**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Answer: 0.2659

Given mean =45, standard deviation =8

As per given condition the work will start after 10 min so mean will be 45+10=55

We have asked customer the car will be ready after 1 hour(x) = 60

Hence,

Z = (60 -55)/ 8= 0.625

From table Z

Z value will be 0.73237

The probability that the service manager cannot meet his commitment = 1-0.73237

0.2676

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Answer:

A .Probability that he employee will be greater than age of 44 = 100-84.1345 =15.86%So so the probability of number of employees between 38.44 years of age = pr(X<44)-0.5=84.1345-0.5 = 34.1345%

Therefore the statement that more employees t the processing center are older than 44 than between 38 is **TURE**

B.So number of employees with probability 0.912 of them being under age 30=0.0912\*400=36.48 (36) Therefore the statement B of the question is also

**TRUE**

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Answer:

As we know that if X ~ N(μ, σ2) and Y ~ N(μ2, σ22) are two independent Random variables than X + Y ~ N(μ1 + μ2, σ12 + σ22)

and X - Y ~ N(μ1 - μ2, σ12+ σ22)Similarly

if Z = ax +by,Where X and Y are as deepened above i.e z is linear combination

X and Y then Z ~ N(aμ1 + bμ2 a² σ1² + b² σ2² )therefore in the queston

2X1~ N(2µ,4σ2) andX1 +X2 ~ N(µ + µ, σ2+ σ2) ~ N(2µ , 2σ2)

2X1 – (X1 +X2) = N(4µ ,6 σ2)

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Answer:

Z\*σ + µ = X

Z(-0.005)\*20+100 = (2.57)\*20+100 = 151.4

Z(0.005)\*20+100 = (-2.57)\*20+100 = 48.6

ANS:- **D. 48.5 ,151.5**

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

Answer:Mean profit is RS 540 Million

 Std deviation is RS 225 Million

Range is RS

state norms interval (0.95, 540, 225)

**Range is RS 99.0081034 , 980.991896**

1. Specify the 5th percentile of profit (in Rupees) for the company

Answer:

formula X= X=μ + Zσ; wherein from z table, 5 percentile = -1.645

X = 540(-1.645)\*225

**X = 169.875**

1. Which of the two divisions has a larger probability of making a loss in a given year?

Answer:

 Probability od division 1 making a loss p(X<0)

 Stats. norm. cdf (0, 5, 3)

**0.0477903**

 Probability od division 2 making a loss p(X<0)

Stats.**norm. cdf(0, 7, 4) 0**

.0400591