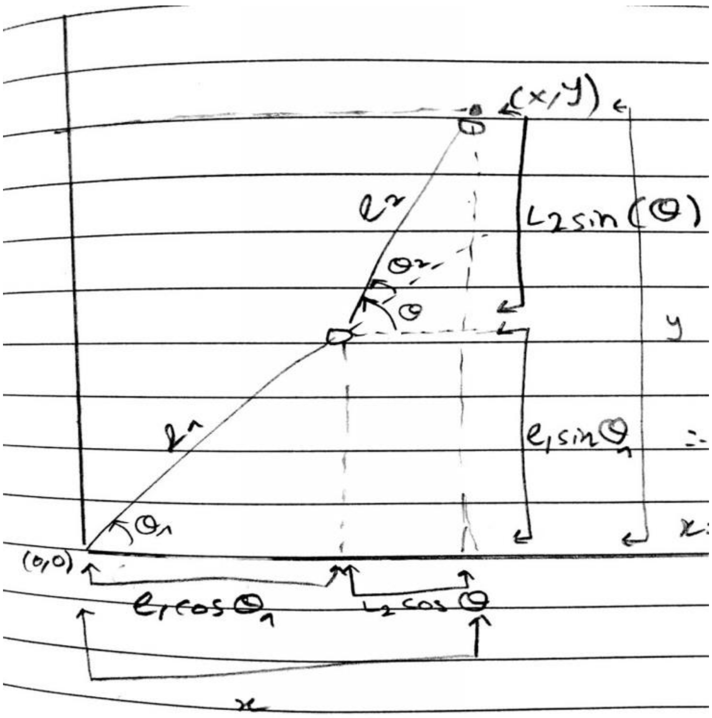
Robot Arm (2 Dimensions 2 Degree of Freedom)

Forward Kinematics



Known L1, L2, Q1, Q2

Required x, y, Q

Using Trigonometric funs:

$$cos(angle) = \frac{adj}{hyp}, sin(angle) = \frac{opp}{hyp}$$

so after sketched two triangles with known Q1 and Q2

$$Q = Q1 + Q2$$

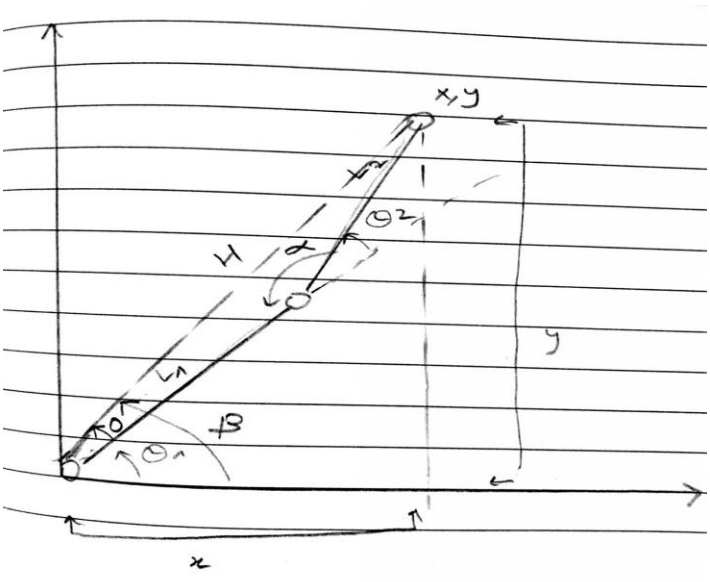
L1 and L2 constituting the hyps of the first and second triangles

We found out

$$x = L1 \times cos(Q1) + L2 \times cos(Q)$$

$$y = L1 \times sin(Q1) + L2 \times sin(Q)$$

Inverse Kinematic

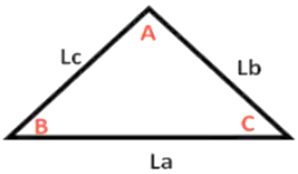


Known x, y, Q

Required Q1, Q2

Sin Law

$$\frac{\sin A}{a} = \frac{Sin\ B}{b} = \frac{Sin\ C}{c}$$



Cos Law

$$c^2 = b^2 + a^2 - 2ba\cos(C)$$

The length of H is the distance between two points Law (0,0) and (x,y)

$$\mathrm{H} = \sqrt{x^2 + y^2}$$

$$alpha = 180 - Q2$$

Using cosine law:

$$H^2 = L1^2 + L2^{\,2} - 2 \times L1 \times L2 \times cos(alpha)$$

$$x^2 + y^2 = L1^2 + L2^{\,2} - 2 \times L1 \times L2 \times cos(alpha)$$

$$cos(alpha) = cos(180 - Q2) = -cos(Q2)$$

$$x^2 + y^2 = L1^2 + L2^{\,2} + 2 \times L1 \times L2 \times cos(Q2)$$

$$Q2 = \cos^{-1}\left(\frac{x^2 + y^2 + L1^2 + L2^2}{2 \times L1 \times L2}\right)$$

To find Q1 with known Q:

$Q1 = Q - Q2$

To find Q1 with unknown Q:

$Q1 = beta - O$

$beta = \tan^{-1}(\frac{y}{x})$

using sin law we can find O:

$$\frac{sin(O)}{L2} = \frac{sin(alpha)}{H} = \frac{sin(180 - Q2)}{\sqrt{x^2 + y^2}} = \frac{sin(Q2)}{\sqrt{x^2 + y^2}}$$

$$O = \sin^{-1}(\frac{L2 \times sin(Q2)}{\sqrt{x^2 + y^2}})$$