

Pandit Deendayal Energy University
Computational Engineering Laboratory
Matlab Case study Report

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March - April 2024

Case Study 1

Problem Statement :-

To Solve a problem on Heat Transfer of Natural Convection in H – Shaped Cavity.

Derivation :-

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho_{hnf}} \frac{\partial p}{\partial x} + \frac{\mu_{hnf}}{\rho_{hnf}} \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \quad (2)$$

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho_{hnf}} \frac{\partial p}{\partial y} + \frac{\mu_{hnf}}{\rho_{hnf}} \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + g\beta_{T,hnf}(T - T_c) + g\beta_{c,hnf}(c - c_c) \quad (3)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_{hnf} \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \quad (4)$$

$$u \frac{\partial c}{\partial x} + v \frac{\partial c}{\partial y} = D \left(\frac{\partial^2 c}{\partial x^2} + \frac{\partial^2 c}{\partial y^2} \right) \quad (5)$$

Introducing the following dimensionless variables:

$$U = \frac{uL}{\alpha_f}, V = \frac{vL}{\alpha_f}, \theta = \frac{(T - T_c)}{\Delta T}, C = \frac{(c - c_c)}{\Delta c}, P = \frac{pL^2}{\rho_f \alpha_f^2} \quad (6)$$

The non-dimensional shape of governing equations becomes:

$$U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = -\frac{\rho_f}{\rho_{hnf}} \frac{\partial P}{\partial X} + \text{Pr} \frac{\rho_f}{\rho_{hnf}} \frac{\mu_{hnf}}{\mu_f} \left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right) \quad (7)$$

$$U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = -\frac{\rho_f}{\rho_{hnf}} \frac{\partial P}{\partial Y} + \text{Pr} \frac{\rho_f}{\rho_{hnf}} \frac{\mu_{hnf}}{\mu_f} \left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) + \frac{\beta_{hnf}}{\beta_f} Ra \text{Pr} (\theta + NC) \quad (8)$$

$$U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \frac{(k_{hnf}/k_f)}{((\rho C_p)_{hnf}/(\rho C_p)_f)} \left(\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \right) \quad (9)$$

$$U \frac{\partial C}{\partial X} + V \frac{\partial C}{\partial Y} = \frac{1}{Le} \left(\frac{\partial^2 C}{\partial X^2} + \frac{\partial^2 C}{\partial Y^2} \right) \quad (10)$$

Where;

$$Ra = \frac{g\beta_f \Delta T L^3}{\nu_f \alpha_f}, \text{Pr} = \frac{\nu_f}{\alpha_f}, Le = \frac{\alpha_f}{D} \quad (11)$$

Boundary conditions:

$$\text{Bottom wall: } \frac{\partial \theta}{\partial Y} = 0, U = 0, V = 0, \Psi = 0, \frac{\partial C}{\partial Y} = 0$$

$$\text{Bottom rib: } \frac{\partial \theta}{\partial n} = -1, U = 0, V = 0, \Psi = 0, \frac{\partial C}{\partial n} = -1$$

$$\text{Right and left walls: } \theta = 0, U = 0, V = 0, \Psi = 0, C = 0$$

$$\text{Top wall: } \frac{\partial \theta}{\partial Y} = 0, U = 0, V = 0, \Psi = 0, \frac{\partial C}{\partial Y} = 0$$

$$\text{Top rib: } \frac{\partial \theta}{\partial n} = -1, U = 0, V = 0, \Psi = 0, \frac{\partial C}{\partial n} = -1$$

$$\text{Baffle: } \frac{\partial \theta}{\partial n} = -1, U = 0, V = 0, \Psi = 0, \frac{\partial C}{\partial n} = -1 \quad (12)$$

The target physical parameters in this paper are the local Nusselt and Sherwood numbers. These parameters are defined as:

$$Nu = \left(\frac{k_{hmf}}{k_f} \right) \left(\frac{1}{\theta} \right) \quad (13)$$

$$Sh = \left(\frac{1}{C} \right) \quad (14)$$

And the average ones are:

$$Nu_{avg} = \frac{1}{s} \int Nu \cdot ds \quad (15)$$

$$Sh_{avg} = \frac{1}{s} \int Sh \cdot ds \quad (16)$$

Where s is the length of the bottom rib.

K , (ρC_p) , μ , $(\rho \beta)$, and ρ of hybrid nanoliquid could be represented as:

$$\frac{k_{hmf}}{k_f} = \left[(1 - \lambda) \varphi (k_f - k_s) + k_s - (1 - \lambda) k_f \right] \times \left[k_s - (1 - \lambda) k_f + \varphi (k_f - k_s) \right]^{-1} \quad (17)$$

$$(\rho C_p)_{hmf} = \varphi_{Cu} (\rho C_p)_{Cu} + \varphi_{Al_2O_3} (\rho C_p)_{Al_2O_3} + (1 - \varphi) (\rho C_p)_f \quad (18)$$

$$\mu_{hmf} = (1 - \varphi)^{-2.5} \mu_f \quad (19)$$

$$(\rho \beta)_{hmf} = (1 - \varphi) (\rho \beta)_f + \varphi_{Cu} (\rho \beta)_{Cu} + \varphi_{Al_2O_3} (\rho \beta)_{Al_2O_3} \quad (20)$$

$$\rho_{hmf} = (1 - \varphi) \rho_f + \varphi_{Cu} \rho_{Cu} + \varphi_{Al_2O_3} \rho_{Al_2O_3} \quad (21)$$

And the correlations of entropy generation would be as:

$$S_{gen} = S_{HT} + S_{FF} + S_{Mass} \quad (22)$$

In which

$$S_{HT} = \frac{k_{hmf}}{k_f} \left(\left(\frac{\partial \theta}{\partial X} \right)^2 + \left(\frac{\partial \theta}{\partial Y} \right)^2 \right) \quad (23)$$

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$$S_{FF} = \varphi_1 \frac{\mu_{hmf}}{\mu_f} \left[2 \left(\frac{\partial V}{\partial Y} \right)^2 + 2 \left(\frac{\partial U}{\partial X} \right)^2 + \left(\frac{\partial U}{\partial Y} + \frac{\partial V}{\partial X} \right)^2 \right] \quad (24)$$

$$S_{Mass} = \varphi_2 \left(\left(\frac{\partial C}{\partial X} \right)^2 + \left(\frac{\partial C}{\partial Y} \right)^2 \right) + \varphi_3 \left(\frac{\partial C}{\partial X} \frac{\partial \theta}{\partial X} + \frac{\partial C}{\partial Y} \frac{\partial \theta}{\partial Y} \right) \quad (25)$$

And the Bejan number is:

$$Be = \frac{S_{HT} + S_{Mass}}{S_{gen}} \quad (26)$$

Code

```
title('Temperature Distribution in H-Shaped Cavity');
% Parameters
L = 1; % Length of cavity
W = 1; % Width of cavity
H = 0.1; % Height of cavity
T_hot = 100; % Temperature of hot wall
T_cold = 0; % Temperature of cold walls
nx = 51; % Number of grid points in x-direction
ny = 51; % Number of grid points in y-direction
dx = L/(nx-1); % Grid spacing in x-direction
dy = W/(ny-1); % Grid spacing in y-direction
max_iter = 1000; % Maximum number of iterations
tolerance = 1e-5; % Convergence tolerance

% Initialization
T = ones(nx, ny) * T_cold; % Initialize temperature array

% Main loop (iterate until convergence)
for iter = 1:max_iter
    % Boundary conditions
    T(:,1) = T_cold; % Left cold wall
    T(:,end) = T_cold; % Right cold wall
    T(1,:) = T_hot; % Hot wall

    % Compute temperature field using simple averaging
    T_new = T;
    for i = 2:nx-1
        for j = 2:ny-1
            % Average of neighboring points
            T_new(i,j) = 0.25*(T(i+1,j) + T(i-1,j) + T(i,j+1) + T(i,j-1));
        end
    end

    % Check for convergence
    if max(abs(T_new(:) - T(:))) < tolerance
        disp(['Converged at iteration ', num2str(iter)]);
        break;
    end
end
```

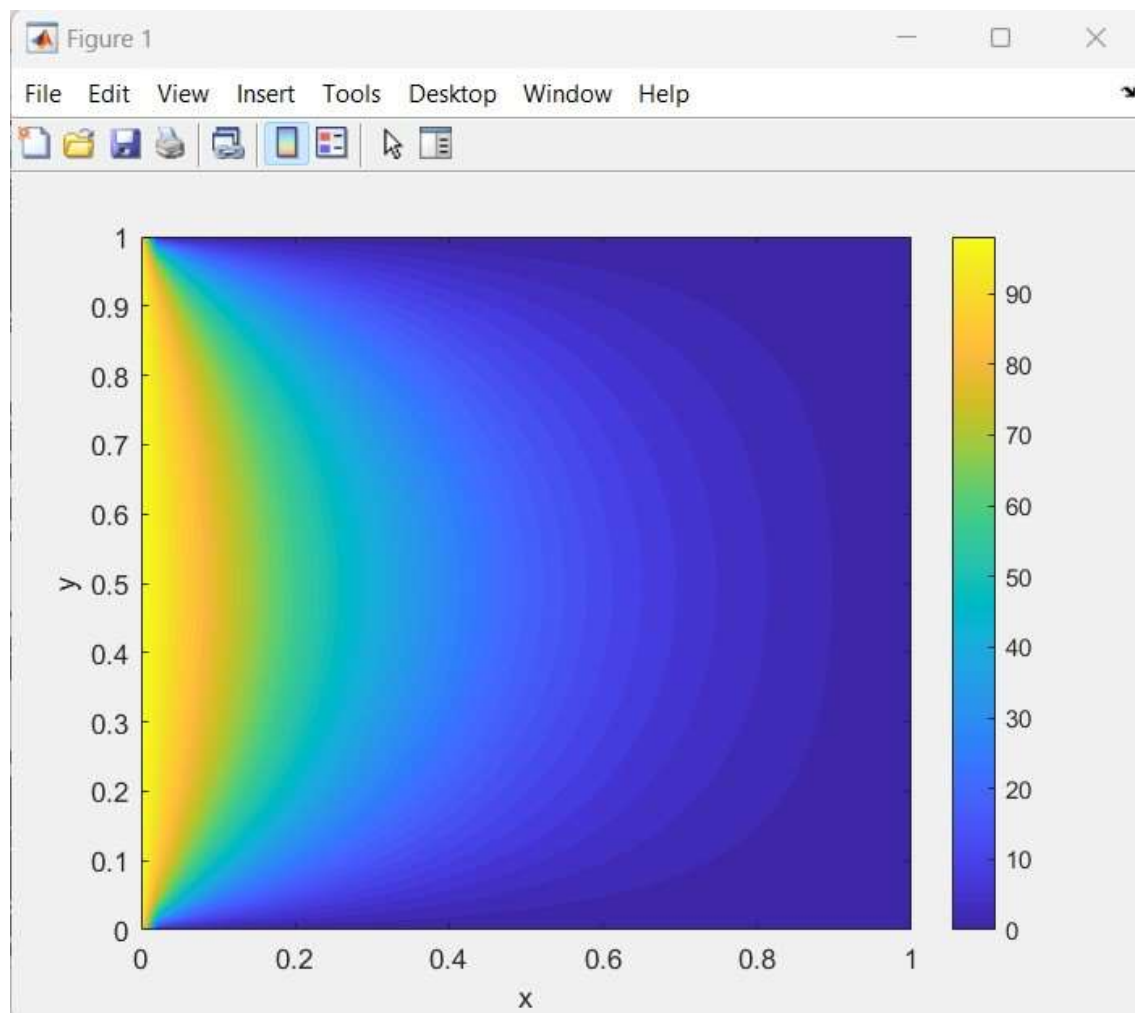
```

    % Update temperature array
    T = T_new;
end

% Plotting the results
[X, Y] = meshgrid(linspace(0,L,nx), linspace(0,W,ny));
contourf(X, Y, T', 50, 'LineColor', 'none');
colorbar;
xlabel('x');
ylabel('y');

```

Results :-



Case Study 2

Problem Statement:-

To Update the code on the problem on Heat Transfer of Natural Convection in H – Shaped Cavity.

Code :-

```
% Parameters
L = 1;           % Length of cavity
W = 1;           % Width of cavity
H = 0.1;         % Height of cavity
T_hot = 100;     % Temperature of hot wall
T_cold = 0;       % Temperature of cold walls
nx = 51;         % Number of grid points in x-direction
ny = 51;         % Number of grid points in y-direction
dx = L/(nx-1);   % Grid spacing in x-direction
dy = W/(ny-1);   % Grid spacing in y-direction
max_iter = 100;  % Maximum number of iterations
tolerance = 1e-5; % Convergence tolerance
alpha = 0.1;     % Thermal diffusivity

% Initialize temperature array
T = ones(nx, ny) * T_cold;

% Set up VideoWriter object

writerObj = VideoWriter('temperature_evolution.avi'); % Specify the file name
writerObj.FrameRate = 10; % Set the frame rate

% Open the video writer object
open(writerObj);

% Create a figure for animation
figure;

% Main loop (iterate until convergence)
for iter = 1:max_iter
    % Boundary conditions
    T(:,1) = T_cold;           % Left cold wall
    T(:,end) = T_cold;         % Right cold wall
    T(1,:) = T_hot;            % Hot wall
    T(ceil(end/2-round(H/(2*dy))):floor(end/2+round(H/(2*dy))), end) = T_cold; % Middle cold wall

    % Compute temperature field using finite difference method
    T_new = T;
    for i = 2:nx-1
        for j = 2:ny-1
            % Finite difference equation
```

```

        T_new(i,j) = T(i,j) + alpha * ( (T(i+1,j) - 2*T(i,j) + T(i-1,j))/dx^2 + (T(i,j+1) - 2*T(i,j) + T(i,j-1))/dy^2 );
    end
end

% Check for convergence
if max(abs(T_new(:) - T(:))) < tolerance
    disp(['Converged at iteration ', num2str(iter)]);
    break;
end

% Update temperature array
T = T_new;

% Plot the current temperature distribution in 3D
[X, Y] = meshgrid(linspace(0,L,nx), linspace(0,W,ny));
surf(X, Y, T, 'EdgeColor', 'none');
colorbar;
xlabel('x');
title(['Iteration: ', num2str(iter)]);
axis([0 L 0 W 0 100]); % Set axis limits for consistency

% Capture the frame
frame = getframe(gcf);

% Write the frame to the video file
writeVideo(writerObj, frame);
end

% Close the video writer object
close(writerObj);

```

Results :-

