Narges Sadat Seyed Haeri – 810100165

Signals & Systems
CA7 Report

- ***Part 1:***

We find the solution to the differential equation like this:

$$R\frac{di(t)}{dt} + l\ \frac{d^2i(t)}{dt} + \frac{1}{c}\int_{-\infty}^{t} i(\tau)d\tau = v_{in}(t)$$

Applying the Laplace transform:

$$RsI(s) + ls^2I(s) + \frac{1}{c}\ I(s) = sV(s)$$

$$ls^2I(s) + RsI(s) + \frac{1}{c}I(s) = sV(s)$$

We know that: $v_c(t) = \frac{1}{c}\int_{-\infty}^{t} i(\tau)d\tau$

The Laplace Transform will be:

$$V_c(s) = \frac{1}{c} * \frac{1}{s} * I(s)$$

$$I(s)\left(ls^2 + Rs + \frac{1}{c}\right) = sV(s)$$

$$I(s) = \frac{sV(s)}{ls^2 + Rs + \frac{1}{c}}$$

Taking $x(t) = V_{in}(t)$ and $y(t) = V_c(t)$:

$$Y(s) = \frac{1}{c}\frac{X(s)}{ls^2 + Rs + \frac{1}{c}}$$

Applying $c = \frac{4}{3}, L\ =\ \frac{1}{4}, R\ =\ 1$:

Also we know that $H(s) = \frac{Y(s)}{X(s)}$

$$H(s) = \frac{\frac{3}{2}}{s+1} + \frac{-\frac{3}{2}}{s+3}$$

We compute the step response in the following manner:

$$H(s) = \frac{3}{(s+1)(s+3)}$$

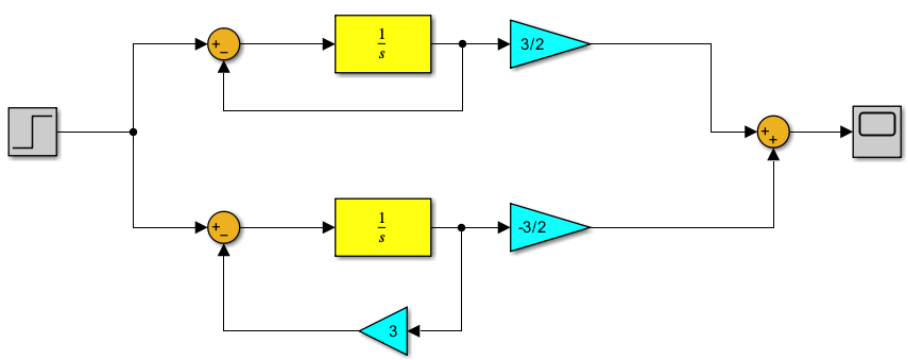
$$U(s) = \frac{1}{s}$$

$$H(s)U(s) = \frac{3}{s(s+1)(s+3)} = \frac{-\frac{3}{2}}{(s+1)} + \frac{\frac{1}{2}}{(s+3)} + \frac{1}{s}$$

Using the inverse Laplace transform we have:

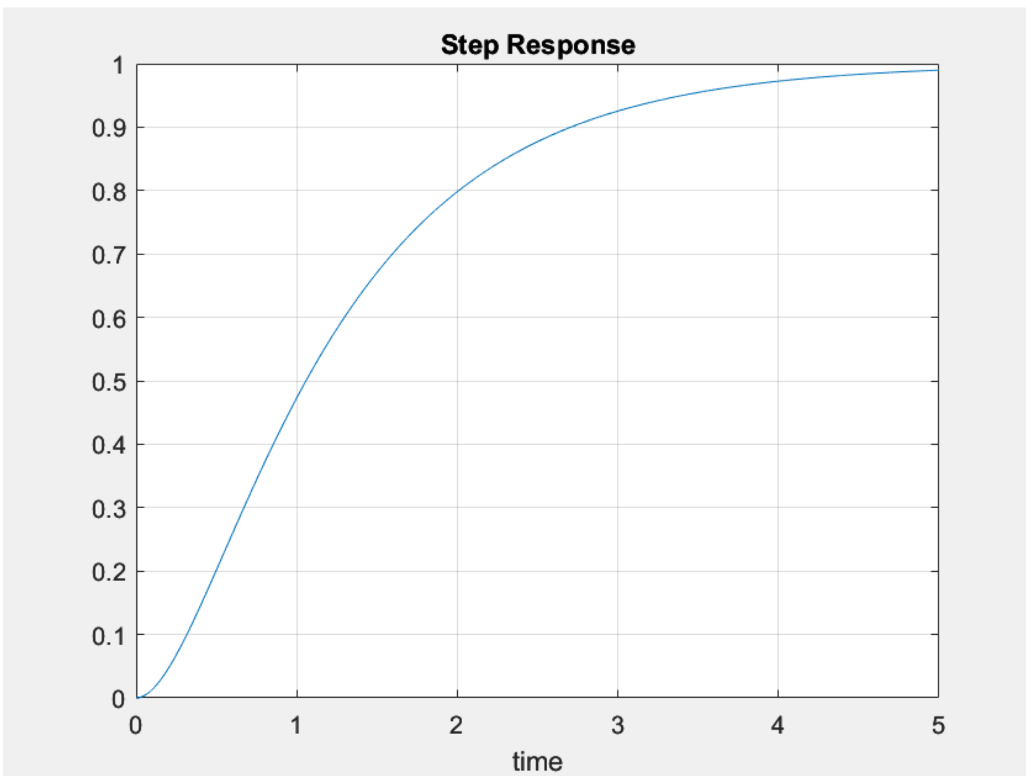
$$S(t) = -\frac{3}{2}e^{-t}u(t) + \frac{1}{2}e^{-3t}u(t) + u(t)$$

We sketch the block diagram as follows:

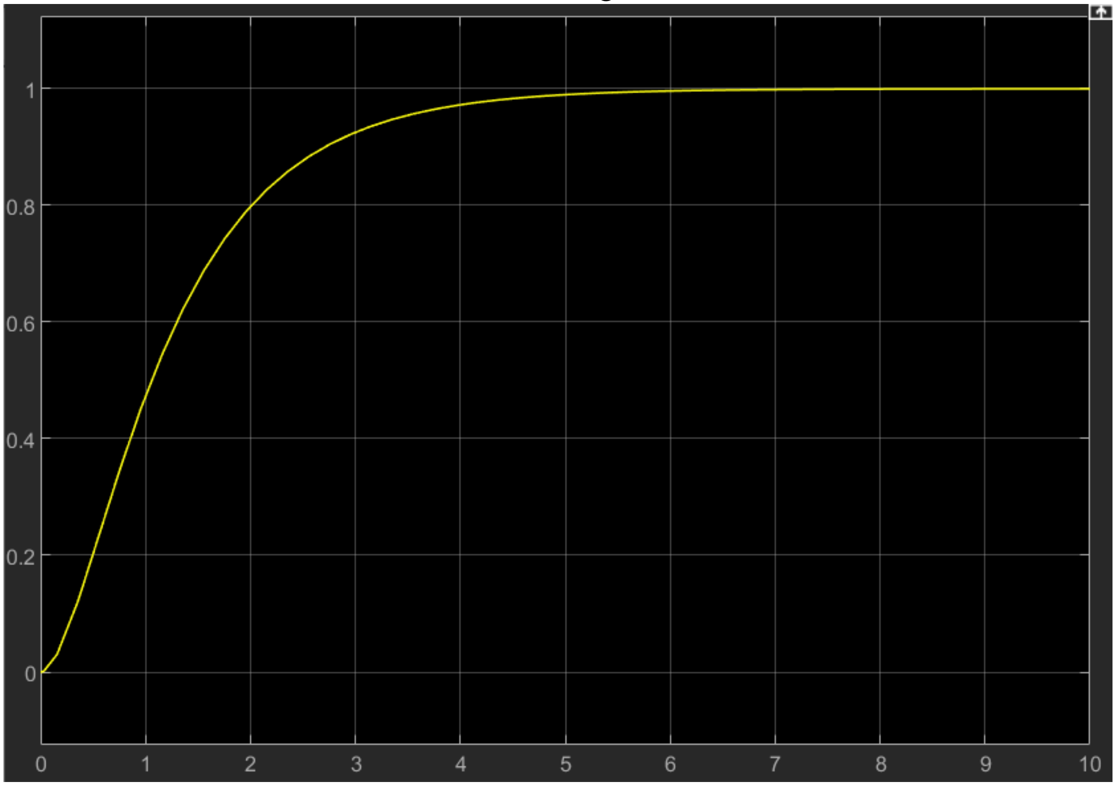


We plot the result in MATLAB using below code:

```
t = 0 : 1/1000 : 5;
u = heaviside(t);

y = -3/2 * exp(-t) .* u + 1/2 * exp(-3*t) .* u + u;

plot(t, y);
title('Step Response');
xlabel('time');
grid on;
```



We observe that the outcome aligns with our calculations.



- ***Part 2:***

$$K\big(x(t) - y(t)\big) + B\left(\frac{dx(t)}{dt} - \frac{dy(t)}{dt}\right) = M\frac{d^2y(t)}{dt^2}$$

Substituting $M = k = 1$ into the formula. We have:

$$\frac{d^2y(t)}{dt^2} + B\frac{dy(t)}{dt} + y(t) = x(t) + B\frac{dx(t)}{dt}$$

Applying the Laplace transform:

$$s^2Y(s) + BsY(s) + Y(s) = X(s) + BsX(s)$$

$$Y(s)(s^2 + Bs + 1) = X(s)(1 + Bs)$$

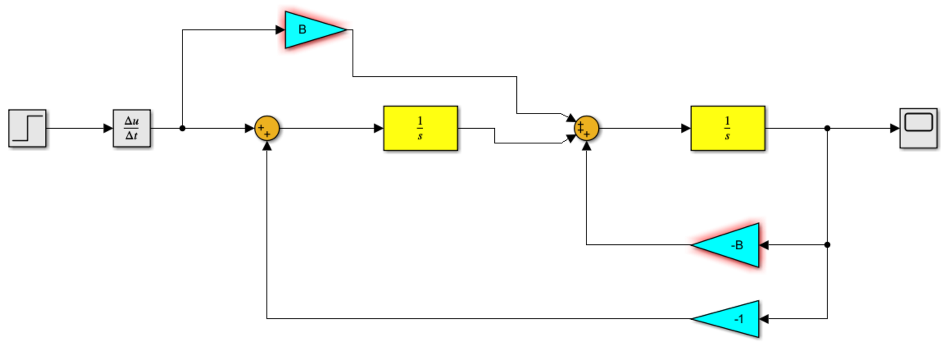
$$H(s) = \frac{Y(s)}{X(s)} = \frac{1 + Bs}{s^2 + Bs + 1}$$

To facilitate the creation of the block diagram for the equation, we express the equation as follows:

$$Y(s) + \frac{BY(s)}{s} + \frac{Y(s)}{s^2} = \frac{X(s)}{s^2} + \frac{BX(s)}{s}$$

$$Y(s) = \frac{1}{s^2}\big(X(s) - Y(s)\big) + \frac{1}{s}\big(BX(s) - BY(s)\big)$$

Next, we illustrate the block diagram:



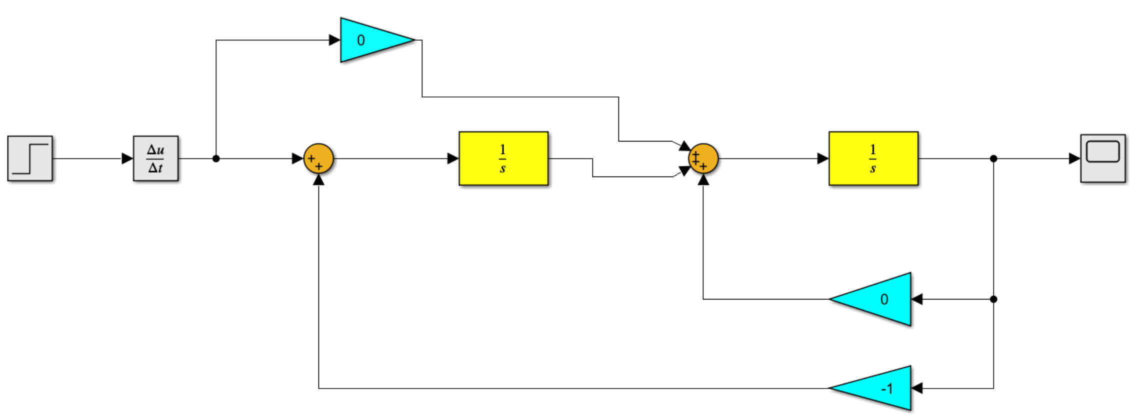
Assuming that the damper does not exert any force (i.e. $B = 0$), we can express the impulse response as follows:

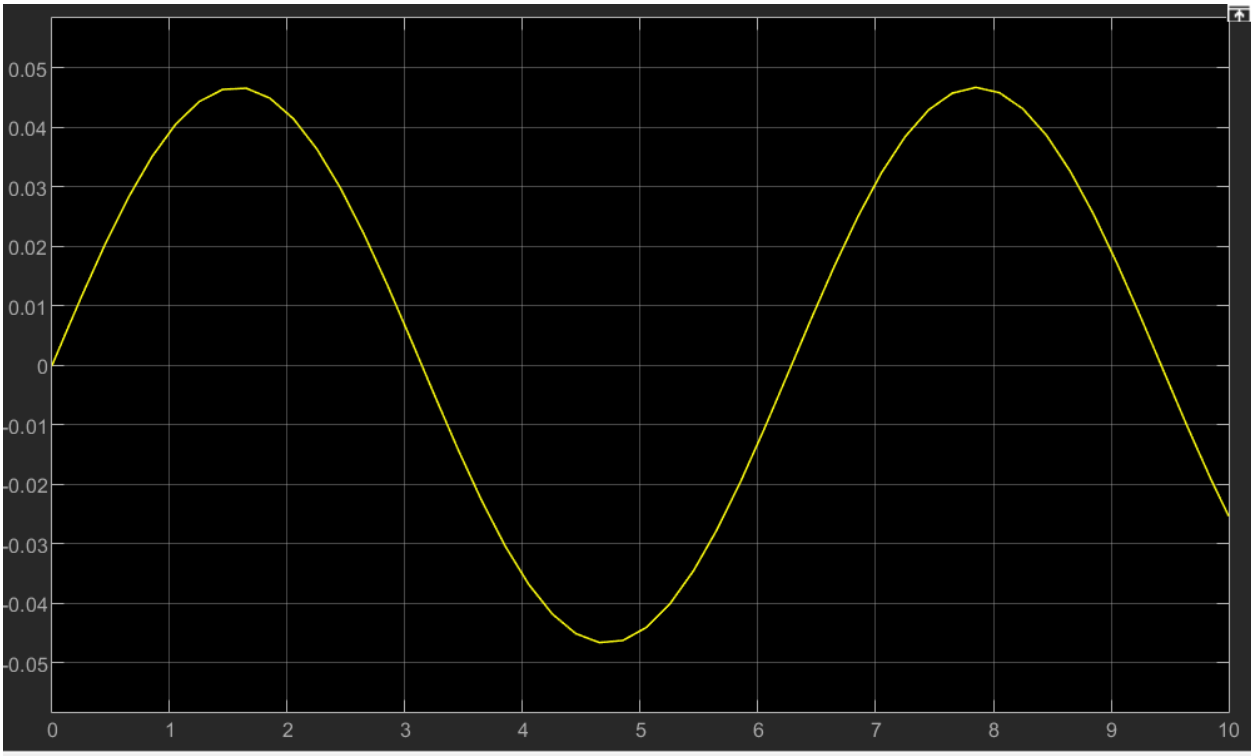
$$H(s) = \frac{1}{s^2 + 1}$$

employing the inverse Laplace transform:

$$h(t) = \sin{(t)}\text{u}(\text{t})$$

Upon sketching the block diagram, we observe that the plot aligns with our calculations, revealing a sine function:





The absence of a damper in the suspension system implies that the car body will undergo repetitive oscillations—rising and falling—after encountering a bump. The sinusoidal nature of the response reflects this behavior.

The poles of $H(s)$ are the answers of the following equation:
$s^2 + Bs + 1 = 0$
If we desire real poles, then the solutions to the equation should also be real.

$\Delta = B^2 - 4 \geq 0$

$|B| \geq 2$

The smallest positive value for B is 2.

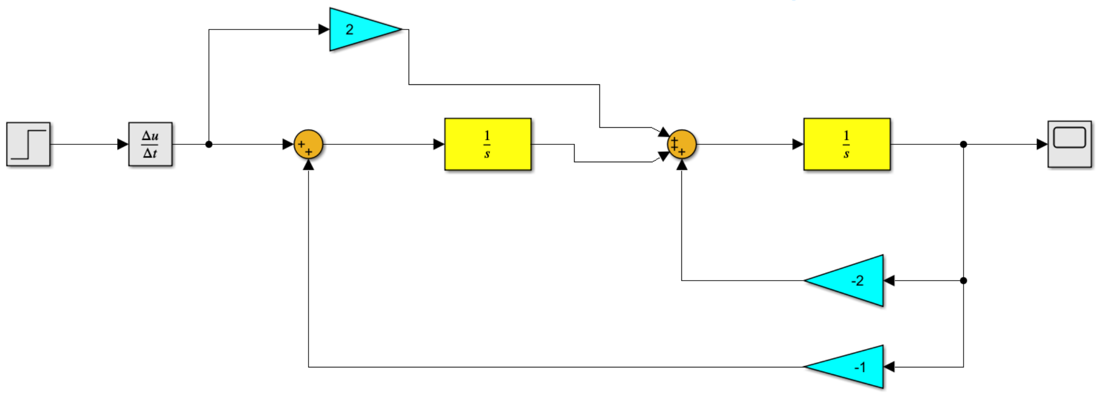
We determine the impulse response of the equation by substituting $B = 2$:

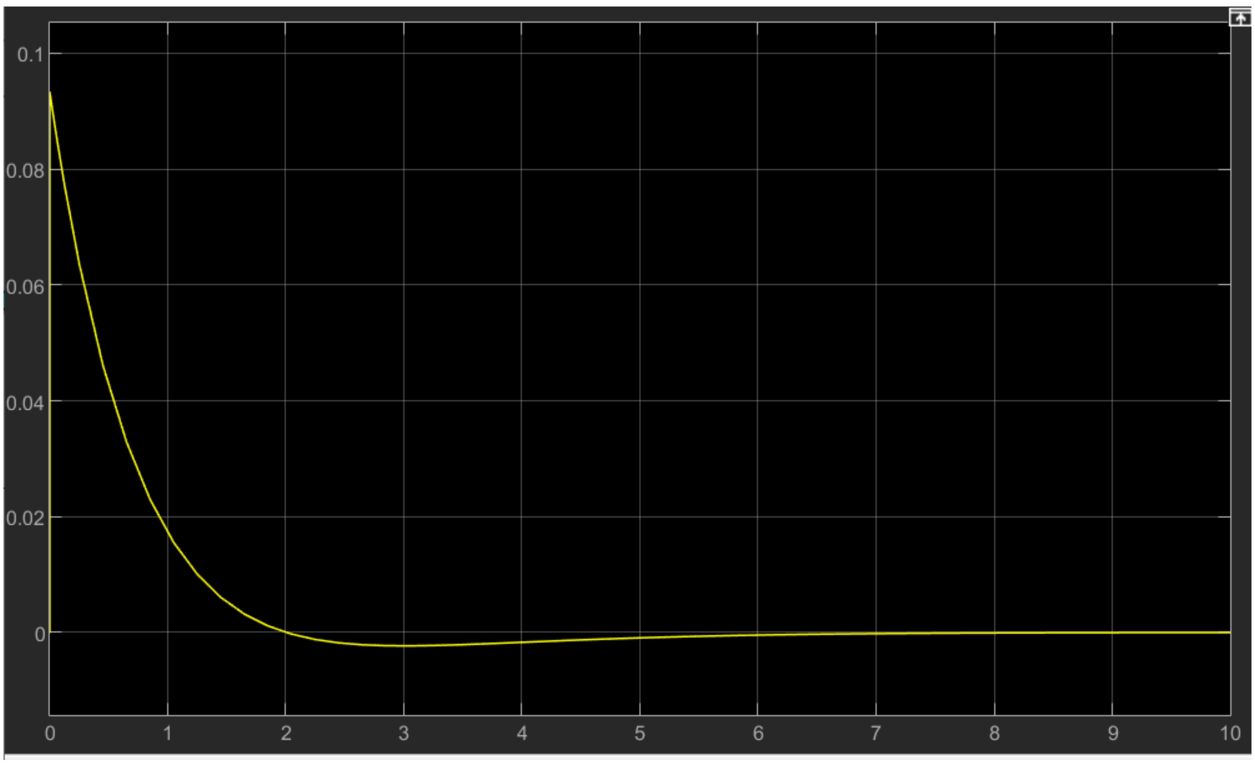
$$H(s) = \frac{1 + 2s}{s^2 + 2s + 1}$$

Using the inverse Laplace transform, we obtain:

$h(t) = \; -e^{-t}(t - 2)u(t)$

Creating a visual representation of the system, we sketch the block diagram as follows:





In this scenario, the car body experiences a sudden upward movement followed by a smooth descent, eventually reaching a stable position. Unlike the previous part, there is no sinusoidal shape in this motion.

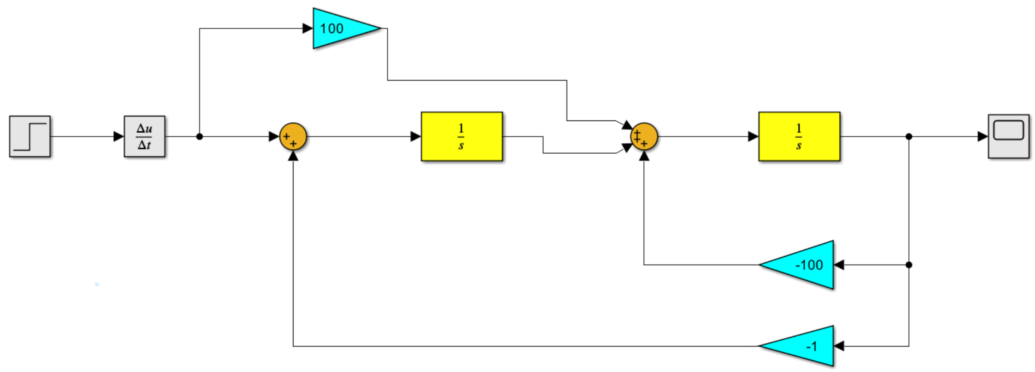
Now, substituting B with 100, we can analyze the impact on the system.

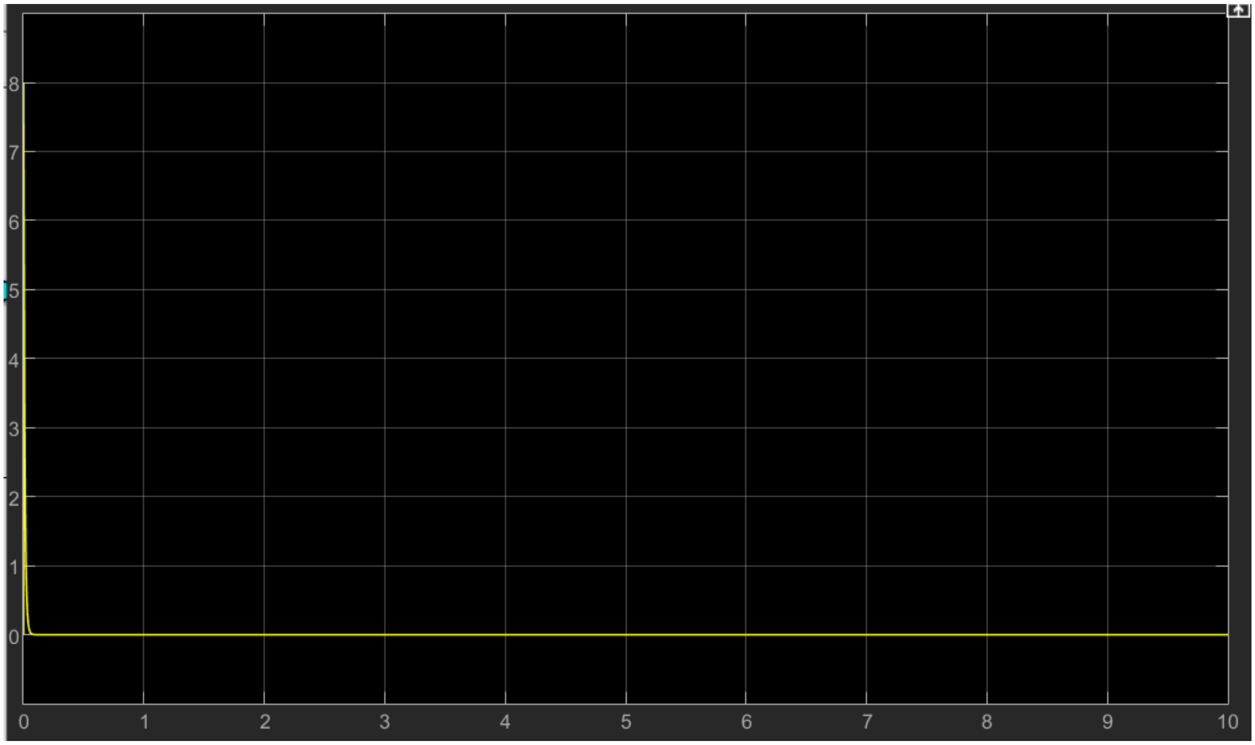
$$H(s) = \frac{1 + 100s}{s^2 + 100s + 1} \approx \frac{1 + 100\mathrm{s}}{(\mathrm{s} + 100)(\mathrm{s} + 0.01)}$$

Using the inverse Laplace transform, we obtain:

$$\mathrm{h(t)} = 100\ \mathrm{e}^{-100\mathrm{t}}u(t)$$

we sketch the block diagram with the updated parameter B set to 100:





In this scenario, with B set to 100, the system damps out oscillations more rapidly, albeit in a more abrupt manner compared to B = 2. Interestingly, the simulation results closely align with the theoretical calculations.

It seems that $B = 2$ was **the preferable choice** among the three options. $B = 2$ provided effective damping, damping out oscillations in a smooth manner. While $B = 100$ also damped out oscillations quickly, the abruptness of the motion and the higher amplitude of the car body movement after hitting the bump may make it less desirable for this specific scenario. $B = 0$ was deemed unsuitable due to the absence of damping, leading to prolonged oscillations.

- ***Part 3:***

$$\frac{d^2y(t)}{dt^2} + 3\frac{dy(t)}{dt} + 2y(t) = x(t)$$

$$y(0^-) = 1, y'(0^-) = 1$$

$$x(t) = 5u(t)$$

We can determine the outcome of the previously stated ODE by using the following approach:

Applying the Laplace transform:

$$s\big(sy(s) + y(0^-)\big) - y'(0^-) + 3sy(s) - 3y'(0^-) + 2y(s) = X(s)$$

$$s^2y(s) - s - 1 + 3sy(s) - 3 + 2y(s) = X(s)$$

$$s^2y(s) + (2 + 3s)y(s) = X(s) + s + 4$$

We can calculate the Laplace transform of $x(t)$:

$$X(s) = \frac{5}{s}$$

Substitute $X(s)$ into the formula. We have:

$$(s^2 + 3s + 2)Y(s) = \frac{5}{s} + s + 4$$

$$Y(s) = \frac{\frac{5}{s} + s + 4}{s^2 + 3s + 2} = \frac{s^2 + 4s + 5}{s(s+1)(s+2)}$$

By employing the inverse Laplace transform, we obtain:

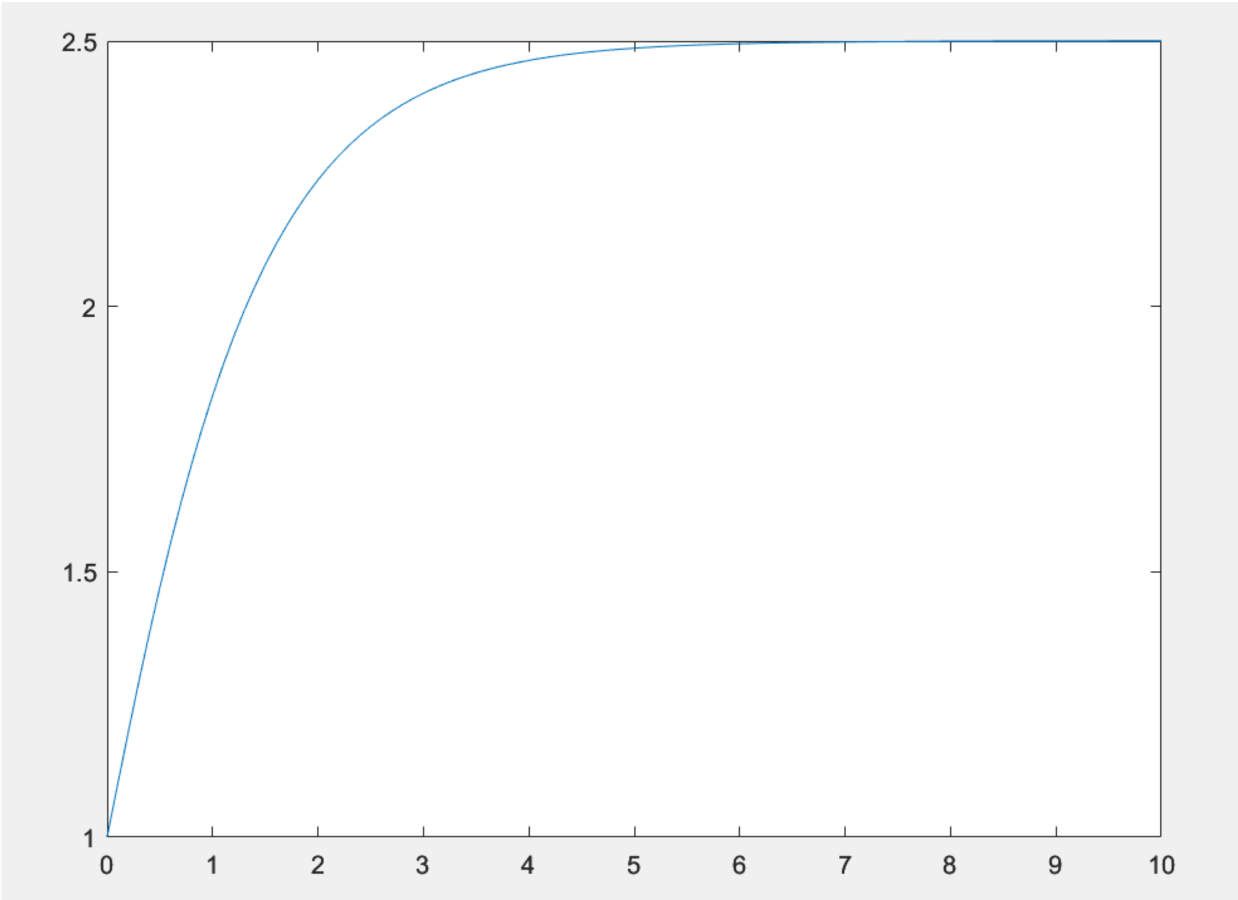
$$y(t) = \frac{e^{-2t}}{2} - 2\,e^{-t} + \frac{5}{2}$$

Below is the MATLAB code that solves the ODE:

```
syms y(t)
syms x(t)
sys = tf(1,1);
Dy = diff(y);

ode = diff(y,t,2) + 3 * diff(y,t,1) + 2*y == 5 * step(sys);
cond1 = y(0) == 1;
cond2 = Dy(0) == 1;
conds = [cond1 cond2];
ySol(t) = dsolve(ode,conds);
ySol = simplify(ySol)

figure
ts = 0 : 0.1 : 10;
plot(ts,ySol(ts));
```

It is evident that the obtained result aligns with our calculations:

```
ySol(t) =

exp(-2*t)/2 - 2*exp(-t) + 5/2
```


2.5
2
1.5
1
0
1
2
3
4
5
6
7
8
9
10