

A quick recap of the material covered in lectures

EFFECTIVE MASS

Effective mass relates the motion of a particle in a crystal to an externally applied force and takes into account the effect of the crystal lattice on the motion of the particle. The effective mass of an electron, m^* , is generally calculated by approximating the band edges by a parabolic equation -

$$E - E_c = C_1 k^2 \quad (1)$$

The energy E_c is the energy at the bottom of the band. Since $E > E_c$, the parameter C_1 is a positive quantity. The effective mass, m^* , is calculated as -

$$\frac{1}{\hbar^2} \frac{d^2 E}{d^2 k} = \frac{2C_1}{\hbar^2} = \frac{1}{m^*} \quad (2)$$

Eq.2 implies that m^* is inversely proportional to the curvature of the energy band. A similar analogy can be applied to the holes in the valence band.

EQUILIBRIUM CARRIER CONCENTRATIONS - INTRINSIC SEMICONDUCTOR

For an intrinsic semiconductor, the thermal equilibrium electron and hole concentrations are given by

$$n_0 = N_c \exp\left(\frac{-(E_c - E_F)}{kT}\right) \quad p_0 = N_v \exp\left(\frac{-(E_F - E_v)}{kT}\right) \quad (3)$$

The intrinsic carrier concentration is given by -

$$n_i^2 = N_c N_v \exp\left(\frac{-E_g}{kT}\right) \quad (4)$$

The intrinsic Fermi level w.r.t the midgap position depends on the effective masses of electron and hole, and is given by -

$$E_{Fi} - E_{midgap} = \frac{3kT}{4} \ln\left(\frac{m_p^*}{m_n^*}\right) \quad (5)$$

EQUILIBRIUM CARRIER CONCENTRATIONS - EXTRINSIC SEMICONDUCTOR

For an extrinsic semiconductor, the thermal equilibrium electron and hole concentrations are given by -

$$\boxed{n_0 = n_i \exp\left(\frac{(E_F - E_{Fi})}{kT}\right)} \quad \boxed{p_0 = n_i \exp\left(\frac{(E_{Fi} - E_F)}{kT}\right)} \quad (6)$$

Alternatively,

$$\boxed{n_0 = \frac{N_d - N_a}{2} + \sqrt{\left(\frac{N_d - N_a}{2}\right)^2 + n_i^2}} \quad \boxed{p_0 = \frac{N_a - N_d}{2} + \sqrt{\left(\frac{N_a - N_d}{2}\right)^2 + n_i^2}} \quad (7)$$

where N_d and N_a are donor and acceptor concentrations, respectively.

The mass action law is given by -

$$\boxed{n_0 p_0 = n_i^2} \quad (8)$$

The position of Fermi level w.r.t intrinsic Fermi level is -

$$\boxed{E_F - E_{Fi} = kT \ln\left(\frac{n_0}{n_i}\right)} \quad \boxed{E_{Fi} - E_F = kT \ln\left(\frac{p_0}{n_i}\right)} \quad (9)$$

CARRIER DRIFT

The drift current density and drift velocity for a semiconductor in an applied electric field, \mathcal{E} , is given by-

$$\boxed{J_{drift} = e(\mu_n n + \mu_p p)\mathcal{E} = \sigma \mathcal{E}} \quad \boxed{v_{drift} = \frac{e\tau \mathcal{E}}{m^*}} \quad (10)$$

where σ is the conductivity, τ is the mean time between collisions, and μ_n , μ_p are mobilities of electrons and holes respectively.

Solve the following questions. There are 18 questions, for a total of 25 marks.

1. (1 mark) Consider a region of Si, which is a perfect single crystal except for the phosphorous atom shown in the figure 1. Note that the P atom has donated one electron to the lattice. The net charge in region A and B is -

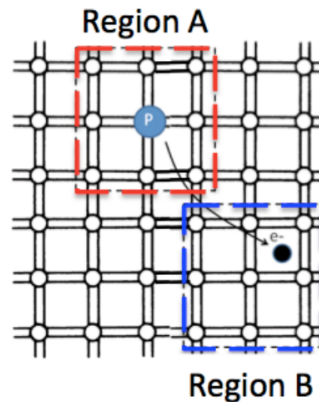


Figure 1: Silicon Lattice

- A. neutral, negative
- B. positive, positive
- C. positive, negative**
- D. neutral, neutral
- E. negative, positive
- F. negative, negative
2. (1 mark) As temperature increases from $T = 5\text{ K}$ to $T = 700\text{ K}$, the carrier concentration goes through three regions. In what order does the transition occur?
- A. intrinsic, extrinsic, partially ionized
- B. extrinsic, partially ionized, intrinsic
- C. partially ionized, intrinsic, extrinsic
- D. intrinsic, partially ionized, extrinsic
- E. partially ionized, extrinsic, intrinsic**
- F. extrinsic, intrinsic, partially ionized

3. (2 marks) A simplified E vs k curve for an electron in the conduction band is shown in figure 2. The value of a is 10 \AA . Determine the relative effective mass $\frac{m^*}{m_0}$. (Hint: Use the $E - E_c$ value at $k = \frac{\pi}{a}$ to compute the constant C_1 .)

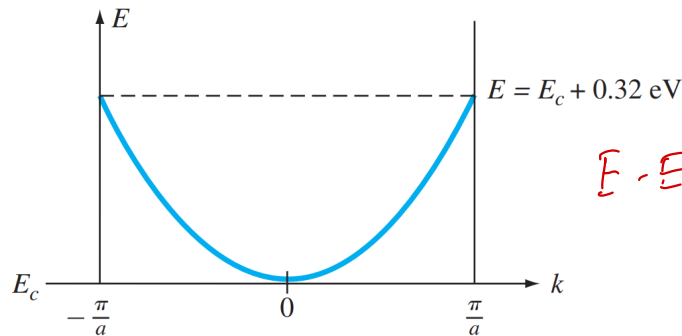


Figure 2: Effective mass

A. 0.2985

B. 1.175

C. 0.597

D. 0.1175

E. 2.985

F. 2.350

$$\text{Sol 1} \Rightarrow E - E_c = C_1 k^2$$

$$\frac{1}{\hbar^2} \frac{d^2 E}{dk^2} = \frac{2C_1}{\hbar^2} = \frac{1}{m^*}$$

$$0.32 \times 1.6 \times 10^{-19} = C_1 \times \frac{\pi^2}{(1 \times 10^{-9})^2}$$

$$C_1 = \frac{0.32 \times 1.6 \times 10^{-19} \times 10^{-18}}{\pi^2}$$

$$= 5.19 \times 10^{-39}$$

$$\frac{m^*}{m_0} = \frac{\hbar^2}{2C_1 m_0} = \frac{(1.05 \times 10^{-34})^2}{2 \times 5.19 \times 10^{-39} \times 9.1 \times 10^{-31}}$$

$$= 1.17$$

4. (1 mark) Assume that the Fermi level is near the valence band. Which of the following is true?

A. $n = p = n_i$

B. $n > p, n \gg n_i$

C. $p \gg n, p > n_i$

D. $n + p = n_i$

E. $n^2 + p^2 = n_i^2$

F. $p = n, p \gg n_i$

Analyze why other options are wrong

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5. (1 mark) Which of the following is the Fermi function?

A. $f = \frac{1}{1 - e^{(E-E_F)/k_B T}}$

B. $f = \frac{1}{1 + e^{(E-E_F)/k_B T}}$

C. $f = 1 - e^{(E+E_F)/k_B T}$

D. $f = \frac{1}{1 + e^{(E+E_F)/k_B T}}$

E. $f = \frac{1}{1 - e^{(E+E_F)/k_B T}}$

F. $f = 1 + e^{(E+E_F)/k_B T}$

6. (2 marks) The probability that a quantum state at energy $E = E_c + kT$ is occupied by an electron, ~~and its electron concentration (n_0)~~, in GaAs at $T = 300\text{ K}$ if the Fermi energy is 0.25 eV below E_c is _____

(take $kT = 0.025\text{ eV}$, $N_{c,\text{GaAs}} = 4.7 \times 10^{17}\text{ cm}^{-3}$ and $\frac{m_n^*}{m_0} = 0.067$)

A. 1.67×10^{-5} , $n_0 = 1.5 \times 10^{10}\text{ cm}^{-3}$

B. 1.67×10^{-3} , $n_0 = 1.9 \times 10^{15}\text{ cm}^{-3}$

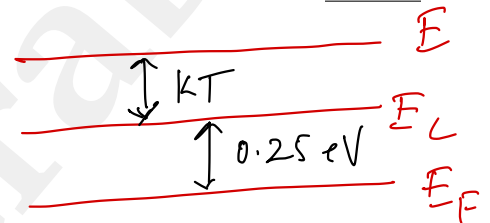
C. 2.13×10^{-3} , $n_0 = 1.67 \times 10^{15}\text{ cm}^{-3}$

D. 1.67×10^{-5} , $n_0 = 2.13 \times 10^{13}\text{ cm}^{-3}$

E. 1.5×10^{-5} , $n_0 = 1.67 \times 10^{10}\text{ cm}^{-3}$

F. 1.67×10^{-3} , $n_0 = 1.5 \times 10^{15}\text{ cm}^{-3}$

Also,
calculate
the electron
concentration
assuming
the material
to be GaAs



$$E = E_F + 0.25 + kT$$

$$f(E) = \frac{1}{1 + e^{(E-E_F)/kT}} = \frac{1}{1 + \exp((kT + 0.25)/kT)} = 1.67 \times 10^{-5}$$

$$n = N_c \exp\left(\frac{-(E_c - E_F)}{kT}\right) = 4.7 \times 10^{17} \times \exp\left(\frac{-0.25}{0.025}\right) = 2.13 \times 10^{13}\text{ cm}^{-3}$$

7. (1 mark) Two semiconductor materials have exactly the same properties except material A has a bandgap energy of 0.90 eV and material B has a bandgap energy of 1.10 eV . The ratio of intrinsic concentration ' n_{iB} ' of material B to that of intrinsic concentration ' n_{iA} ' of material A at $T = 400 \text{ K}$ is _____

A. 18.175

B. 1.8175

C. 0.305

D. 0.055

E. 0.55

F. 3.05×10^{-3}

$$n_i = \sqrt{N_C N_V} \exp \left(\frac{-E_g}{2kT} \right)$$

$$\frac{n_{iB}}{n_{iA}} = \frac{\exp \left(\frac{-1.1}{2kT} \right)}{\exp \left(\frac{-0.9}{2kT} \right)}$$

$$\begin{aligned}
 &= \exp \left(\frac{-0.2}{2kT} \right) \\
 &= \exp \left(\frac{-0.2}{2 \times 0.0345} \right) \\
 &= 0.055
 \end{aligned}$$

kT at 400 K

$$\begin{aligned}
 &= \frac{1.38 \times 10^{-23} \times 400}{1.602 \times 10^{-19}} \\
 &= 0.0345
 \end{aligned}$$

8. (2 marks) A company X aims to "design" a new semiconductor material. The semiconductor is to be p type and doped with $N_a = 5 \times 10^{15} \text{ cm}^{-3}$ acceptor atoms. Assume complete ionization and $N_d = 0$. The effective density of states functions are $N_c = 1.2 \times 10^{19} \text{ cm}^{-3}$ and $N_v = 1.8 \times 10^{19} \text{ cm}^{-3}$ at $T = 300 \text{ K}$, and vary as T^2 . A special semiconductor device fabricated with this material requires that the hole concentration be no greater than $5.08 \times 10^{15} \text{ cm}^{-3}$ at $T = 350 \text{ K}$. The minimum bandgap energy required in this new material is _____ eV (Hint: Use the charge neutrality equation to find n_i at $T = 350 \text{ K}$.)

A. 1.25

B. 0.625

C. 1.625

D. 0.325

E. 1.12

F. 1.42

We are given the required P_0 and asked to calculate E_g .

We know $n_i = \sqrt{N_c N_v} \exp\left(-\frac{E_g}{2kT}\right)$
We need to calculate n_i , N_c and N_v at 350 K .

$$N_{c,350\text{K}} = N_{c,300\text{K}} \cdot \left(\frac{350}{300}\right)^2$$

$$= 1.2 \times 10^{19} \times \left(\frac{350}{300}\right)^2$$

$$= 1.63 \times 10^{19}$$

$$N_{v,350\text{K}} = N_{v,300\text{K}} \cdot \left(\frac{350}{300}\right)^2$$

$$= 2.45 \times 10^{19}$$

But what is n_i ? We will determine this using charge neutrality relationship

Since $n_i^2 = N_c N_v \exp\left(-\frac{E_g}{kT}\right)$

$$4.06 \times 10^{29} = 1.63 \times 2.45 \times 10^{38} \times \exp\left(-\frac{E_g}{kT}\right)$$

Solving $E_g = 0.625 \text{ eV}$

Remember kT @ 350 K is

$$\frac{1.38 \times 10^{-23} \times 350}{1.6 \times 10^{-19}} = 0.03 \text{ eV}$$

Charge neutrality relationship:
 $P_0 + N_D = n_0 + N_A$
Since $N_D = 0$, $P_0 = N_A = 5 \times 10^{15}$
Given $P_0 = 5.08 \times 10^{15}$
Then $n_i^2 = 4.06 \times 10^{29}$

9. (2 marks) Silicon at $T = 300\text{ K}$ is doped with Arsenic atoms such that the concentration of electrons is $n_0 = 7 \times 10^{15}\text{ cm}^{-3}$. The position of Fermi level w.r.t E_c and $E_{F,i}$ (intrinsic Fermi level) is _____ (take $n_i = 1.5 \times 10^{10}\text{ cm}^{-3}$, $kT = 0.025\text{ eV}$, $N_c = 2.8 \times 10^{19}\text{ cm}^{-3}$)

- A. 0.326 eV below E_c and 0.207 eV above $E_{F,i}$
 B. 0.163 eV below E_c and 0.414 eV above $E_{F,i}$
 C. 0.326 eV above E_v and 0.207 eV below $E_{F,i}$
D. 0.207 eV below E_c and 0.326 eV above $E_{F,i}$
 E. 0.207 eV above E_v and 0.326 eV below $E_{F,i}$
 F. 0.414 eV below E_c and 0.163 eV above $E_{F,i}$

$$n_0 = N_c \exp\left(-\frac{E_c - E_F}{kT}\right)$$

$$E_c - E_F = kT \ln\left(\frac{N_c}{n_0}\right)$$

$$= 0.025 \times \ln\left(\frac{2.8 \times 10^{19}}{7 \times 10^{15}}\right)$$

$$= 0.207\text{ eV}$$

Also, $n_0 = n_i \exp\left(\frac{E_F - E_i}{kT}\right)$

$$\therefore E_F - E_i = kT \ln\left(\frac{n_0}{n_i}\right) = 0.025 \ln\left(\frac{7 \times 10^{15}}{1.5 \times 10^{10}}\right)$$

$$= 0.326\text{ eV}$$

10. (1 mark) A silicon sample is 2.5 cm long and has a cross-sectional area of 0.1 cm^2 . The silicon is n type with a donor impurity concentration of $N_d = 2 \times 10^{15} \text{ cm}^{-3}$. The resistance of the sample is measured and found to be 70Ω . The electron mobility (in $\text{cm}^2/(\text{V} \cdot \text{s})$) is _____

A. 44.64

B. 111.6

C. 44.4

D. 1116.1

E. 480

F. 1280

$$R = \frac{\rho L}{A} \Rightarrow 70 = \frac{\rho \cdot 2.5 \text{ cm}}{0.1 \text{ cm}^2}$$

$$\rho = 2.8 \Omega \cdot \text{cm}$$

$$\rho = \frac{1}{n q \mu}$$

$$\Rightarrow \mu = \frac{1}{2 \times 10^{15} \times 1.6 \times 10^{-19} \times 2.8}$$

$$= \frac{1116.1 \text{ cm}^2}{\text{V} \cdot \text{sec}}$$

11. (2 marks) A GaAs semiconductor resistor is doped with donor impurities at a concentration of $N_d = 2 \times 10^{15} \text{ cm}^{-3}$ and has a cross-sectional area of $5 \times 10^{-5} \text{ cm}^2$. A current of $I = 25 \text{ mA}$ is induced in the resistor with an applied bias of 5 V . The length of the resistor and the drift velocity of the electrons is given by _____ respectively. (Use $\mu_n = 8500 \text{ cm}^2/(\text{V} \cdot \text{s})$ and $\mu_p = 400 \text{ cm}^2/(\text{V} \cdot \text{s})$) (Hint: Use the definition of current density.)

A. $L = 0.0272 \text{ cm}, v_d = 1.56 \times 10^3 \text{ cm sec}^{-1}$

B. $L = 0.0156 \text{ cm}, v_d = 2.72 \times 10^5 \text{ cm sec}^{-1}$

C. $L = 0.272 \text{ cm}, v_d = 1.56 \times 10^6 \text{ cm sec}^{-1}$

D. $L = 0.0272 \text{ cm}, v_d = 1.56 \times 10^6 \text{ cm sec}^{-1}$

E. $L = 0.272 \text{ cm}, v_d = 1.56 \times 10^3 \text{ cm sec}^{-1}$

F. $L = 0.156 \text{ cm}, v_d = 2.72 \times 10^3 \text{ cm sec}^{-1}$

$$R = \frac{5 \text{ V}}{25 \text{ mA}} = 200 \Omega$$

$$R = \frac{\rho L}{A} = \frac{L}{nq\mu_n A}$$

$$\Rightarrow L = \frac{200 \times 2 \times 10^{15} \times 1.6 \times 10^{-19}}{8500 \times 5 \times 10^{-5}}$$

Drift velocity $v_d = \mu E$

$$= 0.027 \text{ cm}$$

$$= 8500 \times \frac{5 \text{ V}}{0.027}$$

$$= 1.57 \times 10^6 \text{ cm/sec}$$

12. (2 marks) The ^{electron} carrier density at which a minimum in the conductivity (σ_{min}) of the semiconductor occurs is _____

A. $n_i \sqrt{\frac{\mu_n^2}{\mu_p}}$

B. $n_i \times \frac{\mu_n}{\mu_p}$

C. $n_i \sqrt{\frac{\mu_p}{\mu_n}}$

D. $n_i \sqrt{\frac{\mu_n}{\mu_p^2}}$

E. $n_i^2 \times \frac{\mu_n}{\mu_p}$

F. insufficient information

$$\begin{aligned}\sigma &= n q \mu_n + p q \mu_p \\ &= n q \mu_n + \frac{n_i^2}{n} q \mu_p\end{aligned}$$

Minimum conductivity occurs when $\frac{d\sigma}{dn} = 0 \Rightarrow q \mu_n - \frac{n_i^2 q \mu_p}{n^2} = 0$

$$\Rightarrow n = n_i \sqrt{\frac{\mu_p}{\mu_n}}$$

If you choose $\frac{d\sigma}{dp} = 0$ the options would not match.

Questions adapted from previous GATE examinations

13. (1 mark) **(GATE - ECE 1987)** In an intrinsic semiconductor, the free electron concentration depends on _____

A. Temperature of the semiconductor

B. Effective mass of electrons only

C. Effective mass of holes only

D. Width of the forbidden energy band of the semiconductor

E. Applied bias

F. Difference between the conduction band energy and Fermi level only

14. (1 mark) **(GATE - ECE 2008)** Silicon is doped with boron to a concentration of $4 \times 10^{17} \text{ atoms/cm}^3$. Assuming intrinsic carrier concentration of silicon to be $1.5 \times 10^{10} \text{ cm}^{-3}$ and the value of kT/q to be 25 meV at 300 K . Compared to undoped silicon, the Fermi level of doped silicon _____

A. goes down by 0.13 eV

B. goes up by 0.13 eV

C. goes up by 0.427 eV

D. goes down by 0.427 eV

E. goes up by 0.39 eV

F. goes down by 0.39 eV

$$\begin{aligned} P_0 &= 4 \times 10^{17} \\ E_i - E_F &= kT \ln \left(\frac{4 \times 10^{17}}{1.5 \times 10^{10}} \right) \\ &= 0.427 \text{ eV} \end{aligned}$$

15. (1 mark) **(GATE - ECE 2014)** Consider a silicon sample doped with $N_D = 1 \times 10^{15} \text{ atoms/cm}^3$. Assume the intrinsic carrier concentration $n_i = 1.5 \times 10^{10} \text{ cm}^{-3}$. If the sample is additionally doped with $N_A = 1 \times 10^{18} \text{ atoms/cm}^3$ acceptor atoms, the approximate number of electrons/cm³ in the sample at $T = 300 \text{ K}$ will be _____

- A. $2.25 \times 10^{12} \text{ cm}^{-3}$
 B. $2.25 \times 10^3 \text{ cm}^{-3}$
 C. $2.25 \times 10^5 \text{ cm}^{-3}$
D. $2.25 \times 10^2 \text{ cm}^{-3}$
 E. $2.25 \times 10^8 \text{ cm}^{-3}$
 F. 2.25 cm^{-3}

$$n_0 = 10^{15} \text{ cm}^{-3} \quad p_0 = \frac{(1.5 \times 10^{10})^2}{10^{15}} = 2.25 \times 10^5$$

After additional doping, the sample becomes P type with $p_0 = N_A - N_D$

$$2.25 \times 10^{10} = 10^{18} \text{ cm}^{-3}$$

$$\therefore n_0 = \frac{2.25 \times 10^{10}}{10^{18}} \approx 2.25 \times 10^{-8} \text{ cm}^{-3}$$

- For exact number you need to apply charge neutrality relation.
16. (1 mark) **(GATE - ECE 2014)** A silicon sample is uniformly doped with donor type impurities with a concentration of 10^{16} cm^{-3} . The electron and hole mobilities in the sample are $1200 \text{ cm}^2/(\text{V} \cdot \text{sec})$ and $400 \text{ cm}^2/(\text{V} \cdot \text{sec})$ respectively. Assuming complete ionization of impurities, the resistivity of the sample in $\Omega \cdot \text{cm}$ is _____

- A. 1.92
B. 0.52
 C. 0.95
 D. 5.2
 E. 1.04
 F. 19.2

$$\rho = \frac{1}{n q \mu_n}$$

(Ignoring contribution of holes which will be much smaller in number)

$$= \frac{1}{10^{16} \times 1.6 \times 10^{-19} \times 1200}$$

$$= 0.52 \Omega \cdot \text{cm}$$

17. (1 mark) **(GATE - PHY 2013)** A phosphorous doped silicon semiconductor (doping density : 10^{17} cm^{-3}) is heated from 100°C to 200°C . Which one of the following statements is CORRECT?

- A. Position of Fermi level moves towards conduction band
- B. Position of Fermi level moves towards valence band
- C. Position of dopant level moves towards conduction band
- D. Position of dopant level moves towards middle of energy gap
- E. Position of Fermi level moves beyond conduction band

F. Position of Fermi level moves towards middle of energy gap

18. (2 marks) **(GATE - PHY 2016)** The energy *vs.* wave vector ($E - k$) relationship near the bottom of a band for a solid can be approximated as $E(k) = A(ka)^2 + B(ka)^4$, where the lattice constant $a = 2.1 \text{ \AA}$. The values of A and B are $6.3 \times 10^{-19} \text{ J}$ and $6.3 \times 10^{-20} \text{ J}$, respectively. At the bottom of the conduction band, the ratio of the effective mass of the electron to the mass of free electron is (m^*/m_0) _____

A. 0.22

B. 0.108

C. 2.2

D. 0.022

E. 0.011

F. 22.2

$$\begin{aligned}
 E &= A \cdot k^2 a^2 + B k^4 a^4 \\
 \frac{dE}{dk} &= 2Aa^2 k + 4Ba^4 k^3 \\
 \frac{d^2E}{dk^2} &= 2Aa^2 + 12Ba^4 k^2 \\
 \text{At the bottom of the C.B } k &= 0 \\
 \frac{m^*}{m_0} &= \frac{\hbar^2}{\frac{d^2E}{dk^2}} = \frac{\hbar^2}{2Aa^2 m_0} = \frac{2 \left(\frac{1.054 \times 10^{-34}}{2 \times 6.3 \times 10^{-19} \times (2.1 \times 10^{-10})^2} \right)^2}{\times 9.1 \times 10^{-31}} \\
 &= 0.22
 \end{aligned}$$