

Given a table

$p(x, y)$	$x = 1$	$x = 2$	$x = 3$	$p(y)$
$y = 1$	$\frac{1}{16}$	$\frac{3}{16}$	$\frac{1}{16}$	$\frac{1}{2}$
$y = 2$	$\frac{1}{16}$	$\frac{3}{16}$	$0$	$\frac{1}{4}$
$y = 3$	$0$	$\frac{3}{16}$	$\frac{1}{16}$	$\frac{1}{4}$
$p(x)$	$\frac{1}{8}$	$\frac{3}{4}$	$\frac{1}{8}$	$1$

- Find

1. Entropies  $H(X), H(Y)$

2. Conditional Entropies  $H(X|Y)$  and  $H(Y|X)$

3. Joint Entropy  $H(X, Y)$

4. Mutual Information  $I(X; Y)$

5. Realise that  $p(y)$  and  $p(x)$  is the sum of the row and column respectively. This is not likely to be provided to you. The formula for each is

$$H(X) = \sum_{i=1}^n p(x_i) \log_2 p(x_i)$$

and

$$H(Y) = \sum_{i=1}^n p(y_i) \log_2 p(y_i)$$

in this case,  $i$  is 3 for both as there are 3 rows and columns, this may differ. You will need to manually sum with each  $p(x)$  and  $p(y)$  to get your answers.

6. In order to get  $H(X|Y)$  and  $H(Y|X)$  we can use  $H(X, Y)$ , see step 3 for the answers to step 2 and 3.

7. To find  $H(X, Y)$  sum all values in the table in the form

$$- \sum_{i=1}^n \sum_{j=1}^n p(x_i, y_j) \log_2 p(x_i, y_j)$$

For example,  $p(x_1, y_1)$  would be  $\frac{1}{16}$  (top-left cell) according to the table.

In order to find  $H(X|Y)$  and  $H(Y|X)$  use the formula below:

$$H(X|Y) = H(X, Y) - H(Y)$$

$$H(Y|X) = H(X, Y) - H(X)$$

8. To find  $I(X;Y)$  you can use any of the below formulae

$$\begin{aligned} I(X; Y) &\equiv H(X) - H(X|Y) \\ &\equiv H(Y) - H(Y|X) \\ &\equiv H(X) + H(Y) - H(Y, X) \\ &\equiv H(X, Y) - H(Y|X) - H(X|Y) \end{aligned}$$

$$I(X; Y) = D_{\text{KL}}(P(X, Y) \| P(X)P(Y)) = 0$$