

Week by week

- (1) Introduction to quantum computing
- (2) Single qubit representation and operations
- (3) Two and more qubits
- (4) Simple quantum algorithms
- (5) Quantum search (Grover's algorithm)
- (6) Quantum factorization (Shor's algorithm)
- (7) Quantum supremacy and noise
- (8) Programming real quantum computers (IBM Q)
- (9) Quantum error correction (QEC)
- (10) QUBO problems and Adiabatic Quantum Computation (AQC)
- (11) Variational/hybrid quantum algorithms (QAOA and VQE)
- (12) Solving linear equations, QC computing hardware

Week 2



Lecture 3

- 3.1 The Bloch Sphere representation for qubits
- 3.2 Quantum operations on qubits
- 3.3 Qubit gates in matrix form and the Pauli matrices

Lecture 4

- 4.1 The Pauli gates X, Y and Z and the QUI
- 4.2 Qubit operations around non-cartesian axes – H and R gates
- 4.3 Matrix exponential and arbitrary rotations

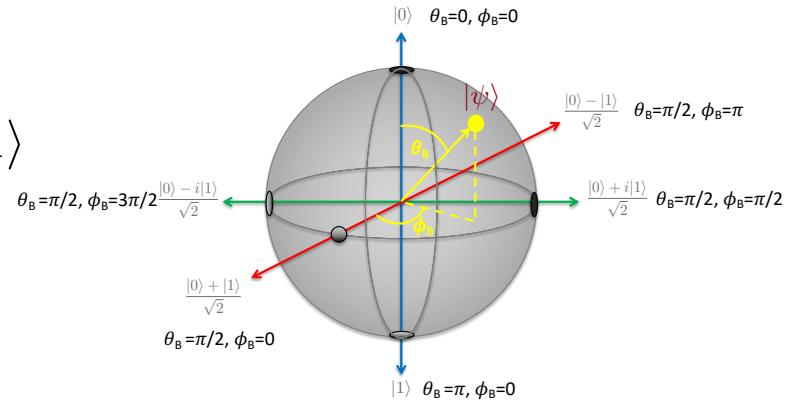
Practice class 2

Bloch sphere and single qubit logic operations on the QUI

Lecture 3 recap

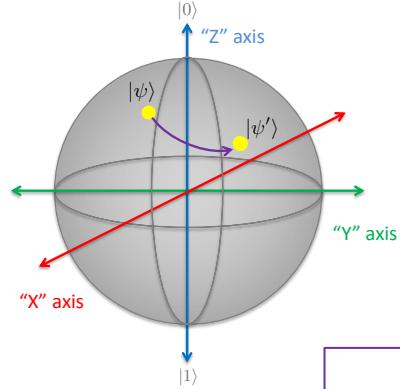
Qubits on the Bloch sphere:

$$|\psi\rangle = a_0 |0\rangle + a_1 |1\rangle \rightarrow \cos \frac{\theta_B}{2} |0\rangle + \sin \frac{\theta_B}{2} e^{i\phi_B} |1\rangle$$



Operations on qubit states:

$$|\psi\rangle \xrightarrow{U} |\psi'\rangle$$



$$|\psi'\rangle = U |\psi\rangle$$

Ket notation:
e.g. X-gate

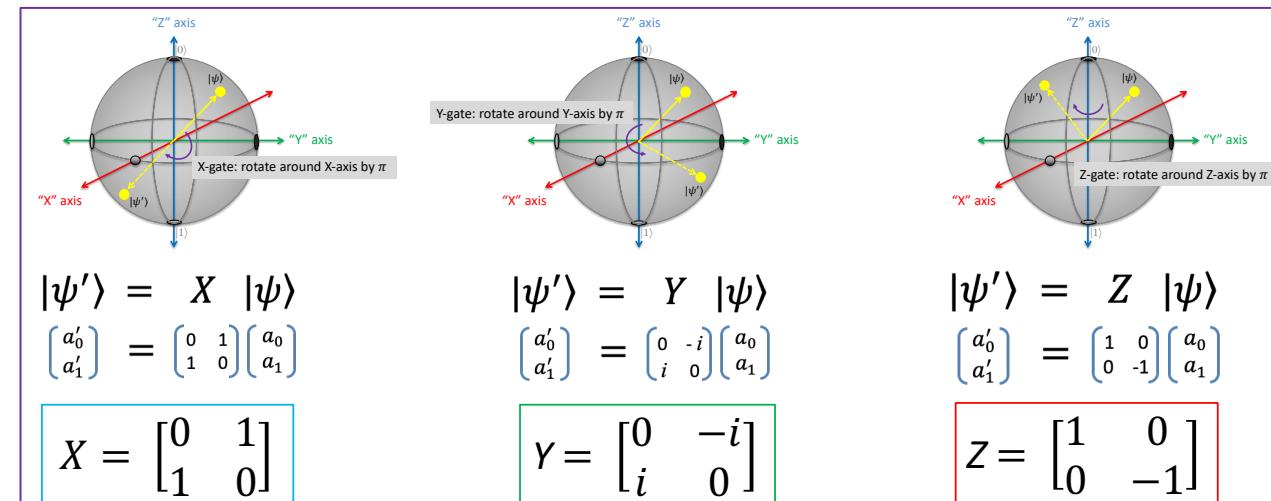
$$a_0 |0\rangle + a_1 |1\rangle \xrightarrow{X} a_1 |0\rangle + a_0 |1\rangle$$

Pauli Matrices

Operations: matrix notation

$$|\psi'\rangle = U |\psi\rangle$$

$$\begin{bmatrix} a'_0 \\ a'_1 \end{bmatrix} = \begin{bmatrix} 2 \times 2 \text{ matrix} \\ \text{matrix} \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix}$$



Recap: Linear Algebra and Dirac notation

$$|\psi\rangle = a_0 |0\rangle + a_1 |1\rangle \rightarrow |\psi\rangle = |a_0|e^{i\theta_0} |0\rangle + |a_1|e^{i\theta_1} |1\rangle$$

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$|1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Computational basis states

$$a_0 |0\rangle + a_1 |1\rangle = \begin{bmatrix} a_0 \\ a_1 \end{bmatrix}$$

$a_0, a_1 \in \mathbb{C}$

General qubit state
 a_0 and a_1 are “amplitudes”

$$a_0 |0\rangle + a_1 |1\rangle = \begin{bmatrix} a_0 \\ a_1 \end{bmatrix}$$

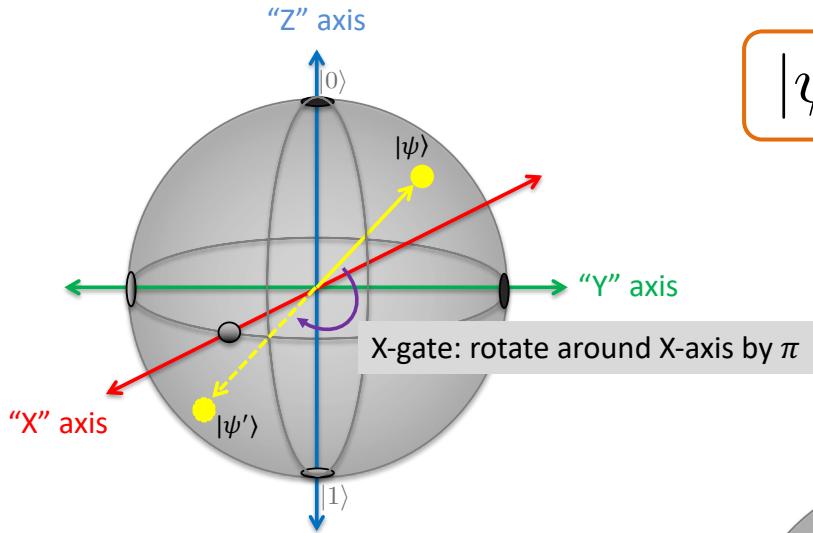
$$\text{Prob[measure “0”]} = |a_0|^2$$

$$\text{Prob[measure “1”]} = |a_1|^2$$

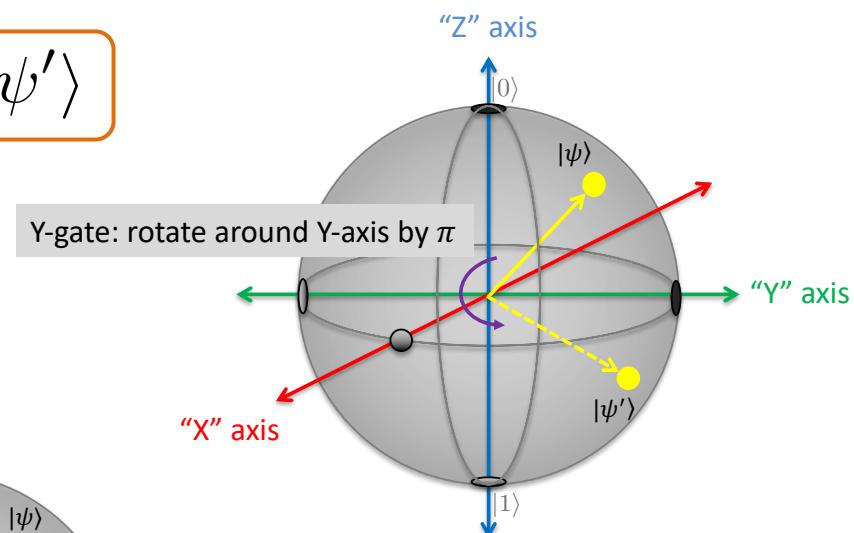
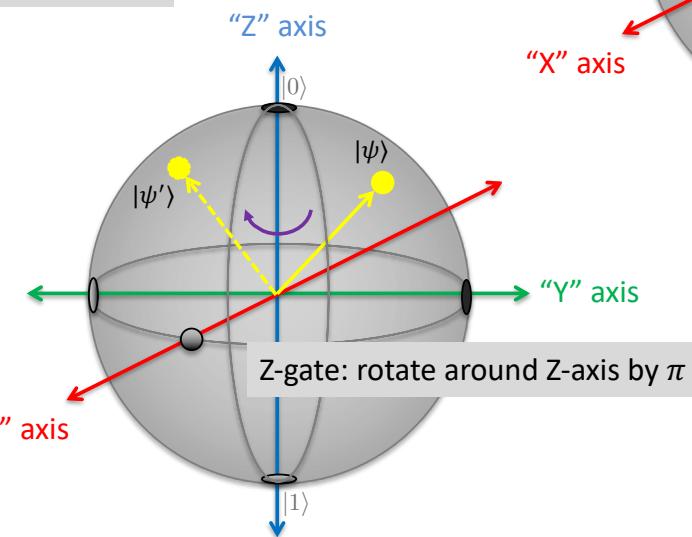
The Pauli gates X, Y and Z and the QUI

Recap: the “Cartesian” quantum operations: X, Y, Z

We can specify the state moving across the Bloch sphere in many ways, but the “Cartesian” operations are very simple – a rotation of π (180°) about any of X, Y, or Z axes:



$$|\psi\rangle \rightarrow |\psi'\rangle$$



N.B. Perspectives not 100% accurate!

The Cartesian rotations are usually referred to as the “Pauli” operators X, Y, Z

Recap: the X gate in matrix form

Action of X-gate in “ket” form: $a_0 |0\rangle + a_1 |1\rangle \xrightarrow{X} a_1 |0\rangle + a_0 |1\rangle$

What is the X-gate in “matrix” form?

Recall “matrix” notation: $|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $|1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ $a_0 |0\rangle + a_1 |1\rangle = \begin{bmatrix} a_0 \\ a_1 \end{bmatrix}$ $|\psi\rangle = \begin{bmatrix} a_0 \\ a_1 \end{bmatrix}$

$$|\psi'\rangle = U |\psi\rangle$$

$$\begin{bmatrix} a'_0 \\ a'_1 \end{bmatrix} = \begin{bmatrix} 2 \times 2 \\ \text{matrix} \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix}$$

Operations in matrix representation:

Action of X-gate in matrix form:

$$a_0 |0\rangle + a_1 |1\rangle = \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} \rightarrow a_1 |0\rangle + a_0 |1\rangle = \begin{bmatrix} a_1 \\ a_0 \end{bmatrix}$$

In matrix notation, in general:

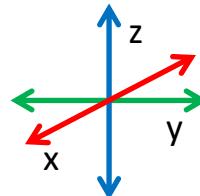
i.e. $\begin{bmatrix} a_0 \\ a_1 \end{bmatrix} \xrightarrow{X} \begin{bmatrix} a_1 \\ a_0 \end{bmatrix}$

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$X \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} a_1 \\ a_0 \end{bmatrix}$$

X Gate (the X operator): π around X-axis

Circuit symbol:



Matrix representation: $X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

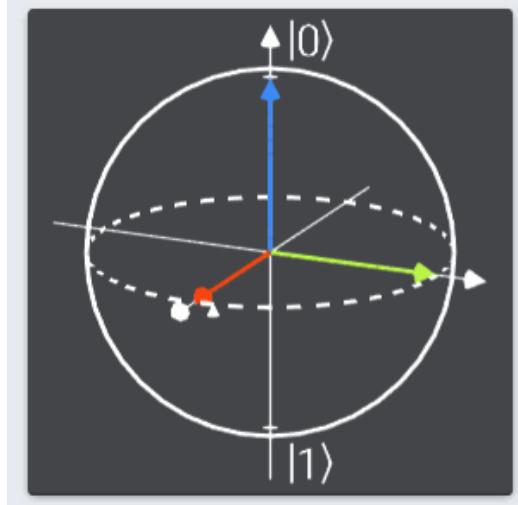
$$X(a_0|0\rangle + a_1|1\rangle) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} a_1 \\ a_0 \end{bmatrix}$$

i.e. $X(a_0|0\rangle + a_1|1\rangle) = a_1|0\rangle + a_0|1\rangle$

Action on ket states: $a_0|0\rangle + a_1|1\rangle \rightarrow a_1|0\rangle + a_0|1\rangle$

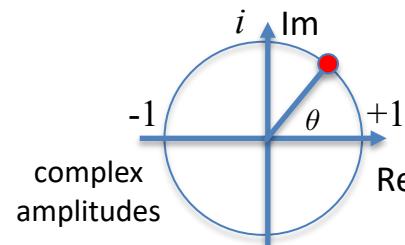
X GATE

Rotate around the X axis by π radians.

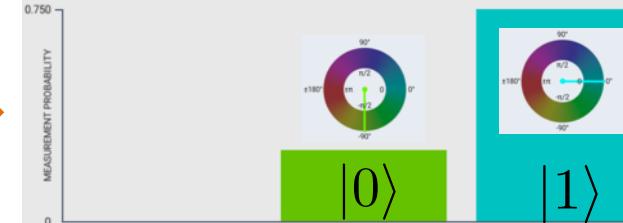
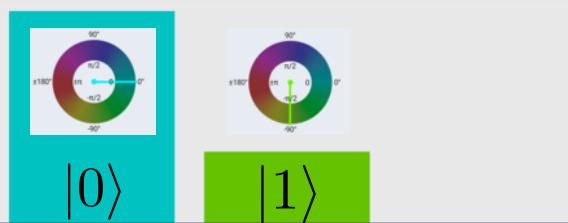


QUI example:

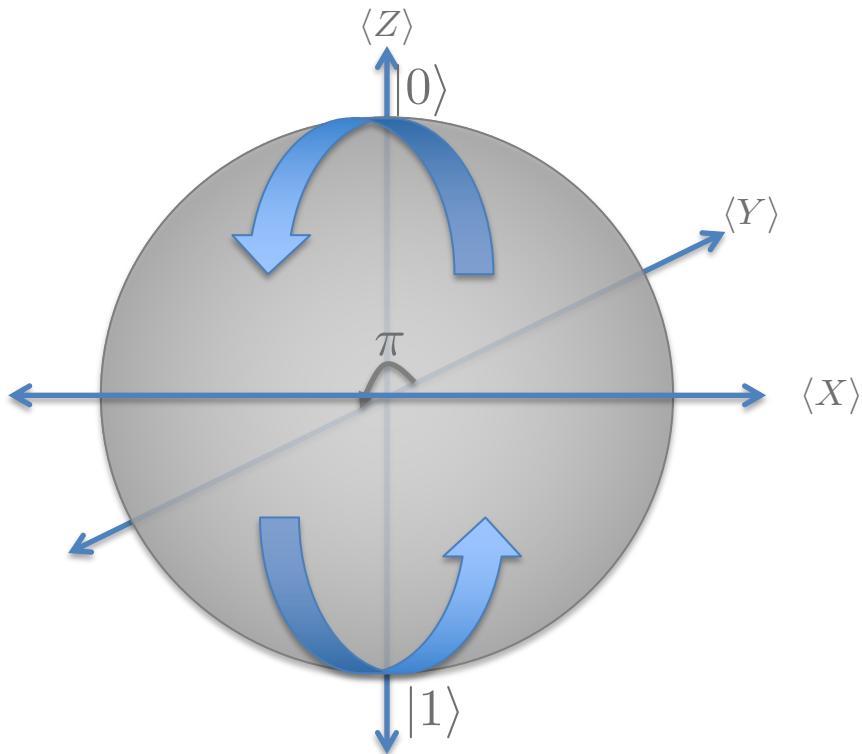
$$\frac{\sqrt{3}}{2}|0\rangle + \frac{-i}{2}|1\rangle$$



$$\frac{-i}{2}|0\rangle + \frac{\sqrt{3}}{2}|1\rangle$$

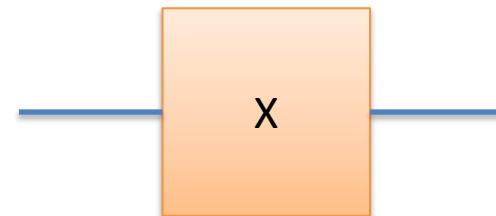


X Gate



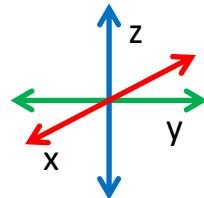
$$\alpha|0\rangle + \beta|1\rangle \rightarrow \alpha|1\rangle + \beta|0\rangle$$

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$



Y Gate (the Y operator): π around Y-axis

Circuit symbol:



Matrix representation: $Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$

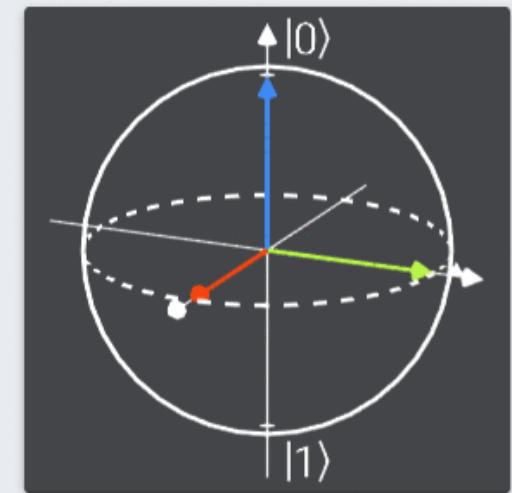
$$Y(a_0|0\rangle + a_1|1\rangle) = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} -ia_1 \\ ia_0 \end{bmatrix}$$

i.e. $Y(a_0|0\rangle + a_1|1\rangle) = -ia_1|0\rangle + ia_0|1\rangle$

Action on ket states: $a_0|0\rangle + a_1|1\rangle \rightarrow -ia_1|0\rangle + ia_0|1\rangle$

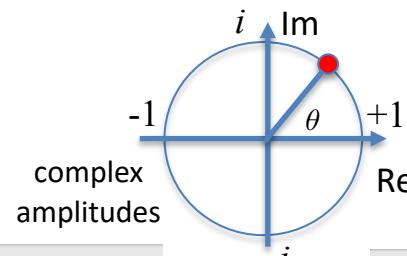
Y GATE

Rotate around the Y axis by π radians.

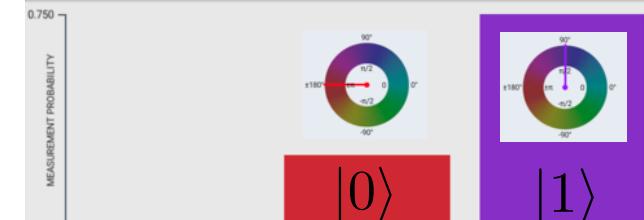
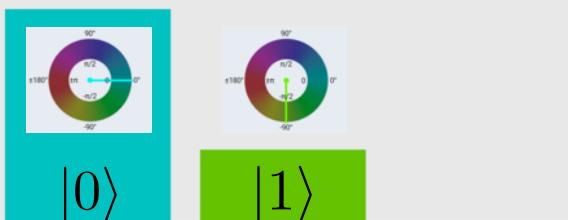


QUI example:

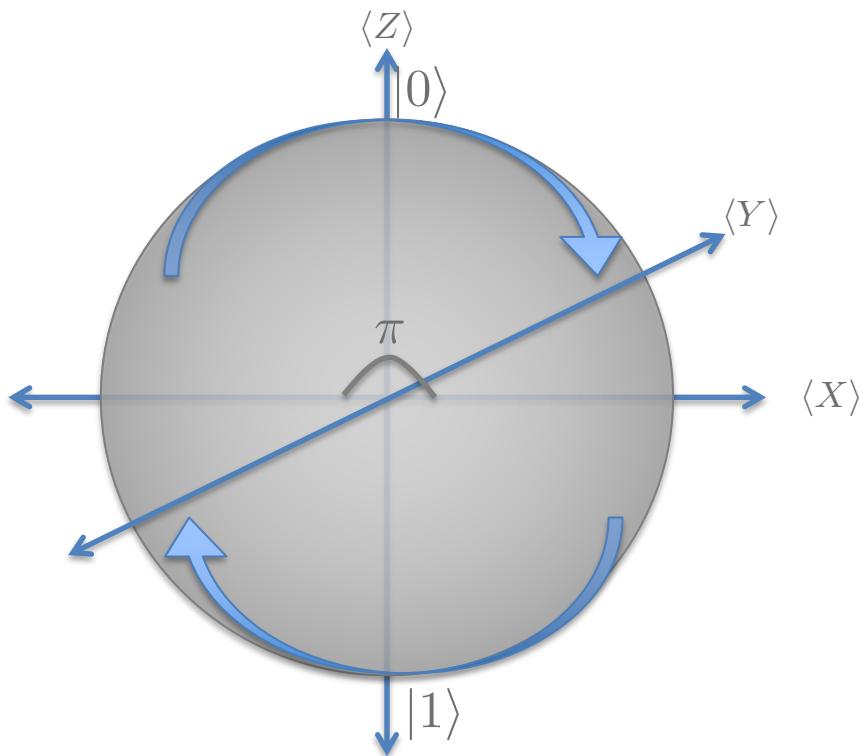
$$\frac{\sqrt{3}}{2}|0\rangle + \frac{-i}{2}|1\rangle$$



$$\frac{-1}{2}|0\rangle + \frac{i\sqrt{3}}{2}|1\rangle$$

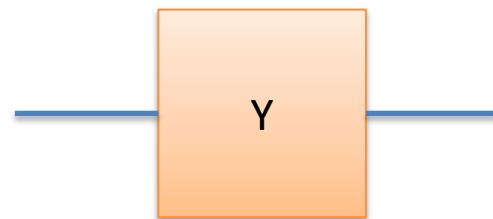


Y Gate



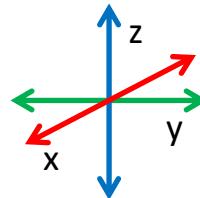
$$\alpha|0\rangle + \beta|1\rangle \rightarrow i\alpha|1\rangle - i\beta|0\rangle$$

$$Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$



Z Gate (the Z operator): π around Z-axis

Circuit symbol:

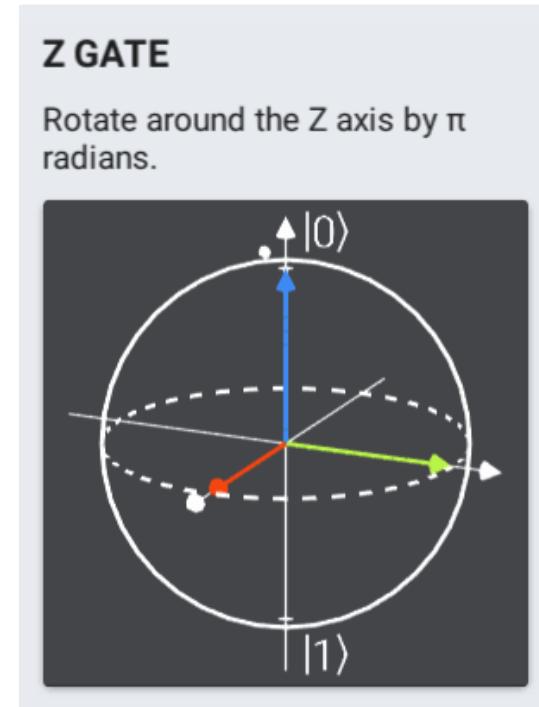


Matrix representation: $Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

$$Z(a_0|0\rangle + a_1|1\rangle) = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} a_0 \\ -a_1 \end{bmatrix}$$

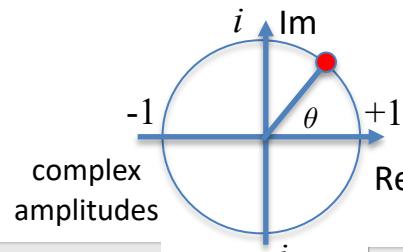
i.e. $Z(a_0|0\rangle + a_1|1\rangle) = a_0|0\rangle - a_1|1\rangle$

Action on ket states: $a_0|0\rangle + a_1|1\rangle \rightarrow a_0|0\rangle - a_1|1\rangle$

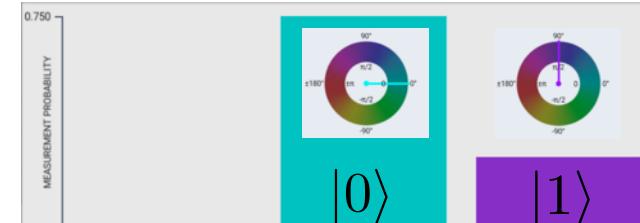
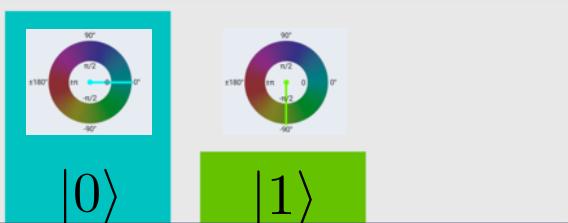


QUI example:

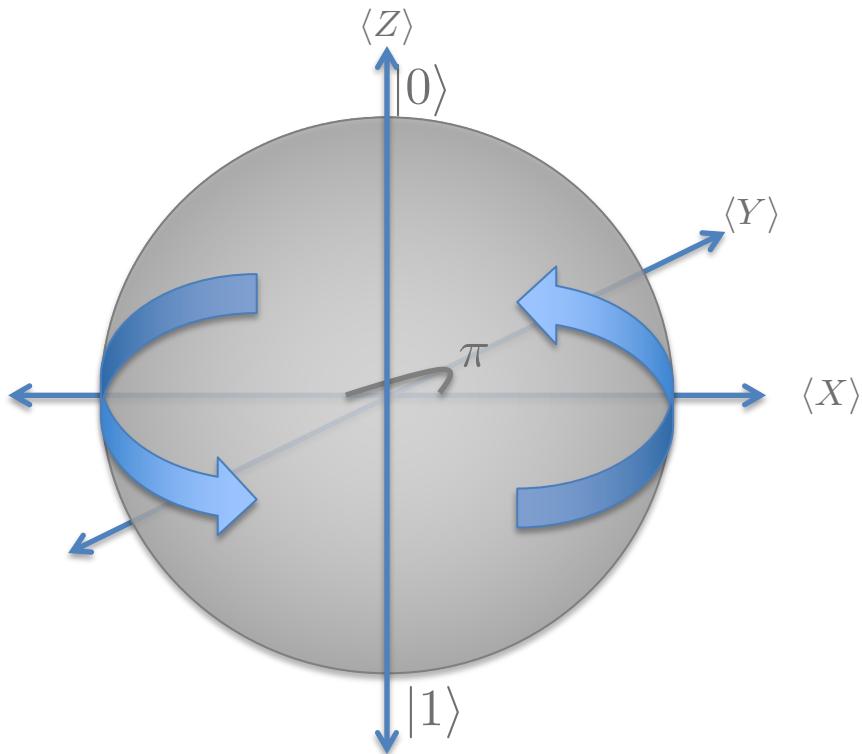
$$\frac{\sqrt{3}}{2}|0\rangle + \frac{-i}{2}|1\rangle$$



$$\frac{\sqrt{3}}{2}|0\rangle + \frac{i}{2}|1\rangle$$

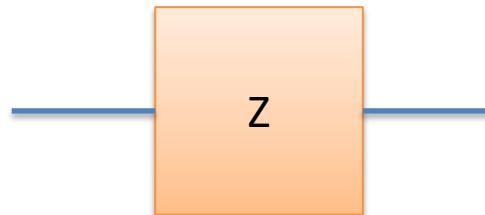


Z Gate



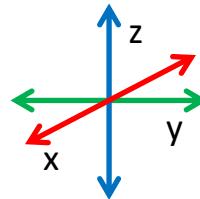
$$\alpha|0\rangle + \beta|1\rangle \rightarrow \alpha|0\rangle - \beta|1\rangle$$

$$Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$



S Gate (the S operator): Z-axis, $\pi/2$ rotation

Circuit symbol:



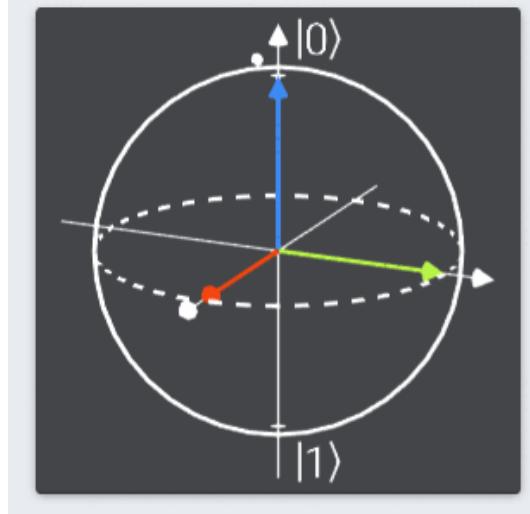
Matrix representation: $S = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$

$$S(a_0|0\rangle + a_1|1\rangle) = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} a_0 \\ ia_1 \end{bmatrix}$$

i.e. $S(a_0|0\rangle + a_1|1\rangle) = a_0|0\rangle + ia_1|1\rangle$

S GATE

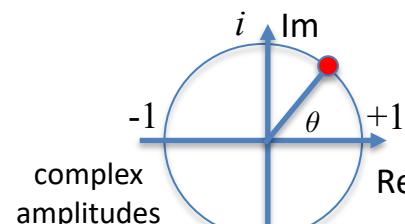
Rotate around the Z axis by $\pi / 2$ radians.



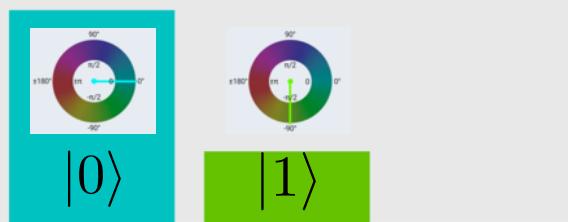
Action on ket states: $a_0|0\rangle + a_1|1\rangle \rightarrow a_0|0\rangle + ia_1|1\rangle$

QUI example:

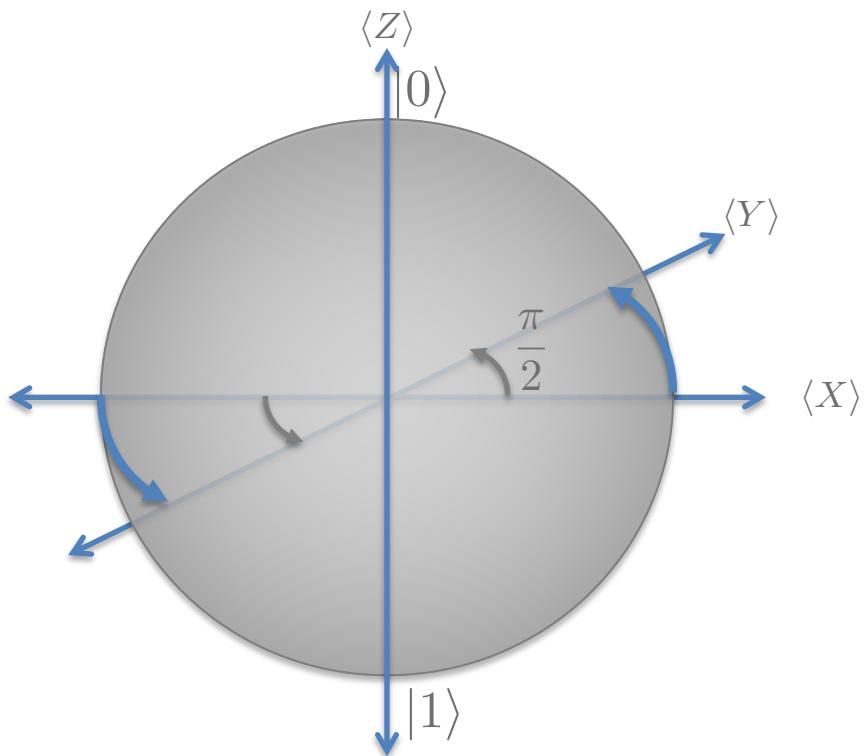
$$\frac{\sqrt{3}}{2}|0\rangle + \frac{-i}{2}|1\rangle$$



$$\frac{\sqrt{3}}{2}|0\rangle + \frac{1}{2}|1\rangle$$

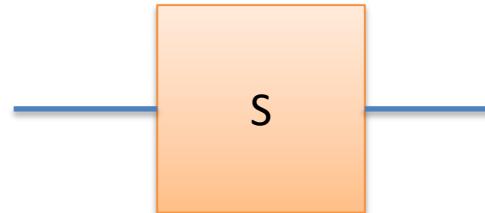


Non- π rotations: S Gate



$$\alpha|0\rangle + \beta|1\rangle \rightarrow \alpha|0\rangle + i\beta|1\rangle$$

$$S = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$$



T Gate (the T operator): Z-axis, $\pi/4$ rotation

Circuit symbol:



Matrix representation:

$$T = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix}$$

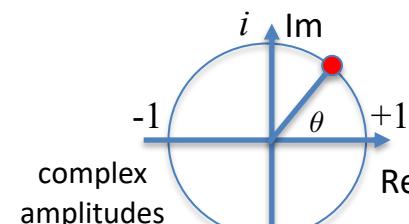
$$T(a_0|0\rangle + a_1|1\rangle) = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} a_0 \\ e^{i\pi/4}a_1 \end{bmatrix}$$

i.e. $T(a_0|0\rangle + a_1|1\rangle) = a_0|0\rangle + e^{i\pi/4}a_1|1\rangle$

Action on ket states: $a_0|0\rangle + a_1|1\rangle \rightarrow a_0|0\rangle + e^{i\pi/4}a_1|1\rangle$

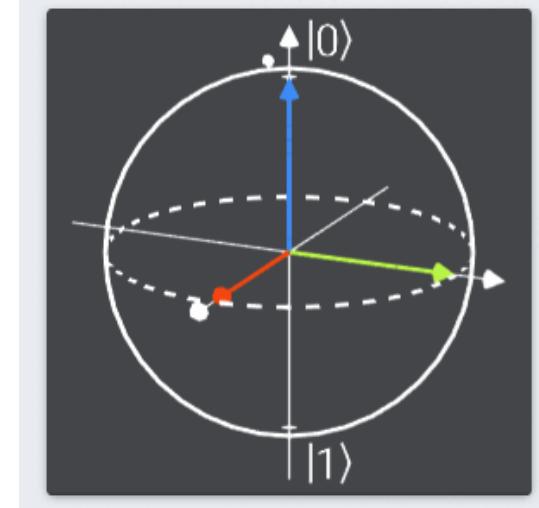
QUI example:

$$\frac{\sqrt{3}}{2}|0\rangle + \frac{-i}{2}|1\rangle$$

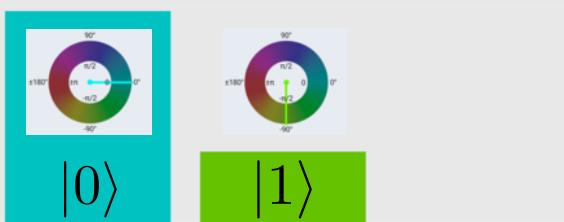


T GATE

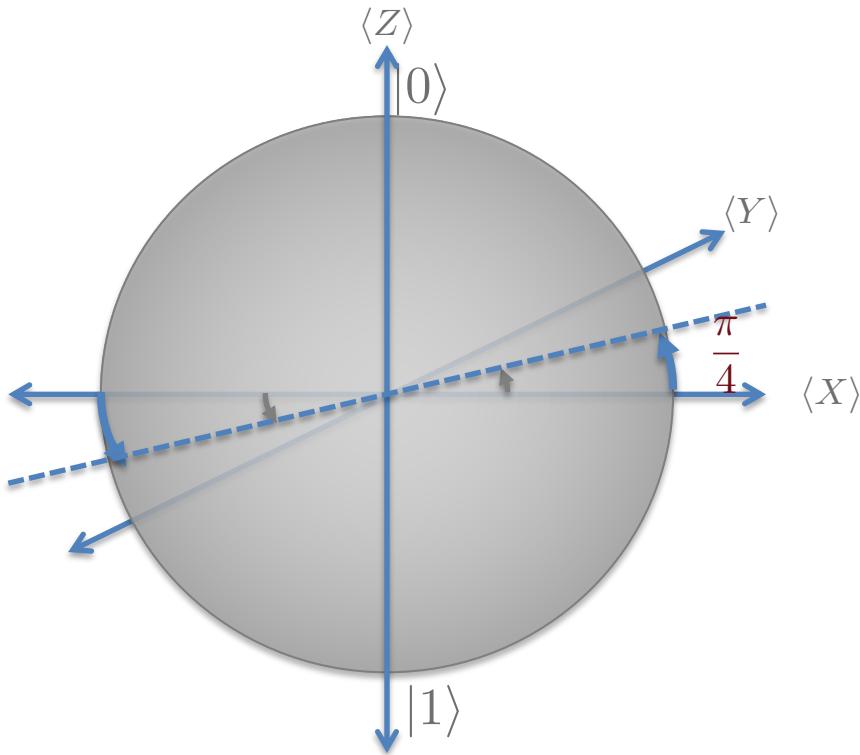
Rotate around the Z axis by $\pi / 4$ radians.



$$\frac{\sqrt{3}}{2}|0\rangle + \frac{-ie^{i\pi/4}}{2}|1\rangle$$

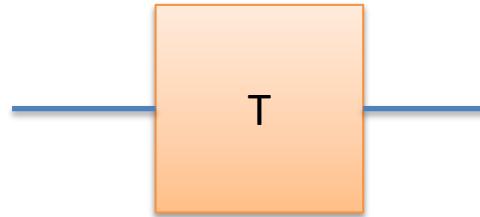


T Gate



$$\alpha|0\rangle + \beta|1\rangle \rightarrow \alpha|0\rangle + e^{i\frac{\pi}{4}}\beta|1\rangle$$

$$T = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix}$$



The first “non-Clifford” gate we’ve encountered – more difficult to perform under some error correction codes.

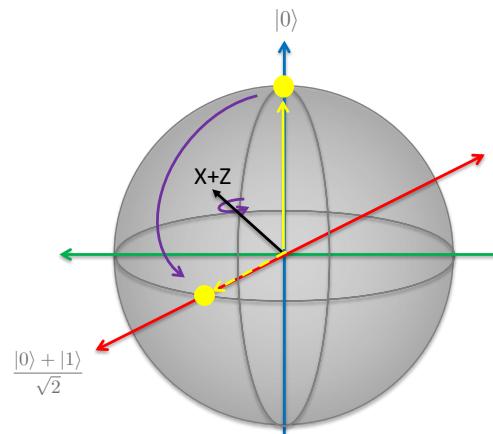
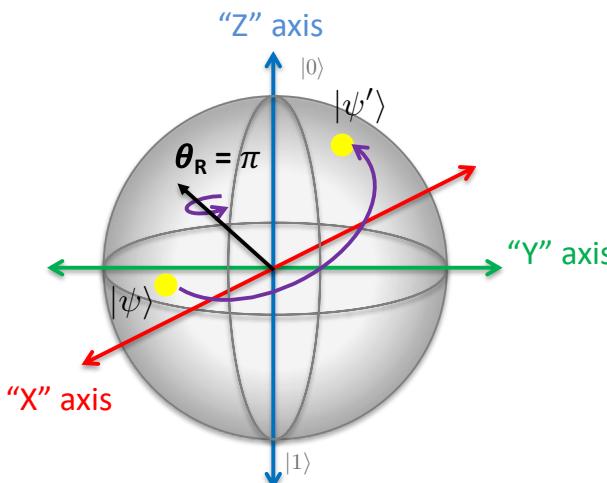
The Hadamard gate

The Hadamard gate H

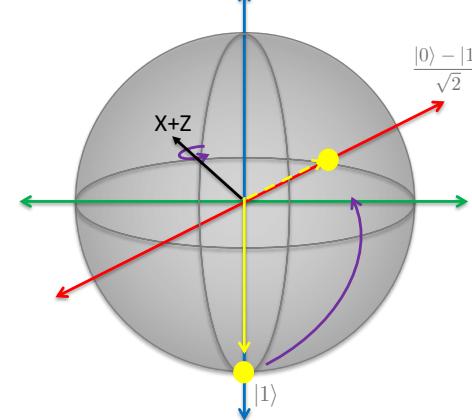
The Hadamard gate H is one of the most important – it generates superposition states.

Unlike the X, Y and Z gates which rotate about one of the cartesian axes, the H-gate rotates about the **X+Z** axis (or unit vector) by an angle π .

Rotation about “X+Z” axis,
given by $n = (1,0,1)/\sqrt{2}$



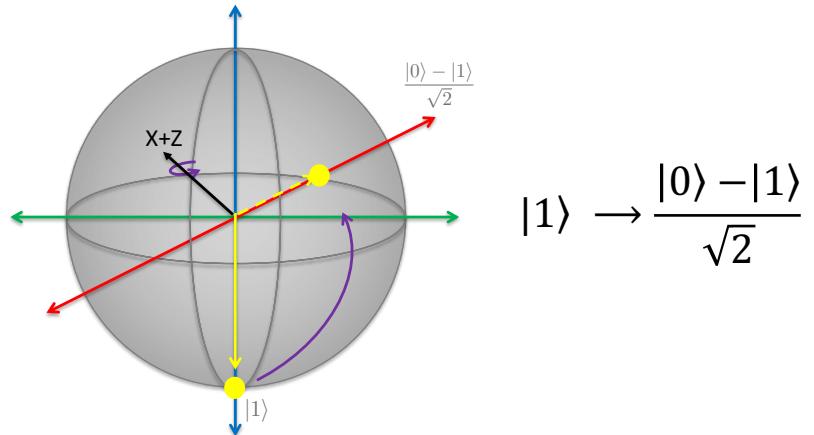
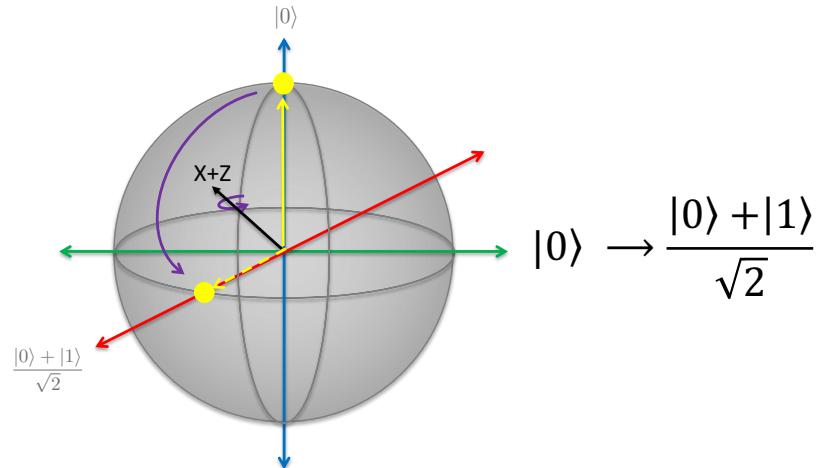
$$|0\rangle \rightarrow \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$



$$|1\rangle \rightarrow \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

The Hadamard gate H

H-gate on the computational states:



H-gate on a general superposition state:

$$a_0|0\rangle + a_1|1\rangle = \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} \rightarrow a_0 \frac{|0\rangle + |1\rangle}{\sqrt{2}} + a_1 \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

$$= \frac{a_0 + a_1}{\sqrt{2}}|0\rangle + \frac{a_0 - a_1}{\sqrt{2}}|1\rangle \rightarrow \frac{1}{\sqrt{2}} \begin{bmatrix} a_0 + a_1 \\ a_0 - a_1 \end{bmatrix}$$

$$|\psi'\rangle = H |\psi\rangle$$

$$\begin{bmatrix} a'_0 \\ a'_1 \end{bmatrix} = \begin{bmatrix} 2 \times 2 \text{ matrix} \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix}$$

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

(Ex. check it)

H Gate (the H operator): π around X+Z-axis

Circuit symbol:



Matrix representation: $H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

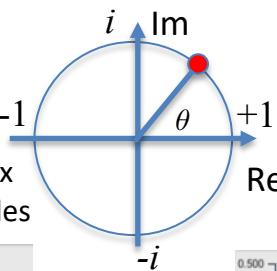
Action on ket states: $|0\rangle \rightarrow \frac{|0\rangle + |1\rangle}{\sqrt{2}}$ $|1\rangle \rightarrow \frac{|0\rangle - |1\rangle}{\sqrt{2}}$

$$a_0 |0\rangle + a_1 |1\rangle \rightarrow \frac{a_0 + a_1}{\sqrt{2}} |0\rangle + \frac{a_0 - a_1}{\sqrt{2}} |1\rangle$$

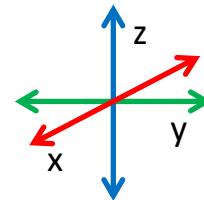
QUI example:

$$\frac{\sqrt{3}}{2} |0\rangle + \frac{-i}{2} |1\rangle$$

complex amplitudes

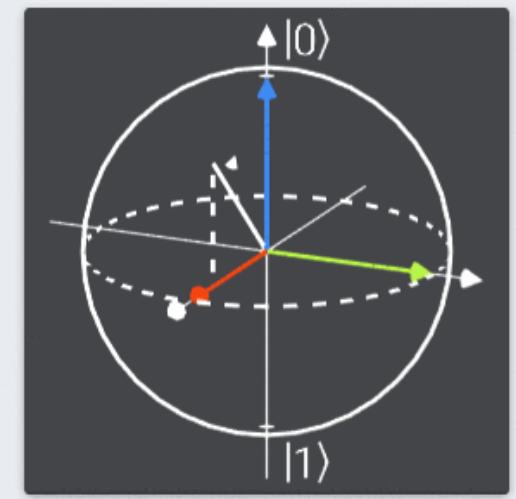


$$\frac{\sqrt{3}-i}{2\sqrt{2}} |0\rangle + \frac{\sqrt{3}+i}{2\sqrt{2}} |1\rangle$$



HADAMARD GATE

Rotate around the X + Z axis by π radians.



Zoo of one-qubit gates

$$\left. \begin{array}{l} X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \\ Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \\ Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \end{array} \right\} \pi \text{ rotation about the x-, y- and z- axes.}$$

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \quad \text{"Hadamard" gate}$$

$$S = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix} \quad \pi/2 \text{ rotation about the z- axis.}$$

$$T = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix} \quad \pi/4 \text{ rotation about the z- axis.}$$

General rotations

The Pauli Matrices

Operations transform states and are equivalent to moving around on the Bloch sphere.

For qubits, important matrices are the Pauli matrices:

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \quad Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

All square to identity (check):

$$X^2 = X X = I \quad Y^2 = Y Y = I \quad Z^2 = Z Z = I$$

Odd and even powers:

$$X^3 = X X X = IX = X \quad X^4 = X X X X = II = I \quad \text{etc}$$

Why is this important? – quantum logic gates are ultimately written as exponentials of Pauli operators...let's see how this works...

Exponential of a Matrix

We can define the exponential of a matrix using the power series for the exponential:

$$\exp(A) = I + A + \frac{A^2}{2!} + \frac{A^3}{3!} + \frac{A^4}{4!} + \dots$$

The exponential of a Pauli Matrix (eg. X) including an angle parameter:

$$\begin{aligned}
 \exp(i\theta X) &= I + i\theta X - \frac{\theta^2}{2!}X^2 - \frac{i\theta^3}{3!}X^3 + \frac{\theta^4}{4!}X^4 + \dots \\
 &= I + i\theta X - \frac{\theta^2}{2!}I - \frac{i\theta^3}{3!}X + \frac{\theta^4}{4!}I + \dots \\
 &= \left(1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} + \dots\right)I + i\left(\theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} + \dots\right)X \\
 &= \cos\theta I + i\sin\theta X
 \end{aligned}$$

(used power series for cos and sin)

General exponentiation of Paulis

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \quad Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

We just proved: $\exp(i\theta X) = I \cos \theta + i X \sin \theta$

to generalise we form a 2×2 matrix Σ from the Pauli matrices as:

$$\hat{n} \cdot \sigma = n_x X + n_y Y + n_z Z$$

$$\Sigma = \hat{n} \cdot \sigma$$

$$\mathbf{X} = (X, Y, Z)$$

$$\hat{n} = \frac{\mathbf{n}}{|\mathbf{n}|}$$

where \hat{n} is a unit spatial 3-vector and σ is a vector of Paulis, a similar proof follows, so

$$\exp(i\theta \hat{n} \cdot \sigma) = I \cos \theta + i \hat{n} \cdot \sigma \sin \theta$$

e.g. $\hat{n} = (1, 0, 0) \rightarrow \hat{n} \cdot \sigma = (1, 0, 0) \cdot \sigma = X$

Recover previous result: $\exp(i\theta X) = I \cos \theta + i X \sin \theta$

Rotation as an exponential

A rotation by angle θ around a (unit vector) axis \hat{n} , is given by:

$$R_n(\theta) = \exp\left(-i\frac{\theta}{2}\hat{n} \cdot \sigma\right)$$

You can write this exponential using this identity:

$$\exp(i\theta \hat{\mathbf{n}} \cdot \underline{\sigma}) = I \cos \theta + i \hat{\mathbf{n}} \cdot \underline{\sigma} \sin \theta$$

Global Phase

There is no experimental difference between

$$|\psi\rangle$$

and

$$\exp(i\theta_{global}) |\psi\rangle$$

This means that a phase which multiplies **all** the amplitudes (and it really does have to be all is not measurable).

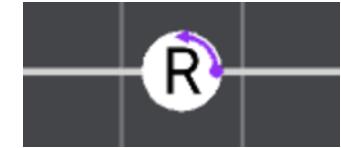
It also means that when you're choosing a gate, you can multiply **the entire matrix** by a global phase factor, and there won't be any measurable difference.

Note: It is common to choose the global phase so that the determinant, $|U| = 1$. Gates in this group are said to be the the group $SU(n)$.

Arbitrary axis rotation – coding in QUI

Coding an arbitrary rotation gate in QUI – R-gate

The “parameters” menu allows you to specify axis and angle:



ARBITRARY ROTATION GATE - PARAMETERS

EDITING 1 GATE

Rotation axis

x: 1 y: 1 z: 1

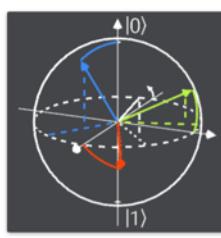
Rotation angle (radians) $1 \cdot \pi$

Global phase (radians) $1 \cdot \pi$

Presets - click to apply:

X	Y	Z	H
\sqrt{X}	\sqrt{Y}	S	T

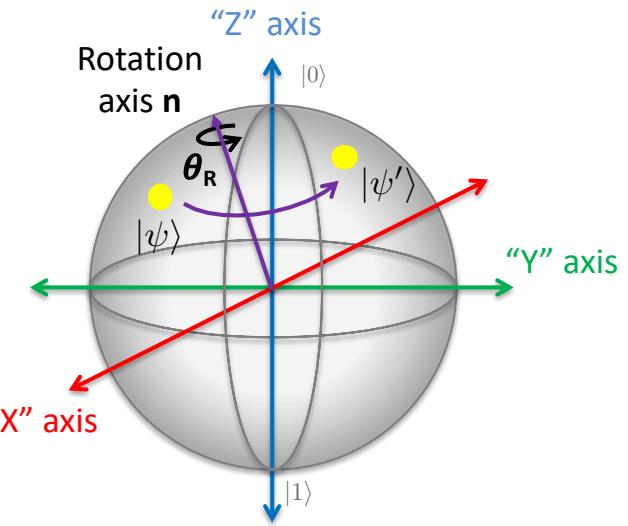
CANCEL OK



Cartesian cords for axis of rotation \mathbf{n}

Angle of rotation θ_R about \mathbf{n}

Global phase θ_g : generally set this to zero unless otherwise directed!



N.B. QUI normalises axis for you:

e.g. X axis, i.e. $\mathbf{n} = (1,0,0)$ -> enter (1,0,0)

Rotation axis

x: 1 y: 0 z: 0

e.g. X+Z axis, i.e. $\mathbf{n} = (1,0,1)/\sqrt{2}$ -> enter (1,0,1)

Rotation axis

x: 1 y: 0 z: 1

(entries can be decimals)

Rotation angle (radians)

$1.4 \cdot \pi$

x: 0.2 y: 1.4 z: 0.4

Useful facts about one qubit operations

Operations are unitary

In quantum mechanics, *unitary* operators acting on quantum states produce new quantum states. These operators can be described by *unitary* matrices.

The new state is given by: $|\psi'\rangle = U |\psi\rangle$

Unitary operations are ones for which: $U^\dagger U = I$

Where the dagger represents taking the transpose (t) and complex conjugate (*).

$$U^\dagger = U^{t*}$$

In quantum mechanics, all unitary operations are **reversible**.

It's possible to efficiently express every classical computation using equivalent reversible logic gates, but there can be a cost in terms of additional bits and operations.

A zoo of one-qubit gates

$$\left. \begin{array}{l} X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \\ Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \\ Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \end{array} \right\} \pi \text{ rotation about the x-, y- and z- axes.}$$

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \quad \text{"Hadamard" gate}$$

$$S = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix} \quad \pi/2 \text{ rotation about the z- axis.}$$

$$T = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix} \quad \pi/4 \text{ rotation about the z- axis.}$$

Operations Don't Commute!

For operators (e.g. matrices), remember

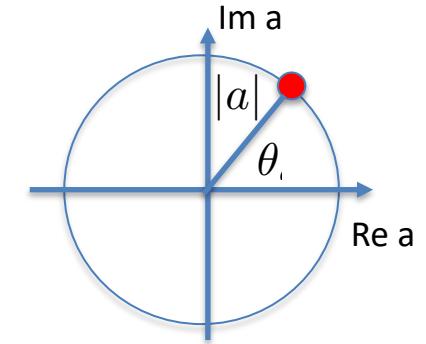
$$AB \neq BA$$

Order matters!

Note angles in context – abundant use of θ

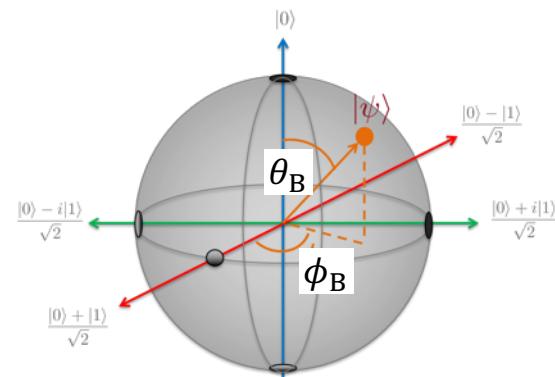
Phase angle of complex amplitudes in polar coordinates:

$$|\psi\rangle = a_0 |0\rangle + a_1 |1\rangle \rightarrow |\psi\rangle = |a_0| e^{i\theta_0} |0\rangle + |a_1| e^{i\theta_1} |1\rangle$$



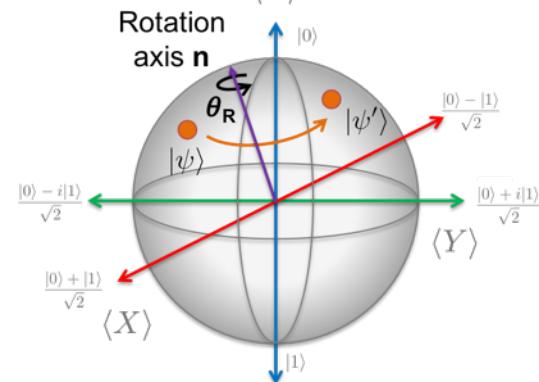
Angle specifying position on the Bloch sphere:

$$|\psi\rangle = a_0 |0\rangle + a_1 |1\rangle \rightarrow \cos \frac{\theta_B}{2} |0\rangle + \sin \frac{\theta_B}{2} e^{i\phi_B} |1\rangle$$



Angle of rotation of a qubit state on the Bloch sphere about a specified axis (unit vector), \mathbf{n} :

$$|\psi'\rangle = R_{\mathbf{n}}(\theta_R) |\psi\rangle$$



Week 2



Lecture 3

- 3.1 The Bloch Sphere representation for qubits
- 3.2 Quantum operations on qubits
- 3.3 Qubit gates in matrix form and the Pauli matrices

Lecture 4

- 4.1 The Pauli gates X, Y and Z and the QUI
- 4.2 Qubit operations around non-cartesian axes – H and R gates
- 4.3 Programming sequences over the qubit logic gate library
- 4.4 Note on the context and use of angles

Practice class 2

Bloch sphere and single qubit logic operations on the QUI

Week by week

- (1) Introduction to quantum computing
- (2) Single qubit representation and operations
- (3) Two and more qubits
- (4) Simple quantum algorithms
- (5) Quantum search (Grover's algorithm)
- (6) Quantum factorization (Shor's algorithm)
- (7) Quantum supremacy and noise
- (8) Programming real quantum computers (IBM Q)
- (9) Quantum error correction (QEC)
- (10) QUBO problems and Adiabatic Quantum Computation (AQC)
- (11) Variational/hybrid quantum algorithms (QAOA and VQE)
- (12) Solving linear equations, QC computing hardware