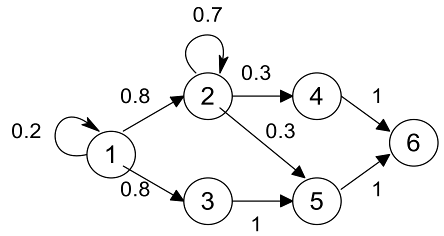
# Appendix 2: Sample examples

## Example A-1: Composition graph summarization without conditional pattern for the first solution of sPop population

Composition graph




Composition graph with 6 abstract web services

$cComp_1=(cc,Q)_1$

$cComp_1.cc$ ={341,797,890,1635,1936,2130}

$cComp_1.Q_1$=[0.7900,0.5684,0.0190]

$cComp_1.Q_2$=[0.8800,0.0349,0.0046]

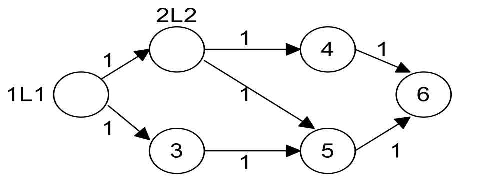
$cComp_1.Q_3$=[0.8500,0.0249,0.0027]

$cComp_1.Q_4$=[0.9100,0.0782,0.0021]

$cComp_1.Q_5$=[1,0.0156,2.4153e-04]

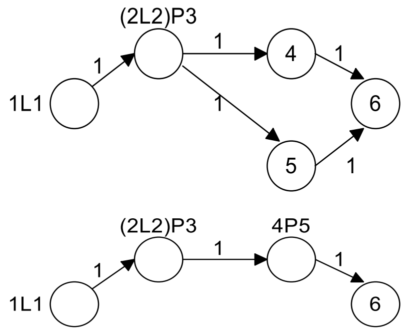
$cComp_1.Q_6$=[0.9600,0.0205,0.0103]

| #web service | Q=[availability,response-time,cost] |
|---|---|
| 1 | [0.7900,0.5684,0.0190] |
| 2 | [0.8800,0.0349,0.0046] |
| 3 | [0.8500,0.0249,0.0027] |
| 4 | [0.9100,0.0782,0.0021] |
| 5 | [1,0.0156,2.4153e-04] |
| 6 | [0.9600,0.0205,0.0103] |



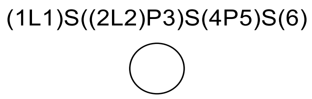

First step: removing the loop pattern (services 1 and 2)

| #web service | Q=[availability,response-time,cost] |
|---|---|
| 1L1 | [0.7506,0.7105,0.0238] |
| 2L2 | [0.6875,0.1164,0.0152] |
| 3 | [0.8500,0.0249,0.0027] |
| 4 | [0.9100,0.0782,0.0021] |
| 5 | [1,0.0156,2.4153e-04] |
| 6 | [0.9600,0.0205,0.0103 |




Second step: removing the parallel pattern

| #web service | Q=[availability,response-time,cost] |
|---|---|
| 1L1 | [0.7506,0.7105,0.0238] |
| (2L2)P3 | [0.5844,0.1164,0.0178] |
| 4P5 | [0.9100,0.0782,0.0023] |
| 6 | [0.9600,0.0205,0.0103] |




Third step: removing the sequence pattern and generating Summary service

| #web service | Q=[availability,response-time,cost] |
|---|---|
| (1L1)S((2L2)P3)S(4P5)S6 | [0.3832,0.9256,0.0542] |

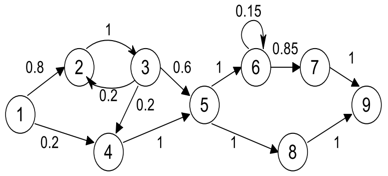
$Q'_1$= [0.3832,0.9256,0.0542]

$sPop_1=(cComp,Q')_1$

**Example A-2: Composition graph summarization with conditional pattern for the first solution of sPop population**

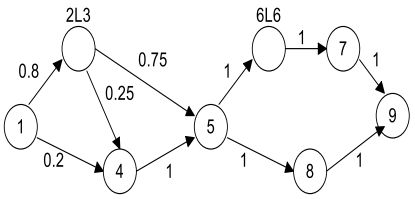
Composition graph

$cComp_1=(cc,Q)_1$
$cComp_1.cc=\{10,401,664,1049,1336,1445,1808,2074,2409\}$



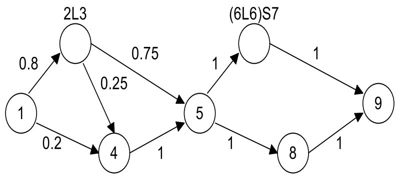

Composition graph with 9 abstract web services

| #web service | Q=[availability,response-time,cost] |
|---|---|
| 1 | [0.9600,0.0187,0.0101] |
| 2 | [0.7200,0.2384,4.8305e-04] |
| 3 | [0.9300,0.0121,0.0022] |
| 4 | [1,0.0392,0.0062] |
| 5 | [0.8800,0.1705,0.1151] |
| 6 | [0.8300,0.0154,7.2458e-04] |
| 7 | [0.9600,0.0397,0.0121] |
| 8 | [0.9300,0.4658,0.3843] |
| 9 | [0.9200,0.0401,0.0020] |



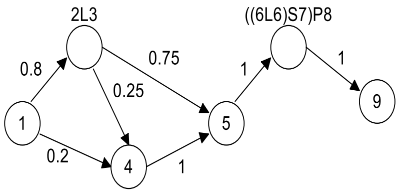

First step: removing the loop patterns

| #web service | Q=[availability,response-time,cost] |
|---|---|
| 1 | [0.9600,0.0187,0.0101] |
| 2L3 | [0.6185,0.3131,0.0033] |
| 4 | [1,0.0392,0.0062] |
| 5 | [0.8800,0.1705,0.1151] |
| 6L6 | [0.8058,0.0328,8.5244e-04] |
| 7 | [0.9600,0.0397,0.0121] |
| 8 | [0.9300,0.4658,0.3843] |
| 9 | [0.9200,0.0401,0.0020] |



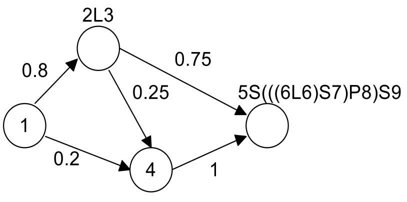

Second step: removing the sequence patterns

| #web service | Q=[availability,response-time,cost] |
|---|---|
| 1 | [0.9600,0.0187,0.0101] |
| 2L3 | [0.6185,0.3131,0.0033] |
| 4 | [1,0.0392,0.0062] |
| 5 | [0.8800,0.1705,0.1151] |
| (6L6)S7 | [0.7736,0.0725,0.0129] |
| 8 | [0.9300,0.4658,0.3843] |
| 9 | [0.9200,0.0401,0.0020] |



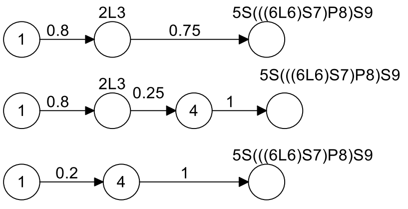

Third step: removing the parallel patterns

| #web service | Q=[availability,response-time,cost] |
|---|---|
| 1 | [0.9600,0.0187,0.0101] |
| 2L3 | [0.6185,0.3131,0.0033] |
| 4 | [1,0.0392,0.0062] |
| 5 | [0.8800,0.1705,0.1151] |
| ((6L6)S7)P8 | [0.7194,0.4658,0.3972] |
| 9 | [0.9200,0.0401,0.0020] |




Fourth step: removing the sequence patterns

| #web service | Q=[availability,response-time,cost] |
|---|---|
| 1 | [0.9600,0.0187,0.0101] |
| 2L3 | [0.6185,0.3131,0.0033] |
| 4 | [1,0.0392,0.0062] |
| 5S((6L6)S7)P8)S9 | [0.5825,0.6765,0.5143] |



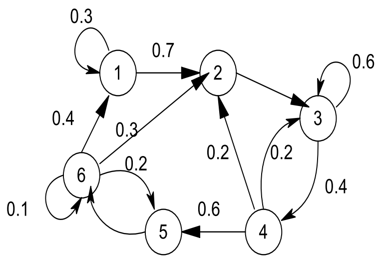

Fifth step: extracting the path and then removing the sequence pattern and generating Summary service for each path

| | | |
|---|---|---|
| Summary $service_1$= 1S(2L3)S(5S((6L6)S7)P8)S9) | $Prob_1$=0.6 | $Q'_1$ =[0.3458,1.0083,0.5277] |
| Summary $service_2$= 1S(2L3)S4S(5S((6L6)S7)P8)S9) | $Prob_2$=0.2 | $Q'_1$ =[0.3458,1.0475,0.5339] |
| Summary $service_3$= 1S4S(5S((6L6)S7)P8)S9) | $Prob_3$=0.2 | $Q'_1$ =[0.5592,0.7344,0.5306] |

**Example A-3. An example of summarizing the loop pattern with 6  services**

In order to make it easier to understand, the beginning of the name of each service is written in find loops. For example, instead of 1L1, only 1 is written in below tables.

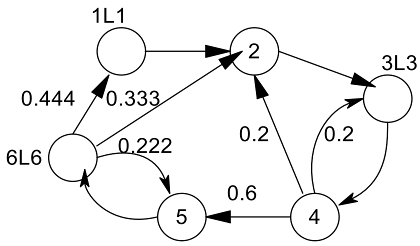
Composition graph




Finding loops from the longest to the smallest ones using Algorithm A-4

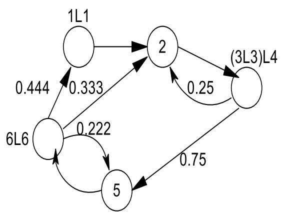
| |
|---|
| {1,2,3,4,5,6} |
| {2,3,4,5,6} |
| {2,3,4} |
| {5,6} |
| {3,4} |
| {6} |
| {1} |
| {3} |

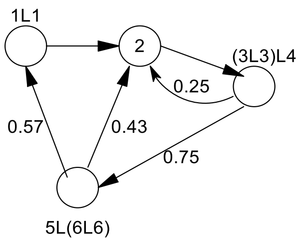
Composition graph

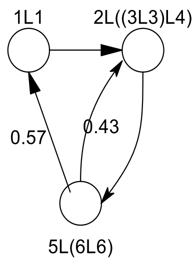



Composition graph after removing loops of length 1

| |
|---|
| {1,2,3,4,5,6} |
| {2,3,4,5,6} |
| {2,3,4} |
| {5,6} |
| {3,4} |



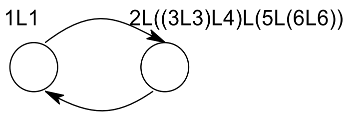



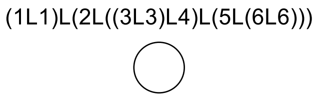

| |
|---|
| {1,2,3,5,6} |
| {2,3,5,6} |
| {2,3} |
| {5,6} |

| |
|---|
| {1,2,3,5} |
| {2,3,5} |
| {2,3} |

| |
|---|
| {1,2,5} |
| {2,5} |

{1,2}     {1}



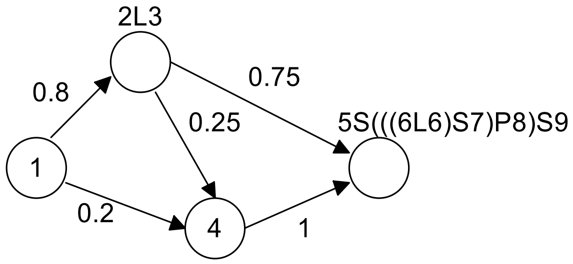




Composition graph after removing loops of length 2

**Example A-4: An example of extracting paths of a composite graph with 6 services after running Algorithm A-5.**

This example shows the Array and Path arrays values in each step of the running the Algorithm A-5 for extracting the path of the below composition graph.

In order to make it easier to understand, the beginning of the name of each service is written in Array or Path values. For example, instead of 2L3, only 2 is written in below arrays.




| Array values | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| [1] | [1] | [1,2] | [1,2] | [1,4] | [1,4] | [1,2,4] | [1,2,4] | [1,2,5] |
| | [1,2] | [1,4] | [1,4] | [1,2,4] | [1,2,4] | [1,2,5] | [1,2,5] | [1,4,5] |
| | [1,4] | | [1,2,4] | [1,2,5] | [1,2,5] | [1,4,5] | [1,4,5] | [1,2,4,5] |
| | | | [1,2,5] | | [1,4,5] | | [1,2,4,5] | |
| Line2 | Lines 5&6 | Line 11 | Lines 5&6 | Line 11 | Lines 5&6 | Line 11 | Lines 5&6 | Line 11 |

| Path values | Array values | Path values | Array values | Path values | Array values |
|---|---|---|---|---|---|
| [1,2,5] | [1,4,5] | [1,2,5] | [1,2,4,5] | [1,2,5] | |
| | [1,2,4,5] | [1,4,5] | | [1,4,5] | |
| | | | | [1,2,4,5] | |
| Line 8 | Line 11 | Line 8 | Line 11 | Line 8 | Line 11 |

**Example A-5. This example shows the quality features values used in Algorithm 5 (Prioritization-Solutions function) step by step**.

Suppose that the random population selected at the beginning of the algorithm consists of four (Npop'=4) solutions: sPop'={$(cComp,Q')_1$, $(cComp,Q')_2$, $(cComp,Q')_3$, $(cComp,Q')_4$}

where each solution in sPop' includes a pair of cComp (a pair of candidate index and corresponding quality features) and Q' (Summary service). In the *Prioritization-Solutions function*, we only deal with Summary services (Q'). We know that each $Q'_i$ contains three quality features ($q'_{i1}$, $q'_{i2}$ , $q'_{i3}$). If the quality features measurements for sPop' solutions are as follows:

$Q'_1$=(0.79,0.23,0.01)

$Q'_2$=(0.82,0.42,0.03)

$Q'_3$=(0.52,0.1,0.1)

$Q'_4$=(0.67,0.3,0.008)

After running line 4 of Algorithm 4 and calling *Prioritization-Solutions* function, the sorted1 set is sorted descending on $q'_{i1}$, and the sorted2 and sorted3 sets are sorted ascending on $q'_{i2}$ and $q'_{i3}$.

sorted1={$q'_{21}$, $q'_{11}$, $q'_{41}$, $q'_{31}$}

sorted2={$q'_{32}$, $q'_{12}$, $q'_{42}$, $q'_{22}$}

sorted3={$q'_{43}$, $q'_{13}$, $q'_{23}$, $q'_{33}$}

$sortedPop_1$= (2+2+2)/3= 2, /* for $Q'_1$,

$sortedPop_2$= (1+4+3)/3= 2.6, /* for $Q'_2$

$sortedPop_3$= (4+1+4)/3 = 3, /* for Q'3

$sortedPop_4$= (3+3+1)/3 = 2.3 /* for $Q'_4$

newsPop'=($(cComp,Q')_1$, $(cComp,Q')_4$, $(cComp,Q')_2$, $(cComp,Q')_3$)

**Example A-6: This example shows the parameters values used in Algorithm 6 (*Discretization-Solutions function*) step by step.**

Suppose that $3 \in C(As_1)$ ،$600 \in C(As_2)$،$1400 \in C(As_3)$ ،$2300 \in C(As_4)$ (see Definition 2) consider for abstract web services $As_1$, $As_2$, $As_3$, $As_4$ respectively and itomega=1. So, we have:

$(w_\omega)_1$.cComp={(3, (0.82,0.2,0.032)), (600, (0.75,0.1,0.001)), (1400,(0.68,0.15,0.003), (2300, (0.78,0.3,0.002)), where 3,600,1400,2300 are candidate numbers from $C(As_1)$, $C(As_2)$, $C(As_3)$, $C(As_4)$, respectively for $As_1$, $As_2$, $As_3$, $As_4$ and (0.82,0.2,0.032), (0.75,0.1,0.001) ,(0.68,0.15,0.003), (0.78,0.3,0.002) are ($Q_{1-4}$ ) the quality features for these candidate services.

In the same way, for alpha, beta and delta wolves (solutions) we also have:
$w_\alpha$.cComp ={(50, (0.92,0.32,0.005)), (400,( 0.73,0.4,0.006)), (1100, (0.88,0.2,0.002)), (2100,(0.96,0.5,0.005))}

$w_\beta$.cComp={(20,(0.88,0.2,0.002)), (740,(0.64,0.35,0.04)), (1800, (0.93,0.1,0.002)),(2050,(0.62,0.1,0.005))}

$w_\delta$.cComp ={(210,(0.96,0.5,0.07)), (531,(0.62,0.01,0.006)), (1920,(0.89,0.18,0.02)), (2150,(0.82,0.25,0.009))}

The quality features of the new omega wolf (generated in line 10 algorithm 4) in continuous space are as follows.
sNewOmeagContinues ={(0.87,0.23,0.006),(0.59,0.018,0.008),(0.91,0.3,0.0012),(0.71,0.1,0.008)}

Now we want the real candidate index and quality features of the new omega wolf in the discrete space.
$sNewOmega_1$.cComp={ (?,?) , (?,?) , (?,?), (?,?)}

So, we should determine the candidate index and the corresponding quality features for each dimension of the sNewOmega shown by (?,?).
Based on the proposed *Discretization-Solutions* function, if the quality features of each dimension of sNewOmeagContinues are within the specified range [0 1], then they are compared with each dimension of delta, beta, alpha and current wolves, and if dominated by any of these wolves, the dominant wolf candidate index is selected. In this example, the quality features of the first dimension of the new wolf in the continuous space are (0.87, 0.23, 0.006), which are first compared with the quality features of the first dimension of the delta wolf (0.96,0.5,0.07), the delta wolf can't dominate it, so it is compared to beta wolf (0.88,0.2,0.002). These quality features in beta wolf are (0.88, 0.2, 0.002), so the beta wolf (with candidate index 20) dominates the new wolf in the first dimension, and the candidate index and the quality features corresponding to it are determined for the first dimension of the new wolf in the discrete space:

$sNewOmega_1$.cComp={ 20,(0.88,0.2,0.002)), (?,?) , (?,?), (?,?)}

The quality features of the second dimension of the new wolf (0.59, 0.018, 0.008) is compared delta wolf (0.62, 0.01, 0.006) and delta wolf dominates it. Therefore, the candidate index and the features corresponding to it for the second dimension of the new wolf are also determined.
$sNewOmega_1$.cComp={ 20,(0.88,0.2,0.002)), (531,(0.62,0.01,0.006)), (?,?), (?,?)}

The quality features of the third dimension of the new wolf (0.91, 0.3, 0.0012) with the quality features of delta (0.89, 0.18, 0.02), beta (0.64, 0.35, 0.04) and alpha (0.88, 0.2, 0.002) is compared and none of them dominates the new wolf. So, all available candidate indexes for this web service are reviewed. The first candidate that could dominate the new wolf is selected. For example, the quality values of the 1990 candidate dominated the quality values of this dimension of the new wolf.
$sNewOmega_1$.cComp={ 20,(0.88,0.2,0.002)), (531,(0.62,0.01,0.006)), (1990,(0.93,0.3,0.0012)), (?,?)}

Now the quality features of the last dimension of new wolf (0.71,0.1,0.008) with the quality features of delta (0.82,0.25,0.009), beta (0.62,0.1,0.005) and alpha (0.96,0.5,0.005) is compared and none of them dominates this dimension of the new wolf. So, all available candidate indexes for this web service are reviewed and none of them dominates it. So, the current wolf candidate index and the quality features corresponding to it (2300, (0.78,0.3,0.002)) are determined for the last dimension of the new wolf in the discrete space:
$sNewOmega_1$.cComp={ 20,(0.88,0.2,0.002)), (531,(0.62,0.01,0.006)), (1990,(0.93,0.3,0.0012)), (2300, (0.78,0.3,0.002)) }

**Appendix 3. Fig. A-1.**

The flowchart of *PBMOA* (the proposed method)