AI1110 ASSIGNMENT 2

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Q.Verify Lagrange's mean value theorem for the function:f(x)=x(1-$\log x$)and find the value of c in the interval [1,2]

Solution:

Given,

f(x) = $x(1-\log x)$

In [1,2]

x and $\log x$ are continuous.

$\therefore$ f(x) is continuous.

We have,

$$f'(x) = (1-\log x) + x\left(-\frac{1}{x}\right) \tag{0.0.1}$$

$$\implies f'(x) = -\log x \tag{0.0.2}$$

As $-\log x$ exists for all x in [1,2],

f(x) is differentiable in [1,2].

By Lagrange's mean value theorem,

$$f'(c) = \frac{f(2)-f(1)}{2-1} \tag{0.0.3}$$

$$\implies -\log c = \frac{(2-2\log 2)-(1-\log 1)}{1} \tag{0.0.4}$$

$$\implies -\log c = 1-2\log 2 \tag{0.0.5}$$

$$\implies \log c = \log 4 - \log e \tag{0.0.6}$$

$$\implies c = \frac{4}{e} \tag{0.0.7}$$

$\therefore$ The value of c for Lagrange's mean value theorem in [1,2] is $\frac{4}{e}$

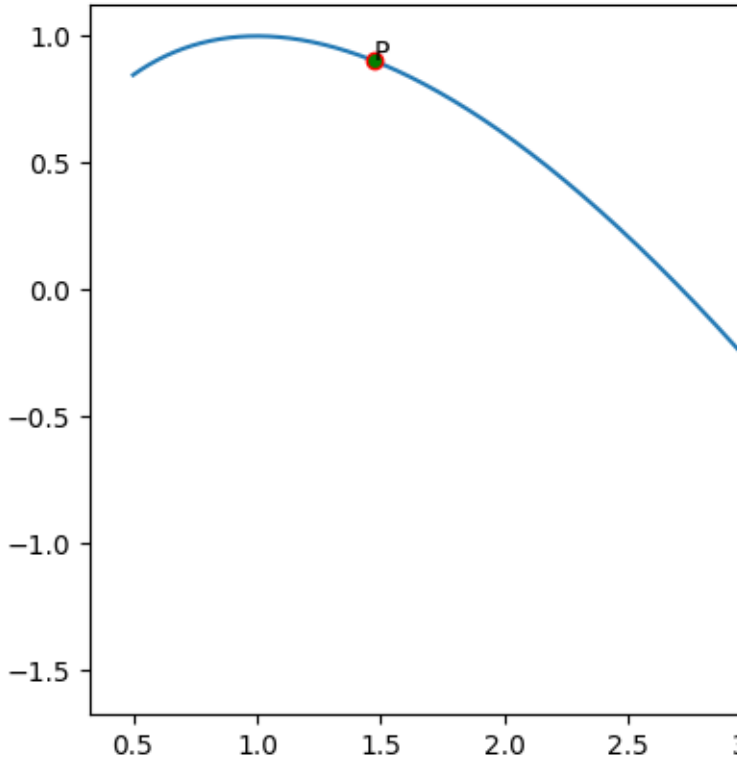

Fig. 0. graph