AI1110
Assignment 10

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Outline

Exercise 12.2

Show that if a process is normal and distribution-ergodic,then it is also mean-ergodic.

Solution

The process X(t) is normal and such that:

$$F(x,x;\tau) \to F^2(x) as \tau \to \infty \tag{1}$$

We shall show that it is mean-ergodic.It suffices to show that:
$C(\tau) \to 0$ as $\tau \to \infty$

Proof

We can assume that $\eta = 0$ and $C(0) = 1$.
With this assumption,
The RVs $X(t+\tau)$ and X(t) are N(0,0;1,1;r) where $r = r(\tau) = C(\tau)$ is the autocovariance of X(t).
Hence,

$$f(x_1, x_2; \tau) = \frac{1}{2\Pi\sqrt{1-r^2}} exp\left\{-\frac{1}{2(1-r^2)}(x_1^2 - 2rx_1x_2 + x_2^2)\right\} \quad (2)$$

$$= \frac{1}{2\Pi\sqrt{1-r^2}} exp\left\{-\frac{1}{2(1-r^2)}(x_1 - rx_2)^2\right\} e^{-\frac{x_2^2}{2}} \quad (3)$$

Clearly, $f(x, y) = f(y, x)$
Hence,

$$F(x + dx, x + dx; \tau) - F(x, x; \tau) = 2 \int_{-\infty}^{x} f(\xi, x)\, d\xi dx \tag{4}$$

$$= \frac{1}{\Pi \sqrt{1 - r^2}} \int_{-\infty}^{x} exp\left\{ -\frac{1}{2(1 - r^2)} (\xi - xr)^2 \right\} d\xi e^{-\frac{x_2^2}{2}} dx \tag{5}$$

Answer

Furthermore,

$$F^2(x+dx) - F^2(x) = 2F(x)f(x)dx \tag{6}$$

From (1) and (6);

$G\left(\dfrac{x - rx}{\sqrt{1 - r^2}}\right) \to G(x)$ as $\tau \to \infty$

Hence,

$r(\tau) \to 0$ as $\tau \to \infty$