A quick recap of the material covered in lectures

Basic paramaters

- Planck's constant : $h = 6.6 \times 10^{-34} \quad J.s$
- Velocity of light in vacuum : $c = 3 \times 10^8 \; m/s$
- Rest mass of electron : $m_0 = 9.1 \times 10^{-31} \; kg$
- Charge of an electron : $q = 1.6 \times 10^{-19} \; C$
- $1 \; eV = 1.6 \times 10^{-19} \;\; J$
- Debroglie wavelength is related to the momentum 'p' and is given by

$$\boxed{\lambda = \frac{h}{p}} \tag{1}$$

- The energy of a free electron is given by

$$\boxed{E = \frac{p^2}{2m_0}} \tag{2}$$

- The energy of a photon is given by

$$\boxed{E = h\nu = \frac{hc}{\lambda} = pc} \tag{3}$$

- For example, a photon of $\lambda = 1.24 \; \mu m$ has an energy of $1.6 \times 10^{-19} \;\; Joule$. However, $Joule$ is not a convenient unit of energy, and hence we often refer to it in terms of eV. The relation is given by

$$\boxed{E[\; eV] = \frac{E[\; J]}{q}} \tag{4}$$

1D Potential box of width 'L'

Consider an infinite potential well with width 'L', where the electron has a mass 'm' and momentum 'p'. The motion of the particle is described by the Schrödinger equation, which in it's time independent form is given by:

$$\boxed{K.E. + P.E. = T.E.} \quad \boxed{-\frac{\hbar^2}{2m}\frac{\partial^2 \Psi}{\partial x^2} + V(x)\Psi(x) = E\Psi(x)} \tag{5}$$

The energy of the particle is quantized, and allowed energy levels are denoted by an integer n. The lowest possible energy state is $n = 1$. E_1 is also called zero point energy, and $E_1 \neq 0$ implies that the particle can never be truly at rest.

- The energy of a particle in its n^{th} state is given by

$$\boxed{E_n = \frac{p^2}{2m} = \frac{\hbar^2 k^2}{2m} = \frac{n^2 h^2}{8mL^2}} = \frac{\hbar^2 n^2 \pi^2}{2mL^2} \tag{6}$$

- The difference in the energy levels is

$$\boxed{E_f - E_i = h\nu = \frac{hc}{\lambda} = \frac{(n_f^2 - n_i^2)h^2}{8mL^2}} \tag{7}$$

- The energy of a particle in a 2D potential box is given by

$$\boxed{E_n = \frac{p^2}{2m} = \frac{\hbar^2(k_x^2 + k_y^2)}{2m} = (\frac{n_x^2}{L_x^2} + \frac{n_x^2}{L_x^2})\frac{h^2}{8m}} \tag{8}$$

Probability of finding a particle

If the wavefunction is denoted by $\psi_n(x)$ in 1D, then $|\psi_n(x)|^2$ represents the probability of finding the particle in the potential well at a particular position for the energy level, E_n .

- The total probability of finding a particle at some x is given by

$$\boxed{P(x) = \int_{-\infty}^{\infty} |\Psi(x)|^2 \, dx = 1} \tag{9}$$

- The probability of finding a particle at a particular x changes with increasing energy of the particle.

Potential Barriers and Quantum Mechanical Tunneling

Excerpts below are taken from chapter 3 of reference text book: Donald A. Neamen, Semiconductor physics and devices: basic principles. New York, NY: McGraw-Hill, 2012. Please refer the same for additional details.

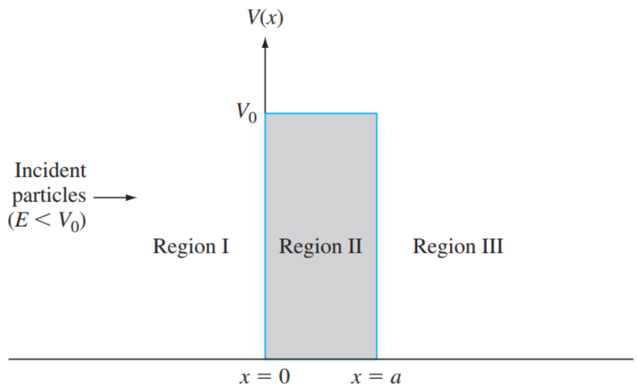

Figure 2.9 | The potential barrier function.

Consider a particle incident on a potential barrier with energy (E) less than the barrier height (V_0) as shown in figure above. Classically the particle cannot reach region III because of the energy barrier. However, quantum mechanics allows a finite non-zero probability of finding the particle in region III. This phenomenon is called tunneling. Tunneling is crucial phenomena in modern MOSFETs as we will see later in the course.

- For energies $E < V_0$, the transmission probability (T) is given by

$$\boxed{T \sim 16\frac{E}{V_0}\left(1-\frac{E}{V_0}\right) exp(-2\kappa a) \simeq exp(-2\kappa a)} \tag{10}$$

  where a is the width of the barrier (in m), κ is the wave vector (in m^{-1}) and η is the penetration depth (in m) in the barrier region.

$$\boxed{\kappa = \sqrt{\frac{2m(V_0 - E)}{\hbar^2}} = \frac{1}{\eta}} \tag{11}$$

- **The tunneling probability may appear to be a small value, but the value is not zero. If a large number of particles impinge on a potential barrier, a significant number can penetrate the barrier.**

Assignment 1

Answer the following questions after reviewing the material covered in week 1 lectures.

1. (2 points) Consider a photon of certain energy and wavelength.

   (a) *(1 point)* The energy of a 500 nm photon is ______ eV .

   A. 1.24

   B. 0.62

   C. 0.25

   D. 2.48

   E. 1.42

   F. 0.5

   $$\lambda [\text{in } eV] = \frac{1.24}{\lambda [\text{in } \mu m]} = \frac{1.24}{0.5} = 2.48\, eV$$

   (b) *(1 point)* The momentum of a 1 eV photon is ______ $kg.m/s$.

   A. 5.3×10^{-34}

   B. 6.6×10^{-34}

   C. 5.5×10^{-25}

   D. 5.3×10^{-28}

   E. 5.3×10^{-24}

   F. 5.3×10^{-26}

   For a photon $E = pc$

   $1\, eV = 1.602 \times 10^{-19}\, J$

   $$p = \frac{1.602 \times 10^{-19}}{3 \times 10^{8}} = 5.3 \times 10^{-28}\ kg\, m/s$$

2. (2 points) The ratio of the momentum of a 2 eV electron and the momentum of a photon of the same energy is approximately ______.

   A. 7.15×10^{2}

   B. 7.15×10^{-2}

   C. 1.75×10^{-2}

   D. 1.75×10^{2}

   E. 7.15×10^{3}

   F. 7.15×10^{0}

   $$P_\lambda = \frac{E_\lambda}{c} \qquad P_{el} = \sqrt{2mE_{el}} \qquad E_{el} = E_\lambda$$

   $$\frac{P_{el}}{P_\lambda} = \frac{c\sqrt{2m \times E_{el}}}{E_\lambda} = \frac{c\sqrt{2m}}{\sqrt{1.602 \times 10^{-19} \times 2}}$$

   [Use Energy in J]

   $= 7.154$

3. (2 points) The wave function $\Psi(x) = C sin(kx)$ for $(0, L)$ where L is the width of the potential well which satisfies the Schrodinger equation for an infinite 1D potential well as discussed in Lecture 1. Choose the correct value of C from the following.

   (Hint: The normalization constant C can be determined by using the fact that the total probability of finding the particle at any x should be one).

   A. $\sqrt{\frac{L}{2}}$

   B. $\sqrt{\frac{2}{L}}$

   C. $\sqrt{\frac{\pi}{L}}$

   D. $\sqrt{\frac{L}{\pi}}$

   E. $\sqrt{\frac{L}{\pi}}$

   F. $\sqrt{\frac{2}{\pi}}$

   Using Eq 9: $\int_0^L C^2 \sin^2(kx) = 1$

   $\frac{C^2}{2} \cdot L = 1 \Rightarrow C = \sqrt{\frac{2}{L}}$

   $\psi(x) = 0$ outside $(0, L)$ due to infinite barrier.

4. (2 points) Consider an electron with an energy of $1\ eV$ impinging on a potential barrier with $V_0 = 5\ eV$ and width $a = 0.5\ nm$. The tunneling probability is approximately given by ______.

   (Hint use the approximate formula of tunneling probability)

   A. 9.2×10^{-5}

   B. 1

   C. 0

   D. 9.2×10^{-1}

   E. 1.05

   F. 9.2×10^{-2}

   From eq 10 & 11

   $K = \sqrt{\frac{2m(V_0 - E)}{\hbar^2}} = \sqrt{\frac{2 \times 9.1 \times 10^{-31} \times 4 \times 1.6 \times 10^{-19}}{(1.05 \times 10^{-34})^2}}$

   $T \sim 16 \times \frac{E}{V_0}\left(1 - \frac{E}{V_0}\right) \exp(-2Ka)$

   $= 16 \times \frac{1}{5} \times \frac{4}{5} \times \exp(-2Ka)$

   $\sim 9.2 \times 10^{-5}$

If you use $T \sim \exp(-2Ka)$ you will get $T \sim 3.5 \times 10^{-5}$. Either way the closest answer in terms of order of magnitude is A.

5. (2 points) Schrödinger wave equation describing the wave function of an electron in space and time is given by ______.

   ($\Psi(x)$ denotes the wave function, V(x) denotes potential, E denotes the total energy, $\hbar = h/2\pi$ is the reduced Planck's constant and m is the rest mass of electron)

   A. $-\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + V(x)\Psi(x) = E\Psi(x)$

   B. $-\frac{\hbar^2}{m}\frac{\partial^2\Psi}{\partial x^2} + V(x)\Psi(x) = E\Psi(x)$

   Recall Eq 5

   C. $-\frac{2\hbar^2}{m}\frac{\partial^2\Psi}{\partial x^2} + V(x)\Psi(x) = E\Psi(x)$

   D. $-\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + 2V(x)\Psi(x) = E\Psi(x)$

   E. $\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + V(x)\Psi(x) = 2E\Psi(x)$

   F. $\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} - V(x)\Psi(x) = E\Psi(x)$

6. (2 points) Heisenberg uncertainty principle is given by ______.

   Δx is the change in position and Δp is the change in momentum of a particle and $\hbar = h/2\pi$ is the reduced Planck's constant.

   A. $\Delta x.\Delta p \leq h/2$

   Recall from +2 physics.

   B. $\Delta x.\Delta p \leq \hbar/2$

   C. $\Delta x.\Delta p \geq \hbar/2$

   D. $\Delta x.\Delta p \geq h/2$

   E. $\Delta x.\Delta p \geq h$

   F. $\Delta x.\Delta p \geq \hbar$

7. (2 points) An electron is confined in a 1D potential box of width $20\ nm$. Calculate the approx. energy (in meV) of the emitted particle when it makes transition from the second excited state ($n = 3$) to the ground state ($n = 1$).

Recall Eq 7

$$E_3 - E_1 = \frac{(9-1)h^2}{8 \times 9.1 \times 10^{-31} \times (20 \times 10^{-9})^2}$$

$$= 1.2 \times 10^{-21} J$$

$$= 7.5\ meV$$

   A. 750
   B. 75
   C. 7.5
   D. 0.75
   E. 0.075
   F. 7500

8. (1 point) Silicon and Germanium are ______.

Recall from lecture

   A. Direct band gap semiconductors
   B. Indirect band gap semiconductors
   C. Metals
   D. Insulators
   E. Semi-metals
   F. None of the above

9. (1 point) Lattice type of Silicon is ______.

Recall from lecture

   A. Diamond type
   B. Wurtzite
   C. Face Centered Cubic
   D. Zinc Blend
   E. Hexagonal
   F. Body Centered Cubic

10. (1 point) Which of the following is correct in terms of conductivity

Insulator conductivity is lowest and metals are highest

   A. Insulator > Semiconductor > Metal
   B. Insulator < Semiconductor < Metal
   C. Insulator = Semiconductor < Metal

D. Insulator < Semiconductor = Metal

E. Insulator < Semiconductor > Metal

F. Insulator = Semiconductor = Metal

11. (2 points) In a 1D potential well, a particle with mass m with zero energy has time-independent wave function $\Psi(x) = Axe^{\frac{-x^2}{L^2}}$. Determine the potential $V(0)$ at x=0. (Use Schrödinger wave equation)

A. $\frac{3\hbar^2}{mL^2}$

B. $\frac{-3\hbar^2}{mL^2}$

C. $\frac{\hbar^2}{mL^2}$

D. $\frac{-\hbar^2}{mL^2}$

E. $\frac{\hbar^2}{3mL^2}$

F. $\frac{-\hbar^2}{3mL^2}$

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V(x)\psi(x) = 0$$

(Since $E=0$)

$$\frac{\partial^2\psi}{\partial x^2} = \frac{\partial}{\partial x}\left(A e^{-x^2/L^2} + Ax\cdot e^{-\frac{x^2}{L^2}} \times \frac{-2x}{L^2}\right)$$

$$= \frac{\partial}{\partial x}\left(A e^{-x^2/L^2} - \frac{2A}{L^2} x^2 e^{-x^2/L^2}\right)$$

$$= -\frac{2x}{L^2} A e^{-x^2/L^2} - \frac{4x}{L^2} A e^{-x^2/L^2} + \frac{4x^3}{L^4} A e^{-x^2/L^2}$$

$$= -\frac{6}{L^2} A x e^{-x^2/L^2} + \frac{4x^2}{L^4} A x e^{-x^2/L^2} = \left(-\frac{6}{L^2} + \frac{4x^2}{L^2}\right)\psi(x)$$

Since $V(x)\psi(x) = \frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2}$

$$V(x) = -\frac{\hbar^2}{2m}\frac{6}{L^2}$$

(at $x=0$)

$$= -\frac{3\hbar^2}{mL^2}$$

Related questions from previous year competitive exams

→ $V(x) = 0$

12. (2 points) **(PH-GATE 2012)** The wave function of particle moving in free space is given by $\Psi(x) = e^{ikx} + 2e^{-ikx}$. The energy of the particle is given by ______. (Hint: Use Schrödinger wave equation)

    A. $\frac{5\hbar^2 k^2}{2m}$

    B. $\frac{3\hbar^2 k^2}{4m}$

    C. $\frac{\hbar^2 k^2}{3m}$

    D. $\frac{3\hbar^2 k^2}{2m}$

    E. $\frac{\hbar^2 k^2}{2m}$

    F. $\frac{\hbar^2 k^2}{m}$

$$\frac{\partial^2 \psi}{\partial x^2} = -k^2 e^{ikx} - 2k^2 e^{-ikx}$$

Substituting in schrodinger equation with $V(x) = 0$

$$\frac{\hbar^2 k^2}{2m} \underbrace{\left(e^{ikx} + 2e^{-ikx}\right)}_{\psi(x)} = E\,\psi(x)$$

13. (1 point) **(EC-GATE 2014)** Cut off wavelength (in μm) of light that can be used for intrinsic excitation of a semiconductor material of band gap $1.1\ eV$.

    A. 0.85

    B. 1.125

    C. 1.45

    D. 2.25

    E. 0.67

    F. 1.24

$$\lambda\,[\text{in } \mu m] = \frac{1.24}{E_g\,[\text{in } eV]} = \frac{1.24}{1.1\,eV}$$

$\sim 1.125\ \mu m$

Any $\lambda > 1.125\ \mu m$ would have energy smaller than E_g and cannot excite an electron from VB to C.B.

14. (2 points) **(EC-ISRO 2012)** The band gap of elements arranged in ascending order is ______. (Ge, Si indicates - Germanium, Silicon respectively)

    A. Diamond, Ge, Si

    B. Si, Ge, Diamond

    C. Ge, Diamond,Si

    D. Diamond, Si, Ge

    E. Si, Diamond, Ge

    F. Ge, Si, Diamond

Ge and Si were mentioned in lecture.
Diamond is an insulator with bandgap of $\sim 5.5\ eV$ (lookup wikipedia!)

15. (1 point) **(ESE 2015)** The band gap of a material is $9\ eV$ at room temperature. The material can be a/an ______.

A. Insulator

B. Metal

C. Semi-metal

D. Direct band gap semiconductor

E. Indirect band gap semiconductor

F. cannot determine

Bandgap is much larger than typical insulators. SiO_2 which is an insulator has $E_g \sim 9$ eV