LINEAR ALGEBRA

HOMEWORK ASSIGNMENTS

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Homework 1 in 18.06 Due by Sunday night, February 12

1. Problem 1.1.10: (Not easy) How could you decide if the vectors $\boldsymbol{u} = (1,1,0)$ and $\boldsymbol{v} = (0,1,1)$ and $\boldsymbol{w} = (a,b,c)$ are linearly independent or dependent?

2. Problem 1.1.24: How many corners $(\pm 1, \pm 1, \pm 1, \pm 1)$ does a cube of side 2 have in 4 dimensions? What is its volume? How many 3D faces? How many edges? Find one edge.

3. Problem 1.2.20: The ***triangle inequality*** says: (length of $\boldsymbol{v} + \boldsymbol{w}$) $\leq$ (length of $\boldsymbol{v}$) + (length of $\boldsymbol{w}$).

(a) Show that $\|\boldsymbol{v} + \boldsymbol{w}\|^2 = \|\boldsymbol{v}\|^2 + 2\boldsymbol{v} \cdot \boldsymbol{w} + \|\boldsymbol{w}\|^2$.

(b) Increase that $\boldsymbol{v} \cdot \boldsymbol{w}$ to $\|\boldsymbol{v}\| \; \|\boldsymbol{w}\|$ to show that $\|$**side 3**$\|$ cannot exceed $\|$**side 1**$\|$ + $\|$**side 2**$\|$:

Triangle inequality $\quad \|\boldsymbol{v} + \boldsymbol{w}\|^2 \leq (\|\boldsymbol{v}\| + \|\boldsymbol{w}\|)^2$ **or** $\boxed{\|\boldsymbol{v} + \boldsymbol{w}\| \leq \|\boldsymbol{v}\| + \|\boldsymbol{w}\|}$

4. Problem 1.3.14: Which numbers q would leave A with two independent columns?

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 3 & 1 & 9 \\ 5 & 0 & q \end{bmatrix} \qquad A = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & q \end{bmatrix} \qquad A = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 2 & 4 \\ 0 & 0 & q \end{bmatrix}$$

5. Problem 1.3.19: If (a,b) is a multiple of (c,d) with $abcd \neq 0$, *show that* (a,c) *is a multiple of* (b,d). This is surprisingly important; two columns are falling on one line. You could use numbers first to see how a, b, c, d are related. The question will lead to:

If $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ has dependent rows, then it also has dependent columns.

6. Problem 1.4.5: Why is it impossible for a matrix A with 7 columns and 4 rows to have 5 independent columns? This is not a trivial or useless question.

7. Problems 1.4.6 and 1.4.7 : Going from left to right, put each column of A into the matrix C if that column is not a combination of earlier columns :

$$A = \begin{bmatrix} 2 & -2 & 1 & 6 & 0 \\ 1 & -1 & 0 & 2 & 0 \\ 3 & -3 & 0 & 6 & 1 \end{bmatrix} \qquad C = \begin{bmatrix} 2 & & & \\ 1 & & & \\ 3 & & & \end{bmatrix}$$

Find R in Problem 6 so that $A = CR$. If your C has r columns, then R has r rows. The 5 columns of R tell how to produce the 5 columns of A from the columns in C.

8. Problem 1.4.14 : Complete these 2 by 2 matrices to meet the requirements printed underneath :

$$\underset{\text{rank one}}{\begin{bmatrix} 3 & 6 \\ 5 & \end{bmatrix}} \qquad \underset{\text{orthogonal columns}}{\begin{bmatrix} 6 & 7 \\ 7 & \end{bmatrix}} \qquad \underset{\text{rank 2}}{\begin{bmatrix} 2 & \\ 3 & 6 \end{bmatrix}} \qquad \underset{A^2 = I}{\begin{bmatrix} 3 & 4 \\ & -3 \end{bmatrix}}$$

9. Problem 1.4.17 : True or false, with a reason (not easy) :

(a) If 3 by 3 matrices A and B have rank 1, then AB will always have rank 1.

(b) If 3 by 3 matrices A and B have rank 3, then AB will always have rank 3.

(c) Suppose $AB = BA$ for every 2 by 2 matrix B. Then $A = \begin{bmatrix} c & 0 \\ 0 & c \end{bmatrix} = cI$ for some number c. Only those matrices $A = cI$ commute with every B.

10. Problem 1.4.20 : How many small multiplications for $(AB)C$ and $A(BC)$ if those matrices have sizes $ABC = (4 \times 3)\,(3 \times 2)\,(2 \times 1)$? The two counts are different.