Homework 2 in 18.06 Due on Gradescope by Sunday night, February 19

1. What multiple ℓ of equation 1 should be subtracted from equation 2 to remove c ?

$$\begin{aligned} ax + by &= f \\ cx + dy &= g. \end{aligned}$$

The first pivot is a (assumed nonzero). Elimination produces what formula for the second pivot ? What is y ? The second pivot is missing when $ad = bc$: singular.

This is Problem 2.1.4 on page 46.

2. For which three numbers k does elimination break down ? Which is fixed by a row exchange ? Is the number of solutions 0 or 1 or ∞ ? **Draw the 3 row pictures**.

3 pictures from
3 particular k's

$$\begin{aligned} kx + 3y &= 6 \\ 3x + ky &= -6. \end{aligned}$$

This is Problem 2.1.8 on page 46.

3. Which number d forces a row exchange, and what is the triangular system (not singular) for that d? Which d makes this system singular (no third pivot) ?

$$\begin{aligned} 2x + 5y + z &= 0 \\ 4x + dy + z &= 2 \\ y - z &= 3. \end{aligned}$$

This is Problem 2.1.13 on page 47.

4. Write down the 3 by 3 matrices that produce these elimination steps :

(a) E_{21} subtracts 5 times row 1 from row 2.

(b) E_{32} subtracts -7 times row 2 from row 3.

(c) P exchanges rows 1 and 2, then rows 2 and 3.

This is Problem 2.2.1 on page 53.

5. Which three matrices E_{21}, E_{31}, E_{32} put A into triangular form U ?

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 4 & 6 & 1 \\ -2 & 2 & 0 \end{bmatrix} \quad \text{and} \quad E_{32}E_{31}E_{21}A = EA = U.$$

Multiply those E's to get one elimination matrix E. What is $E^{-1} = L$?

Include $\boldsymbol{b} = (1, 0, 0)$ as a fourth column to produce $[\,A \;\; \boldsymbol{b}\,]$. Carry out the elimination steps on this augmented matrix to solve $A\boldsymbol{x} = \boldsymbol{b}$.

These are problems 2.2.3 and 2.2.4 on pages 53-54.

6. Suppose A is invertible and you exchange its first two rows to reach B. Is the new matrix B invertible? How would you find B^{-1} from A^{-1}?

This is Problem 2.2.18 on page 55.

7. (a) What 3 by 3 matrix E has the same effect as these three steps? Subtract row 1 from row 2, subtract row 1 from row 3, then subtract row 2 from row 3.

   (b) What single matrix L has the same effect as these three reverse steps? Add row 2 to row 3, add row 1 to row 3, then add row 1 to row 2.

   This is Problem 2.2.25 on page 55.

8. (Recommended) Prove that A is invertible if $a \neq 0$ and $a \neq b$ (find the pivots or A^{-1}). Then find three numbers c so that C is not invertible:

$$A = \begin{bmatrix} a & b & b \\ a & a & b \\ a & a & a \end{bmatrix} \qquad C = \begin{bmatrix} 2 & c & c \\ c & c & c \\ 8 & 7 & c \end{bmatrix}.$$

   This is Problem 2.2.34 on page 56.

9. What three elimination matrices E_{21}, E_{31}, E_{32} put A into its upper triangular form $E_{32}E_{31}E_{21}A = U$? Multiply by E_{32}^{-1}, E_{31}^{-1} and E_{21}^{-1} to factor A into L times U:

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 2 & 2 \\ 3 & 4 & 5 \end{bmatrix} \quad \text{and} \quad L = E_{21}^{-1}E_{31}^{-1}E_{32}^{-1}.$$

   This is Problem 2.3.5 on page 62.

10. (a) How many entries of S can be chosen independently, if $S = S^{\mathrm{T}}$ is 5 by 5?

   (b) How do L and D (still 5 by 5) give the same number of choices in LDL^{T}?

   (c) How many entries can be chosen if A is *skew-symmetric*? $(A^{\mathrm{T}} = -A)$.

   (d) Why does $A^{\mathrm{T}}A$ have no negative numbers on its diagonal?

   This is Problem 2.4.17 on page 72.