AI1110
Assignment 9

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Outline

Exercise 10.25

If $R_n(\tau) = N\delta(\tau)$ and
$x(t) = A\cos\omega_0 t + n(t)$
$H(\omega) = \dfrac{1}{\alpha + j\omega}$
$y(t) = B\cos(\omega_0 + t + \phi) + y_n(t)$
where $y_n(t)$ is the component of the output y(t) due to n(t), find the value of α that maximises the signal to noise ratio $\dfrac{B^2}{E(y_n^2(t))}$

Solution

We have,

$$B = A|H(\omega_0)| = \frac{A}{\sqrt{\alpha^2 + \omega_0^2}} \tag{1}$$

$$S_{y_n}(\omega) = \frac{N}{\alpha^2 + \omega_0^2} \tag{2}$$

$$R_{y_n}(\tau) = \frac{N}{2\alpha} e^{-\alpha|\tau|} \tag{3}$$

$$Ey_n^2(t) = R_{y_n}(0) = \frac{N}{2\alpha} \tag{4}$$

Answer

Hence,
$\dfrac{B^2}{E(y_n^2(t))} = \dfrac{2A^2}{N}\dfrac{\alpha}{\alpha^2+\omega_0^2}$
Differentiating, we get

$$\frac{1(\alpha^2+\omega_0^2) - \alpha(2\alpha)}{(\alpha^2+\omega_0^2)^2} = 0 \tag{5}$$

$$\omega_0^2 - \alpha^2 = 0 \tag{6}$$

$$\alpha = \omega_0 \tag{7}$$

Also, $f''(\alpha) < 0$
$\therefore \alpha = \omega_0$ is the maxima value for given ratio.