AMERICAN JOURNAL of PHYSICS

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Citation: American Journal of Physics **65**, 553 (1997); doi: 10.1119/1.18595
View online: http://dx.doi.org/10.1119/1.18595
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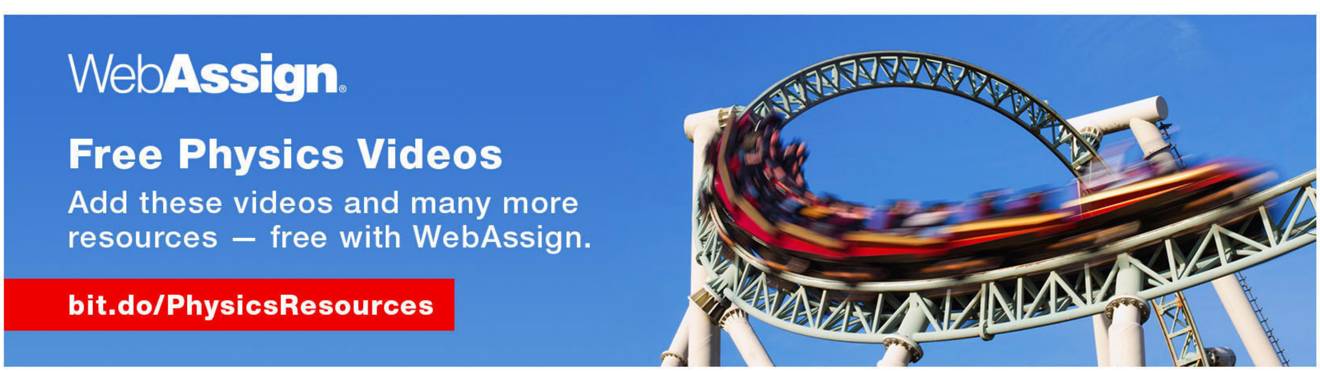

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The human discharge chain

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(Received 9 December 1996; accepted 16 January 1997)

The common classroom demonstration of a human chain, charged by a Van de Graaff generator, and then discharged via the person at the end of the chain touching ground, is analyzed as a capacitor and resistor circuit model. The energy deposited in each person in the chain is determined. Further, the effect of increasing energy deposited in the person who touched ground, as the number of people in the chain is increased, is shown and quantified. © *1997 American Association of Physics Teachers.*

I. INTRODUCTION

A common classroom demonstration involves a chain of people being charged with a Van de Graaff generator. The charging generally takes place through a person at the end of the chain. Once charged, another person may touch ground to discharge the chain. This is illustrated in Fig. 1.

The effect noticed by the people performing this demonstration is that the person who grounds out the charged chain receives the biggest shock. The effect diminishes the further one is (in the chain) from the person who grounded out the chain, to the point where there is no noticeable effect. This shock, felt by the person touching ground, gets larger as the number of people in the chain increases.

We modeled this demonstration as a capacitor and resistor network, where we view each body as one ''plate'' of a capacitor, and ground as the other ''plate.'' Each of these bodies (plates) is connected to the other via a resistive element.

Important note: Within this model results are presented which relate to the safety of the demonstration. It is stressed that this is only a model, and should not be construed as a full analysis of all safety factors. One must always evaluate the safety of a real demonstration.

II. THE MODEL

A circuit model for charging and discharging is shown in Fig. 2. Applying Kirchhoff's laws to the circuit model yields a system of first-order differential equations (N-coupled first-order equations for N people in the chain). For example, if we let Q_1 be the charge on the person who touches ground, then for discharging a four-person chain we have

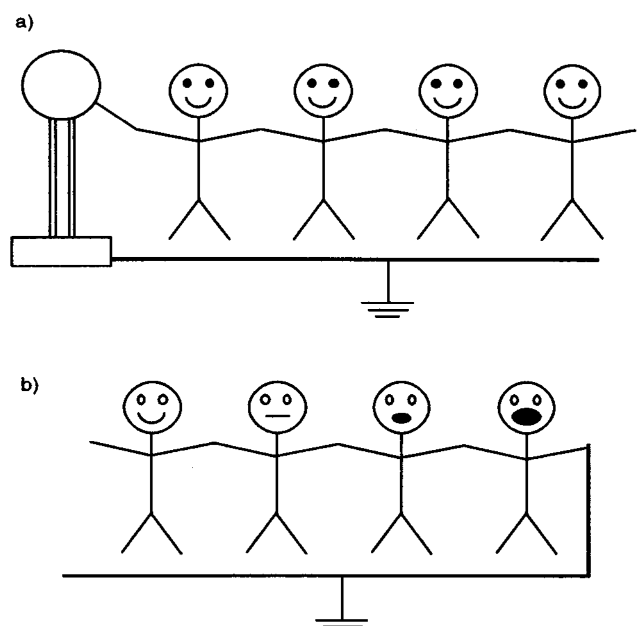

Fig. 1. A stickman model for charging a chain of people with a Van de Graaff generator, and for discharging.

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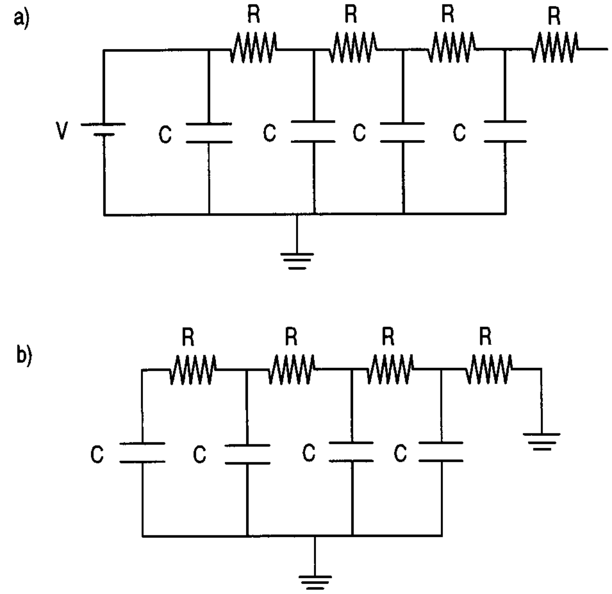

Fig. 2. The equivalent circuit model for the situation depicted in Fig. 1.

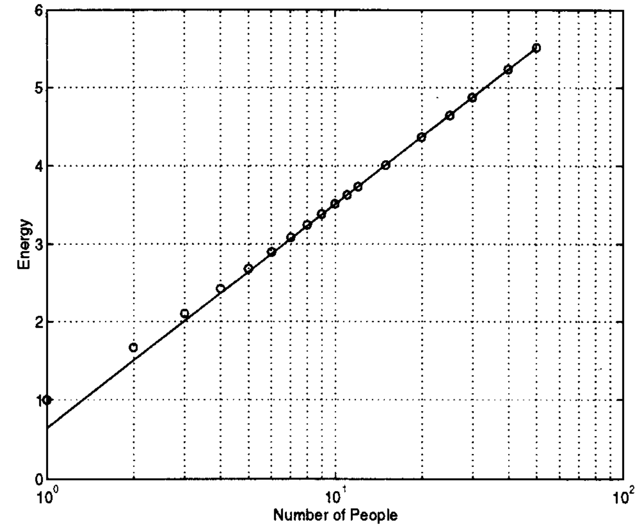

Fig. 3. In this figure the energy deposited in the grounding person is plotted vs the number of people in the chain. The energy unit is given in terms of the initial stored energy per person. The circles represent actual model calculations. The line represents a fit to the 9 points with $N \geqslant 10$. The equation of the line is $E = 2.86 \log_{10}(N) + 0.645$.

$$\begin{aligned}
&Q_4(t)+\dot{Q}_4(t)=Q_3(t),\\
&Q_3(t)+\dot{Q}_4(t)+\dot{Q}_3(t)=Q_2(t),\\
&Q_2(t)+\dot{Q}_4(t)+\dot{Q}_3(t)+\dot{Q}_2(t)=Q_1(t),\\
&Q_1(t)+\dot{Q}_4(t)+\dot{Q}_3(t)+\dot{Q}_2(t)+\dot{Q}_1(t)=0,\\
&Q_1(0)=Q_2(0)=Q_3(0)=Q_4(0)=1,
\end{aligned} \tag{1}$$

where we have let $C=R=V_{\text{Van de Graaff}}=1$. (Note, this normalization does not effect the generality of our results as all the numbers presented in the tables and plots are independent of the actual values of the capacitance, resistance, and charging voltage.)

We analyzed the discharging of such a circuit using Mathematica. In this discharge, we focus on the energy being deposited in each resistive element; this, we feel, correlates to the shock which a person in an actual chain feels. We assume that each person initially has one unit of stored energy ($\frac{1}{2}CV^2$, where C is an individual's capacitance and V the Van de Graaff charging voltage). For an N-person chain this means we track where the N units of energy are dissipated. This involves solving the circuit for $I_i(t)$, the current in the ith element as a function of time, and then evaluating $\int_0^\infty I_i(t)^2 R dt$ to obtain the total energy dissipated in each resistive element (body). The result of this analysis is shown in Table I.

As is seen in the table (and Fig. 3), the person touching ground receives more and more energy as the number of people in the chain increases—just over twice that of the 1 person chain at 3 people and four times at 15 people, etc. Fortunately, for safety reasons, it appears that this energy grows rather slowly. This effect is shown in Fig. 3. If this dependence extrapolates to higher values of N, then we estimate that for a factor of 10 increase in dissipated energy, the chain would need to be over 1000-people long.

Anecdotal safety information: The authors again wish to stress that they have not performed a thorough safety analysis. It is their experience that though this demonstration certainly can deliver a sizable static shock—of sufficient size that some people prefer not to repeat the experience—there have not been any medical problems. In decades of collected experience they are aware of one instance where a small female child's arms broke into a mild rash after touching a standard classroom Van de Graaff generator to perform the

Table I. The left column is the number of people in the chain. The next column is the amount of energy dissipated in the person who grounded out the chain, etc., on down the line. Since each person stores one unit of energy, each row adds to N, the total number of people in the chain.

1	1.000 00									
2	1.666 67	0.333 33								
3	2.103 46	0.724 14	0.172 41							
4	2.427 30	1.037 73	0.429 52	0.105 44						
5	2.684 86	1.292 08	0.665 29	0.286 71	0.071 07					
6	2.898 74	1.504 73	0.870 57	0.469 47	0.205 38	0.051 12				
7	3.081 72	1.687 13	1.049 59	0.638 01	0.350 62	0.154 44	0.038 51			
8	3.241 58	1.846 71	1.207 46	0.790 51	0.490 98	0.272 33	0.120 37	0.030 05		
9	3.383 56	1.988 53	1.348 34	0.928 41	0.622 10	0.390 70	0.217 80	0.096 46	0.024 09	
10	3.511 26	2.116 14	1.475 38	1.053 71	0.743 38	0.504 36	0.318 77	0.178 23	0.079 02	0.019 74

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Table II. This table focuses on the energy being deposited in the grounding person only. It shows what percent of the total energy which that person will receive has received within 1, 5, 10, and 50 RC time constants from the start of the discharge.

% energy deposited within	1 RC	5 RC	10 RC	50 RC
for a 1-person chain	86.5	100.0	100.0	100.0
for a 5-person chain	38.0	73.6	88.4	100.0
for a 10-person chain	29.1	56.3	68.7	95.1
for a 20-person chain	23.4	45.3	55.2	78.6
for a 50-person chain	18.5	35.9	43.7	62.2

‘‘hair flying apart’’ demonstration. The authors strongly recommend against any individual with health problems or electrical implants, such as pacemakers, performing this demonstration.

Loosely speaking, these effects may be viewed as the discharging person receiving all the energy stored on himself, plus a (diminishing) fraction of all the energy stored on people ‘‘upstream.’’ For the individual farthest from the discharge, the greatly diminished energy is due to the fact that this individual is discharging through N people (or resistors). Thus the energy stored on this individual is distributed across all other people in the chain. But the actual numbers involved are not trivial and have thus far eluded any simple explanation.

III. ITEMS OF FURTHER INTEREST

A. Estimates of pertinent numbers

If one estimates the body as a spherical capacitor of radius 1 m, then C is on the order of 100 pF. Typical Van de Graaff voltages vary, but are on the order of 100 000 V. Therefore, our unit of energy is somewhere under 1 J. We estimate the body’s resistance at 1 MΩ or less. With this estimate the characteristic time scale of the discharge is less than 0.1 ms. As a point of reference, a typical electrical heart defibrillation unit deposits a few hundred joules of energy over a period of roughly 10 ms (see Ref. 1).

B. Rate of energy dissipation

As the chain of people gets longer and longer, so too does the discharging time. This effect is summarized in Table II. Logically, the longer it takes to dissipate the energy the less reaction the body would have. However, we feel that since even 1 ms is short compared to neural response times, this instantaneous power lowering effect probably does not significantly alter the shock a person feels.

Table III. Same as Table I, except that the results are exact.

1	1				
2	$\frac{5}{3}$	$\frac{1}{3}$			
3	$\frac{61}{29}$	$\frac{21}{29}$	$\frac{5}{29}$		(speculated)
4	$\frac{2187}{901}$	$\frac{935}{901}$	$\frac{387}{901}$	$\frac{95}{901}$	(speculated)

C. Exact solutions

The numbers given in Table I are approximate numbers evaluated using Mathematica’s numerical algorithms. The fractions listed in Table III are exact numbers for the 2-person chain, and we speculate (though we have not shown) on the exact fractions for the 3- and 4-person case. It is interesting that this problem yields rational fractions after the power integration. We were not able to find a general term for these fractions. In theory, at least the 3- and 4-person cases should yield exact solutions. This is due to the fact that N-coupled first-order equations, with constant coefficients, are equivalent to an Nth order ordinary differential equation with constant coefficients whose solution would be exponentials raised to the powers of the roots of the ‘‘characteristic equation.’’ Thus the 3- and 4-person model is exactly solvable (since up through fourth order polynomials have exact analytic solutions). However, we did not (exactly) solve the 3- and 4-person case.

IV. CONCLUSION

We modeled the discharge of a human chain as a resistor capacitor (RC) network. The interesting effect that we focus on is the energy deposited in the person who grounds out the chain versus the number of people in the chain. This energy increases with the number of people in the chain. Fortunately the increase is rather slow. Our results show that the person touching ground receives all the energy stored on himself plus a (diminishing) fraction of every other person’s stored energy. However, the actual fractions are not trivial and lead to some interesting unanswered questions.

ACKNOWLEDGMENTS

The authors would like to thank the cadets taking Physics 215, who conducted many ‘‘shocking’’ experiments, and Perry R. Malcolm and James H. Head for valuable discussions.

[1] American Heart Association, *Textbook of Advanced Cardiac Life Support* (American Heart Association, Dallas, 1987), 2nd ed., pp. 89–95.

EXPONENTIAL DECAY

Personal computers are notorious for having a half-life of about two years. In scientific terms, this means that two years after you buy the computer, half of your friends will sneer at you for having an outdated machine. In households with children the half-life is closer to six months.

Peter H. Lewis, ‘‘Leasing as One Means of Avert Obsolescence,’’ The New York Times, 21 November 1995.

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