HW1 Report: Solve a Puzzle with SMT solver

Discrete Mathematics 1p.m.

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0. Introduction

0-1.Number Cross is a puzzle that has MxN grid. The goal of the game id to determine the color of the square as either Black(1) or White(0).

1. Constraints

1-1. Number Cross consists of a M x N grid where every square, every row, and every column and is labeled with a positive integer.

1-2. The number of each column is the same as the sum of the numbers in the Black squares of the column.

1-3. The number of each row is the same as the sum of the numbers in the White squares of the row.

1-4. The number of a square label is greater than or equal to 1, and not greater than 9.

2. Definition and Logic formula

2-1. $p(x, y)$ is the integer value that represents the color of grid whether black or white(black is 1, white is 0). x is the row of the grid and y is the column of the grid $(1 \leq x \leq M,\ 1 \leq y \leq N)$.

2-2. $num(x, y)$ is the positive integer number of the grids$(1 \leq x \leq M,\ 1 \leq y \leq N,\ 1 \leq num(x, y) \leq 9)$.

2-3. $q(n)$ is the positive integer value that represents the $n\text{-}th$ column label $(1 \leq n \leq N)$.

2-4. $r(n)$ is the positive integer value that represents 'the sum of the numbers of $n\text{-}th$ row' minus 'the $n\text{-}th$ row label $(1 \leq n \leq M)$.

$$\textbf{Q1.}\quad \bigwedge_{x=1}^{M} \bigwedge_{y=1}^{N} (p(x,y) = 0)\ \mathrm{V}\ (p(x,y) = 1)$$

$$\textbf{Q2.}\quad \bigwedge_{y=1}^{N} \left[\sum_{x=1}^{M} p(x,y) \times num(x,y)\right] = q(y)$$

$$\textbf{Q3.}\quad \bigwedge_{x=1}^{M} \left[\sum_{y=1}^{N} [p(x,y) \times num(x,y)\right] = r(x)$$

$$\textbf{Solution} \equiv \boldsymbol{Q1} \wedge \boldsymbol{Q2} \wedge \boldsymbol{Q3}$$

3. Test and Result

```
s21800001@peace:~/19-2DiscreteMath$ cat inputExample.txt
20 23 30 29 34 6 9 21 19
8 2 1 8 1 3 5 7 6 18
9 1 4 2 5 6 3 1 7 28
3 5 1 4 9 1 3 9 1 8
8 6 6 3 5 1 1 4 1 4
8 6 6 2 6 8 3 3 9 31
8 7 8 8 4 5 2 1 1 18
4 8 3 5 5 2 1 2 8 24
1 2 8 8 8 3 7 2 7 22
8 3 9 5 9 2 1 4 9 35
```

```
s21800001@peace:~/19-2DiscreteMath$ ./hw inputExample.txt
1 1 0 1 0 0 1 0 0
0 1 1 0 1 0 0 0 0
1 1 1 0 1 0 0 1 1
1 1 1 1 1 1 1 0 1
0 0 0 1 1 0 0 1 1
0 1 1 1 0 0 1 1 0
0 0 1 0 0 0 1 1 1
1 1 1 1 0 1 0 1 0
0 0 0 0 1 1 0 1 0
```

4. Discussion

4-1. After changing logic formula of Q1

$$\bigwedge_{x=1}^{M} \bigwedge_{y=1}^{N} (p(x,y) \geq 0)\ \mathrm{V}\ (p(x,y) \leq 1)\ \text{to}$$

$$\bigwedge_{x=1}^{M} \bigwedge_{y=1}^{N} (p(x,y) = 0)\ \mathrm{V}\ (p(x,y) = 1),$$

the processing speed of the z3 has increased noticeably. Perhaps it's because the second logic formula specified the range of the value $p(x,y)$ more clearly.

4-2. While solving this homework, I found that the processing speed of this program was significantly slower than the programs in PA1. It is hard to know whether this speed problem was caused by the algorithm I had devised or the character of this puzzle.