5\ cm$ long and has a cross-sectional area of $0.1\ cm^2$. The silicon is n type with a donor impurity concentration of $N_d = 2 \times 10^{15}\ cm^{-3}$. The resistance of the sample is measured and found to be $70\ \Omega$. The electron mobility $(in\ cm^2/(V - s))$ is ______

A. 44.64

B. 111.6

C. 44.4

D. 1116.1

E. 480

F. 1280

$$R = \frac{\rho L}{A} \Rightarrow 70 = \rho \frac{2.5\ cm}{0.1\ cm^2}$$

$$\rho = 2.8\ \Omega\text{-}cm$$

$$\rho = \frac{1}{nq\mu}$$

$$\Rightarrow \mu = \frac{1}{2\times10^{15} \times 1.6\times10^{-19} \times 2.8}$$

$$= 1116.1\ \frac{cm^2}{V\text{-}sec}$$

11. (2 marks) A GaAs semiconductor resistor is doped with donor impurities at a concentration of $N_d = 2 \times 10^{15}\ cm^{-3}$ and has a cross-sectional area of $5 \times 10^{-5}\ cm^{-2}$. A current of $I = 25\ mA$ is induced in the resistor with an applied bias of $5\ V$. The length of the resistor and the drift velocity of the electrons is given by ______ respectively. (Use $\mu_n = 8500\ cm^2/(V-s)$ and $\mu_p = 400\ cm^2/(V-s)$) (*Hint:* Use the definition of current density.)

A. $L = 0.0272\ cm$, $v_d = 1.56 \times 10^3\ cm\ sec^{-1}$

B. $L = 0.0156\ cm$, $v_d = 2.72 \times 10^5\ cm\ sec^{-1}$

C. $L = 0.272\ cm$, $v_d = 1.56 \times 10^6\ cm\ sec^{-1}$

D. $L = 0.0272\ cm$, $v_d = 1.56 \times 10^6\ cm\ sec^{-1}$

E. $L = 0.272\ cm$, $v_d = 1.56 \times 10^3\ cm\ sec^{-1}$

F. $L = 0.156\ cm$, $v_d = 2.72 \times 10^3\ cm\ sec^{-1}$

$$R = \frac{5\ V}{25\ mA} = 200\ \Omega$$

$$R = \frac{\rho L}{A} = \frac{L}{nq\mu_n A}$$

$$\Rightarrow L = \frac{200 \times 2\times10^{15} \times 1.6\times10^{-19}}{8500 \times 5\times10^{-5}} = 0.027\ cm$$

Drift velocity $V_d = \mu \mathcal{E}$

$$= \frac{8500 \times 5V}{0.027}$$

$$= 1.57 \times 10^{6}\ cm/sec$$

12. (2 marks) The ~~carrier~~ electron density at which a minimum in the conductivity (σ_{min}) of the semiconductor occurs is ______

A. $n_i\sqrt{\frac{\mu_n^2}{\mu_p}}$

B. $n_i \times \frac{\mu_n}{\mu_p}$

C. $n_i\sqrt{\frac{\mu_p}{\mu_n}}$

D. $n_i\sqrt{\frac{\mu_n}{\mu_p^2}}$

E. $n_i^2 \times \frac{\mu_n}{\mu_p}$

F. insufficient information

$$\sigma = nq\mu_n + pq\mu_p$$

$$= nq\mu_n + \frac{n_i^2}{n} q\mu_p$$

Minimum conductivity occurs when $\frac{d\sigma}{dn} = 0$ $\Rightarrow \cancel{q}\mu_n - \frac{n_i^2 \cancel{q}\mu_p}{n^2} = 0$

$$\Rightarrow n = n_i\sqrt{\frac{\mu_p}{\mu_n}}$$

If you choose $\frac{d\sigma}{dp} = 0$ the options would not match.

Questions adapted from previous GATE examinations

13. (1 mark) **(GATE - ECE 1987)** In an intrinsic semiconductor, the free electron concentration depends on ______

    A. Temperature of the semiconductor

    B. Effective mass of electrons only

    C. Effective mass of holes only

    D. Width of the forbidden energy band of the semiconductor

    E. Applied bias

    F. Difference between the conduction band energy and Fermi level only

14. (1 mark) **(GATE - ECE 2008)** Silicon is doped with boron to a concentration of $4 \times 10^{17}\ atoms/cm^3$. Assuming intrinsic carrier concentration of silicon to be $1.5 \times 10^{10}\ cm^{-3}$ and the value of kT/q to be $25\ meV$ at $300\ K$. Compared to undoped silicon, the Fermi level of doped silicon ______

    A. goes down by $0.13\ eV$

    B. goes up by $0.13\ eV$

    C. goes up by $0.427\ eV$

    D. goes down by $0.427\ eV$

    E. goes up by $0.39\ eV$

    F. goes down by $0.39\ eV$

$P_0 = 4 \times 10^{17}$

$E_i - E_F = kT \ln\left(\frac{4 \times 10^{17}}{1.5 \times 10^{10}}\right)$

$= 0.427\ eV$

15. (1 mark) **(GATE - ECE 2014)** Consider a silicon sample doped with $N_D = 1 \times 10^{15}\ atoms/cm^3$. Assume the intrinsic carrier concentration $n_i = 1.5 \times 10^{10}\ cm^{-3}$. If the sample is additionally doped with $N_A = 1 \times 10^{18}\ atoms/cm^3$ acceptor atoms, the approximate number of $electrons/cm^3$ in the sample at $T = 300\ K$ will be ______

    A. $2.25 \times 10^{12}\ cm^{-3}$

    B. $2.25 \times 10^{3}\ cm^{-3}$

    C. $2.25 \times 10^{5}\ cm^{-3}$

    D. $2.25 \times 10^{2}\ cm^{-3}$

    E. $2.25 \times 10^{8}\ cm^{-3}$

    F. $2.25\ cm^{-3}$

$n_0 = 10^{15}\ cm^{-3}$ $\quad p_0 = \frac{(1.5\times10^{10})^2}{10^{15}} = 2.25\times10^{5}$

After additional doping, the sample becomes p type with $p_0 = N_A - N_D = 10^{18}\ cm^{-3}$

$\therefore n_0 = \frac{2.25\times10^{20}}{10^{18}} \sim 2.25\times10^{2}\ cm^{-3}$

For exact number you need to apply charge neutrality relation.

16. (1 mark) **(GATE - ECE 2014)** A silicon sample is uniformly doped with donor type impurities with a concentration of $10^{16}\ cm^{-3}$. The electron and hole mobilities in the sample are $1200\ cm^2/(V - sec)$ and $400\ cm^2/(V - sec)$ respectively. Assuming complete ionization of impurities, the resistivity of the sample in $\Omega - cm$ is ______

    A. 1.92

    B. 0.52

    C. 0.95

    D. 5.2

    E. 1.04

    F. 19.2

$\rho = \frac{1}{n q \mu_n}$ (Ignoring contribution of holes which will be much smaller in number)

$= \frac{1}{10^{16}\times1.6\times10^{-19}\times1200}$

$= 0.52\ \Omega - cm$

17. (1 mark) **(GATE - PHY 2013)** A phosphorous doped silicon semiconductor (doping density : $10^{17}\ cm^{-3}$) is heated from $100\ °C$ to $200\ °C$. Which one of the following statements is CORRECT?

   A. Position of Fermi level moves towards conduction band

   B. Position of Fermi level moves towards valence band

   C. Position of dopant level moves towards conduction band

   D. Position of dopant level moves towards middle of energy gap

   E. Position of Fermi level moves beyond conduction band

   F. Position of Fermi level moves towards middle of energy gap

18. (2 marks) **(GATE - PHY 2016)** The energy *vs.* wave vector $(E-k)$ relationship near the bottom of a band for a solid can be approximated as $E(k) = A(ka)^2 + B(ka)^4$, where the lattice constant $a = 2.1\ \mathring{A}$. The values of A and B are $6.3 \times 10^{-19}\ J$ and $6.3 \times 10^{-20}\ J$, respectively. At the bottom of the conduction band, the ratio of the effective mass of the electron to the mass of free electron is (m^*/m_0) ______

   A. 0.22

   B. 0.108

   C. 2.2

   D. 0.022

   E. 0.011

   F. 22.2

$$E = A \cdot k^2 a^2 + B k^4 a^4$$

$$\frac{dE}{dk} = 2Aa^2 k + 4Ba^4 k^3$$

$$\frac{d^2E}{dk^2} = 2Aa^2 + 12Ba^4 k^2$$

At the bottom of the C.B $k=0$

$$\frac{m^*}{m_0} = \frac{\hbar^2}{\frac{d^2E}{dk^2}} = \frac{\hbar^2}{2Aa^2 m_0} = \frac{(1.054 \times 10^{-34})^2}{2 \times 6.3 \times 10^{-19} \times (2.1 \times 10^{-10})^2 \times 9.1 \times 10^{-31}}$$

$$= 0.22$$