AI1110 ASSIGNMENT-1

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Exercise 1

1.1

The sample generated is

https://github.com/NareshBandaru13/ASSIGNMENT1/tree/main/example%201/1.1

1.2

The python code is

https://github.com/NareshBandaru13/ASSIGNMENT1/tree/main/example%201/1.2

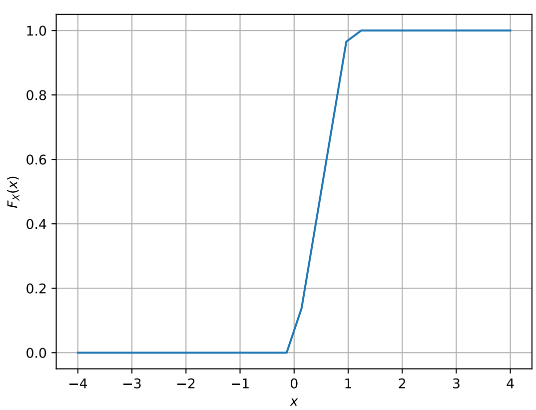

Fig. 1: uni-cdf

1.3

Given that,
U is uniform random variable.
So,

$$f_X(x) = \begin{cases} 1 & 0 \le x \le 1 \\ 0 & otherwise \end{cases}$$

$$\begin{aligned} F_X(x) &= \int_{-\infty}^{x} f_X(x)dx \\ &= 0 + \int_0^x 1dx \\ &= \begin{cases} 1 & x > 1 \\ x & 0 \le x \le 1 \\ 0 & x < 0 \end{cases} \end{aligned}$$

1.4

The c code to find mean and variance is

https://github.com/NareshBandaru13/ASSIGNMENT1/tree/main/example%201/1.4

1.5

Given

$$E[U^k] = \int_{-\infty}^{\infty} x^k dF_X(x)$$

1) k=1

$$\begin{aligned} E[U] &= \int_{-\infty}^{\infty} x^k f_X(x)dx \\ &= \int_0^1 x \times 1dx \\ &= \left[\frac{x^2}{2}\right]_0^1 = \frac{1}{2} \end{aligned}$$

2) k=2

$$\begin{aligned} E[U^2] &= \int_0^1 x^2 \times 1dx \\ &= \left[\frac{x^3}{3}\right]_0^1 = \frac{1}{3} \end{aligned}$$

$$\begin{aligned} variance &= E[u - E[u]]^2 \\ &= E[U^2] - E^2[U] \\ &= \frac{1}{3} - \frac{1}{4} = 0.0833 \end{aligned}$$

Exercise 2

2.1

The gau.dat file is given by

https://github.com/NareshBandaru13/ASSIGNMENT1/tree/main/example%202/2.1

2.2

cdf plot of gau.dat

https://github.com/NareshBandaru13/ASSIGNMENT1/tree/main/example%202/2.2

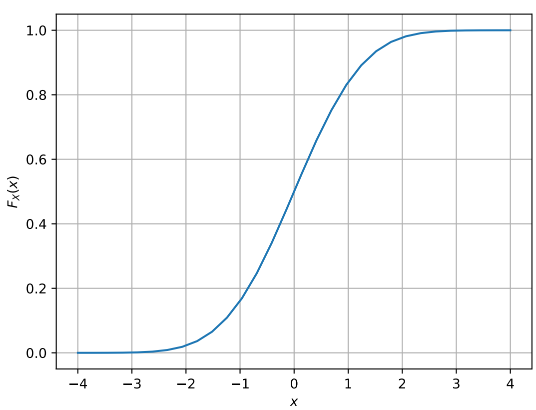

Fig. 2: gauss-cdf

2.3

PDF of X is given by

https://github.com/NareshBandaru13/ASSIGNMENT1/tree/main/example%202/2.3

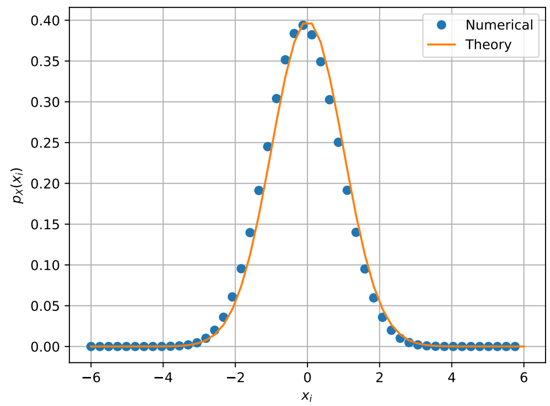

Fig. 3: gauss-pdf

2.4

C code to find the mean and variance

https://github.com/NareshBandaru13/ASSIGNMENT1/tree/main/example%202/2.4

2.5

Given that,

$$p_X(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}, -\infty < x < \infty$$

By property of probability:

$$\int_{-\infty}^{\infty} p_X(x)dx = 1$$

$$\begin{aligned} F_X(x) &= \int_{-\infty}^{x} p_X(x)dx \\ &= \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \\ &= G(x - \mu/\sigma) \text{ where} \mu = 0, \sigma = 1 \\ &= G(x) \end{aligned}$$

Here the random variable $X \sim N(0, 1)$
Comparing the random variable in 2.1 :

$$X_0 = \sum_{i=1}^{12} U_i - 6$$

U_i are independent.
We get:
$\mu_0 = 0.0002$
$\sigma_0 = 0.666$
As i increases, we approach normal distribution.

Exercise 3

3.1

Generated samples of V and its cdf is given by

https://github.com/NareshBandaru13/ASSIGNMENT1/tree/main/example%203/3.1

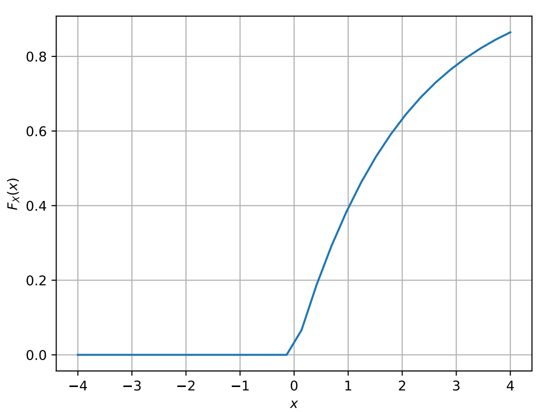

Fig. 4: V-cdf

3.2

$$
\begin{aligned}
F_V(x) &= P\{V \leq x\} \\
&= P\{-2 \times \ln(1 - U) \leq x\} \\
&= P\{U \leq 1 - e^{(-\frac{x}{2})}\} \\
&= F_U\{1 - e^{(-\frac{x}{2})}\} \\
&= \begin{cases} 1 - e^{(-\frac{x}{2})} & 0 \leq x < \infty \\ 0 & x < 0 \end{cases}
\end{aligned}
$$