AI1110
Assignment 11

Bandaru Naresh Kumar
AI21BTECH11006

Outline

Exercise 15.12

In the genetic model in (15-31),consider the possibility that prior to the formation of a new generation each gene can spontaneously mutate into a gene of the other kind with probabilities
$P(A \rightarrow B) = \alpha(> 0)$ and
$P(B \rightarrow A) = \beta(> 0)$
Thus for a system in state e_j, after mutation there are
$N_A = j(1 - \alpha) + (N - 1)\beta$ genes of type A and $N_B = j\alpha = (N - 1)(1 - \beta)$
genes of type B.

Exercise 15.12

Hence the modified probabilities prior to forming a new generation are
$p_j = \frac{N_A}{N} = \frac{j}{N}(1-\alpha) + (1-\frac{j}{N})\beta$ and
$q_j = \frac{N_B}{N} = \frac{1}{N}\alpha + (1-\frac{j}{N})(1-\beta)$
for the A and B genes,respectively.

Exercise 15.12

This gives
$p_{jk} = \binom{N}{k} p_j^k q_j^{N-k}$ where j,k = 0,1,2,....,N
to be the modified transition probabilities for the Morkov chain with mutation.Derive the steady state distribution for this model,and show that,unlike the models in (15-30) and (15-31),fixation to ”the pure gene states” does not occur in this case.

Solution

In this case,the chain is irreducible and aperiodic and there are no absorption states.
The steady state distribution $\{u_k\}$ satisfies:
$u_k = \sum_j u_j p_{jk} = \sum_{j=0}^{N} u_j \binom{N}{k} p_j^k q_j^{N-k}$
So if $\alpha > 0$ and $\beta > 0$,then "fixation to pure genes" does not occur.