A quick recap of the material covered in lectures

Interpretation of Energy Band Diagram

When an electric field ($\mathscr{E}$) exists inside a material or applied as an external stimuli, the band energies E_c and E_v become a function of position. The resulting variation of E_c and E_v with position on band diagram is popularly known as "band bending" and serves as a powerful tool to interpret electric field $\mathscr{E}$, electrostatic potential V, $K.E.$, $P.E$ of carriers, carrier density n and p in the semiconductor by mere visual inspection. The following formulation provides a means of readily deducing the electrostatic variables associated with the"band bending"-

- Potential energy of $-q$ charged particle:

$$\boxed{P.E. = -qV = E_c - E_{ref}} \tag{1}$$

- Electrostatic potential V:

$$\boxed{V = -\frac{1}{q}(E_c - E_{ref})} \tag{2}$$

- Electric field $\mathscr{E}$:

$$\boxed{\mathscr{E} = \frac{1}{q}\frac{dE_c}{dx} = \frac{1}{q}\frac{dE_v}{dx} = \frac{1}{q}\frac{dE_i}{dx}} \tag{3}$$

Minority Carrier Diffusion Equation (MCDE)

Diffusion is migration from regions of high particle concentration to regions of low particle concentration due to the random thermal motion of the carriers. The minority carrier diffusion equation is a partial differential equation in space and time depicting the evolution of carrier-action when a semiconductor system is perturbed. It is derived from the continuity equations under following assumptions -

- System under analysis is *one-dimensional*
- Analysis is restricted to *minority carriers*
- Electric field $\mathscr{E} \simeq 0$ in the semiconductor
- Perturbation is mostly the *photogeneration* of carriers
- *Low-level* injection conditions prevail, *i.e.*, $\Delta p \ll n_0$ for n-type semiconductor or $\Delta n \ll p_0$ for p-type semiconductor

The minority carrier diffusion equations for electrons in p-type materials and holes in n-type materials are given as -

$$\boxed{\frac{\partial \Delta n_p}{\partial t} = D_n \frac{\partial^2 \Delta n_p}{\partial x^2} - \frac{\Delta n_p}{\tau_n} + G_L} \qquad \boxed{\frac{\partial \Delta p_n}{\partial t} = D_p \frac{\partial^2 \Delta p_n}{\partial x^2} - \frac{\Delta p_n}{\tau_p} + G_L} \tag{4}$$

The complexity of MCDE can be drastically reduced by applying additional simplifications depending on the conditions of problem given.

Minority Carrier Diffusion Lengths

The diffusion length wherein excess carrier concentration falls off exponentially to $1/e$ of to that at the point of injection is termed as *minority carrier diffusion length*. It is given as -

$$\boxed{L_p \equiv \sqrt{D_p \tau_p}} \qquad \boxed{L_n \equiv \sqrt{D_n \tau_n}} \tag{5}$$

for holes in an n-type material and electrons in p-type material, respectively. In short, $L_{p/n}$ is the average distance a hole or electron diffuses before recombining.

Quasi-Fermi Levels

Quasi-Fermi levels are energy levels used to specify the carrier concentrations inside a semiconductor under *non*equilibrium conditions. By definition, they are related to the non-equilibrium carrier concentrations in the same way E_F is related to the equilibrium carrier concentrations. E_{Fn} (the quasi-Fermi level for electrons) and E_{Fp} (the quasi-Fermi level for holes) are given as below -

$$\boxed{E_{Fn} \equiv E_i + kT\ ln\left(\frac{n}{n_i}\right)} \qquad \boxed{E_{Fp} \equiv E_i - kT\ ln\left(\frac{p}{n_i}\right)} \tag{6}$$

Solve the following questions. There are 13 questions, for a total of 25 marks.

1. (1 mark) When is $np = n_i^2$?

   A. Under steady–state conditions.

   B. Under low-level injection.

   C. When an electric field is absent.

   D. Only for a nondegenerate semiconductor in equilibrium.

2. (5 marks) A silicon sample maintained at 300 K is characterized by the energy band diagram as shown in the figure 1. Answer the following questions by choosing the correct diagram. Consider E_F as reference energy level.

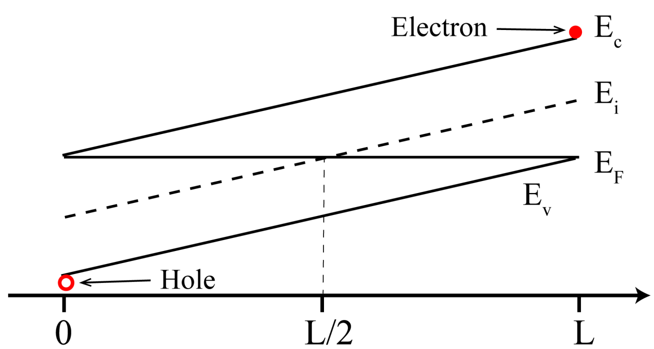


Figure 1: Band diagram - Problem 2

(a) *(1 mark)* Do equilibrium conditions prevail? Why?

   A. Yes. Because Fermi level E_F is constant along x

   B. No. Because energy bands E_c, E_v are bending.

   C. Yes. Because energy difference between E_c and E_i, E_i and E_v is same.

   D. No. Because E_F is not parallel to E_c

(b) *(2 marks)* Referring to figure 1, the sketch of electric field ($\mathscr{E}$) inside the Si as a function of x is best described by _______

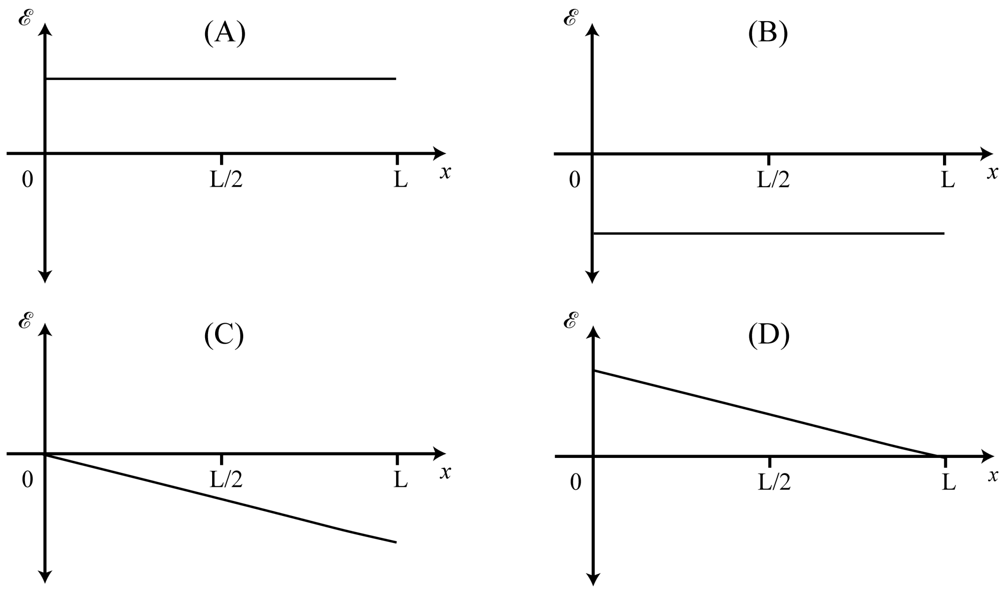


Figure 2: E-fields - Problem 2(b)

A. A

Refer Eq 3

B. B

C. C

D. D

(c) *(2 marks)* Referring to figure 1, the sketch of $K.E.$ of electrons and holes inside the Si as a function of x is best described by ______

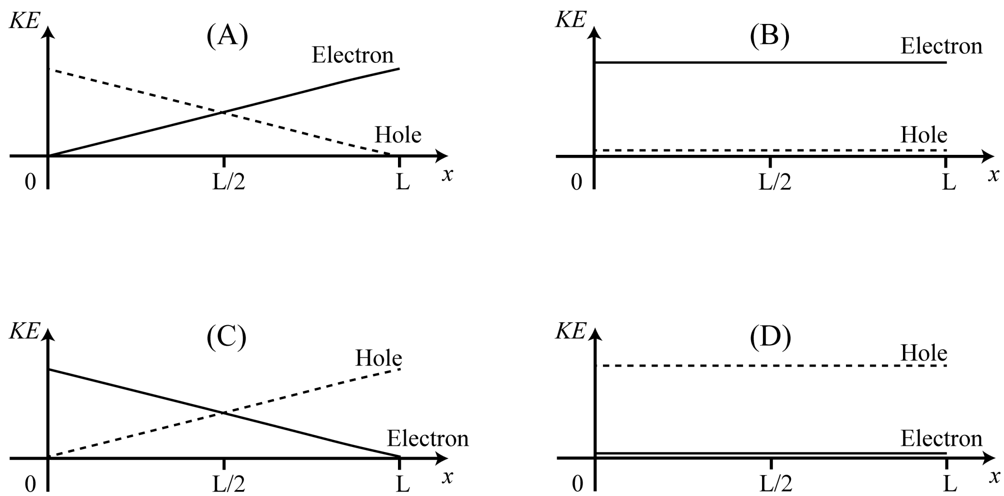


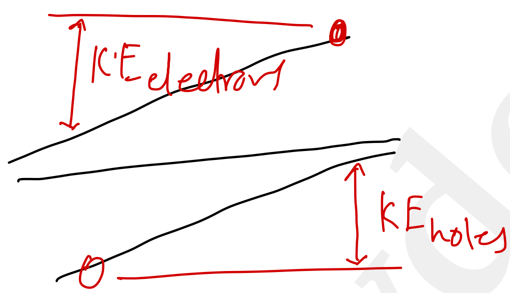
Figure 3: Kinetic Energy - Problem 2(c)

A. A

B. B

C. C

D. D



KE is the distance from E_C for electrons. Electrons at E_C have zero KE. KE of holes is the distance from E_V. Remember hole energy increases as you move deeper into valance band.

3. (1 mark) For a p-type semiconductor, the minority carrier diffusion length is given by ______

A. $L_n \equiv \sqrt{D_n \tau_n}$

B. $L_p \equiv \sqrt{D_p \tau_p}$

C. $L_n \equiv \sqrt{D_n / \tau_n}$

D. $L_p \equiv \sqrt{D_p / \tau_p}$

Electrons are minority carriers. Check if units of $\sqrt{D\tau}$ are consistent with length.

4. (5 marks) Consider a bar of p-type silicon that is uniformly doped to a value of $N_A = 2 \times 10^{16}\ cm^{-3}$ at $T = 300$ K. The applied electric field is zero. Light is incident on the end of the semiconductor as shown in figure 4. The steady-state concentration of excess carriers generated at $x = 0$ is $\delta p(0) = \delta n(0) = 2\times 10^{14}\ cm^{-3}$. Assume the following parameters: $\mu_n = 1200\ cm^2/V-s$, $\mu_p = 400\ cm^2/V-s$, $\tau_n = 10^{-6}\ s$, $\tau_p = 5 \times 10^{-7}\ s$, and $n_i = 10^{10}\ cm^{-3}$.

$N_A = 2\times10^{16}\ cm^{-3}$

$\therefore P_0 = 2\times10^{16}\ cm^{-3}$

$n_0 = \frac{n_i^2}{P_0} = 5\times10^{3}\ cm^{-3}$

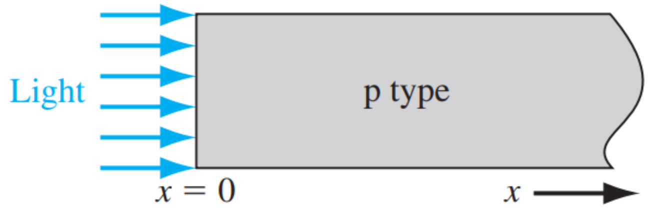


Figure 4: Side-illumination - Problem 4

(a) *(1 mark)* What is the steady-state electron and hole concentrations at $x = 0$?

A. $n = 2.02 \times 10^{16}\ cm^{-3}$ and $p = 2 \times 10^{14}\ cm^{-3}$

B. $n = 0.5 \times 10^{4}\ cm^{-3}$ and $p = 2 \times 10^{16}\ cm^{-3}$

C. $n = 2 \times 10^{16}\ cm^{-3}$ and $p = 0.5 \times 10^{4}\ cm^{-3}$

D. $n = 2 \times 10^{14}\ cm^{-3}$ **and** $p = 2.02 \times 10^{16}\ cm^{-3}$

$n = n_0 + \delta n(0)$

$p = P_0 + \delta p(0)$

(b) *(2 marks)* Calculate the minority carrier diffusion constant in $cm^2/\cancel{V}-s$ and length in μm.

A. $D_p = 10.4$ and $L_p = 22.8$

B. $D_p = 31.2$ and $L_p = 55.85$

C. $D_n = 31.2$ **and** $L_n = 55.85$

D. $D_n = 10.4$ and $L_n = 22.8$

$D - cm^2/s$

$\mu - cm^2/V\text{-}s$

$\frac{D}{\mu} = \frac{kT}{q}$ (Volts)

$D_n = 1200 \times 0.0259 = 31.2\ cm^2/s$

$L_n = \sqrt{D_n \tau_n} = 55.85\ \mu m$

(c) *(1 mark)* The electron concentration at $x = 30\ \mu m$ is ______

A. $1.17 \times 10^{14}\ cm^{-3}$

B. $1.18 \times 10^{16}\ cm^{-3}$

C. $5.42 \times 10^{15}\ cm^{-3}$

D. $5.36 \times 10^{13}\ cm^{-3}$

$n(x) = n(0)\exp(-x/L_n)$

$= 2\times10^{14} \times \exp\left(\frac{-30}{55.85}\right)$

$= 1.17\times10^{14}\ cm^{-3}$

-ve sign indicates current direction is opposite to diffusion of electrons.

(d) *(1 mark)* What is electron diffusion current J_n at $x = 30\ \mu m$?

A. $J_n = -0.138\ kA/cm^2$

B. $J_n = +0.138\ kA/cm^2$

C. $J_n = -0.138\ A/cm^2$

D. $J_n = +0.138\ A/cm^2$

$J_n = -q D_n \frac{dn}{dx}$

$= 1.602\times10^{-19} \times 31.2 \times \left(\frac{2\times10^{14} - 1.17\times10^{14}}{30\times10^{-4}}\right)$

$= -0.138\ A/cm^2$

5. (2 marks) Consider a doped Si sample at $T = 300$ K. Figure 5(a) depicts the energy band diagram corresponding to thermal equilibrium while figure 5(b) shows the energy band diagram under the non-equilibrium condition. Assume $n_i = 10^{10}\ cm^{-3}$, $E_g = 1.1\ eV$, $kT = 26\ meV$ and looking at the corresponding values of energy levels, answer the following questions

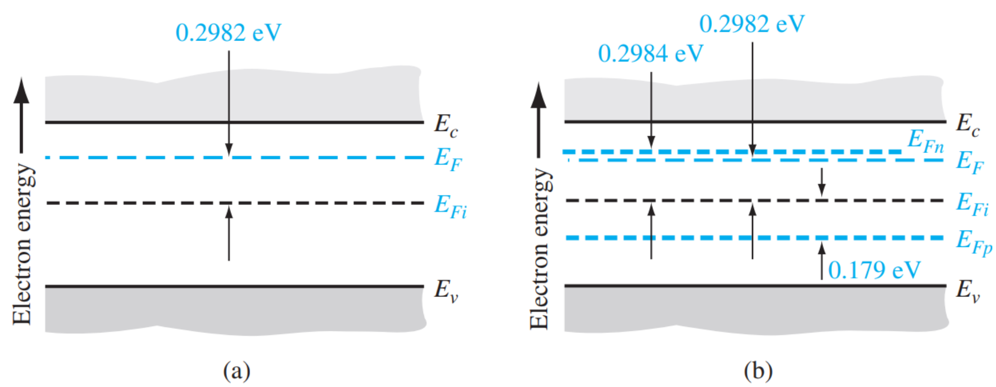


Figure 5: Equilibrium and non-equilibrium band diagrams - Problem 5

(a) *(1 mark)* What are the electron and hole densities in equilibrium condition?

A. $n_0 = 9.57\times10^{14}\ cm^{-3}$ and $p_0 = 0.1\times10^6\ cm^{-3}$

B. $n_0 = 0.62\times10^6\ cm^{-3}$ and $p_0 = 1.6\times10^{14}\ cm^{-3}$

C. $n_0 = 1.6\times10^{14}\ cm^{-3}$ and $p_0 = 0.62\times10^6\ cm^{-3}$

D. $n_0 = 0.1\times10^6\ cm^{-3}$ and $p_0 = 9.57\times10^{14}\ cm^{-3}$

$E_F - E_i = kT\ln\left(\frac{n_0}{n_i}\right)$

$n_0 = 9.57\times10^{14}\ cm^{-3}$

$p_0 = \frac{n_i^2}{n_0} = 0.1\times10^6\ cm^{-3}$

(b) *(1 mark)* What are the excess carrier densities generated?

A. $\delta n = 0.1\times10^8\ cm^{-3}$ and $\delta p = 9.77\times10^{12}\ cm^{-3}$

B. $\delta n = 7.64\times10^{12}\ cm^{-3}$ and $\delta p = 9.77\times10^{12}\ cm^{-3}$

C. $\delta n = 9.64\times10^{14}\ cm^{-3}$ and $\delta p = 0.1\times10^6\ cm^{-3}$

D. $\delta n = \delta p \equiv 10^{13}\ cm^{-3}$

$\delta n = n - n_0$

$= n_i\exp\left(\frac{E_{Fn}-E_i}{kT}\right) - n_i\exp\left(\frac{E_F-E_i}{kT}\right)$

$= 7.39\times10^{12}\ cm^{-3}$

$\delta p = n_i\exp\left(\frac{E_i-E_{Fp}}{kT}\right) = 9.77\times10^{12}\ cm^{-3}$

6. (1 mark) If the quasi-Fermi level splitting is $E_{Fn} - E_{Fp} = 10\ kT$, then what is np ?

   A. $10\ n_i^2$

   B. $ln(10)\ n_i^2$

   C. $e^{10}\ n_i^2$

   D. n_i^2

Use equation 6

$$np = n_i^2 \exp\left(\frac{E_{Fn} - E_{Fp}}{kT}\right)$$

$$= n_i^2 e^{10}$$

Questions adapted from previous GATE examinations

$N_A = 10^{16}\ cm^{-3}$

7. (2 marks) **(EC-GATE 2021)** A bar of silicon is doped with boron concentration of $10^{16}\ cm^{-3}$ and assumed to be fully ionized. It is exposed to light such that electron-hole pairs are generated throughout the volume of the bar at the rate of $10^{20}\ cm^{-3}s^{-1}$. If the recombination lifetime is $100\ \mu s$ and intrinsic carrier concentration of silicon is $10^{10}\ cm^{-3}$ and assuming 100% ionization of boron, then the approximate product of steady-state electron and hole concentrations due to this light exposure is ______

   A. $10^{20}\ cm^{-6}$

   B. $2 \times 10^{20}\ cm^{-6}$

   C. $10^{32}\ cm^{-6}$

   D. $2 \times 10^{32}\ cm^{-6}$

$\delta n = \delta p = g\tau = 10^{20}\ cm^{-3} s^{-1} \times 100 \times 10^{-6} s$

$= 10^{16}\ cm^{-3}$

$n = n_0 + \delta n = 10^4 + 10^{16} = 10^{16}\ cm^{-3}$

$p = p_0 + \delta p = 2 \times 10^{16}$

$np = 2 \times 10^{32}\ cm^{-6}$

8. (1 mark) **(EC-GATE 2005)** Under low level injection assumption, the injected minority carrier current for an extrinsic semiconductor is essentially the

   A. Drift current

   B. Recombination current

   C. Induced current

   D. Diffusion current

9. (1 mark) **(EC-GATE 2021)** The energy band diagram of a p-type semiconductor bar of length L under equilibrium condition ($i.e.$, the Fermi energy level E_F is constant) is shown in the figure 6. The valence band E_V is sloped since doping is non-uniform along the bar. The difference between the energy levels of the valence band at the two edges of the bar is Δ.

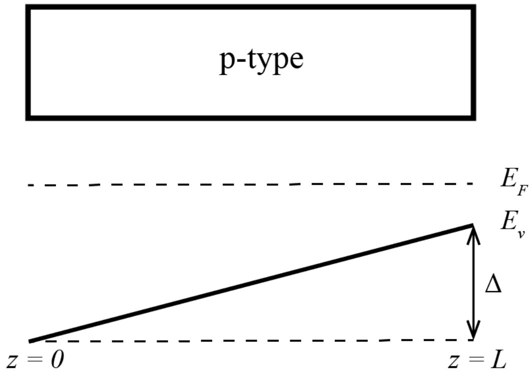


Figure 6: Non-uniform doping - Problem 9

If the charge of an electron is q, then the magnitude of the electric field developed inside this semiconductor bar is

Review Eq 3 and answer.

A. $\frac{\Delta}{qL}$

B. $\frac{2\Delta}{qL}$

C. $\frac{\Delta}{2qL}$

D. $\frac{3\Delta}{2qL}$

10. (2 marks) **(EC-GATE 2017)** As shown in figure 7, a uniformly doped Silicon (Si) bar of length $L = 0.1\ \mu m$ with a donor concentration $N_D = 10^{16}\ cm^{-3}$ is illuminated at $x = 0$ such that electron and hole pairs are generated at the rate of $G_L = G_{L0}(1 - \frac{x}{L}),\ 0 \leq x \leq L$, where $G_{L0} = 10^{17}\ cm^{-3}s^{-1}$. Hole lifetime is $10^{-4}\ s$, electronic charge is $q = 1.6 \times 10^{-19}\ C$, hole diffusion coefficient $D_p = 100\ cm^2/s$ and low level injection condition prevails.

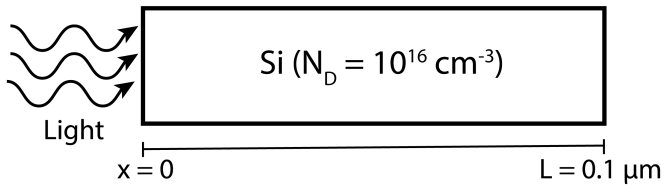


Figure 7: Side illumination - Problem 10

Assuming a linearly decaying steady state excess hole concentration that goes to 0 at $x = L$, the magnitude of the diffusion current density at $x = L/2$, in A/cm^2, is ______

A. 16000

B. 1.6

C. 16

D. 1600

At $x = 0$

$\delta p = 10^{17}\ cm^{-3} s^{-1} \times 10^{-4}$

$= 10^{13}\ cm^{-3}$

At $x = L/2$

$\delta p = G_L \tau$

$= 10^{17}\left(1 - \frac{\cancel{L}}{2\cancel{L}}\right) \times 10^{-4}$

$= 0.5 \times 10^{13}\ cm^{-3}$

$J_{diff} = q D_p \frac{dp}{dx}$

since we are asked to assume a linear decay

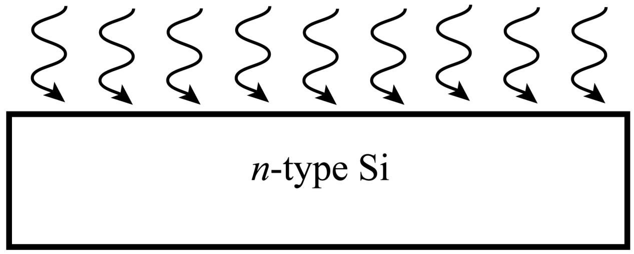
$\frac{dp}{dx} = \frac{10^{13} - 0.5 \times 10^{13}}{0.05 \times 10^{-4}} = \frac{5 \times 10^{12}}{5 \times 10^{-6}} = 1 \times 10^{18}$

$J_{p,diff} = 1.602 \times 10^{-19} \times 100 \times 10^{18}$

$= 16\ A/cm^2$

11. (2 marks) **(EC-GATE 2016)** Consider a silicon sample at $T = 300$ K, shown in figure 8 with a uniform donor density $N_d = 5 \times 10^{16}\ cm^{-3}$ illuminated uniformly such that the optical generation rate is $G_{opt} = 1.5 \times 10^{20}\ cm^{-3}s^{-1}$ throughout the sample. The incident radiation is turned off at $t = 0$. Assume the condition of low-level injection to be valid and ignore surface effects. The carrier lifetimes are $\tau_{po} = 0.1\ \mu s$ and $\tau_{no} = 0.5\ \mu s$.

For $t<0$ sample is in steady state



$\Delta p = G_L \tau_p$
$= 1.5 \times 10^{20} \times 0.1 \times 10^{-6}$
$= 1.5 \times 10^{13}$

Figure 8: Top illumination - Problem $\boxed{11}$

The hole concentration at $t = 0$ and the hole concentration at $t = 0.3\ \mu s$, respectively, are

A. $1.5 \times 10^{13}\ cm^{-3}$ **and** $7.47 \times 10^{11}\ cm^{-3}$

B. $7.5 \times 10^{13}\ cm^{-3}$ and $3.73 \times 10^{11}\ cm^{-3}$

C. $1.5 \times 10^{13}\ cm^{-3}$ and $8.23 \times 10^{11}\ cm^{-3}$

D. $7.5 \times 10^{13}\ cm^{-3}$ and $4.12 \times 10^{11}\ cm^{-3}$

For $t>0$ since excitation is switched off the Δp decays exponentially as

$1.5 \times 10^{13} \exp\left(\frac{-0.3\ \mu s}{0.1\ \mu s}\right)$

$= 1.5 \times 10^{13} \times \exp(-3)$
$= 7.47 \times 10^{11}\ cm^{-3}$

12. (1 mark) **(EC-GATE 2014)** Assume electronic charge $q = 1.6 \times 10^{-19}\ C$, $kT/q = 25\ mV$ and electron mobility $\mu_n = 1000\ cm^2/V-s$. If the concentration gradient of electrons injected into a p-type silicon sample is $1 \times 10^{21}\ cm^{-3}$, the magnitude of electron diffusion current density (in A/cm^2) is ______

Should be cm^{-4}

A. 40

B. 4000

C. 25

D. 25×10^{21}

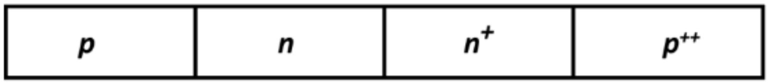
$D_n = 0.0259 \times 1000\ cm^2/V\text{-}s$
$= 25.9\ cm^2/s$

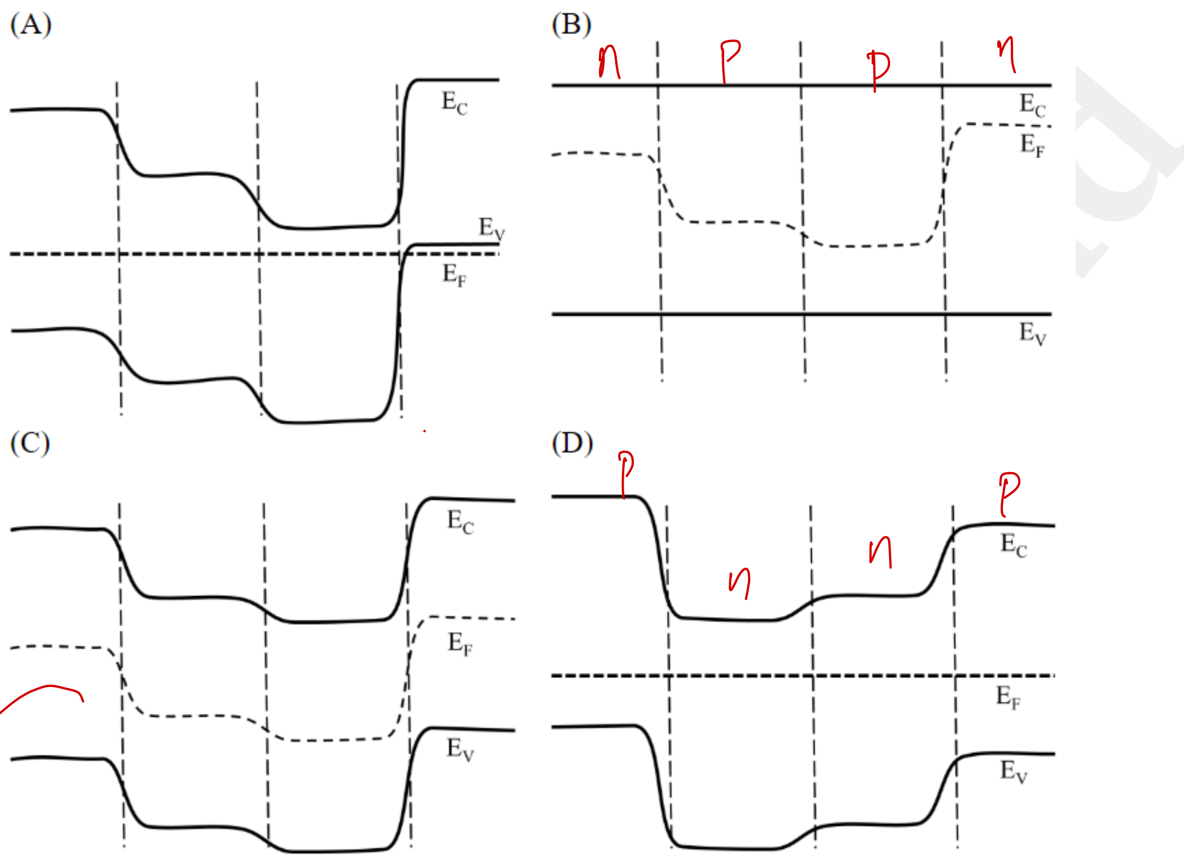
$J_n = q D_n \nabla n$
$= 1.6 \times 10^{-19}\ C \times 25.9 \frac{\cancel{cm^2}}{s} \times 10^{21} / \cancel{cm^4}\, cm^2$

$= 4144\ A/cm^2$

Choose closest answer.

13. (1 mark) **(EC-GATE 2020)** Which one of the following options describes correctly the equilibrium band diagram at $T = 300$ K of a Silicon pnn^+p^{++} configuration shown in the figure?





A. A

B. B

C. C

D. D

Analyze why these are wrong

E_F is close to E_i in all regions.
Intrinsic semiconductor!