HW 1

11.09.2022 17:25

(2) $x^2 - 8x + 32 = 0$

$2^6 \quad 2^3$

$D = 64 - 4 \cdot 1 \cdot 32 = 64 - 128 = -64$

$x_{1/2} = \frac{8 \pm \sqrt{-64}}{2} = \left[\begin{array}{l} \frac{8 - 8i}{2} = \frac{8(1-i)}{2} = 4(1-i) = 4 - 4i \\ \frac{8 + 8i}{2} = 4(1+i) = 4 + 4i \end{array} \right.$

(1) $\sqrt[2]{6(\cos(4{,}185) + i\sin(4{,}185))} =$

$6^{\frac{1}{2}}(\cos(4{,}185 \cdot \frac{1}{2}) + i\sin(4{,}185 \cdot \frac{1}{2}) =$

$= \sqrt{6}(\cos(2{,}09) + i\sin(2{,}09)) = 2{,}44(\cos(2{,}09) + i\sin(2{,}09))$

(3)! $\sqrt[5]{\frac{2i}{-1-3i}}$

$\frac{2i}{-1-3i} = 2i \cdot \left(\frac{-1}{10} + \frac{3i}{10}\right) =$

$= -\frac{1}{10} \cdot 0 - \frac{3}{10} \cdot 2 + i\left(~~0 \cdot \frac{3}{10}~~ + \left(-\frac{1}{10}\right) \cdot 2\right) =$

$= -\frac{3}{10} \cdot 2 + \frac{1}{5}i = -\frac{3}{5} - \frac{1}{3}i$

$r = \sqrt{\frac{9}{25} + \frac{1}{25}} = \sqrt{\frac{10}{25}}$

$\varphi = \mathrm{arctg}\left(\frac{-\frac{1}{5}}{-\frac{3}{5}}\right) = \mathrm{arctg}\left(-\frac{1}{5} \cdot \left(-\frac{5}{3}\right)\right) = \mathrm{arctg}\left(\frac{1}{3}\right)$

$z^1 = \sqrt{\frac{10}{25}}\left(\cos\left(\mathrm{arctg}\left(\frac{1}{3}\right)\right) + i\sin\left(\mathrm{arctg}\left(\frac{1}{3}\right)\right)\right) =$

$z^{\frac{1}{5}} = \left(\frac{10}{25}\right)^{\frac{1}{10}}\left(\cos\left(\mathrm{arctg}\left(\frac{1}{3}\right)/5\right) + i\sin\left(\mathrm{arctg}\left(\frac{1}{3}\right)/5\right)\right) =$

$= 0{,}9124\,(\cos(-0{,}58397) + i\sin(-0{,}56397)) =$

$= 0{,}771139 + 0{,}487741i$

(4)! $\begin{cases} (-5-4i)z + (-5-2i)w = -2-i \\ (-5+2i)z + (-5+4i)w = -4+3i \end{cases}$

$z = \frac{-2-i-(-5-2i)w}{-5-4i}$

$(-5+2i) \cdot \frac{(-2-i)-(-5-2i)w}{-5-4i} + (-5+4i)w = -4+3i$

$\frac{(-5+2i)((-2-i)-(-5-2i)w)}{-5-4i} - \frac{(-5+4i)(-5-4i)w}{(-5-4i)} = -4+3i$

$\frac{(-5+2i)((-2-i)-(-5-2i)w)}{-5-4i} - \frac{(-5+4i)(-5-4i)w}{(-5-4i)} = -4+3i$

$\frac{(-5+2i)(-2-i)-(-5+2i)(-5-2i)w}{-5-4i} - \frac{(5+4i)(-5-4i)w}{(-5-4i)} = -4+3i$

$\frac{10+5i-4i-2i^2-(25+10i-10i-4i)w}{-5-4i} - \frac{(-25-20i-20i-16i^2)w}{-5-4i} = -4+3i$

$+2 \qquad 16$

$\frac{10+5i-4i-2i^2-(25+~~10i~~-~~10i~~-4i)w}{-5-4i} - \frac{(-25-20i-20i-16i^2)w}{-5-4i} = -4+3i$

$\frac{12+i-(25-4i)w}{-5-4i} - \frac{(-9-40i)w}{-5-4i} = -4+3i$

$\frac{12+i-(25-4i)w-(-9-40i)w}{-5-4i} = -4+3i$

$\frac{(12+i)\,w(-25+4i~~7~~9+40i)}{-5-4i} = -4+3i$

$\frac{(12+i)+(-16+44i)w}{-5-4i} = -4+3i$

$(12+i)+(-16+44i)w = (-4+3i)(-5-4i)$

$w = \frac{12}{17} + \frac{8}{51}i$ из вырах $z = \frac{2}{51} - \frac{3i}{17}$

$w \approx 0{,}705 + 0{,}156i$ $\qquad z \approx 0{,}039 - 0{,}176i$

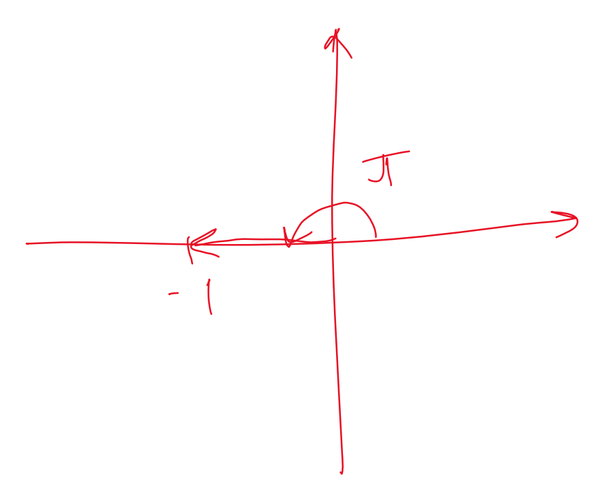
(5)

$z = -1 \qquad (-1;0)$

$r = \sqrt{1+0} = 1$

$\varphi_0 = \mathrm{arctg}\,\frac{y}{x} = \mathrm{arctg}\,0 = 0$

$\varphi = \pi$



(6) $\frac{2(\cos(\frac{3\pi}{2}) + i\sin(\frac{3\pi}{2}))}{2(\cos(\frac{7\pi}{4}) + i\sin(\frac{7\pi}{4}))} - \frac{(2(\cos(\frac{7\pi}{4}) + i\sin(\frac{7\pi}{4})))^2}{(2(\cos(\frac{3\pi}{2}) + i\sin(\frac{3\pi}{2})))^2} =$

$= \frac{2(\cos(\frac{3\pi}{2}) + i\sin(\frac{3\pi}{2}))}{2(\cos(\frac{7\pi}{4}) + i\sin(\frac{7\pi}{4}))} - \frac{4(\cos(2 \cdot \frac{7\pi}{4}) + i\sin(2 \cdot \frac{7\pi}{4}))}{4(\cos(2 \cdot \frac{3\pi}{2}) + i\sin(\frac{3\pi}{2} \cdot 2))} =$

$= \frac{~~2~~(\cos(\frac{3\pi}{2}) + i\sin(\frac{3\pi}{2}))}{~~2~~(\cos(\frac{7\pi}{4}) + i\sin(\frac{7\pi}{4}))} - \frac{~~4~~(\cos(\frac{7\pi}{2}) + i\sin(\frac{7\pi}{2}))}{~~4~~(\cos(3\pi) + i\sin(3\pi))} =$

$= \frac{0 + i \cdot (-1)}{\frac{\sqrt{2}}{2} + i(-\frac{\sqrt{2}}{2})} - \frac{0 - i1}{-1 + i \cdot 0} = \frac{-i}{\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i} - \frac{-i}{-1} =$

$= -i \cdot \left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i\right)^{-1} - (-i) \cdot (-1)^{-1} =$

$= -i \cdot \left(\frac{\frac{\sqrt{2}}{2}}{\sqrt{\left(\frac{\sqrt{2}}{2}\right)^2 + \left(\frac{\sqrt{2}}{2}\right)^2}} + \frac{\frac{\sqrt{2}}{2}}{\sqrt{\left(\frac{\sqrt{2}}{2}\right)^2 + \left(\frac{\sqrt{2}}{2}\right)^2}}i\right) - (-i) \cdot (-1) = -i \cdot \left(\frac{\frac{\sqrt{2}}{2}}{1} + \frac{\sqrt{2}}{2}i\right) - (-i) \cdot (-1) =$

$= -i \cdot \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right) - (-i) \cdot (-1) = 0 \cdot \frac{\sqrt{2}}{2} - (-1) \cdot \frac{\sqrt{2}}{2} + i\left(0 \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \cdot (-1)\right) - ((-i) \cdot (-1)) =$

$(0; -1) \quad (-1; 0)$

$0 \quad 0 \quad 0 \quad +1$

$= \left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i\right) - (0 \cdot (-1) - (-1) \cdot 0 + i(0 \cdot 0 + (-1) \cdot (-1)) =$

$= \left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i\right) - (i) = \frac{\sqrt{2}}{2} - \left(\frac{\sqrt{2}}{2} + \frac{2}{2}\right)i = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}+2}{2}i \approx 0{,}707106 - 1{,}707106i$

(7)

$\frac{2+6i}{3-4i} - \frac{(3-4i)^2}{(2+6i)^3}$

$\frac{2+6i}{3+4i} = (2+6i) \cdot \left(\frac{3}{25} - \frac{4}{25}i\right) = 2 \cdot \frac{3}{25} - 6 \cdot \left(\frac{4}{25}\right) + i\left(2 \cdot \left(-\frac{4}{25}\right) + 6 \cdot \frac{3}{25}\right) = \frac{6}{25} - \frac{24}{25} + i\left(-\frac{8}{25} + \frac{18}{25}\right) = -\frac{18}{25} + \frac{10}{25}i$

$\frac{(3-4i)^2}{(2+6i)^3} = \frac{(5(\cos(\mathrm{arctg}(-\frac{4}{3})) + i\sin(\mathrm{arctg}(-\frac{4}{3}))))^2}{(\sqrt{40}(\cos(\mathrm{arctg}(3)) + i\sin(\mathrm{arctg}(3))))^3} = \frac{25(\cos(\mathrm{arctg}(-\frac{4}{3})/2) + i\sin(\mathrm{arctg}(-\frac{4}{3})/2))}{40^{\frac{3}{2}}(\cos(\mathrm{arctg}(3)/3) + i\sin(\mathrm{arctg}(3)/3))} = 0{,}965597 - 0{,}0484910 4i$

$-\frac{18}{25} + \frac{10}{25}i - (0{,}965597 - 0{,}0484910i) = -0{,}79675 + 0{,}97775i$