In this chapter we develop some techniques for determining without direct enumeration the number of possible outcomes of a particular experiment or the number of elements in a particular set.

Factorial Notation:

The product n (n – 1) (n – 2) …… 3.2.1 is called factorial n and is denoted by the symbol n! where n is a positive whole number.

$$
\begin{aligned}
0! &= 1 \\
1! &= 1 \\
2! &= 2 \times 1 = 2 \\
3! &= 3 \times 2 \times 1 = 6 \\
4! &= 4 \times 3 \times 2 \times 1 = 24
\end{aligned}
$$

Multiplication Rule:

Example-1:

How many sample points are in the sample space when a pair of dice is thrown once?

Solution:

$$n_1 \times n_2 = 6 \times 6 = 36 \text{ ways}$$

Example-2:

How many lunches are possible consisting 4 soups, 3 kinds of sandwiches, 5 desserts and 4 drinks?

Solution:

The total number of lunches would be:

$$4 \times 3 \times 5 \times 4 = 240$$

Permutations:

A permutation is an ordered arrangement of objects.

OR

An arrangement of a set of n objects in a given order is called a Permutation.

The number of permutations of n distinct objects taken r at a time is

$$\text{i.e.}\quad {}^{n}P_{r} = \frac{n!}{(n-r)!}$$

Example-3:

The lottery tickets are drawn from 10 for first and second prizes. Find the number of permutations.

Solution:

$${}^{n}P_{r} = {}^{10}P_{2} = 90 \text{ Ways}$$

Example 4:

In how many different ways can the three letters a, b and c be arranged by taking two at a time?

Solution:

$${}^{n}P_{r} = {}^{3}P_{2} = 6 \text{ Ways}$$

Example-5:

In how many ways the letters of the word QUIT can be arranged?

Solution:

$${}^{n}P_{r} = {}^{4}P_{4} = 24 \text{ Ways}$$

OR

$$n! = 4! = 24 \text{ Ways}$$

Example-6:
In how many ways can 6 men be seated in a row having 4 seats?

Solution:

$$^{n}P_{r} = {}^{6}P_{4} = 360 \text{ Ways}$$

Example-7:
How many two digit numbers can be formed from the digits 1, 3, 6, 7, 9 when the digits are:

(a) not repeated
(b) repeated

Solution:

(a) $^{n}P_{r} = {}^{5}P_{2} = 20$ Numbers

OR

$\underline{5} \times \underline{4} = 20$ numbers (First place can be filled by any 5 digits and the second place can be filled by remaining 4 digits)

(b) $^{n}P_{r} = {}^{5}P_{2} = 5^{2} = 25$ Numbers

OR

$\underline{5} \times \underline{5} = 25$ numbers (First and second place can be filled by any 5 digits)

Example-8:

How many three digit numbers can be formed from the digits 1, 3, 6, 7, 9 when the digits are:

(a) Not repeated

(b) Repeated

Solution:

(a) $^{n}P_{r} = {}^{5}P_{3} = 60$

OR

$\underline{5} \times \underline{4} \times \underline{3} = 60$ numbers

(b) $^{n}P_{r} = {}^{5}P_{3} = 5^{3} = 125$ numbers

OR

$\underline{5} \times \underline{5} \times \underline{5} = 125$ numbers

Example-9: How many odd three-digit numbers can be formed from the digits 1, 2, 3, 5, 6 if each digit can be used only once?

Solution:

We have 3 choices (odd numbers) for the units position for each of three we have 4 choices for the hundreds position and 3choices for the tens position.

$$\frac{4}{} \times \frac{3}{} \times \frac{3}{1,\ 3,\ 5} = 36$$ odd three digit numbers

Example-10:

A person travels from Karachi to Lahore and back. 10 buses are available. In how many ways he can commute such that:

(i) He does not like to return by the same bus.

(ii) He does not mind to return by the same bus.

Solution:

(i) He has 10 choices from Karachi to Lahore

And 9 choices from Lahore to Karachi

So $\underline{10} \times \underline{9} = 90$ Ways

(ii) He has 10 choices from Karachi to Lahore

And 10 choices from Lahore to Karachi

So $\underline{10} \times \underline{10} = 100$ Ways

The number of distinct permutations of n things of which n_1 are of one kind, n_2 of a second kind, …… n_k of a k^{th} kind is:

$$\frac{n!}{n_1! \cdot n_2! \cdot \cdots \cdot n_k!}$$

Example-11:

In how many ways can the letters of the word CALL be arranged?

Solution:

$n = 4$

$n_1 = c = 1$

$n_2 = a = 1$

$n_3 = l = 2$

$$\frac{4!}{1! \times 1! \times 2!} = \frac{4 \times 3 \times 2 \times 1}{1 \times 1 \times 2 \times 1} = \frac{24}{2} = 12 \text{ ways}$$

Example-12:

How many different ways can 3 red, 4 yellow and 2 blue bulbs be arranged?

Solution:

$n = 3 + 4 + 2 = 9$

n_1 = red = 3

n_2 = yellow = 4

n_3 = blue = 2

$$\frac{9!}{3!\times4!\times2!} = \frac{362880}{6\times24\times2} = \frac{362880}{288} = 1260 \text{ Ways}$$

Example-13:

In how many ways can a student answer 8 questions true-false examination if he marks half the questions true and half the questions false?

Solution:

$n = 8$

$n_1 = \text{True} = 4$

$n_2 = \text{False} = 4$

$$\frac{n!}{n_1!\times n_2!} = \frac{8!}{4!\times4!} = 70 \text{ Ways}$$

The number of permutations of n distinct objects arranged in a circle is (n – 1)!

Example-14:

In how many ways can 5 different trees be planted in a circle?

Solution:

$n = 5$

$(n - 1)! = (5 - 1)! = 4! = 24$ ways

Example-15:

In how many ways can seven people sit at a round table?

Solution:

$n = 7$

$(n - 1)! = (7 - 1)! = 6! = 720$ ways

Combinations:

A combination is a selection of objects considered without regard to their order.

Example-16:

How many ways are there to select 3 students from 8 students for a party.

Solution:

${}^{n}C_{r} = {}^{8}C_{3} = 56$ ways

Example-17:

From a group of 4 men and 5 women, how many committees of size 3 are possible.

(a) With no restriction?

(b) With 1 man and 2 women?

Solution:

(a) $^{n}C_{r} = {}^{9}C_{3} = 84$ Ways

(b) $^{4}C_{1} \times {}^{5}C_{2} = 4 \times 10 = 40$ Ways

Example-18:

A shipment of 15 computer sets contains 3 defective sets. In how many ways can a person purchase 5 of these sets and receive 2 defective sets?

Solution:

12 Non Defective Sets

3 Defects Sets

$^{12}C_{3} \times {}^{3}C_{2} = 220 \times 3 = 660$ Ways