Week by week

(1) Introduction to quantum computing
(2) Single qubit representation and operations
(3) Two and more qubits
(4) Simple quantum algorithms
(5) Quantum search (Grover's algorithm)
(6) Quantum factorization (Shor's algorithm)
(7) Quantum supremacy and noise
(8) Programming real quantum computers (IBM Q)
(9) Quantum error correction (QEC)
(10) QUBO problems and Adiabatic Quantum Computation (AQC)
(11) Variational/hybrid quantum algorithms (QAOA and VQE)
(12) Solving linear equations, QC computing hardware

Week 2

Lecture 3

3.1 The Bloch Sphere representation for qubits

3.2 Quantum operations on qubits

3.3 Qubit gates in matrix form and the Pauli matrices

Lecture 4

4.1 The Pauli gates X, Y and Z and the QUI

4.2 Qubit operations around non-cartesian axes – H and R gates

4.3 Matrix exponential and arbitrary rotations

Practice class 2

Bloch sphere and single qubit logic operations on the QUI

Opportunity to be a student representatives

Your feedback is valuable to us.

If you would like to representative for this subject, email me at cdhill@unimelb.edu.au.

Lecture 2 recap

A qubit in "ket" notation:

$|a_0|^2 + |a_1|^2 = 1$

Prepared state $\|\psi\rangle$		Outcome	$\|\psi\rangle$ "collapsed" to	Outcome probability
$\|\psi\rangle = a_0 \|0\rangle + a_1 \|1\rangle$	measurement ?	"0"	$\|\psi\rangle \to \|0\rangle$	Prob["0"] = $\|a_0\|^2$
	or	"1"	$\|\psi\rangle \to \|1\rangle$	Prob["1"] = $\|a_1\|^2$

$$\to |\psi\rangle = |a_0|e^{i\theta_0}|0\rangle + |a_1|e^{i\theta_1}|1\rangle$$ Amplitudes in polar notation

"matrix" notation: $|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $|1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ $a_0|0\rangle + a_1|1\rangle = \begin{bmatrix} a_0 \\ a_1 \end{bmatrix}$ $|\psi\rangle = \begin{bmatrix} a_0 \\ a_1 \end{bmatrix}$

dual "bra" notation: $\langle\psi| = [\, a_0^* \quad a_1^* \,]$

inner product "bra-ket": $|\psi\rangle = \begin{bmatrix} a \\ b \end{bmatrix}$ $|\phi\rangle = \begin{bmatrix} c \\ d \end{bmatrix}$ $\langle\psi|\phi\rangle \equiv \langle\psi||\phi\rangle$ $= [\, a^* \quad b^* \,]\begin{bmatrix} c \\ d \end{bmatrix}$

$= a^*c + b^*d$

More detail about measurement

Projective measurement

What is the state of the system after a measurement is made?

Two useful operators, known as projectors:

$$P_0 = |0\rangle\langle 0| = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$P_1 = |1\rangle\langle 1| = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

Can also easily construct other projectors (using outer product).

One-qubit measurement

Measurement on a two-qubit state:

(1) Apply projector into the measured state
(2) Renormalize the state

$$|\psi'\rangle = \frac{P_m |\psi\rangle}{\sqrt{p(m)}}$$

$$|\psi\rangle = a|0\rangle + b|1\rangle$$

Apply Born's rule to work out the measurement probabilities.

If state is measured to be to be "0", apply P_0 and renormalize to get the collapsed state:

$$|\psi'\rangle = \frac{a|0\rangle}{\sqrt{|a|^2}} = e^{i\theta_0}|0\rangle$$

If state is measured to be "1", apply P_1 and renormalize to get the collapsed state:

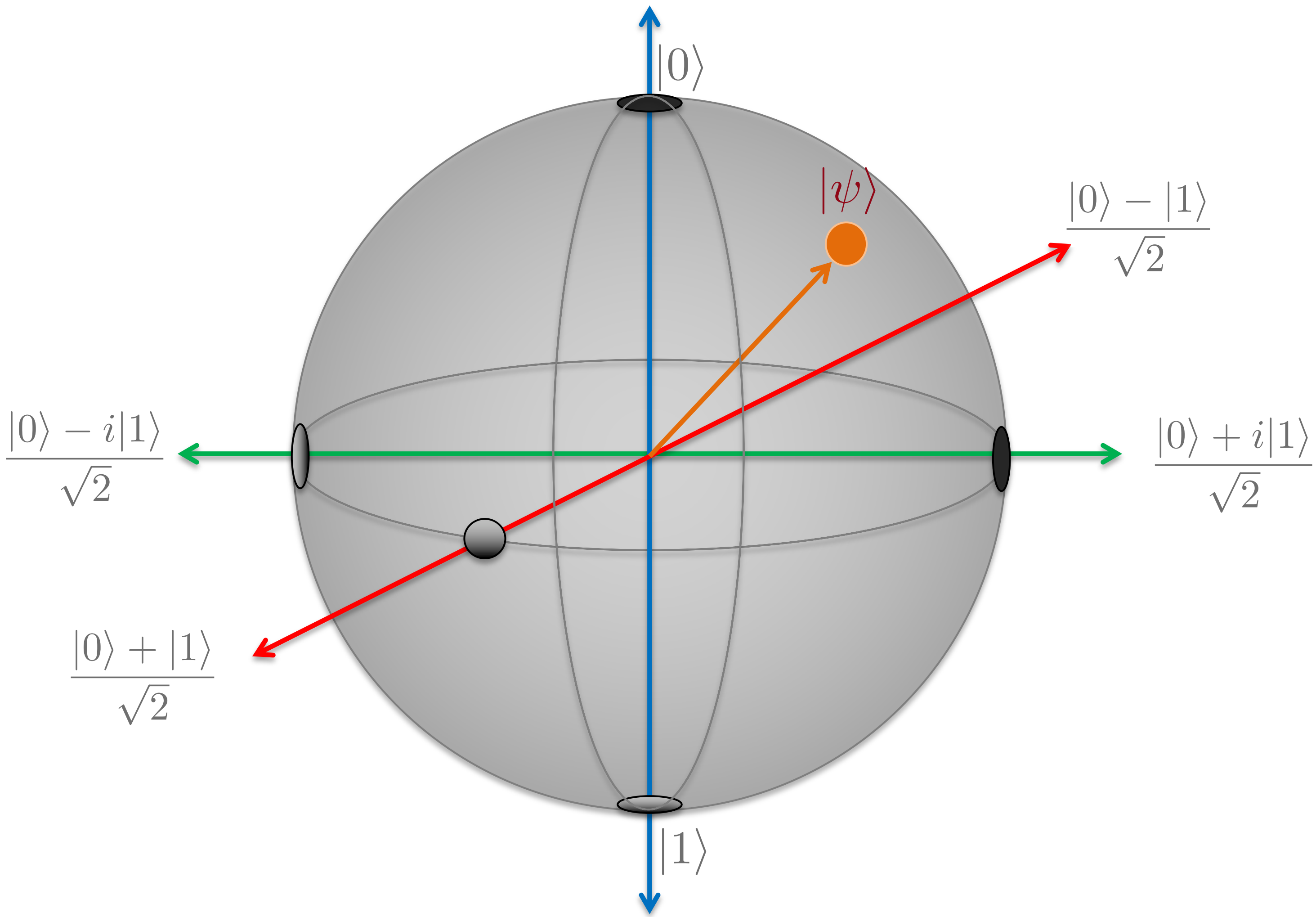
$$|\psi'\rangle = \frac{b|1\rangle}{\sqrt{|b|^2}} = e^{i\theta_1}|1\rangle$$

In quantum mechanics measurement necessarily disturbs the state you are measuring! Taking a superposition and measuring it, a superposition of many different states becomes just one. This is known as the **"collapse"** of the wavefunction.

The Bloch Sphere representation for qubits

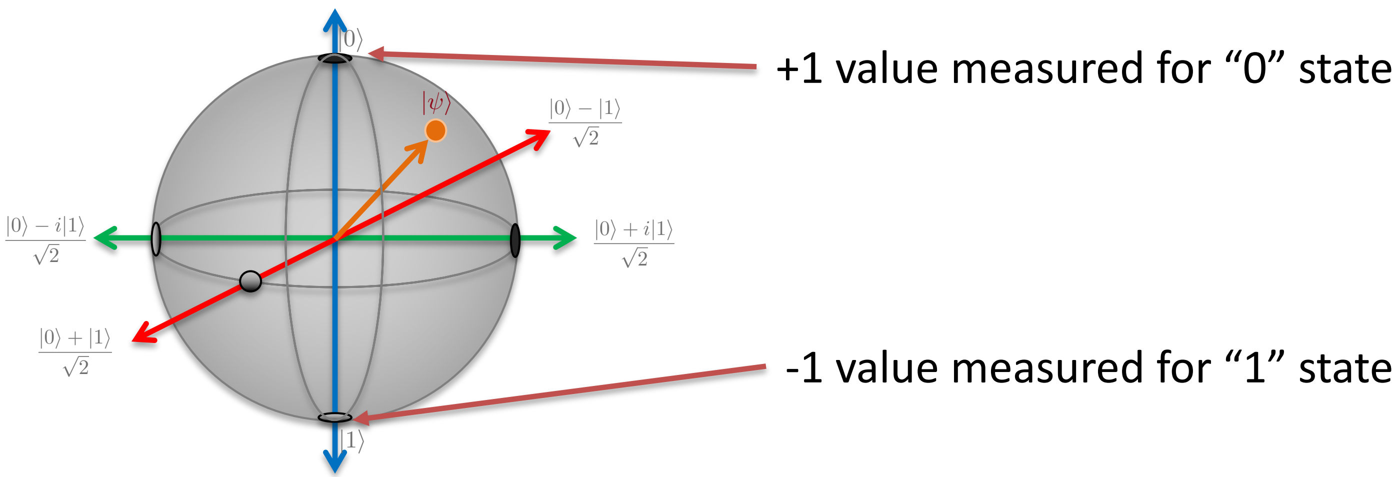
The Bloch Sphere

A convenient geometric representation of single qubit states is the Bloch sphere:



What is the z-projection?

Let's relate the Bloch sphere to states. Average, or "expectation" of the measured z component:



Average, or "expectation" value of the z (after measurement):

$$\langle Z \rangle = (+1)P_0 + (-1)P_1$$

Z-projection continued

Expressing P_0 and P_1 in terms of the state:

$$P_0 = |a_0|^2 = a_0^* a_0 = \langle\psi|0\rangle\langle 0|\psi\rangle$$

$$P_1 = |a_1|^2 = a_1^* a_1 = \langle\psi|1\rangle\langle 1|\psi\rangle$$

Therefore,

$$\begin{aligned}\langle Z\rangle &= (+1)P_0 + (-1)P_1 \\ &= (+1)\langle\psi|0\rangle\langle 0|\psi\rangle + (-1)\langle\psi|1\rangle\langle 1|\psi\rangle \\ &= \langle\psi(|0\rangle\langle 0| - |1\rangle\langle 1|)\psi\rangle \\ &= \langle\psi|\, Z\, |\psi\rangle\end{aligned}$$

Where

$$Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

Expectation Values

The *expectation value* of an operator is given by:

$$\langle A \rangle = \langle \psi | A | \psi \rangle$$

Hermitian operators correspond to physical observables – the expectation value then gives the average value of that quantity when measured in a given state.

For example, consider measuring the the total energy of the system represented by the energy operator, i.e. the "Hamiltonian" $\mathcal{H}$ which in matrix representation is

$$\mathcal{H} = \begin{pmatrix} E_0 & 0 \\ 0 & E_1 \end{pmatrix}$$

Expectation values:

$$\langle 0 | \mathcal{H} | 0 \rangle = E_0 \qquad \langle 1 | \mathcal{H} | 1 \rangle = E_1$$

Expectation values in a superposition state

For example, consider measuring the the total energy of the system

$$\mathcal{H} = \begin{pmatrix} E_0 & 0 \\ 0 & E_1 \end{pmatrix}$$

For the equal superposition state:

$$|+\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

The expectation value is:

$$\langle +| H |+\rangle = \frac{E_0 + E_1}{2}$$

The Pauli Matrices

Can think about the expectation along any of the cartesian axes. Equivalent to taking expectation values of any of these matrices:

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

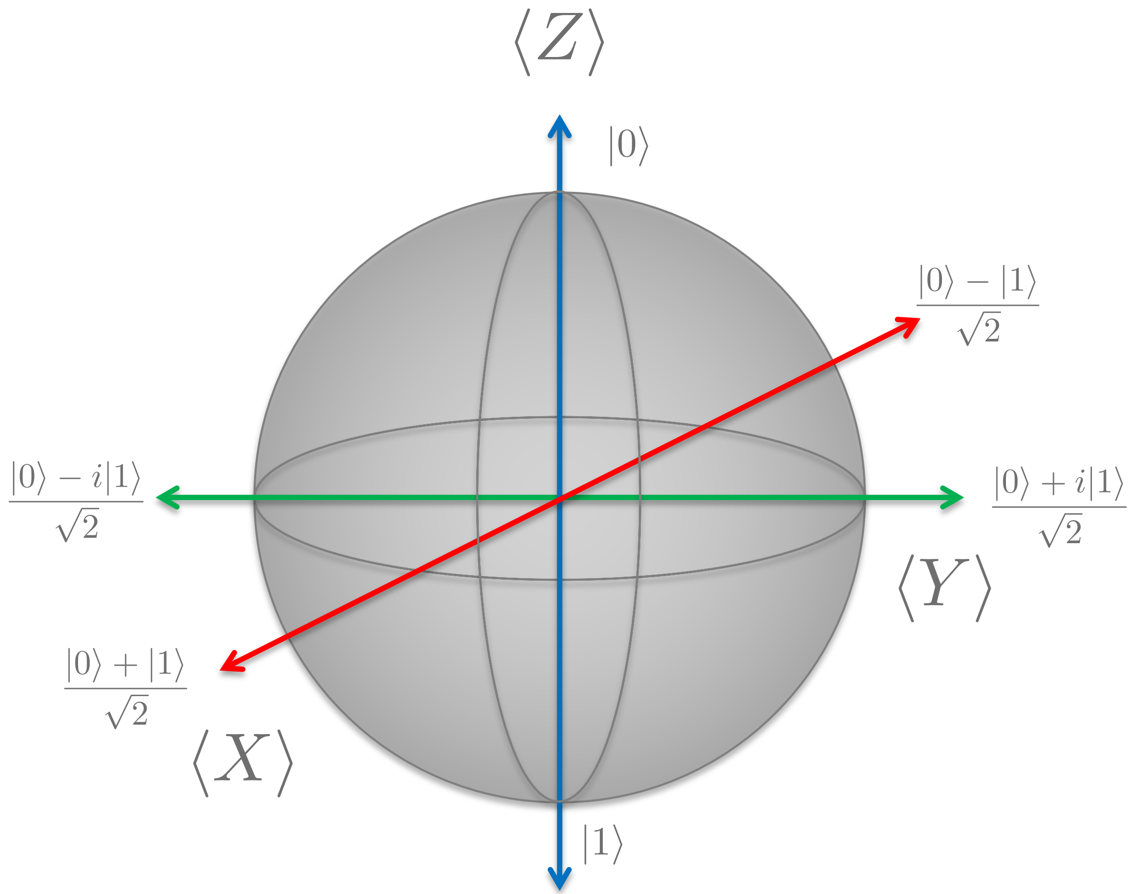
$$Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

$$Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

These matrices are known as the Pauli matrices. Worth remembering!

Pauli matrices & Bloch sphere axes

The expectation values of the Pauli matrices (operators) define the axes on the Bloch sphere.



Expectation of Pauli Matrices

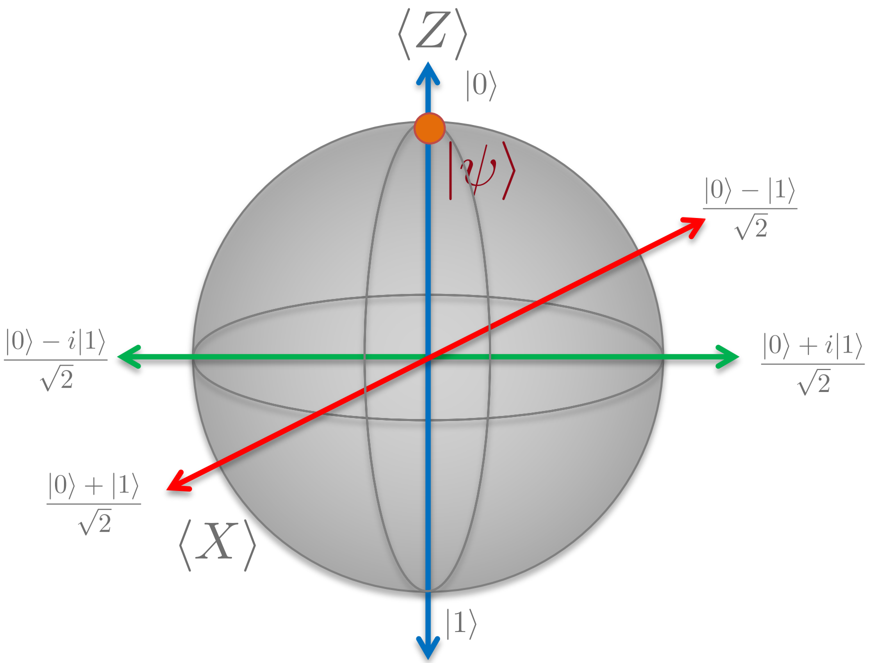
Recall: expectation value for an operator: $\langle A \rangle = \langle \psi | A | \psi \rangle$

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\langle X \rangle = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = 0$$

$$\langle Y \rangle = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = 0$$

$$\langle Z \rangle = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = 1$$



Expectation of Pauli Matrices

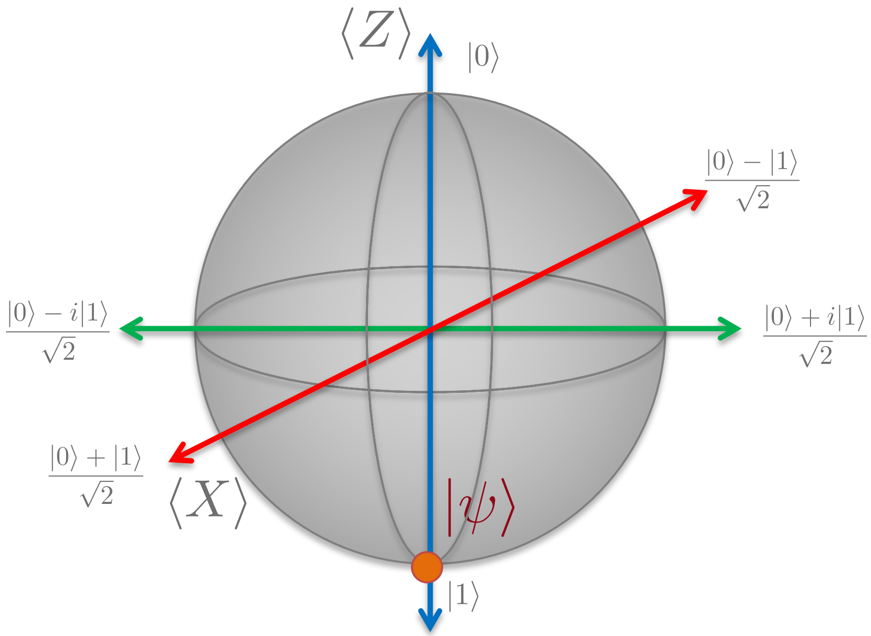
Recall: expectation value for an operator: $\langle A \rangle = \langle \psi | A | \psi \rangle$

$$|1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\langle X \rangle = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = 0$$

$$\langle Y \rangle = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = 0$$

$$\langle Z \rangle = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = -1$$



Expectation values of Pauli matrices

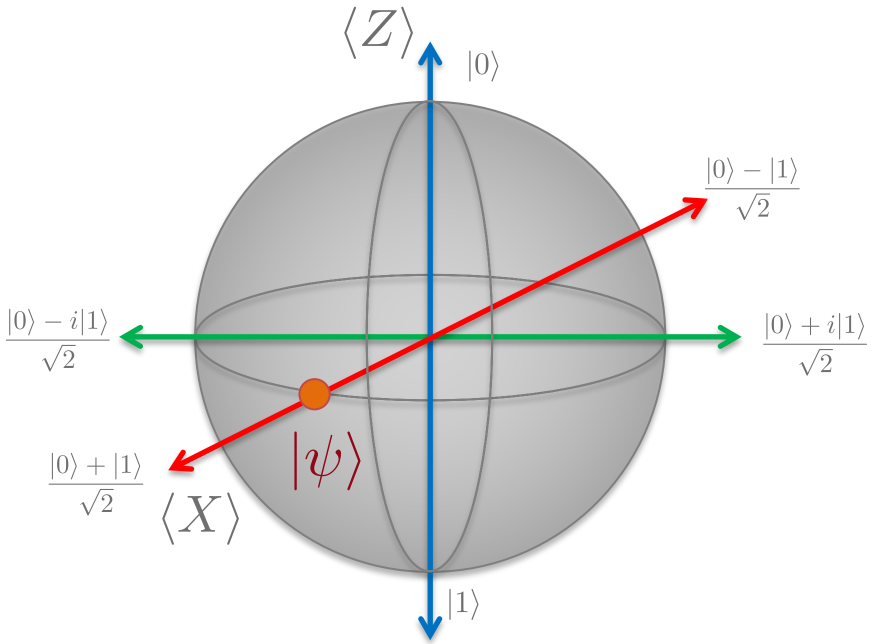
Recall: expectation value for an operator: $\langle A \rangle = \langle \psi | A | \psi \rangle$

$$|+\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

$$\langle X \rangle = \frac{1}{2} \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 1$$

$$\langle Y \rangle = \frac{1}{2} \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 0$$

$$\langle Z \rangle = \frac{1}{2} \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 0$$



Transformation to Bloch form

Recall, arbitrary qubit state: $|\psi\rangle = a_0|0\rangle + a_1|1\rangle = |a_0|e^{i\theta_0}|0\rangle + |a_1|e^{i\theta_1}|1\rangle$

Probability normalisation: $|a_0|^2 + |a_1|^2 = 1$

We can rearrange as:

$$|\psi\rangle = e^{i\theta_0}\left(|a_0|\,|0\rangle + |a_1|e^{i(\theta_1-\theta_0)}|1\rangle\right) = e^{i\theta_{\text{global}}}\left(\cos\frac{\theta_{\text{B}}}{2}|0\rangle + \sin\frac{\theta_{\text{B}}}{2}e^{i\phi_{\text{B}}}|1\rangle\right)$$

Where: $\phi_{\text{B}} \equiv \theta_1 - \theta_0$

$$\left.\begin{aligned} |a_0| &\equiv \cos\frac{\theta_{\text{B}}}{2} \\ |a_1| &\equiv \sin\frac{\theta_{\text{B}}}{2} \end{aligned}\right\}$$

NB: $\left(\cos\frac{\theta_B}{2}\right)^2 + \left(\sin\frac{\theta_B}{2}\right)^2 = 1$

i.e. $|a_0|^2 + |a_1|^2 = 1$

For single qubit we ignore global phase θ_{global} → state expressed via "Bloch" angles θ_{B} , ϕ_{B}

Suggests a representation on a sphere – the Bloch Sphere

Re-writing the state...

So, a qubit state (global phase set to zero) can be represented by two angles:

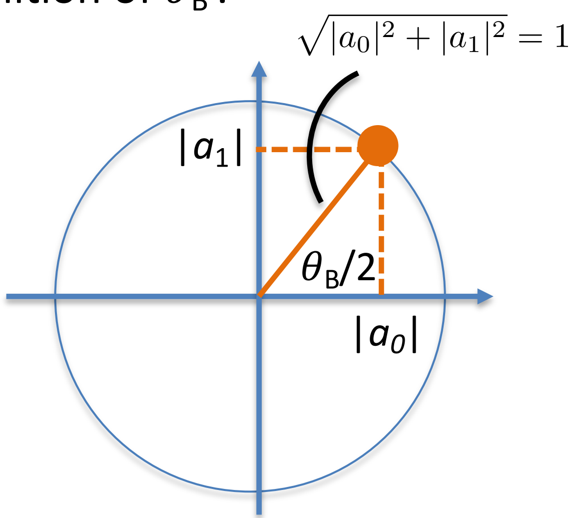
$$|\psi\rangle = a_0 |0\rangle + a_1 |1\rangle \;\rightarrow \cos\frac{\theta_\mathrm{B}}{2}|0\rangle + \sin\frac{\theta_\mathrm{B}}{2}e^{i\phi_\mathrm{B}}|1\rangle$$

The real variables θ_B and ϕ_B dictate the position of this state on the Bloch sphere.

$$|\psi\rangle = a_0 |0\rangle + a_1 |a_1\rangle$$

$$\phi_\mathrm{B} \equiv \theta_1 - \theta_0 \qquad |a_0| \equiv \cos\frac{\theta_\mathrm{B}}{2} \qquad |a_1| \equiv \sin\frac{\theta_\mathrm{B}}{2}$$

Definition of θ_B:



And the azimuthal angle ϕ_B runs from 0 to 2π

NB. Bloch Sphere angles distinct from amplitude polar angles...

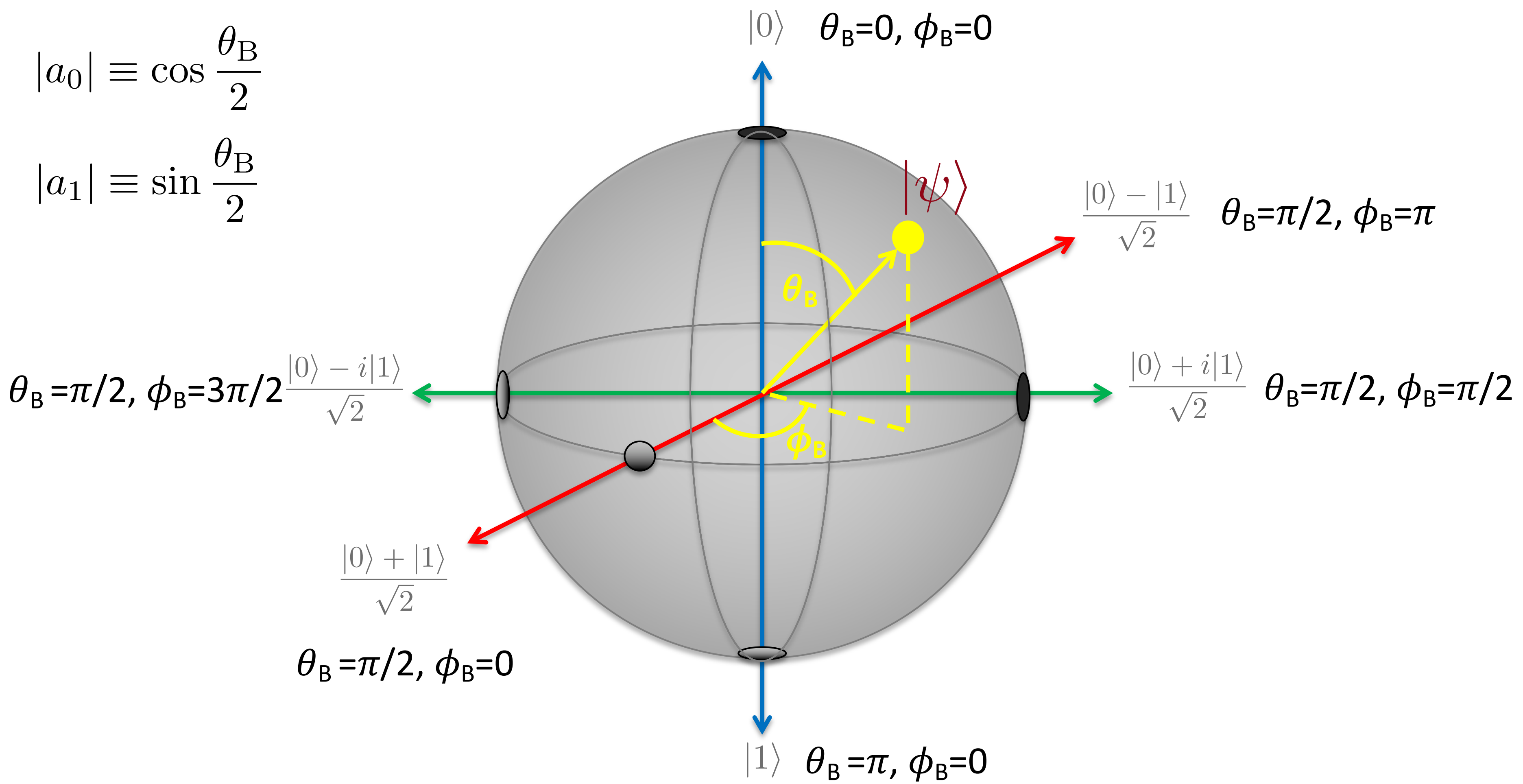
Qubit states on the Bloch sphere

$$|\psi\rangle = a_0|0\rangle + a_1|1\rangle \rightarrow \cos\frac{\theta_B}{2}|0\rangle + \sin\frac{\theta_B}{2}e^{i\phi_B}|1\rangle$$

$$\phi_B \equiv \theta_1 - \theta_0$$

$$|a_0| \equiv \cos\frac{\theta_B}{2}$$

$$|a_1| \equiv \sin\frac{\theta_B}{2}$$



Polar co-ordinates and global phase

Recall, an arbitrary qubit state: $|\psi\rangle = a|0\rangle + b|1\rangle = |a|e^{i\theta_a}|0\rangle + |b|e^{i\theta_b}|1\rangle$

Which we can rearrange to be:

$$|\psi\rangle = e^{i\theta_a}\left(|a|\,|0\rangle + |b|e^{i(\theta_b-\theta_a)}|1\rangle\right) = e^{i\theta_a}\left(|a|\,|0\rangle + |b|e^{i(\theta_b-\theta_a)}|1\rangle\right)$$

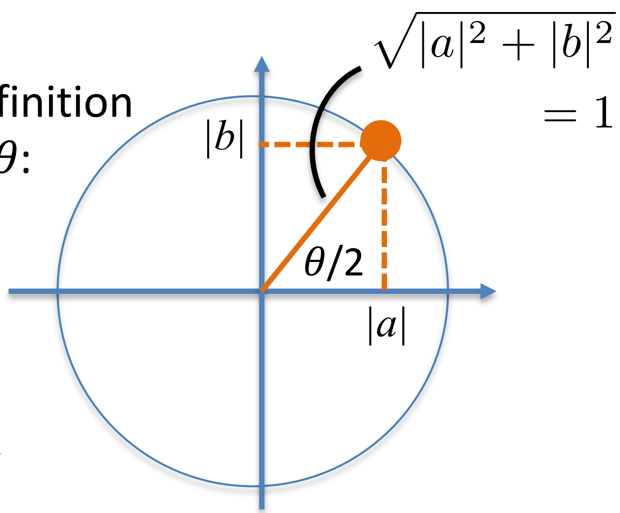
$$|\psi\rangle = e^{i\theta_a}\left(\cos\frac{\theta}{2}|0\rangle + \sin\frac{\theta}{2}e^{i(\theta_b-\theta_a)}|1\rangle\right) = e^{i\theta_{\text{global}}}\left(\cos\frac{\theta}{2}|0\rangle + \sin\frac{\theta}{2}e^{i\phi}|1\rangle\right)$$

The global phase is unimportant, and can never be measured in experiment.

The other variables θ and ϕ relate to the position of this state on the Bloch sphere:

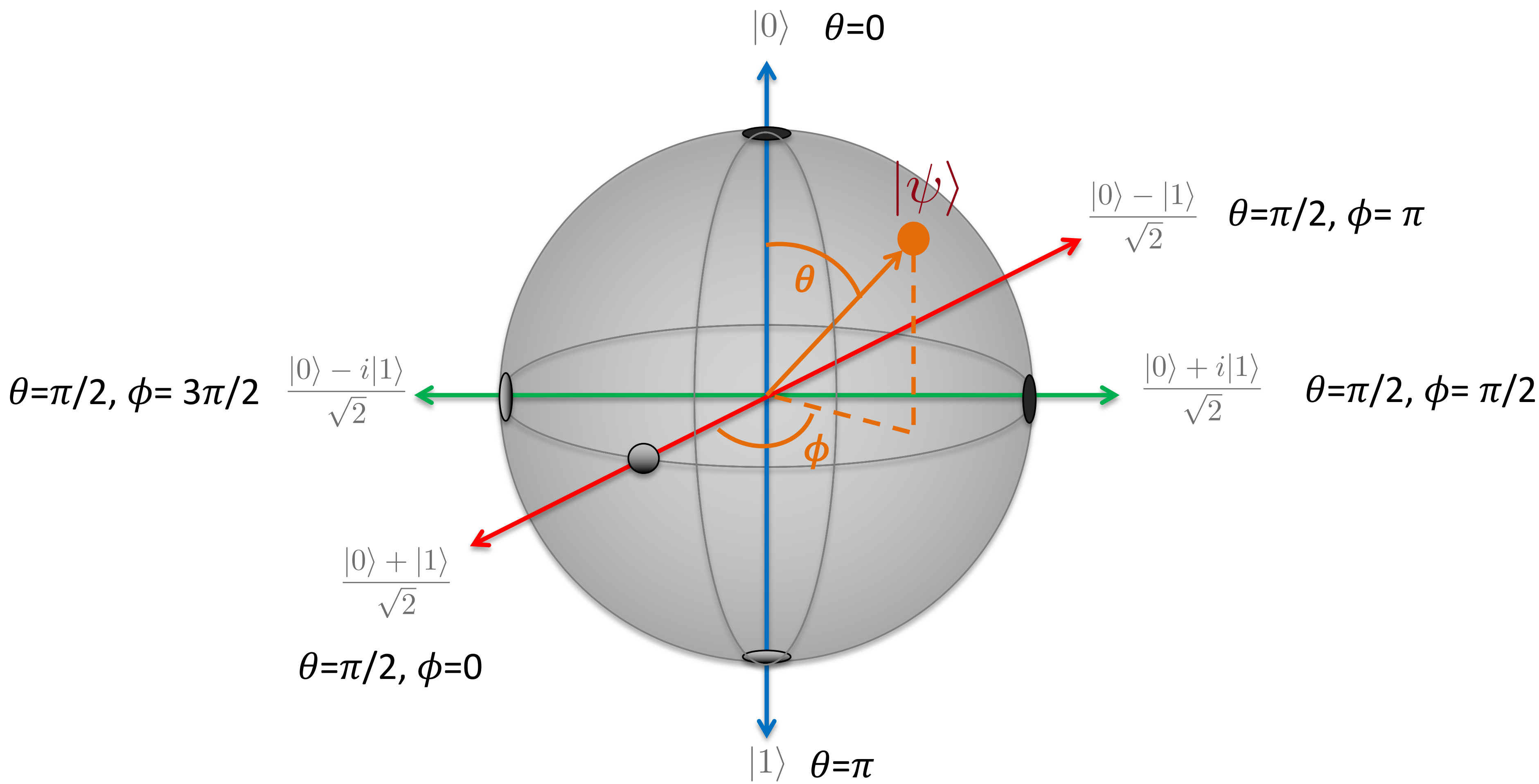
$$|\psi\rangle = a|0\rangle + b|1\rangle \to \cos\frac{\theta}{2}|0\rangle + \sin\frac{\theta}{2}e^{i\phi}|1\rangle$$

Definition of θ:



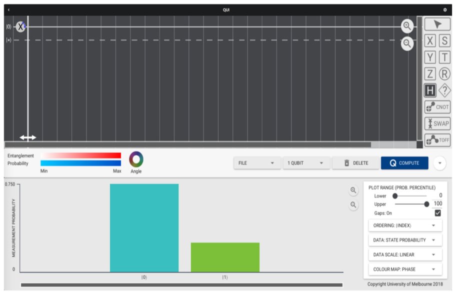
States on the Bloch sphere

$$|\psi\rangle = a\,|0\rangle + b\,|1\rangle \rightarrow \cos\frac{\theta}{2}\,|0\rangle + \sin\frac{\theta}{2}e^{i\phi}\,|1\rangle$$



Qubit states on the Bloch sphere – example (Prac Class 1)

Recall, in the prac class we produced the state: $|\psi\rangle = \frac{\sqrt{3}}{2}|0\rangle + \frac{-i}{2}|1\rangle$



$a_0 = \frac{\sqrt{3}}{2}$
i.e. $|a_0|$ = 0.866, $\theta_0 = 0$

$a_1 = \frac{-i}{2}$
i.e. $|a_1|$ = 0.500, θ_1 = - $\pi/2$

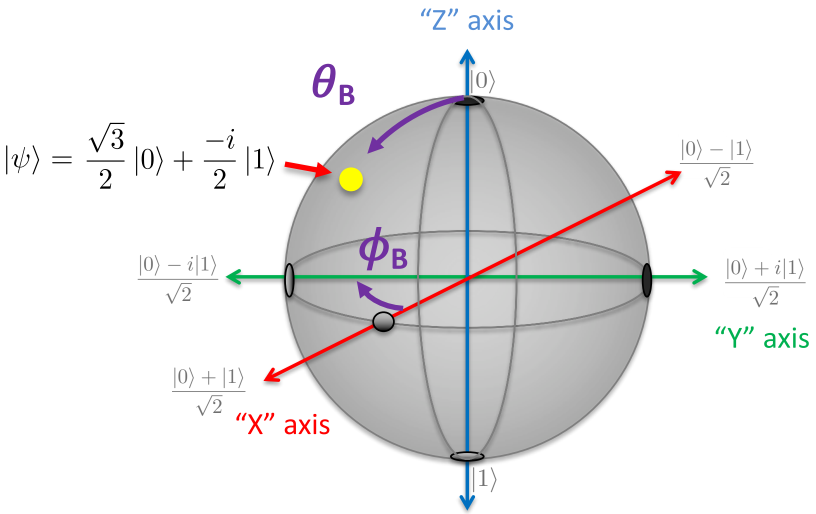
We want to express this on the Bloch Sphere…

$$\phi_B = \theta_1 - \theta_0 = -\frac{\pi}{2} = -90^o$$

$$|\psi\rangle = a_0|0\rangle + a_1|1\rangle \rightarrow \boxed{\cos\frac{\theta_B}{2}}|0\rangle + \boxed{\sin\frac{\theta_B}{2}e^{i\phi_B}}|1\rangle$$

$$\boxed{\cos\frac{\theta_B}{2} = \frac{\sqrt{3}}{2} \longrightarrow \theta_B = 2\cos^{-1}\frac{\sqrt{3}}{2} = 60^o}$$

$$\boxed{\begin{array}{c}\sin\frac{\theta_B}{2} = \sin 30 = \frac{1}{2} \longrightarrow e^{i\phi_B} = -i \\ \longrightarrow \phi_B = -90^o\end{array}}$$



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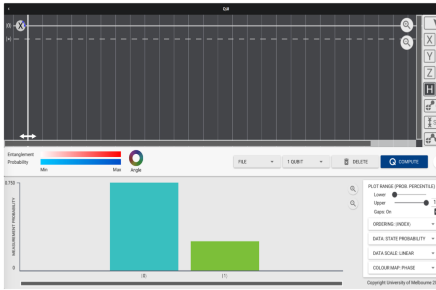

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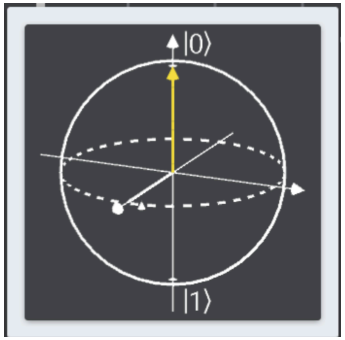
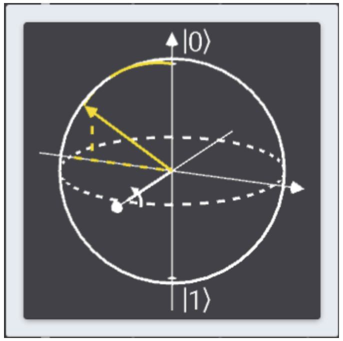
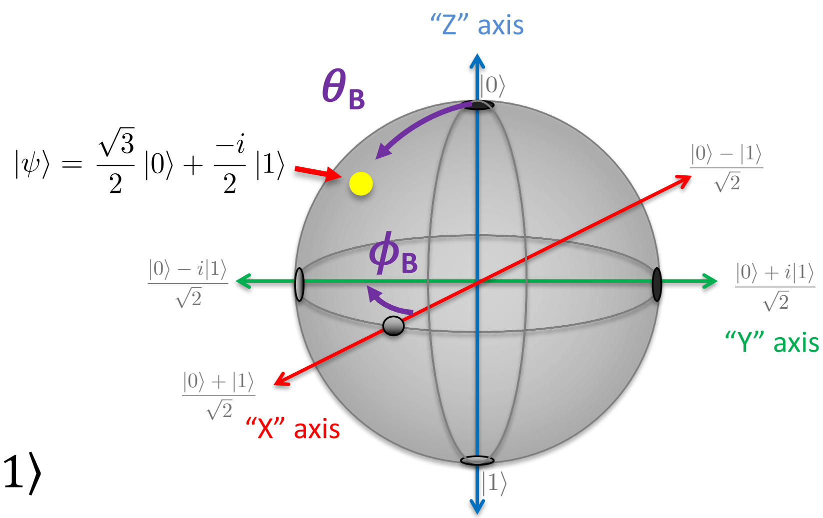
QUI Bloch spheres:



$|\psi\rangle=|0\rangle$



$|\psi'\rangle = \frac{\sqrt{3}}{2}|0\rangle + \frac{-i}{2}|1\rangle$



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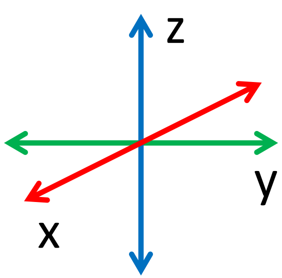
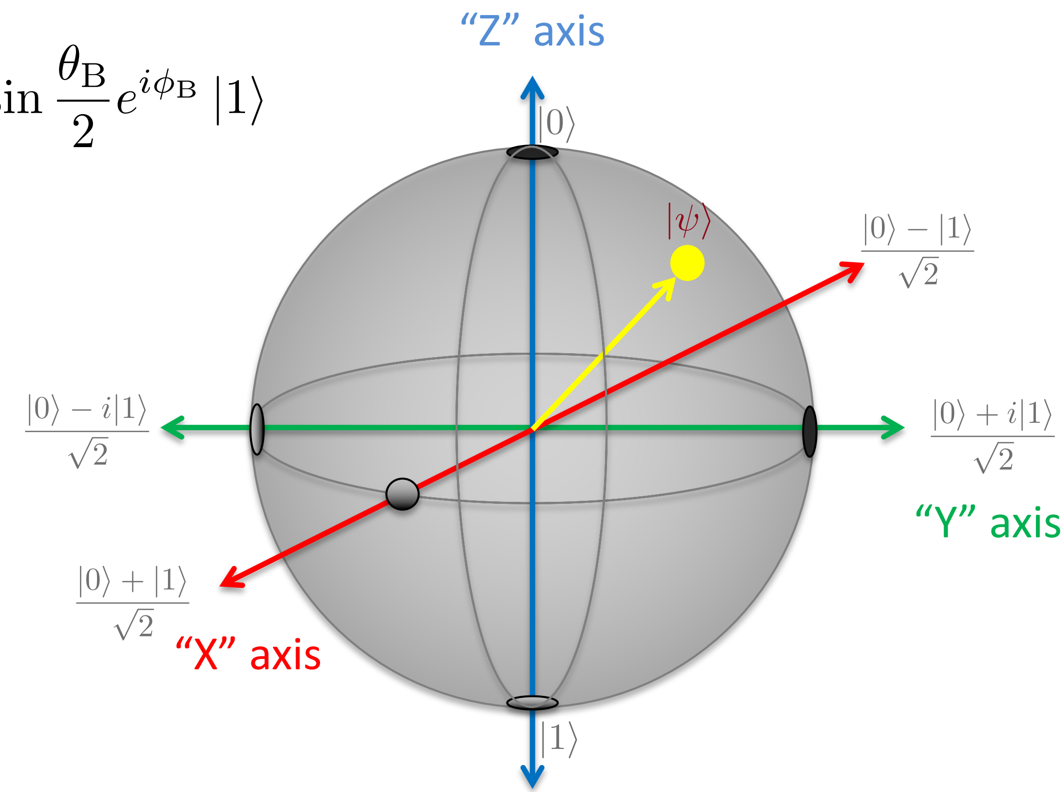
The Bloch Sphere – Cartesian Axes

Bloch sphere representation:

$$|\psi\rangle = a_0 |0\rangle + a_1 |1\rangle \rightarrow \cos\frac{\theta_{\mathrm{B}}}{2} |0\rangle + \sin\frac{\theta_{\mathrm{B}}}{2} e^{i\phi_{\mathrm{B}}} |1\rangle$$

C/f cartesian axes labelling:





These X, Y and Z axes are not our usual "real space" cartesian axes…these are a representation of the space in which the single-qubit states live…

…but they are used a lot in describing <u>quantum logic operations</u>!

Quantum operations on qubits

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Quantum operations in general

A *quantum operation* "U" takes a state $|\psi\rangle$ and changes it to a new state $|\psi'\rangle$

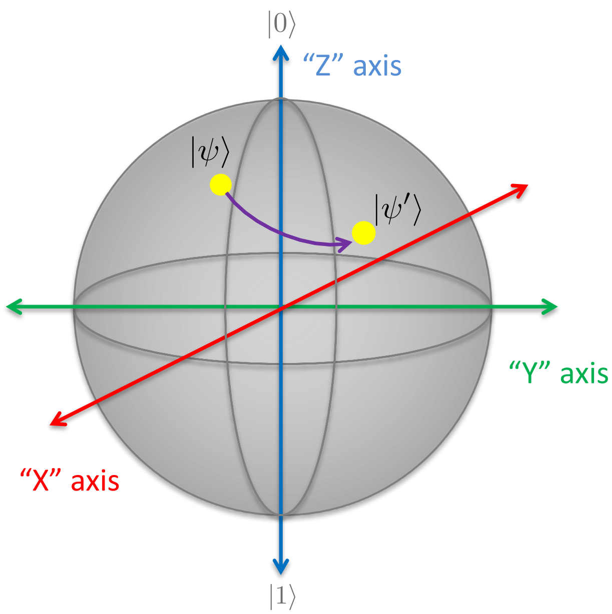
$$|\psi\rangle \xrightarrow{U} |\psi'\rangle$$

Mathematically, we write this as: $U|\psi\rangle = |\psi'\rangle$

i.e. U acting on the state $|\psi\rangle$ gives the new state $|\psi'\rangle$

$$|\psi'\rangle = U\,|\psi\rangle$$

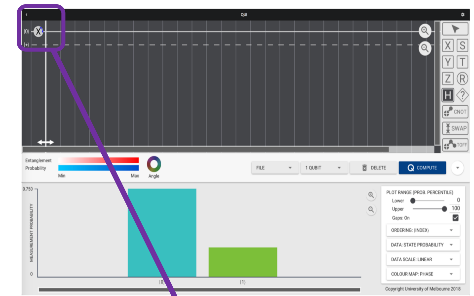
The Bloch sphere is a useful way to visualise how these operations work…(and how it's done in the QUI)



The operation "U" moves the state across the surface of the Bloch sphere.

Qubit operations/Bloch sphere – example, (Prac Class 1)

Recall, in the prac class we produced the state: $|\psi\rangle = \frac{\sqrt{3}}{2}|0\rangle + \frac{-i}{2}|1\rangle$



Operation U

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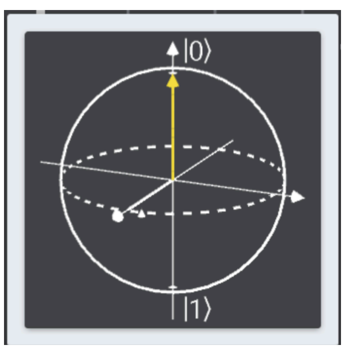
$a_1 = \frac{-i}{2}$
i.e. $|a_1|$ = 0.500, θ_1 = - $\pi/2$

We want to express this on the Bloch Sphere…

$\phi_B = \theta_1 - \theta_0 = -\frac{\pi}{2} = -90^o$

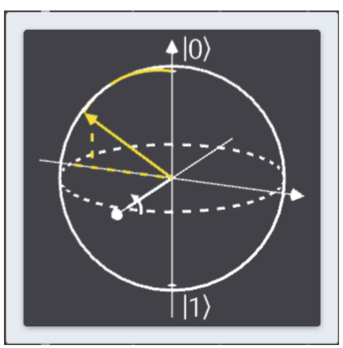
$|\psi\rangle = a_0|0\rangle + a_1|1\rangle \rightarrow \boxed{\cos\frac{\theta_B}{2}}|0\rangle + \boxed{\sin\frac{\theta_B}{2}e^{i\phi_B}}|1\rangle$

QUI animations:

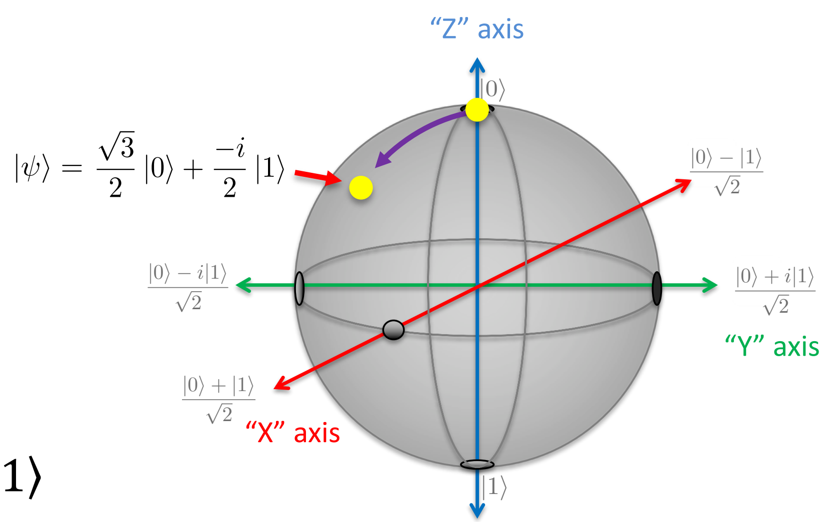


$|\psi\rangle$=$|0\rangle$

U



$|\psi'\rangle = \frac{\sqrt{3}}{2}|0\rangle + \frac{-i}{2}|1\rangle$

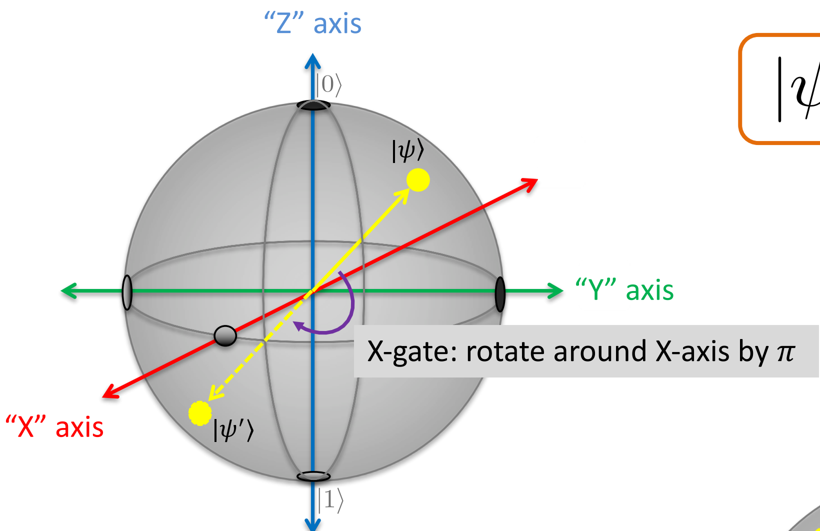


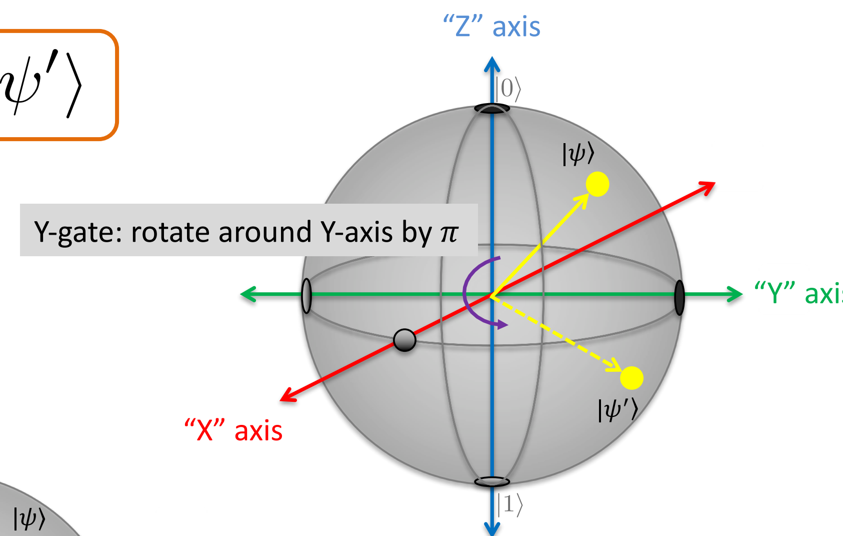
Qubit gates in matrix form and the Pauli matrices

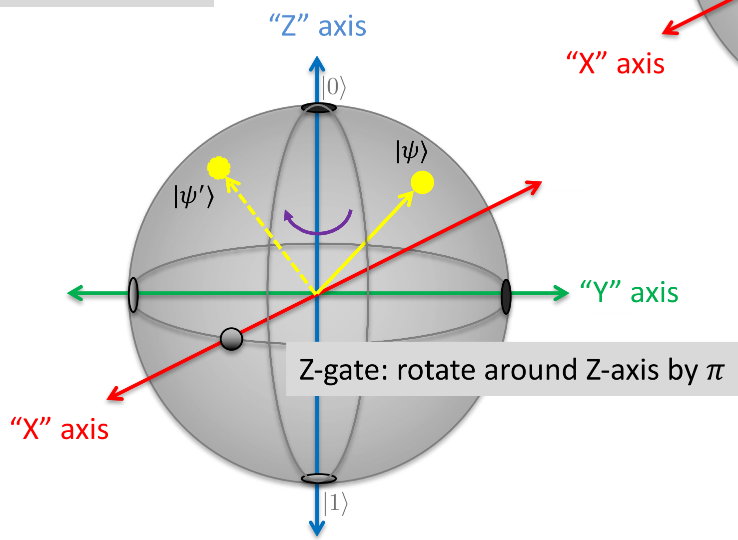
The "Cartesian" quantum operations: X, Y, Z

We can specify the state moving across the Bloch sphere in many ways, but the "Cartesian" operations are very simple – a rotation of π (180°) about any of X, Y, or Z axes:

$$\boxed{|\psi\rangle \rightarrow |\psi'\rangle}$$







NB. Perspectives not 100% accurate!

The cartesian rotations are usually referred to as the "Pauli" operators X, Y, Z

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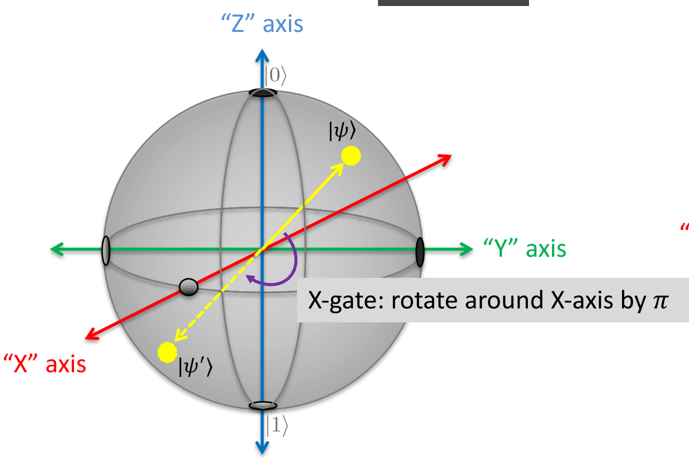
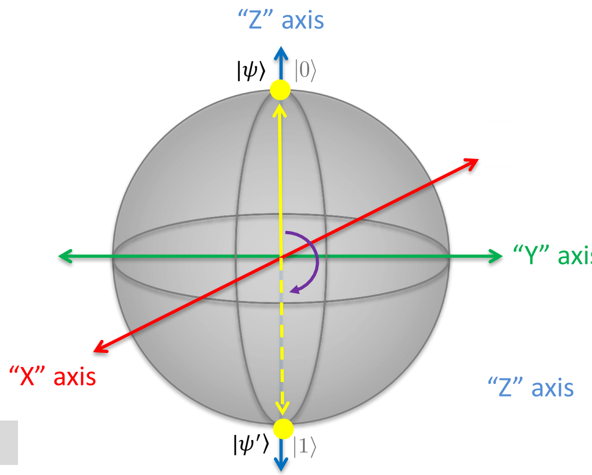
The X gate – acting on the basis states $|0\rangle$ and $|1\rangle$

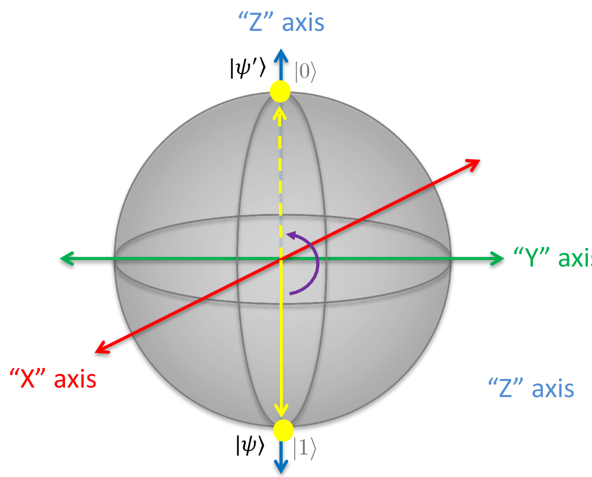
Circuit symbol:





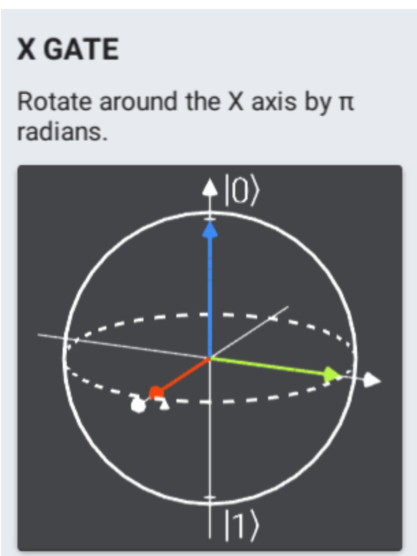
e.g. system starts in: $|\psi\rangle = |0\rangle$ (north pole)

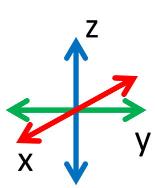
X gate: rotation about X-axis by π (180°) sends to $|\psi'\rangle = |1\rangle$ (south pole)



e.g. system starts in: $|\psi\rangle = |1\rangle$ (south pole)

X gate: rotation about X-axis by π (180°) sends to $|\psi'\rangle = |0\rangle$ (north pole)





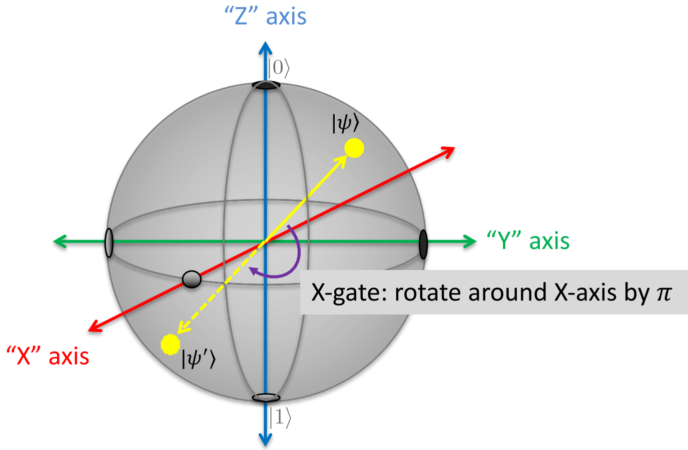
We write $X|\psi\rangle = |\psi'\rangle$

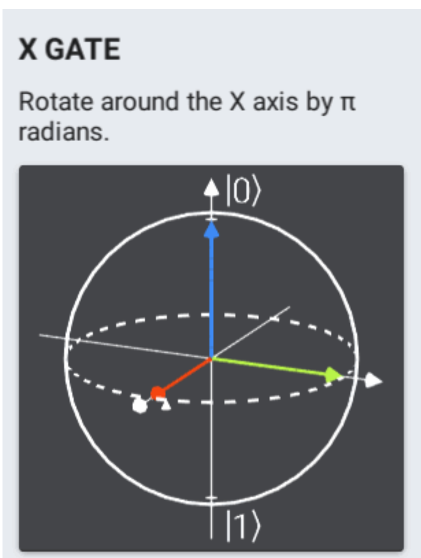
$$\begin{cases} X|0\rangle = |1\rangle \\ X|1\rangle = |0\rangle \end{cases}$$

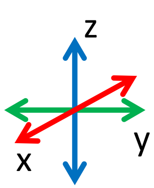
hence: "flip gate"

The X gate – acting on general states $a_0|0\rangle + a_1|1\rangle$

Circuit symbol: 







On the computational states the X-gate operation is a "bit flip":

$$\begin{cases} X|0\rangle = |1\rangle \\ X|1\rangle = |0\rangle \end{cases}$$

Quantum mechanics is inherently linear, so the X-gate acting on a linear superposition is:

$$X\left(a_0\left|0\right\rangle + a_1\left|1\right\rangle\right) = a_1\left|0\right\rangle + a_0\left|1\right\rangle$$

Or we can write in "ket" notation:

$$a_0\left|0\right\rangle + a_1\left|1\right\rangle \xrightarrow{X} a_1\left|0\right\rangle + a_0\left|1\right\rangle$$

The X gate in matrix form

Action of X-gate in "ket" form: $$a_0 |0\rangle + a_1 |1\rangle \xrightarrow{X} a_1 |0\rangle + a_0 |1\rangle$$

What is the X-gate in "matrix" form?

Recall "matrix" notation: $|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $|1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ $a_0 |0\rangle + a_1 |1\rangle = \begin{bmatrix} a_0 \\ a_1 \end{bmatrix}$ $|\psi\rangle = \begin{bmatrix} a_0 \\ a_1 \end{bmatrix}$

Operations in matrix representation:

$$|\psi'\rangle = U \; |\psi\rangle$$

$$\begin{pmatrix} a_0' \\ a_1' \end{pmatrix} = \begin{pmatrix} \text{2 x 2} \\ \text{matrix} \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \end{pmatrix}$$

Action of X-gate in matrix form:

$$a_0|0\rangle + a_1|1\rangle = \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} \longrightarrow a_1|0\rangle + a_0|1\rangle = \begin{bmatrix} a_1 \\ a_0 \end{bmatrix}$$

i.e. $\begin{bmatrix} a_0 \\ a_1 \end{bmatrix} \xrightarrow{X} \begin{bmatrix} a_1 \\ a_0 \end{bmatrix}$

$$\boxed{X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}}$$

In matrix notation, in general:

$$X\begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} a_1 \\ a_0 \end{bmatrix}$$

The X gate in matrix form – the Pauli matrices

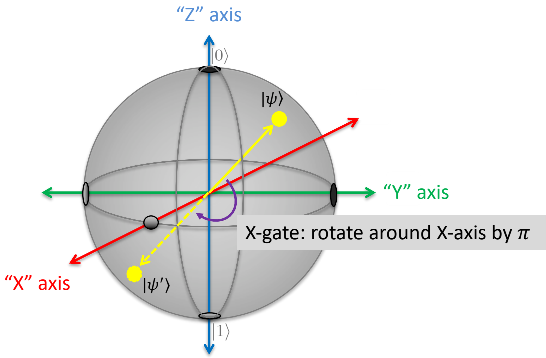
$$|\psi'\rangle = X \; |\psi\rangle$$

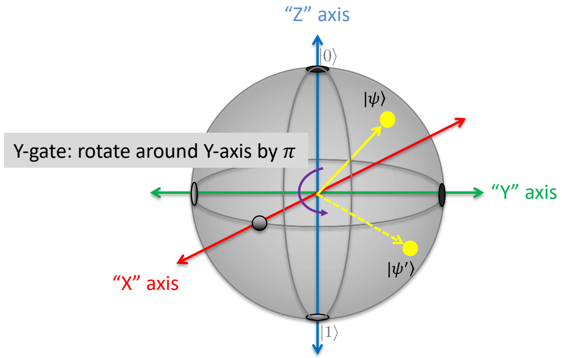
$$\begin{pmatrix} a_0' \\ a_1' \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \end{pmatrix}$$

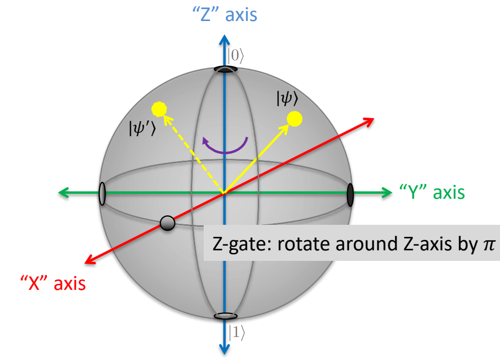
$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

This is the so-called Pauli X matrix…one of three Pauli matrices representing X, Y and Z operations…

All cartesian axes – the Pauli matrices for X, Y and Z:







$$|\psi'\rangle = X \; |\psi\rangle$$

$$\begin{pmatrix} a_0' \\ a_1' \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \end{pmatrix}$$

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$|\psi'\rangle = Y \; |\psi\rangle$$

$$\begin{pmatrix} a_0' \\ a_1' \end{pmatrix} = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \end{pmatrix}$$

$$Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

$$|\psi'\rangle = Z \; |\psi\rangle$$

$$\begin{pmatrix} a_0' \\ a_1' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \end{pmatrix}$$

$$Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

Week 2

Lecture 3

3.1 The Bloch Sphere representation for qubits

3.2 Quantum operations on qubits

3.3 Qubit gates in matrix form and the Pauli matrices

Lecture 4

4.1 The Pauli gates X, Y and Z and the QUI

4.2 Qubit operations around non-cartesian axes – H and R gates

4.3 Programming sequences over the qubit logic gate library

4.4 Note on the context and use of angles

Practice class 2

Bloch sphere and single qubit logic operations on the QUI