

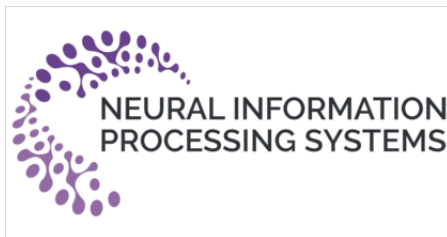


How not to loose \$100.000

Alex Vigneron



Constrained Cross-Entropy Method for Safe Reinforcement-Learning (2018)



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1 Appetisers

2 Crash-course

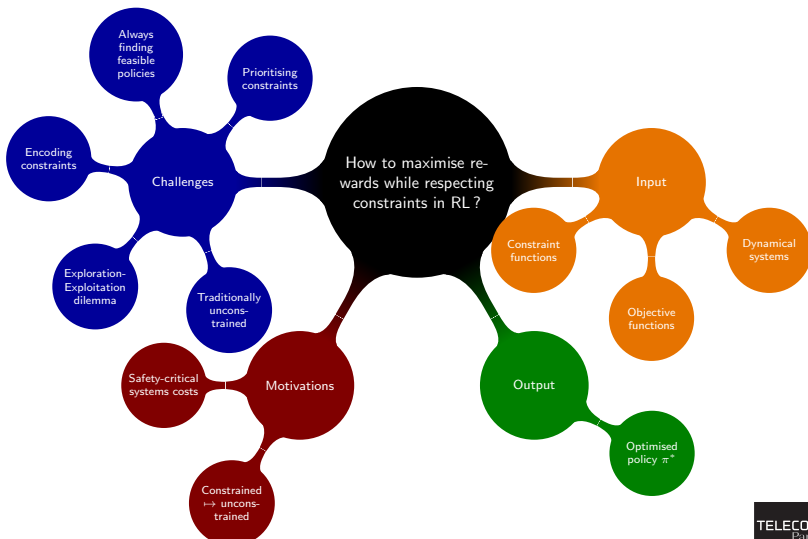
3 Main course : CCE

4 Experiments & Performance

5 Conclusion

6 Extras

Problem Overview 🍷



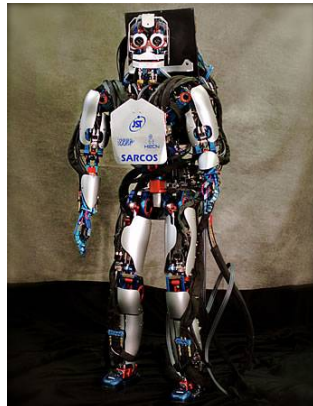
Applications : safety-critical systems 🍀

Definition

Safety-critical systems are those systems whose failure could result in loss of life, significant property damage, or damage to the environment. (Knight et al. 2002)

Safety Critical Systems using RL

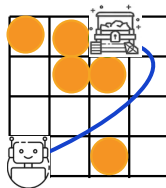
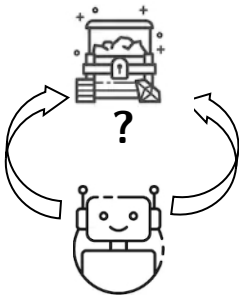
- Cooling systems (Li et al. 2018)
- Autonomous-driving vehicles (survey by Kiran et al. 2020)
- Industry robots Sarcos' Humanoid DB used RL to learn a pole-balancing task (Schaal, 1996).
Today Sarcos' Guardian suits are rented \$ 150.000 a year



Source : Brain controlled robots (Kawato, 2008)

Imagine...

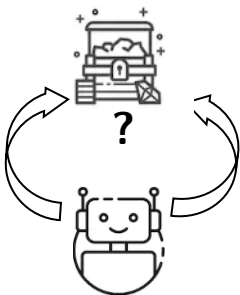
A brave robot having to look for a marvelous treasure kept in a solid chest inside volcanic caves filled with lava pits...



It needs to **get the treasure** while **avoiding lava**. **Where should it tread ?**

- Get treasure = objective
- Avoid lava = constraint
- Where to tread = policy

Reinforcement Learning Paradigm 🍷



Policy π

A probability distribution parameterised by a vector θ over actions given states.

$$\pi_{\theta}(s_i) \mapsto \{(a_{i0}, p_{i0}) \dots (a_{in}, p_{in})\}$$

Concept

Agent interacts with environment looking for optimal behaviour, receives rewards, modifies its actions to maximise reward

- **Input** : MDP
- **Output** : Policy $\Pi = \max(\sum_{r \in \mathcal{R}} \mathcal{R}(s, a))$

Markov Decision Process (Sutton & Barto 2017)

MDP characterised as a tuple $(\mathcal{S}, \mathcal{A}, \mathcal{P}, \mathcal{R})$

- \mathcal{S} , set of states $\mathcal{S} = \{s_0, s_1 \dots s_n\}$
- \mathcal{A} , set of actions $\mathcal{A} = \{a_0, a_1 \dots a_n\}$
- \mathcal{R} , reward function $\mathcal{R}(s, a) \mapsto \mathbb{R}$
- \mathcal{P} , transition probability function $\mathcal{P}(s, a) \mapsto \mathbb{R}^+$ where for a fixed s , $\sum_a \mathcal{P}(s, a) = 1$
- Initial distribution state

Constraints in traditional RL : an example 🍷

Problem

Avoiding lava while obtaining the treasure

Solution

Lava is encoded as a (major) negative reward

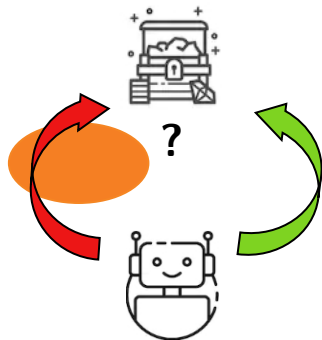
Effect

First iterations : agent explores any state (including lava states)

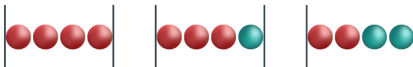
Following iterations : agent learns it *should* not tread lava

Problem

Lava **is** still an option... → no guarantee on lava constraint



Constrained Cross-Entropy



Source : Medium/Udacity Serrano

Entropy (Shannon's)

Entropy is a measure of our lack of information about the microstate of a system (Machta *et al.* 1999).

Cross-Entropy

Cross-entropy is commonly used to quantify the difference between two probability distribution.

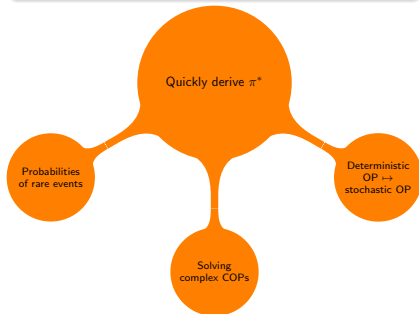
Constrained Cross-Entropy

This functions incorporates the constraints on the system in a generic fashion, irrespective of the form or even the number of the constraints (Niven *et al.*,

CE : an intuition 🍷

Idea

Probabilistic approach to maximise possibility of an unlikely event happening.



Algorithm

1. Sample from a distribution of policies
2. Select a set of elite samples and use them to update policy distribution.

Example

Unlikely event = having a policy which both maximises rewards and respects all constraints.

CCE Algorithm

Input: G , H , upper bound b , set of parameterised policies Π_θ ,
NEF family F

Output: Π^*

```
1 Initialise (random, feasible, unfeasible)
  while stopping rule is not satisfied do
2     Sample over parameters  $\theta$  of policy
      for each policy  $\pi \in \Pi$  with  $\theta$  parameters do
3         Simulate  $\pi_\theta$ 
          Compute and store  $G(\pi_\theta)$  and  $H(\pi_\theta)$ 
4     end
5     Sort  $\theta$  in ascending order of  $H$ -value
      Let  $\psi$  be the first  $k$  elements
      if  $H_{\theta_\psi} \leq b$  then
6         Sort  $\theta_\pi$  with  $H_{\pi_\theta} b$  in descending order of  $G$ 
7     end
8     Estimate through CE
      Ensure new p.d.f. still  $\in$  initial NEF family
      Update
9 end
```

L-function

The L function

Optimised through learning !

$$L(\mathbf{v}, \rho) = \begin{cases} \text{if } \xi_H(\rho, \mathbf{v}) > d \longrightarrow \mathbb{E}_{\theta \sim f_v} [\mathcal{G}(\pi_\theta) \delta(\mathcal{H}(\pi_\theta) \leq \xi_H(\rho, \mathbf{v}))] \\ \text{else} \longrightarrow \mathbb{E}_{\theta \sim f_v} [\mathcal{U}(\pi_\theta) \delta(\mathcal{U}(\pi_\theta) \geq \xi_U(1 - \rho, \mathbf{v}))] \end{cases} \quad (1)$$

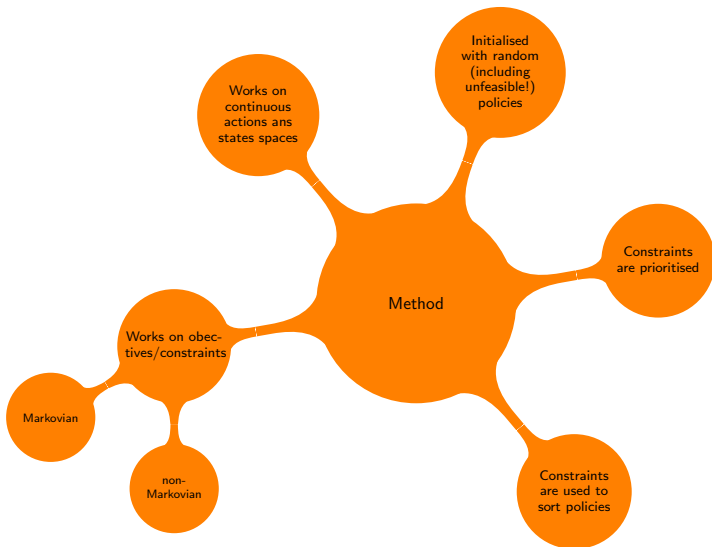
- \mathbf{v} , parameter for probability distribution
- $f_v(\theta)$, probability distribution over parameter θ
- d : acceptability threshold
- $\mathcal{G}(\pi_\theta)$, expected gain with regards to policy π with parameters θ
- $\mathcal{H}(\pi_\theta)$ expected cost of constraints with regards to π with parameters θ
- $\xi_F(\rho_i, \mathbf{v})$, p-quantile of the p.d.f. parameterised by \mathbf{v}
- $\delta(F)$... maps result to a boolean
- $\mathcal{U}(\pi_\theta)$: expected reward $\mathcal{G}(\pi_\theta)$ if respects constraints, 0 otherwise



Source : theproducemoms



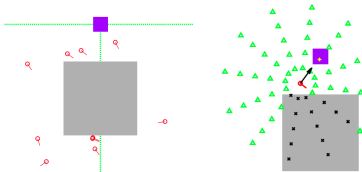
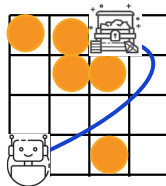
CCE recap 🍷



The problem at hand

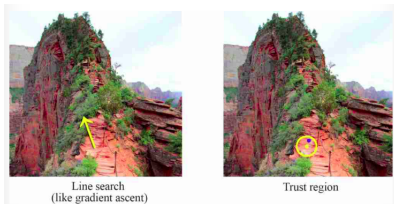
Task : robot navigation with only local sensors

Setting : compact goal region \mathcal{G} and a non-overlapping compact bad region \mathcal{B}



- $\mathcal{S}, \mathcal{A}, \Pi$ sets of states, actions and policies
- Objective function $\mathcal{O} : (\mathcal{S} \times \mathcal{A})^n \rightarrow \mathbb{R}^+$
- Cost function $\mathcal{C} : (\mathcal{S} \times \mathcal{A})^n \rightarrow \mathbb{R}$
- Deterministic Transition function
- $\mathcal{G} : \Pi \rightarrow \mathbb{R}^+$ expected value of rewards over Π
- $\mathcal{H} : \Pi \rightarrow \mathbb{R}$ expected value of constraints over Π

Trust Region Policy Optimisation 🍷



Source : MC.AI

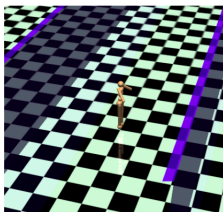
TRPO (Schulman *et al.* 2015)

- Unconstrained RL
- Used as a safeguard against policy gradient disasters (especially in high-curvature functions).
- Restricts policy moves so changes are not too aggressive.
- Allows one policy update per iteration.

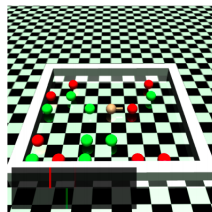
Constrained Policy Optimisation 🍏

CPO (Achiam *et al.* 2017)

- Constrained RL
- Defines a risk function to ensure agent's security
- Both reward and risk are taken into account at each step
- Promotes safe exploration



(a) Humanoid-Circle

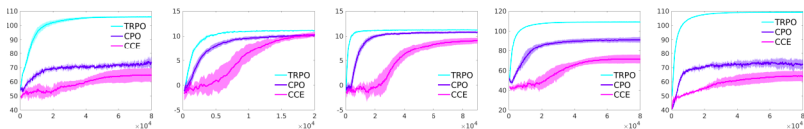


(b) Point-Gather

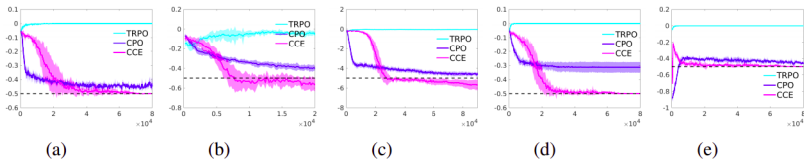
Source : Achiam (2017)

Baseline

Objective value $G_{J_i}(\pi_\theta)$



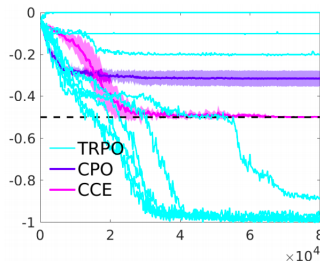
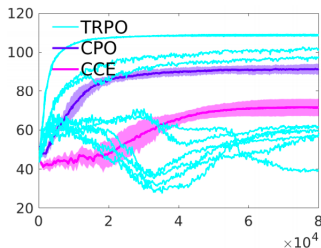
Constraint value $H_{Z_i}(\pi_\theta)$ (Feasible regions are below dashed lines)



x-axis : number of sample trajectories

y-axis : expectation of cumulated reward/punishment for CPO/TRPO and learned policy distribution for CCE

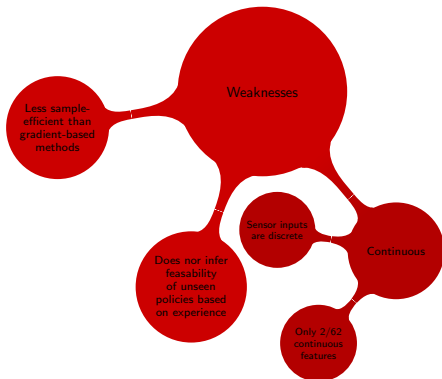
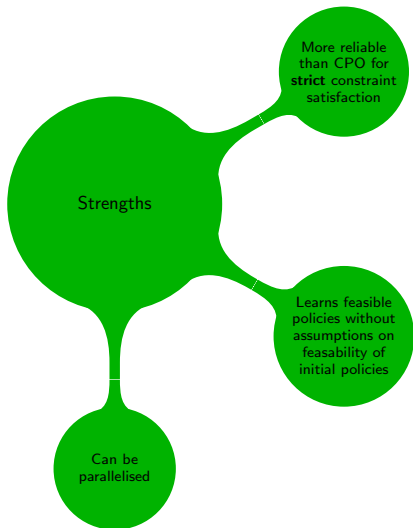
Baseline 🍷



TRPO's unconstrained approach is insufficient, merely optimising objectives is not enough to output feasible policy.

CPO needs more samples to find a feasible policy or converges to infeasible policy (especially if constraint is non-Markovian).

Performance analysis 🍷



Limitations of the paper 🍏

Perspective

Postulate that it is very hard to encode constraints as reward functions, but though hard, this is feasible (Geibel *et al.* 2005, Abe *et al.* 2010, Tong *et al.* 2000)

Experimental setting

Both goal and danger region are compact. This simplification does not take into account the difficulty of proper exploration in sparse settings.

Thank you for your time

In a nutshell

- CCE uses a probabilistic approach to solve safe RL problems.
- CCE can work while initialised with unfeasible policy, which is often not the case in safe RL.
- The key function is $\mathcal{L}(\mathbf{v}, \rho)$ since it takes into account both \mathcal{G} the objective function and \mathcal{H} the constraint function.

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This presentation together with the bibtex file of references are available on github at
github.com/Narmondil/CCE_for_safe_RL

Constrained Cross-Entropy

Entropy (Shannon's)

Skewed Probability Distribution (unsurprising) : Low entropy. Balanced Probability Distribution (surprising) : High entropy.

$$\mathcal{H}(X) = - \sum_{i=1}^n P(x_i) \log(P(x_i))$$

$$X = (x_1, \dots, x_n)(2)$$

Cross-Entropy

$$\mathcal{H}(p, q) = -\mathbb{E}_p[\log(q)] = \mathcal{H}(p) + \mathcal{D}_{KL}(p||q) \quad (3)$$

Constrained Cross-Entropy

This functions incorporates the constraints on the system in a generic fashion, irrespective of the form or even the number of the constraints (Niven *et al.*, 2003).

Remembering why we're doing CCE 🍷



FIGURE – RL-based self-driving car