AI1110 ASSIGNMENT 1

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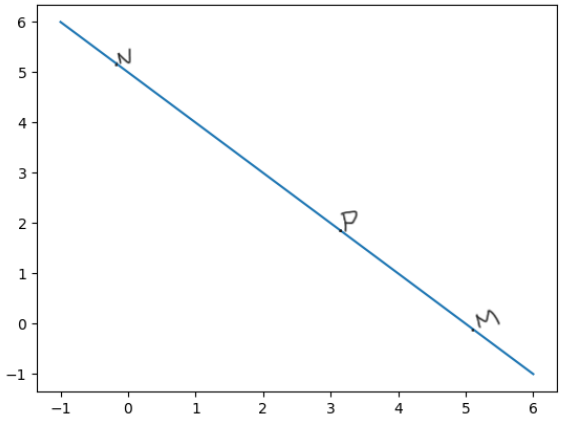
Q3 (b): $\mathbf{M}$ and $\mathbf{N}$ are two points on the X axis and Y axis respectively. $\mathbf{P}(3,2)$ divides the line segment MN in the ratio 2:3.
Find:
(i)the coordinates of $\mathbf{M}$ and $\mathbf{N}$
(ii)the slope of MN.



Solution:
Given,
$\mathbf{M}$ and $\mathbf{N}$ are two points on X and Y axes respectively.
Define:

$$\mathbf{e_1} = \begin{pmatrix}1\\0\end{pmatrix} \tag{0.0.1}$$

$$\mathbf{e_2} = \begin{pmatrix}0\\1\end{pmatrix} \tag{0.0.2}$$

Let

$$\mathbf{M} = a\mathbf{e_1} \tag{0.0.3}$$

$$\mathbf{N} = b\mathbf{e_2} \tag{0.0.4}$$

$\mathbf{P}$ divides MN in the ratio 2:3.

According to Section formula, If $\mathbf{P}$ divides MN in the ratio k:1,then:

$$\mathbf{P} = \frac{k(\mathbf{N}) + 1(\mathbf{M})}{k+1} \tag{0.0.5}$$

$$\mathbf{P} = \frac{bk\mathbf{e_2} + a\mathbf{e_1}}{k+1} \tag{0.0.6}$$

$$\mathbf{P} = \left(\frac{a}{k+1}\right)\mathbf{e_1} + \left(\frac{bk}{k+1}\right)\mathbf{e_2} \tag{0.0.7}$$

But we have,

$$\mathbf{P} = \begin{pmatrix}3\\2\end{pmatrix}$$

Therefore,

$$\left(\frac{a}{k+1}\right)\mathbf{e_1} + \left(\frac{bk}{k+1}\right)\mathbf{e_2} = \begin{pmatrix}3\\2\end{pmatrix} \tag{0.0.8}$$

$$\left(\frac{a}{k+1}\right)\mathbf{e_1} + \left(\frac{bk}{k+1}\right)\mathbf{e_2} = 3\mathbf{e_1} + 2\mathbf{e_2} \tag{0.0.9}$$

$$\implies \frac{a}{k+1} = 3 \text{ and } \frac{bk}{k+1} = 2$$

$$\implies a = 3(k+1) \text{ and } b = \frac{2(k+1)}{k}$$

Substituting $k = \frac{2}{3}$, we get:
$a = 5$ and $b = 5$

(i)$\mathbf{M} = 5\mathbf{e_1}$ and $\mathbf{N} = 5\mathbf{e_2}$

(ii) Slope of MN = $\frac{5-0}{0-5}$

= -1