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GIVEN: Samples $0, 1, 0, 0, 1, 0$ from a binomial distribution which has the form: P(x=0)=(1-μ), P(x=1)=μ

REQUESTED: What is the maximum likelihood estimate of μ Hint: you can use SymPy to compute the derivities symboliclly

Let's say we have $X_1, \ldots, X_n$ iid Bernoulli random variable with probability p as it is a binomial distribution

The PMF of a Bernoulli random variable X is:

$$p_{X_i}(k) = p^k(1-p)^{1-k} \; where \; k \in 0, 1$$

The likelihood function for this distribution for n samples assuming all the sample are from same distribution with probability p

$$L(p) = \prod_{i=1}^{n} p^{x_i}(1-p)^{x_i}$$

Likelihood function in log scale:

$$l(p) = logL(p) = \log p \sum_{n=1}^{n} X_i + \log(1-p) \sum_{n=1}^{n} (1 - X_i)$$

Maximum Likelihood estimator of parameter p can be caluclated by differentiating the above equation wrt p and equating it to zero

$$l'(p) = \frac{\sum_{n=1}^{n} X_i}{p} - \frac{sum_{n=1}^{n}(1-X_i)}{1-p} = 0$$
$$\frac{\sum_{n=1}^{n} X_i}{p} = \frac{sum_{n=1}^{n}(1-X_i)}{1-p}$$

Solving for p

$$p = \frac{X_i}{n}$$

Given the samples $0, 1, 0, 0, 1, 0$

$n = 6$

$P(X_i = 0) = (1 - \mu)$

$P(X_i = 1) = \mu$
The probability p is the summation of all the sample divided by number of samples $p = \frac{X_i}{n}$ $p = \mu = \frac{2}{6}$

$\mu = 0.33$

Reference:

https://stats.stackexchange.com/questions/275380/maximum-likelihood-estimation-for-bernoulli-distribution (https://stats.stackexchange.com/questions/275380/maximum-likelihood-estimation-for-bernoulli-distribution)

In [3]:
```
import sympy as sp
import numpy as np
from sympy.abc import x, z
p=sp.symbols('p')
y=sp.Function('y')(p)
# the likelihood fucntion as a product of Bernoulli variable
y=(p**2)*((1-p)**4)
# taking log
logliklihood = sp.expand_log(sp.log(y))
mu_estimate = sp.solve(sp.diff(logliklihood,p),p)[0]

# printing the solution found through maximum likelihood
mu_estimate
```

Out[3]: 1/3

References

https://ocw.mit.edu/courses/mathematics/18-05-introduction-to-probability-and-statistics-spring-2014/readings/MIT18_05S14_Reading10b.pdf (https://ocw.mit.edu/courses/mathematics/18-05-introduction-to-probability-and-statistics-spring-2014/readings/MIT18_05S14_Reading10b.pdf)