กันต์ธินันท์ กุมทเก้า 65090500461

1. จงแก้ระบบสมการโดยใช้ Gauss Elimination Method

$$2x_1 + 6x_2 + x_3 = 7$$
$$x_1 + 2x_2 - x_3 = -1$$
$$5x_1 + 7x_2 - 4x_3 = 9$$

$$\left[\begin{array}{ccc|c} 2 & 6 & 1 & 7 \\ 1 & 2 & -1 & -1 \\ 5 & 7 & -4 & 9 \end{array}\right]$$

1st forward elimination

$m_{21} = \frac{1}{2}$

$m_{31} = \frac{5}{2}$

$$\rightarrow \left[\begin{array}{ccc|c} 2 & 6 & 1 & 7 \\ 0 & -1 & -1.5 & -4.5 \\ 0 & -8 & -6.5 & -8.5 \end{array}\right]$$

$S_2^{(1)} = -1-1.5-4.5 = -7$

$S_2^{*} = 1 - \frac{1}{2} \times 16 = -7$ ✓

$S_3^{(1)} = -8-6.5-8.5 = -23$

$S_3^{*} = 17 - \frac{5}{2} \times 16 = -23$ ✓

2nd forward elimination

$m_{32} = \frac{-8}{-1} = 8$

$$\rightarrow \left[\begin{array}{ccc|c} 2 & 6 & 1 & 7 \\ 0 & -1 & -1.5 & -4.5 \\ 0 & 0 & 5.5 & 27.5 \end{array}\right]$$

$-8-(8)(-1) = 0$

$-6.5-(8)(-1.5) =$

$S_3^{(2)} = 5.5 + 27.5 = 33$

$S_3^{*} = -23-(8)(-7) = 33$ ✓

Back Substitution

$$x_3 = \frac{27.5}{5.5} = 5$$

$$x_2 = \frac{-4.5 - (-1.5)(5)}{-1} = -3$$

$$x_1 = \frac{7-(6)(-3)-(1)(5)}{2} = 10$$

$$t_3^* = (8)(10) + (15)(-3) + (-4)(5) = 15$$

$$t_4 = 7-1+9 = 15 \checkmark$$

$$\therefore \; x_1 = 10 \;, \; x_2 = -3 \;, \; x_3 = 5$$

2. จงหาคำตอบของระบบสมการโดยใช้ Gauss-Jordan Elimination Method

$$x_1 - x_2 - x_3 - x_4 = 2$$
$$-2x_1 + 4x_2 + 3x_3 = -3$$
$$-4x_2 + 2x_3 + 3x_4 = -1$$
$$2x_1 + 2x_2 - 5x_4 = 5$$

Sol^n

$$\left[\begin{array}{cccc|c} 1 & -1 & -1 & -1 & 2 \\ -2 & 4 & 3 & 0 & -3 \\ 0 & -4 & 2 & 3 & -1 \\ 2 & 2 & 0 & -5 & 5 \end{array}\right]$$

Step 1:
eliminate

$$\left[\begin{array}{cccc|c} 1 & -1 & -1 & -1 & 2 \\ 0 & 2 & 1 & -2 & 1 \\ 0 & -4 & 2 & 3 & -1 \\ 0 & 4 & 2 & -3 & 1 \end{array}\right]$$

$a_{22}^{(1)} = 4 - (-2)(-1) = 2$

$a_{23}^{(1)} = 3 - (-2)(-1) = 1$

$a_{24}^{(1)} = 0 - (-2)(-1) = -2$

$a_{25}^{(1)} = -3 - (-2)(2) = 1$

$a_{32}^{(1)} = 2 - (2)(-1) = 4$

$a_{33}^{(1)} = 0 - (2)(-1) = 2$

$a_{34}^{(1)} = -5 - (2)(-1) = -3$

$a_{35}^{(1)} = 5 - (2)(2) = 1$

Step 2:

$$\left[\begin{array}{cccc|c} 1 & -1 & -1 & -1 & 2 \\ 0 & 1 & 0.5 & -1 & 0.5 \\ 0 & -4 & 2 & 3 & -1 \\ 0 & 4 & 2 & -3 & 1 \end{array}\right]$$

$a_{23}^{(2)} = \frac{1}{2} = 0.5$

$a_{24}^{(2)} = -\frac{2}{2} = -1$

$a_{25}^{(2)} = \frac{1}{2} = 0.5$

eliminate

$$\left[\begin{array}{cccc|c} 1 & 0 & -0.5 & -2 & 2.5 \\ 0 & 1 & 0.5 & -1 & 0.5 \\ 0 & 0 & 4 & -1 & 1 \\ 0 & 0 & 0 & 1 & -1 \end{array}\right]$$

$a_{13}^{(2)} = -1 - (-1)(0.5) = -0.5$

$a_{14}^{(2)} = -1 - (-1)(-1) = -2$

$a_{15}^{(2)} = 2 - (-1)(0.5) = 2.5$

$a_{43}^{(2)} = 2 - (4)(0.5) = 0$

$a_{44}^{(2)} = -3 - (4)(-1) = 1$

$a_{45}^{(2)} = 1 - (4)(0.5) = -1$

$a_{33}^{(2)} = 2 - (-4)(0.5) = 4$

$a_{34}^{(2)} = 3 - (-4)(-1) = -1$

$a_{35}^{(2)} = -1 - (-4)(0.5) = 1$

Step 3:

$$\left[\begin{array}{cccc|c} 1 & 0 & -0.5 & -2 & 2.5 \\ 0 & 1 & 0.5 & -1 & 0.5 \\ 0 & 0 & 1 & -0.25 & 0.25 \\ 0 & 0 & 0 & 1 & -1 \end{array}\right]$$

$a_{34}^{(3)} = -\frac{1}{4} = -0.25$

$a_{35}^{(3)} = \frac{1}{4} = 0.25$

eliminate

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -2.125 & 2.625 \\ 0 & 1 & 0 & -0.875 & 0.375 \\ 0 & 0 & 1 & -0.25 & 0.25 \\ 0 & 0 & 0 & 1 & -1 \end{array}\right]$$

$a_{14}^{(3)} = -2 - (\frac{1}{4})(-0.25) = -2.125$

$a_{15}^{(3)} = 2.5 - (\frac{1}{4})(0.25) = 2.625$

$a_{24}^{(3)} = -1 - (\frac{1}{4})(-0.25) = 0.875$

$a_{25}^{(3)} = 0.5 - (\frac{1}{4})(0.25) = 0.375$

Step 4:

eliminate

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0.5 \\ 0 & 1 & 0 & 0 & -0.5 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 \end{array}\right]$$

$a_{15}^{(4)} = 2.625 - (-1)(-2.125) = 0.5$

$a_{25}^{(4)} = 0.375 - (-1)(-0.875) = -0.5$

$a_{35}^{(4)} = 0.25 - (-1)(-0.25) = 0$

ดังนั้น $x_1 = 0.5$, $x_2 = -0.5$, $x_3 = 0$, $x_4 = -1$

3. Find an LU factorization for

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ -2 & 3 & -2 \end{bmatrix}$$

วิธีทำ ให้ $A^{(1)} = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ -2 & 3 & -2 \end{bmatrix}$

หาตัวคูณ $m_{21} = \frac{2}{1} = 2$ $\qquad m_{31} = \frac{-2}{1} = -2$

จะได้ $A^{(1)} = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ -2 & 3 & -2 \end{bmatrix} \xrightarrow[R_3 + 2R_1 \Rightarrow R_3]{R_2 - 2R_1 \Rightarrow R_2} \begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & -5 \\ 0 & 7 & 4 \end{bmatrix} = A^{(2)}$

หาตัวคูณ $m_{32} = \frac{7}{-1} = -7$

จะได้ $A^{(2)} = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & -5 \\ 0 & 7 & 4 \end{bmatrix} \xrightarrow[R_3 + 7R_2 \Rightarrow R_3]{} \begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & -5 \\ 0 & 0 & -31 \end{bmatrix} = A^{(3)}$

ดังนั้น $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -2 & -7 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & -5 \\ 0 & 0 & -31 \end{bmatrix}$

4. จงหาผลเฉลยของระบบสมการโดยใช้ LU factorization

$$2x_1 + x_2 + x_3 = -3$$

$$4x_1 + 4x_2 + 3x_3 = -3$$

$$8x_1 + 10x_2 + 13x_3 = -45$$

วิธีทำ ให้ $A^{(1)} = \begin{bmatrix} 2 & 1 & 1 \\ 4 & 4 & 3 \\ 8 & 10 & 13 \end{bmatrix}$

หาตัวคูณ $m_{21} = \frac{4}{2} = 2$ $\quad m_{31} = \frac{8}{2} = 4$

จะได้

$$A^{(1)} = \begin{bmatrix} 2 & 1 & 1 \\ 4 & 4 & 3 \\ 8 & 10 & 13 \end{bmatrix} \xrightarrow[R_3 - 4R_1 \Rightarrow R_3]{R_2 - 2R_1 \Rightarrow R_2} \begin{bmatrix} 2 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 6 & 9 \end{bmatrix} = A^{(2)}$$

หาตัวคูณ $m_{32} = \frac{6}{2} = 3$

จะได้

$$A^{(2)} = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 6 & 9 \end{bmatrix} \xrightarrow[R_3 - 3R_2 \Rightarrow R_3]{} \begin{bmatrix} 2 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 6 \end{bmatrix} = A^{(3)}$$

$$A = LU = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 6 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 4 & 3 & 1 \end{bmatrix}$$

จาก $A\bar{x} = \bar{b}$

$L(U\bar{x}) = \bar{b}$

ให้ $U\bar{x} = \bar{Y}$ —— ①

จะได้ $L\bar{Y} = \bar{b}$ —— ②

จาก $L\bar{Y} = \bar{b}$ จะได้

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 4 & 3 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} -3 \\ -3 \\ -45 \end{bmatrix}$$

Forward Substitution

$$y_1 = -3$$

$$y_2 = -3-(2)y_1 = -3-(2)(-3) = 3$$

$$y_3 = -45-(4)y_1-(3)y_2 = -45-(4)(-3)-(3)(3) = -42$$

$$\bar{Y} = \begin{bmatrix} -3 \\ 3 \\ -42 \end{bmatrix}$$

แทนค่า $\bar{Y}$ ใน $U\bar{X} = \bar{Y}$

$$\begin{bmatrix} 2 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -3 \\ 3 \\ -42 \end{bmatrix}$$

Back substitution

$$x_3 = \frac{-42}{6} = -7$$

$$x_2 = \frac{3-1(-7)}{2} = 5$$

$$x_1 = \frac{-3-1(5)-1(-7)}{2} = -0.5$$