Projects for Computational Physics

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The Kosterlitz-Thouless phase transition and the XY model

Preparation(**), Implementation(**), Analysis(***)

Project Description

The XY model is a generalisation of the Ising model with 2-dimensional continuous spin vectors $\vec{s} = (s_x, s_y)$ of fixed unit norm $\vec{s}^2 = s_x^2 + s_y^2 = 1$ and a Hamiltonian

$$\mathcal{H}(\vec{\mathbf{s}}) = -J \sum_{\langle i,j \rangle} \vec{s}_i \cdot \vec{s}_j$$

with nearest-neighbour coupling denoted by $\langle i, j \rangle$. Note, that the space dimension (where sites i, j reside) need not be 2-dimensional itself.
Like the Ising model it exhibits a phase transition, which has, however, quite different and interesting properties, which can be studied using the Monte-Carlo approach, when put in a heat-bath at temperature T with partition sum

$$\mathcal{Z} = \sum_{\vec{\mathbf{s}}} \exp(-\mathcal{H}/(k_B T)) \,.$$

Tasks / hints / interesting problems[1]

- investigate the temperature dependence of magnetisation and susceptibility
- determine the critical temperature T_c
- compare the (single spin-flip) Metropolis and the Cluster algorithm
- apply the method of finite size scaling to determine critical exponents
- investigate higher dimensions

Literature

[1] J. M. Kosterlitz, D. J. Thouless, "Ordering, metastability and phase transitions in two-dimensional systems", J. Phys. C: Solid State Phys. 7 1181

[2] J. M. Kosterlitz, "The critical properties of the two-dimensional xy model", 1974 J. Phys. C: Solid State Phys. 7 1046

[3] U. Wolff, "Collective Monte Carlo Updating for Spin Systems," Phys. Rev. Lett. **62** (1989) 361

[1] These tasks should rather be considered a general guideline than a strict requirement. Besides, the list is by no means complete and could easily be extended