AI1110
Assignment 5

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Outline

Exercise 6.33

Let x and y be jointly normal random variables with parameters $\mu_x, \mu_y, \sigma_x^2, \sigma_y^2$ and r.Find a necessary and sufficient condition for x+y and x-y to be independent.

Solution

We have,
z = x + y
w = x - y
are jointly normal random variables.
Hence if they are uncorrelated,then they are also independent.

$$\begin{aligned} Cov(z,w) &= E[(z-\mu_z)(w-\mu_w)] && (1) \\ &= E[(x-\mu_x)+(y-\mu_y)(x-\mu_x)-(y-\mu_y)] && (2) \\ &= Var(x) - Var(y) && (3) \\ &= \sigma_x^2 - \sigma_y^2 && (4) \end{aligned}$$

Answer

The random variables z and w are uncorrelated.

$$\implies Cov(z, w) = 0 \tag{5}$$

$$\implies \sigma_x^2 = \sigma_y^2 \tag{6}$$

Hence $\sigma_x^2 = \sigma_y^2$ is the necessary and sufficient condition for the independence of x+y and x-y.