Beautiful Tiling Domination

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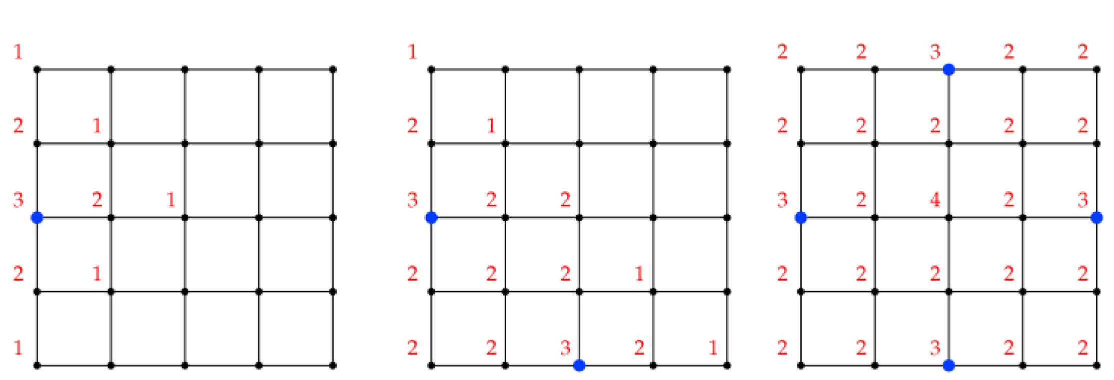
Abstract

Domination theory was first introduced by Claude Berge in 1958 when considering the optimal way to build a communication network between multiple cities. Since then, domination and broadcast domination of the coordinate and isometric planes has been found, bounded and optimized. In this research project, we search for optimal domination and broadcast domination of other tilings of the plane. In addition to creating aesthetically pleasing graphs, finding optimal subpatterns of unexplored tilings will hopefully provide insight into generating subpatterns for non-regular tilings that can be directly applied to real-world domination problems, such as city planning and resource distribution.

What is Broadcast Domination?

Broadcasting Vertex

We say a vertex $v \in G$ is broadcasting vertex of transmission strength t if it sends a signal of strength $t - d(u, v)$ to every vertex u with $d(u, v) < t$.



Reception Strength

We define the reception strength $r(u)$ at a vertex $u \in G$ to be the sum of the transmission strengths from all surrounding broadcasting vertices.

Given a pair of desired transmission and reception strengths (t, r) we look for subpatterns of broadcasting vertices of transmission strength t that dominate each vertex with a reception strength r.

Finding the Patterns

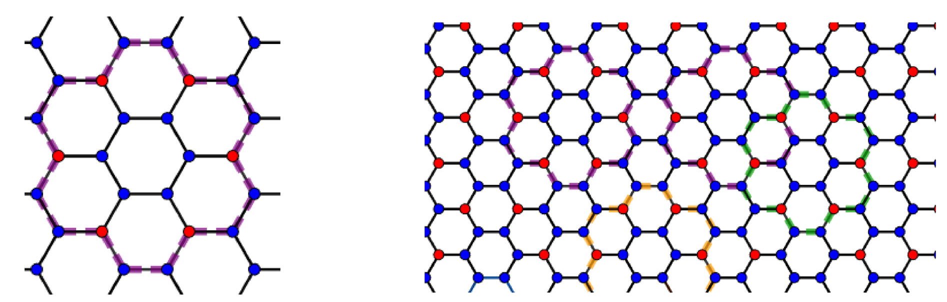


Figure 1: Hexa (2,1) Flower and Overlap

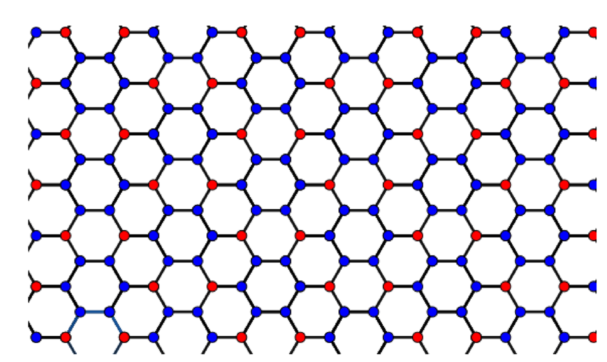


Figure 2: Hexa (2,1)

However, some domination patterns do not stitch together perfectly. This is where we begin prioritizing based on the parameters chosen. For example, Figure 4 shows "holes" in the sub-pattern. These holes represent vertices that cannot be dominated without over-dominating the adjacent ones.

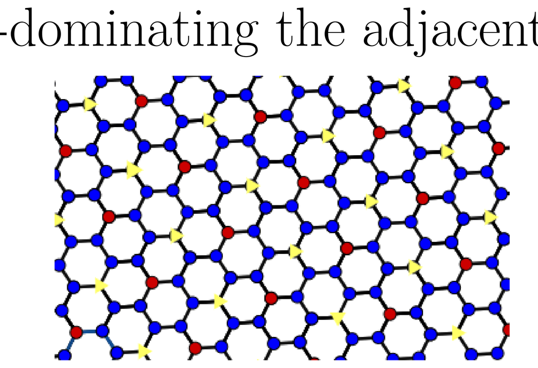


Figure 3: Hexa (3,2)

In some cases, the patterns made by the non-dominated vertices are interesting on their own. Looking at Figure 5, for instance, the non-dominated vertices of the Hexa tiling with (4,3) domination are the vertices that form the main subpattern of Hexa's (2,1) domination and the contained vertices.

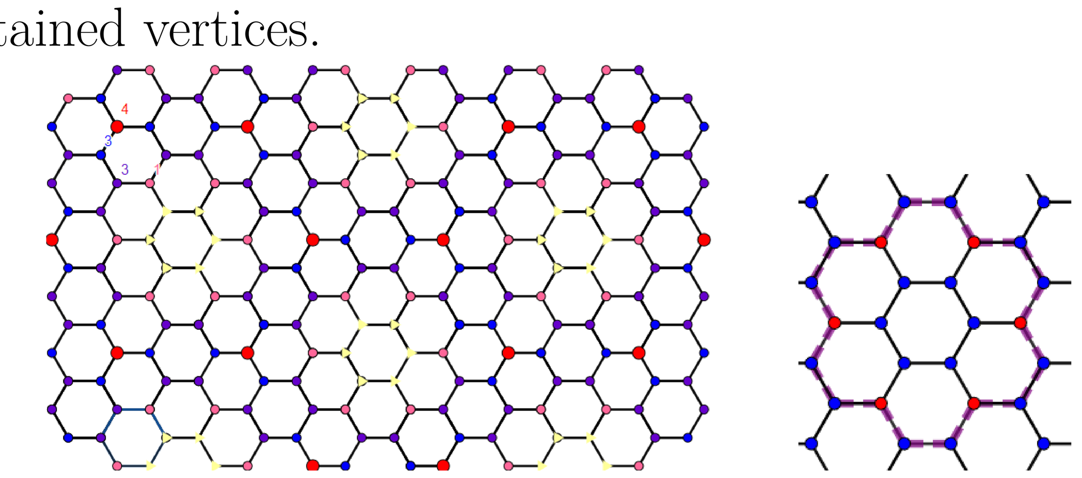


Figure 4: Hexa (4,3) and Hole Pattern

Fixing Grids

Since the purpose of the project is to find completely dominated tilings, we then try "fixing" the tiling to fit the subpattern that most dominates it. Note that on Figure 7 the vertices with green circles receive no reception from our broadcasting vertices, and thus form holes in the subpattern.

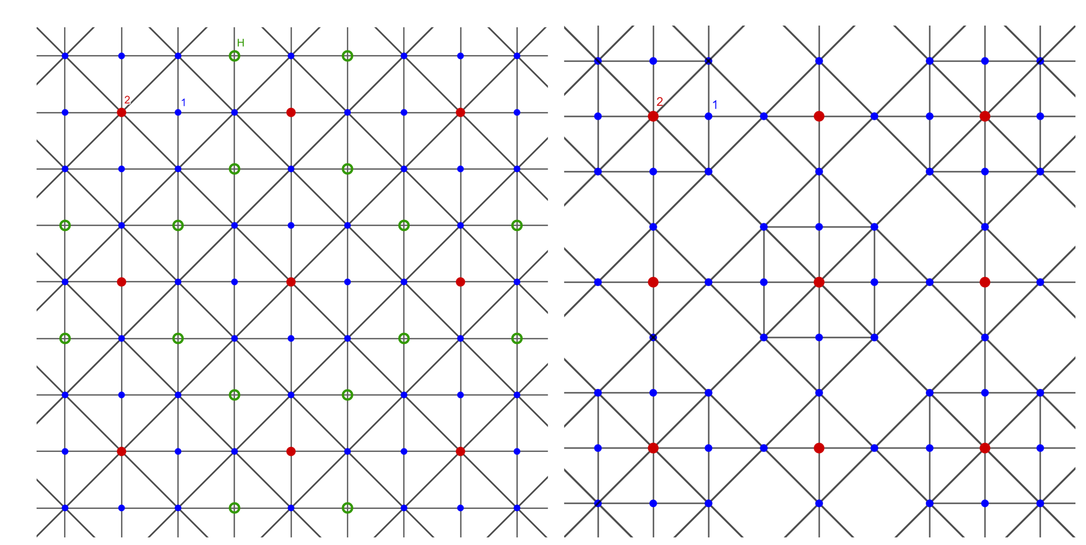


Figure 5: Diamond (2,1) and Its Fixed Tiling

If one were to remove the vertices (and their edges) from the tiling all together, the remaining tiling and pattern would be dominated and dominating, respectively. Although this changes the original tiling, we have created a new tiling that can be efficiently dominated, and can thus be useful.

Once we are confident we've found the optimal dominating sub-pattern to a tiling, we can use this "fixing" method to create a tiling that is efficiently dominated.

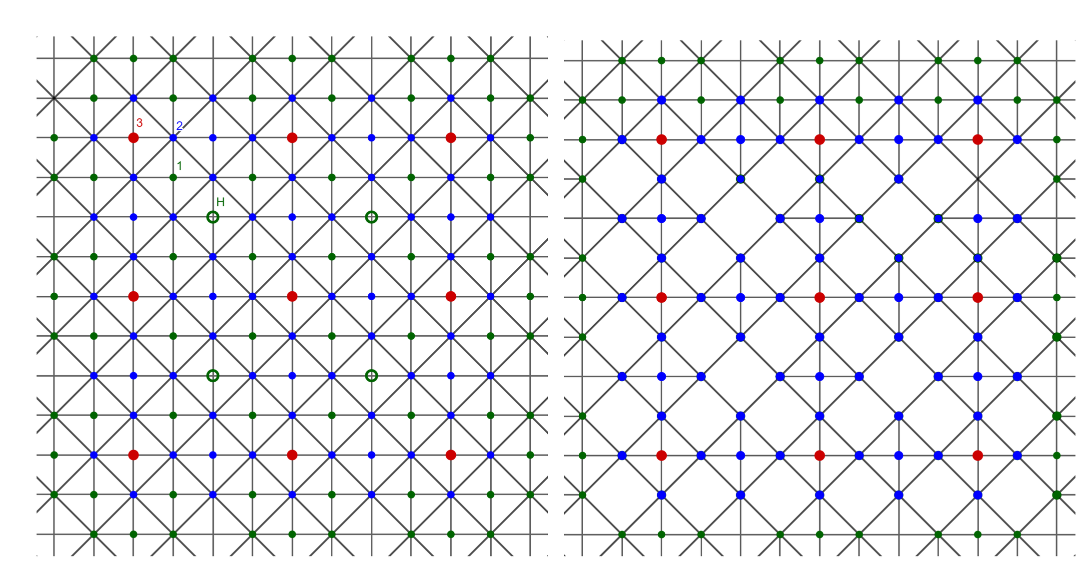


Figure 6: Diamond (3,2) and Its Fixed Tiling

Coloring Conventions: The red vertices in the figures above are broadcasting vertices; the blue vertices are dominated vertices. Green vertices are under-dominated.

Sample Application

Suppose your telephone company is looking to expand into a developing city. You've mapped out the areas of the city with a minimum population p. Your cell towers can provide coverage within a 10 mile radius, with the call quality decreasing 10% every mile. If you want every area with population p or higher to have a call quality of 90%, where should you place the towers in the most cost efficient way?

Open Questions

- Can all tilings generated from regular polygons be (2,1) dominated?
- For the Hexa tiling, would optimal subpatterns for $t = 2^n$ be fixed by removing the main patterns and containing vertices of a 'smaller' domination?
- Can we generalize whether a tiling will have an efficient subpattern based on its main degree sequence?
- Can we generalize whether a tiling will have an efficient subpattern based on the central eccentricity of its main pattern?
- Can we generate a negative test for efficient subpatterns?

Acknowledgements

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References

- David Blessing, Katie Johnson, Christie Mauretour, Erik Insko, On (t,r) broadcast domination numbers of grids, Discrete Appl. Math. 187 (2015) 19-40
- Benjamin F. Drews, Pamela E. Harris , Timothy W. Randolph, Optimal (t,r) broadcasts on the infinite grid, Discrete Appl. Math. (2018)