# A quick recap of the material covered in Week 4 lecture

## PN Junction Electrostatics (Depletion approximation)

Consider a step graded (abrupt) pn junction at thermal equlibrium, where the p-side doping density is $N_a$ and the n-side doping density is $N_d$ (typically $10^{15} - 10^{20}\ cm^{-3}$). Diffusion of the charge carriers creates a depletion region which is formed across the junction with width '$W$' (typically $1 - 10\ \mu m$). The depletion width on the p-side is '$x_p$' and on the n-side is '$x_n$' and $\boxed{W = x_p + x_n}$.
Due to the net space charge density, there is an electric field exists in the depletion region and its direction is from the $n \rightarrow p$. A maximum field ($\mathscr{E}_{max}$) is created exactly at the junction. A potential difference also known as the built-in potential formed across the region is $V_{bi}$.

- At equilibrium condition, $\mathscr{E}$ field is continuous at the junction (x=0) :

$$\boxed{qN_a x_p = qN_d x_n} \tag{1}$$

- The maximum electric field, $\mathscr{E}$ field:

$$\boxed{\mathscr{E}_{max} = -\frac{qN_a}{\epsilon_{si}} x_p = -\frac{qN_d}{\epsilon_{si}} x_n} \tag{2}$$

- Built-in potential across the junction:

$$\boxed{V_{bi} = -\frac{q}{2\epsilon_{si}}(N_d x_n^2 + N_a x_p^2)} \quad \boxed{V_{bi} = \frac{kT}{q} \ln\left(\frac{N_a N_d}{n_i^2}\right)} \tag{3}$$

- The width of the depletion region:

$$\boxed{W = \sqrt{\frac{2\epsilon}{q}\left(\frac{1}{N_a} + \frac{1}{N_d}\right) V_{bi}}} \tag{4}$$

- The width of the depletion region on the p-side and on the n-side:

$$\boxed{x_p = \frac{N_d}{N_a + N_d} W} \quad \boxed{x_n = \frac{N_a}{N_a + N_d} W} \tag{5}$$

### One sided junction

Consider a step graded (abrupt) $pn^+$ junction at thermal equilibrium, where the p-side dopant density is $N_a$ and the n-side dopant density is $N_d$. $(N_d >> N_a)$ The depletion width on the n-side is so small such that $W \simeq x_p$. The depletion region will cover almost in the lightly doped region $(N_a)$.
Similarly, $p^+n$ junction will have $N_a >> N_d$ and $W \simeq x_n$.

### Junction profiles

- In an abrupt junction,
$$\boxed{W \propto \sqrt{V_{bi}}} \tag{6}$$
- In a linearly graded junction,
$$\boxed{W \propto \sqrt[3]{V_{bi}}} \tag{7}$$

### PN Junction Nano Hub tool

Please refer to the additional document (**Week 4 - nanoHub Exercise**) attached separately. The nanoHub assignment will not be graded, but students are advised to attempt it to understand the concepts better.

## Solve the following 11 questions for a total of 25 marks.

1. (3 marks) A silicon pn junction at $T = 300\ K$ with zero applied bias has doping concentrations $N_a = 2 \times 10^{17}\ cm^{-3}$ and $N_d = 1 \times 10^{16}\ cm^{-3}$. (Use $n_i = 1.5 \times 10^{10}\ cm^{-3}$, $\epsilon_{Si} = 11.9$, $kT = 0.0259\ eV$)

   (a) Determine the built-in potential voltage in the pn junction (in $V$).

   Recap Eq 3

   A. 1.20

   **B. 0.77**

   C. 0.62

   D. 0.5

   $$V_{bi} = \frac{kT}{q} \ln\left(\frac{N_a N_d}{n_i^2}\right)$$

   $$= 0.0259 \times \ln\left(\frac{2\times10^{17} \times 1\times10^{16}}{(1.5\times10^{10})^2}\right) = \underline{0.772\ V}$$

   (b) Determine the width of the depletion region on the n-side in $\mu m$.

   Eq 4 and 5

   A. 0.326

   **B. 0.311**

   C. 0.015

   D. 0.64

   $$W = \sqrt{\frac{2\epsilon_{Si}}{q}\left(\frac{N_a + N_d}{N_a N_d}\right) V_{bi}} = \sqrt{\frac{2\times11.9\times8.85\times10^{-14}}{1.602\times10^{-19}} \times \frac{2\times10^{17}+1\times10^{16}}{2\times10^{31}} \times 0.77}$$

   $$\underline{W = 3.26\times10^{-5}\ cm}$$

   $$x_n = \frac{N_a}{N_a + N_d} W$$

   $$= 3.1\times10^{-5}\ cm$$

   $$\underline{= 0.31\ \mu m}$$

   (c) Determine the magnitude of maximum Electric field at the junction in $V/\mu m$.

   Eq. 2

   A. $4.72 \times 10^4$

   B. 47.2

   **C. 4.72**

   D. $47.2 \times 10^4$

   $$\mathcal{E}_{max} = \frac{q}{\epsilon_{Si}} N_d |x_n|$$

   $$= \frac{1.602\times10^{-19}}{\underbrace{11.9\times8.85\times10^{-14}}_{F/cm}} \times 1\times10^{16} \times \underline{3.1\times10^{-5}\ cm}$$

   Make sure to use consistent units for lengths.

   $$= 4.72\times10^{4}\ V/cm$$

   $$\underline{= 4.72\ V/\mu m}$$

2. (1 mark) Consider a pn junction, where there is a difference in the doping on both the sides. Then the space charge region will be extended more into ______ region.

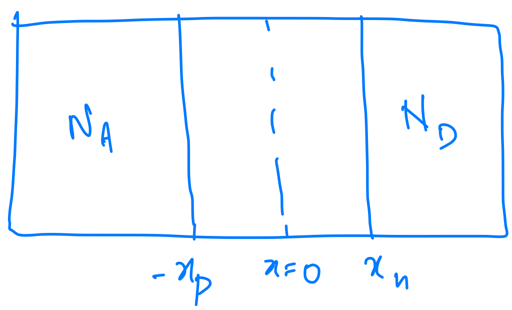
   **A. lightly doped**

   B. heavily doped

   C. middle of the junction

   D. can't determine

Recap Eq 1




$N_a x_p = N_d x_n$

Why?

Under equilibrium total charge has to be balanced on either side of junction.

If p is lightly doped

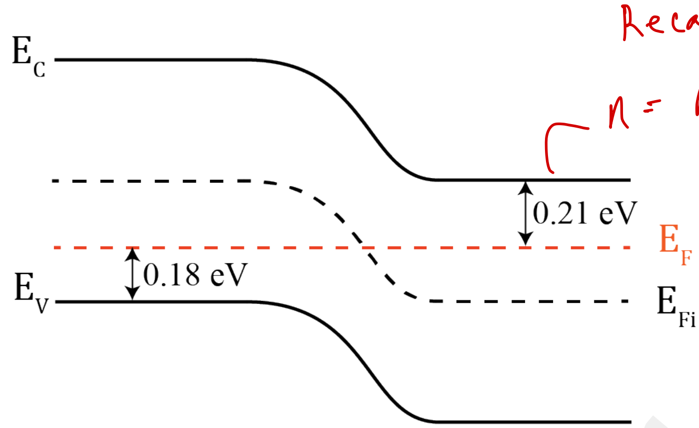
$N_a << N_d$

$x_p = \frac{N_d}{N_a} x_n$

$x_p >> x_n$ Junction will extend more into lightly doped region.

Analyze what happens when n is lightly doped.

3. (3 marks) Consider a step pn junction made of silicon under zero bias ($T = 300\ K$), and whose energy band diagram is as shown in the Figure 1. (Use $kT = 0.0259\ eV$, $n_i = 1.5 \times 10^{10}\ cm^{-3}$, band gap of silicon $E_g = 1.12\ eV$)




Recall $n = N_C \exp\left(-\frac{(E_C - E_F)}{kT}\right)$

But you are not given $N_C$. Need to find another way.

Recall $n = n_i \exp\left(\frac{E_F - E_i}{kT}\right)$

Figure 1: PN Junction Energy band diagram - Problem 3

This can be used!

(a) Find the doping density on the n-side in $cm^{-3}$.

A. $1.1 \times 10^{15}$

B. $3.5 \times 10^{15}$

**C. $1.1 \times 10^{16}$**

D. $3.5 \times 10^{16}$

$$E_F - E_i = \frac{E_g}{2} - (E_C - E_F)$$
$$= 0.56 - 0.21$$
$$= 0.35\ eV$$

$$n = 1.5 \times 10^{10} \times \exp\left(\frac{0.35}{0.0259}\right)$$
$$= 1.1 \times 10^{16}\ cm^{-3}$$

(b) Find the doping density on the p-side in $cm^{-3}$.

A. $1.1 \times 10^{15}$

B. $3.5 \times 10^{15}$

C. $1.1 \times 10^{16}$

**D.** $3.5 \times 10^{16}$

Repeat for P side

$$p = n_i \exp\left(\frac{E_i - E_F}{kT}\right)$$

$$= 1.5 \times 10^{10} \times \exp\left(\frac{0.38}{0.0259}\right)$$

$$= 3.5 \times 10^{16}\ cm^{-3}$$

(c) Find the built-in potential in $V_{bi}$.

A. 0.21

B. 0.35

**C. 0.73**

D. 0.38

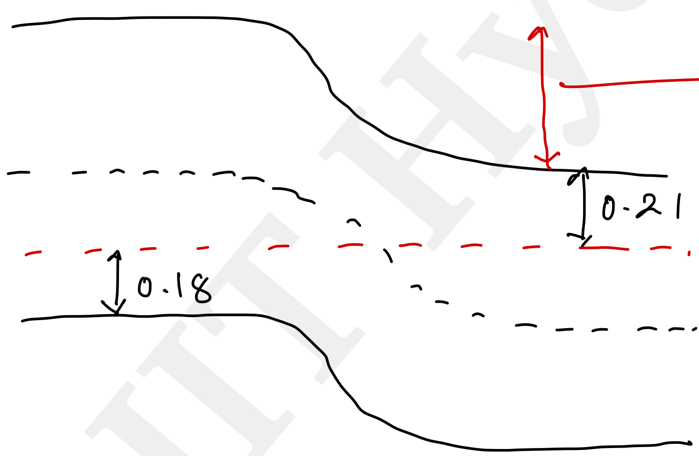
Recall Eq 3

$$V_{bi} = 0.0259 \ln\left(\frac{1.1 \times 10^{16} \times 3.5 \times 10^{16}}{(1.5 \times 10^{10})^2}\right)$$

$$= 0.729\,V$$

This approach requires you to get $N_a$ and $N_d$ right!

You can verify your answer by recalling $V_{bi}$ is difference $E_{i,p} - E_{i,n}$




This is $V_{bi}$

Verify that $V_{bi}$

$= (0.56 - 0.18) +$

$(0.56 - 0.21)$

$= 0.73\,V$!

This approach does not require accurate calculation of $N_a$ and $N_d$!

4. (1 mark) Consider an $n^+p$ Si-diode, $w_n$, $w_p$ are the depletion regions on n-side, p-side respectively. Which of the following is true?

   A. $w_n = w_p$

   B. $w_n >> w_p$

   C. $w_n << w_p$

   D. $w_n < w_p$

$n^+p$ means n-side is highly doped compared to p-side

$\therefore N_D >> N_a \Rightarrow w_n << w_p$

Refer to question 2 and compare.

5. (5 marks) Consider a step pn junction made of GaAs at $T = 300\ K$. At zero bias, only 20% of the total depletion region width is in the p-side. The built-in potential $V_{bi} = 1.20\ V$. (Use $n_i = 2 \times 10^6\ cm^{-3}$, $kT = 0.0259\ eV$, $\epsilon_{GaAs}$=12.88, $\epsilon_0 = 8.85 \times 10^{-14}\ F/cm$). Choose the closest answer while answering the questions below.

   (a) *(2 mark)* Determine the donor density in the n-side in $cm^{-3}$.

      A. $4.6 \times 10^{19}$

      B. $1.1 \times 10^{19}$

      C. $4.6 \times 10^{16}$

      D. $1.1 \times 10^{16}$

$x_p = \frac{1}{5} W$ (20% of W is in p side)

$x_n = \frac{4}{5} W$ $\Rightarrow x_n = 4 x_p$

$\Downarrow$

$N_a = 4 N_d$ (recall eq 1 & 5)

Aside: Can $V_{bi}$ be 1.2 V for a Silicon PN junction? Why?

$V_{bi} = 1.2\ V = 0.0259 \times \ln\left(\frac{N_a N_d}{n_i^2}\right)$

$\Rightarrow \frac{4 N_d^2}{n_i^2} = \exp\left(\frac{1.2}{0.0259}\right) = 1.32 \times 10^{20}\ cm^{-3}$

$\Rightarrow N_d = 1.15 \times 10^{16}\ cm^{-3}$

$N_a = \frac{N_d}{4} \Rightarrow 0.2875 \times 10^{16}\ cm^{-3}$

(b) *(2 marks)* Determine the depletion region width in $\mu m$ .

Recall Eq 4

A. 0.34

**B. 0.43**

C. 0.08

D. 1.2

$$W = \sqrt{\frac{2 \times 12.88 \times 8.85 \times 10^{-14}}{1.602 \times 10^{-19}} \times \frac{N_d + 4N_d}{4N_d^2} \times 1.2}$$

$$= 4.31 \times 10^{-5}\ cm = \underline{0.431\ \mu m}$$

(c) *(1 mark)* Determine maximum electric field in $kV/cm$.

Recall Eq 2

A. $5.56 \times 10^4$

B. 5.56

**C. 55.6**

D. $55.6 \times 10^4$

$$\mathcal{E}_{max} = \frac{q}{\epsilon_{GaAs}} \times |x_n| N_d$$

$$= \frac{1.6 \times 10^{-19}}{12.88 \times 8.85 \times 10^{-14}} \times \frac{4}{5} \times 4.31 \times 10^{-5} \times 1.15 \times 10^{16}$$

$$= 5.57 \times 10^4\ V/cm$$

$$= \underline{55.7\ kV/cm}$$

Choose closest answer!

6. (1 mark) The doping concentrations in a uniformly doped silicon pn junction are $N_a = 4 \times 10^{16}\ cm^{-3}$ and $N_d = 2 \times 10^{15}\ cm^{-3}$. Approximate the temperature (in $K$) at which the built-in potential barrier is $V_{bi} = 0.95\ V$. (Use at $T = 300\ K$, $kT = 0.0259\ eV$, $n_i = 1.5 \times 10^{10}\ cm^{-3}$)

   A. 300

   B. $<$ 300

   **C. $>$ 300**

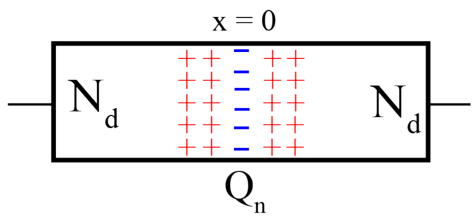
   D. 0

$V_{bi}$ at $300K = 0.0259 \times \ln\left(\frac{4\times10^{16}\times2\times10^{15}}{(1.5\times10^{10})^2}\right)$

$= 0.688\ V$

Since given $V_{bi} > 0.688$ $T > 300K$.

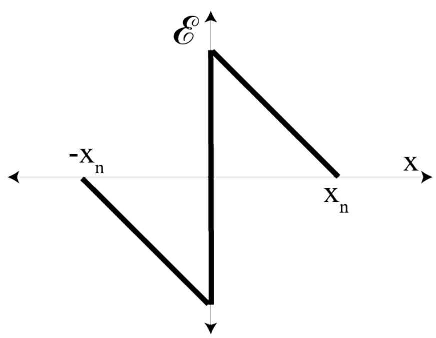
7. (5 marks) Consider a uniform n-type bar within which a thin sheet of negative charge has been embedded. This charge is fixed in the lattice and cannot move. This forces the majority carrier electrons to rearrange and satisfy Poisson's equation. Assume sheet charge density $Q_n$ in it as shown in the figure.



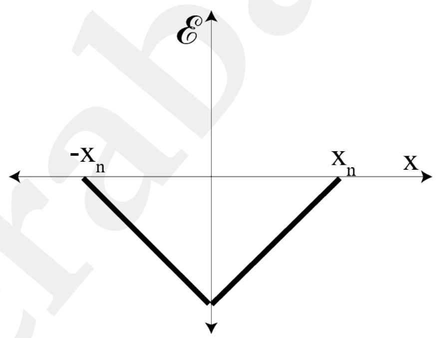

For $x > 0$
$\mathcal{E}$ is right to left
∴ -ve

$x < 0$
$\mathcal{E}$ is left to right
∴ +ve

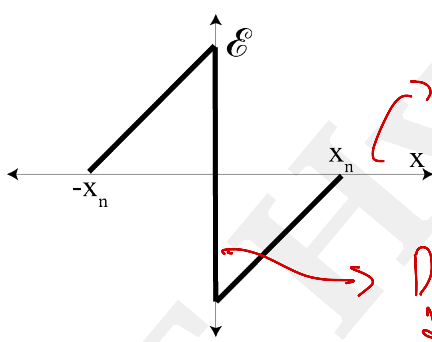
(a) Which of the following figures represent the electric field in it?



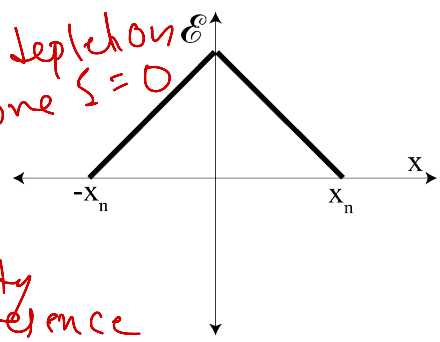

A.




C.




B.




D.

Outside depletion region $\mathcal{E} = 0$

Discontinuity due to presence of negative charge.

Correct Answer.

(b) The condition for the depletion region width which satisfies the charge neutrality is ______.

A. $W = \frac{Q_n}{2qN_d}$

**B.** $W = \frac{Q_n}{qN_d}$

C. $W = \frac{2Q_n}{qN_d}$

D. $W = \frac{qN_d}{Q_n}$

(c) The built in potential across the junction is ______ $V$.

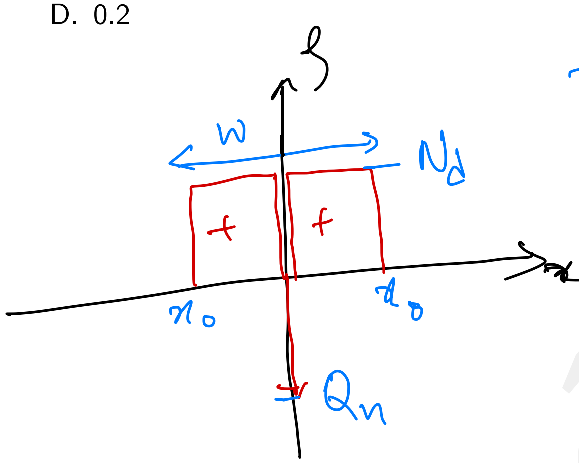
A. 1

**B. 0**

C. 1.2

D. 0.2

(b)




Total charge has to be zero.

$Q_n = q N_d W$

Total negative charge. Total positive charge

(c) The builtin potential is $\int_{\text{Depletion region.}} \mathcal{E}\, dx = 0$

$$\mathcal{E}(x<0) = \frac{qN_a}{\epsilon_{si}}(x+x_0)$$

$$\mathcal{E}(x>0) = -\frac{qN_a}{\epsilon_{si}}(x_0-x)$$

Verify that these are correct and $V_{bi} = 0$

8. (1 mark) What is the physical meaning of area under the curve $\mathscr{E}(x)$ vs $x$?

   A. Band gap of the semiconductor

   B. Net space charge density in the transition region

   **C. Built in potential of the junction**

   D. Total doping density in the transition region

Refer solution of 7(c)

9. (1 mark) In a one-sided pn junction ($N_d >> N_a$), the width of the depletion region varies with

   A. $\sqrt{V_{bi}}$, $\sqrt{N_a}$

   **B. $\sqrt{V_{bi}}$, $\frac{1}{\sqrt{N_a}}$**

   C. $\sqrt{V_{bi}}$, $N_a$

   D. $V_{bi}$, $\sqrt{N_a}$

Refer equation 6 and 7

## Previous year GATE questions

10. (2 marks) **(EC-GATE 2016)** Consider a silicon p-n junction with a uniform acceptor doping concentration of $10^{17}\ cm^{-3}$ on the p-side and a uniform donor doping concentration of $10^{16}\ cm^{-3}$ on the n-side. No external voltage is applied to the diode. Assume, $n_i = 1.5 \times 10^{10}\ cm^{-3}$, $\epsilon_{Si} = 12 \times \epsilon_0 = 12 \times 8.85 \times 10^{-14}\ F/cm$. The magnitude of charge per unit junction area (in $nC/cm^2$) in the depletion region on the p-side is ______.

    A. 100

    B. 500

    ~~C. 50~~ C. 500

    D. 20

Correct choice

Charge neutrality requires

$$Q_{donors} = Q_{acceptors}$$

$$\Rightarrow qN_d x_d = qN_a x_p$$

Charge on p side $= qN_a x_p$

$$x_p = \frac{N_d}{N_a+N_d} W = \frac{10}{11} W$$

$$V_{bi} = 0.0259 \times \ln\left(\frac{10^{16} \times 10^{17}}{(1.5\times10^{10})^2}\right) = 0.754\ V$$

$$W = \sqrt{\frac{2\epsilon_{Si}}{q} \frac{N_a+N_d}{N_a N_d} \times 0.754} = 3.3\times10^{-5}\ cm$$

$$Q = 1.602\times10^{-19} \times 10^{17} \times \frac{10}{11} \times 3.3\times10^{-5}$$

$$= 4.82\times10^{-7}\ C/cm^2$$

$$= 482\ nC/cm^2 \quad \boxed{\sim 500\ nC/cm^2}$$

Correct answer.

11. (2 marks) **(EC-GATE 2018)** A junction is made between $p^-$ - Si with doping density $N_{a1} = 10^{15}\ cm^{-3}$ and $p$ -Si with doping density $N_{a2} = 10^{17}\ cm^{-3}$. Assume $T = 300K$ and calculate the magnitude of built in potential (in $V$) across the junction.

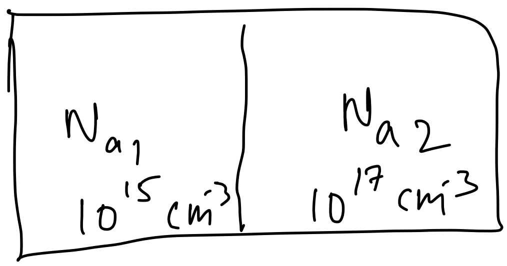
   A. 0.08

   B. 0.2

   **C. 0.12**

   D. 1.02

Here the junction is formed between 2 p type semiconductors




Notation

$p_p$ – holes in p type

$p_n$ – holes in n type

$n_n$ – electrons in n type

$n_p$ – electrons in p type

Rewriting

$$V_{bi} = KT \ln\left(\frac{N_a}{n_i^2/N_d}\right)$$

$$= KT \ln\left(\frac{p_p}{p_n}\right) = KT \ln\left(\frac{n_n}{n_p}\right)$$

We can generalize the above and write an expression for builtin potential between similar semiconductors as

$$|V_{bi}| = KT \ln\left(\frac{p_1}{p_2}\right) = KT \ln\left(\frac{n_1}{n_2}\right)$$

In this problem $p_1 = 10^{17}\ cm^{-3}$ $p_2 = 10^{15}\ cm^{-3}$

$$\therefore V_{bi} = KT \ln\left(\frac{10^{17}}{10^{15}}\right) = 0.12\ V$$