Problem Set 6

Due Date: Sat Nov 7, 2015
by 9 PM

Name: Solutions

Lab Section & TF: _____

Collaborators: _____

For Grading Purposes Only:

Q1: _____ / 16
Q2: _____ / 10
Q3: _____ / 10
Q4: _____ / 20
Q5: _____ / 10
Q6: _____ / 14

Total: _____ / 80

Problem 1: Combinational Logic (16 points)

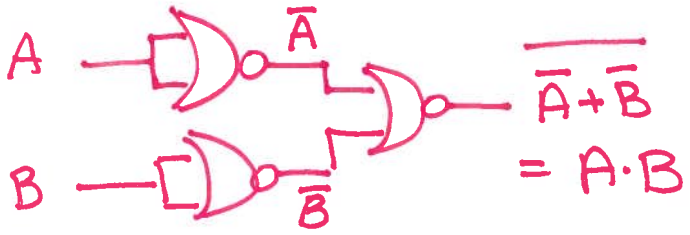
NAND and NOR gates are called universal logic gates because they can be used to implement any other gate or combination thereof. Using only 2-input NOR gates, implement (that is show the logic circuit) the following logic gates.

a. (2 points) AND gate

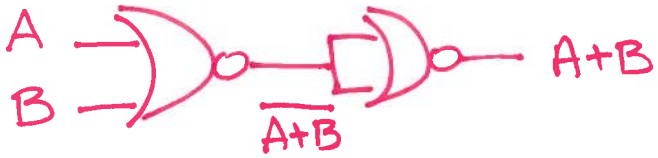
Note:

De Morgan's

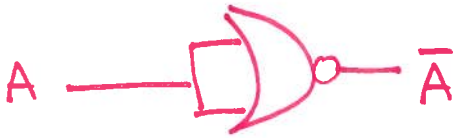
$\overline{A+B} \Leftrightarrow \bar{A}\cdot\bar{B}$



b. (2 points) OR gate



c. (2 points) NOT gate



d. (2 points) Simplify the following equations

$$Y = \overline{\bar{A}(A+\bar{B}) + B(\bar{A}+C) + C(\bar{B}+\bar{C})}$$

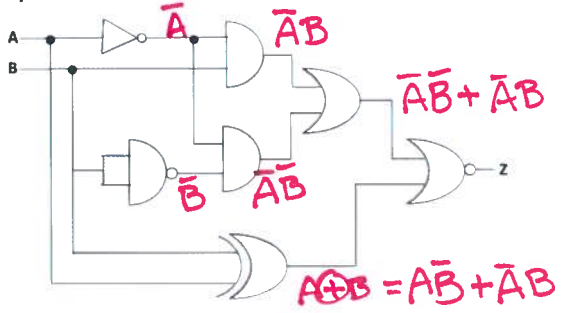
$\bar{Y} = \bar{A}\bar{A} + \bar{A}\bar{B} + B\bar{A} + BC + C\bar{B} + C\bar{C}$

$= \bar{A}(\bar{B}+B) + C(B+\bar{B})$

$= \bar{A}+C$

$Y = \overline{\bar{A}+C} = A\cdot\bar{C}$

e. (2 points) Construct the truth table for the following circuit:



A	B	Z
0	0	0
0	1	0
1	0	0
1	1	1

f. (2 points) Using the circuit from part (e), write a corresponding logic equation for output Z.

$\bar{Z} = \bar{A}\bar{B} + \bar{A}B + A\bar{B} + \bar{A}B$

$= \bar{A}\bar{B} + \bar{A}B + A\bar{B}$

$Z = \overline{\bar{A}\bar{B} + \bar{A}B + A\bar{B}}$

$= \overline{\bar{A}(\bar{B}+B) + A\bar{B}}$

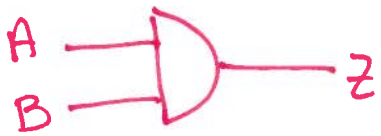
$Z = \overline{\bar{A} + A\bar{B}}$ ↓ DeMorgan's

$= A \cdot (\bar{A}+B)$

$= A\bar{A} + AB$

$Z = AB$

g. (2 points) Using the circuit from part (e), design a circuit that produces the same output using at most two logic gates. You may use only AND, OR, and NOT gates.



h. (2 points) Find and simplify the logic expression for *Out* that corresponds to the truth table below.

A	B	C	Out
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

$\overline{Out} = \bar{A}BC + A\bar{B}C = (\bar{A}B + A\bar{B})C$

$Out = \overline{(A \oplus B)C}$

Problem 2: Practical Combinational Logic (10 points)

Using logic gates of your choice, with as many inputs as you wish, create a circuit that turns on red (R), green (G) or blue (B) light-emitting diodes, depending on the number of 1's in the 3-bit input number $X_2X_1X_0$:

- The red LED is on only when exactly one of the three input bits is 1 ($X_2X_1X_0 = 010$)
- The green LED is on when exactly two of the three input bits are 1 ($X_2X_1X_0 = 110$)
- The blue LED is on when all three input bits are 1 ($X_2X_1X_0 = 111$)
- All LEDs are off when all the bits are zero ($X_2X_1X_0 = 000$)

a.

a. (4 points) Write the truth table for this circuit.

X_2	X_1	X_0	R	G	B
0	0	0	0	0	0
0	0	1	1	0	0
0	1	0	1	0	0
0	1	1	0	1	0
1	0	0	1	0	0
1	0	1	0	1	0
1	1	0	0	1	0
1	1	1	0	0	1

b. (3 points) Write the logic equations for output R, G, and B signals.

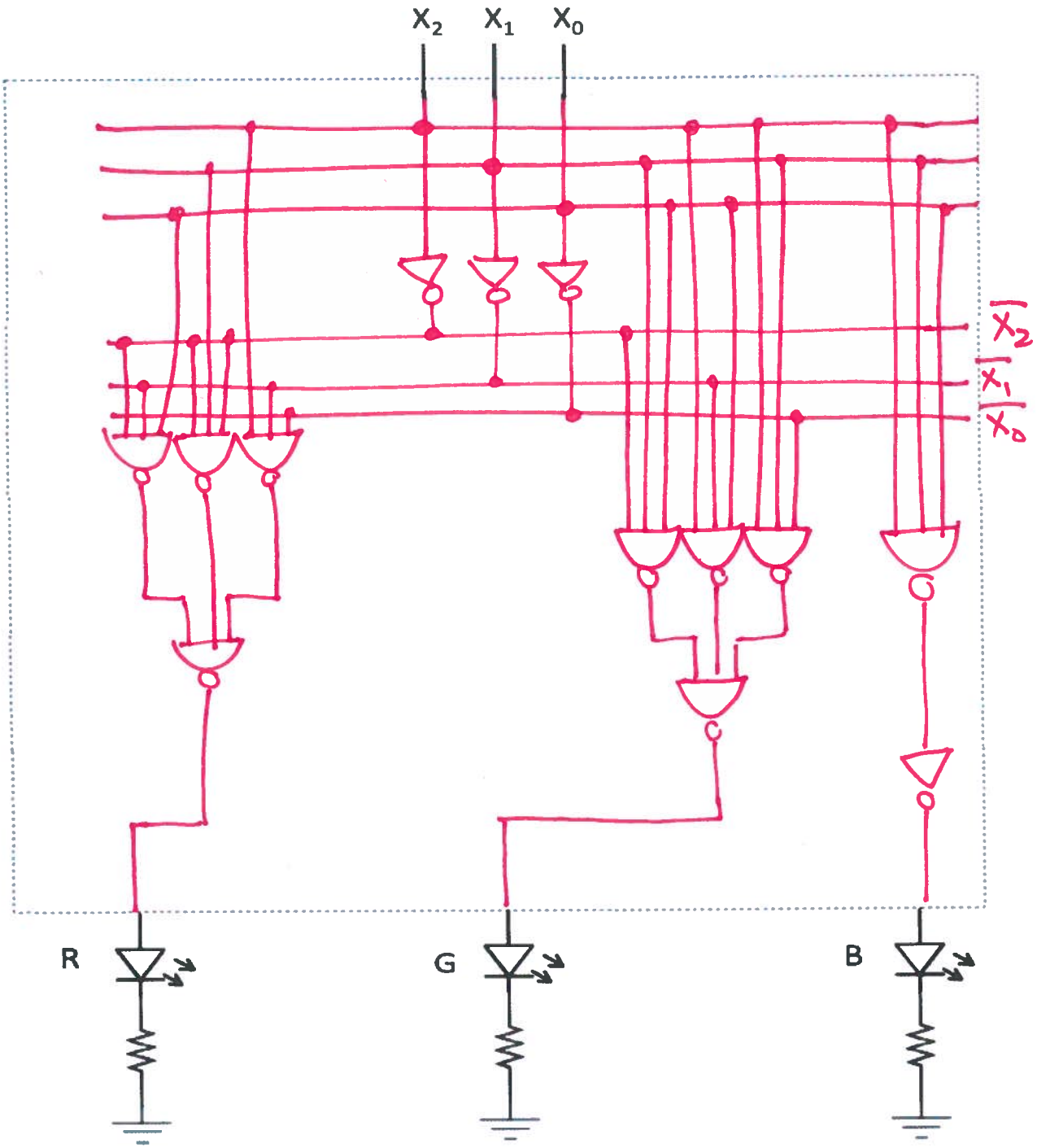
$$R = \bar{X}_2\bar{X}_1X_0 + \bar{X}_2X_1\bar{X}_0 + X_2\bar{X}_1\bar{X}_0$$

$$G = \bar{X}_2X_1X_0 + X_2\bar{X}_1X_0 + X_2X_1\bar{X}_0$$

$$B = X_2X_1X_0$$

c. (3 points) Draw the corresponding circuit diagram in the space below.

ONLY USING NAND, NOR + INVERTERS

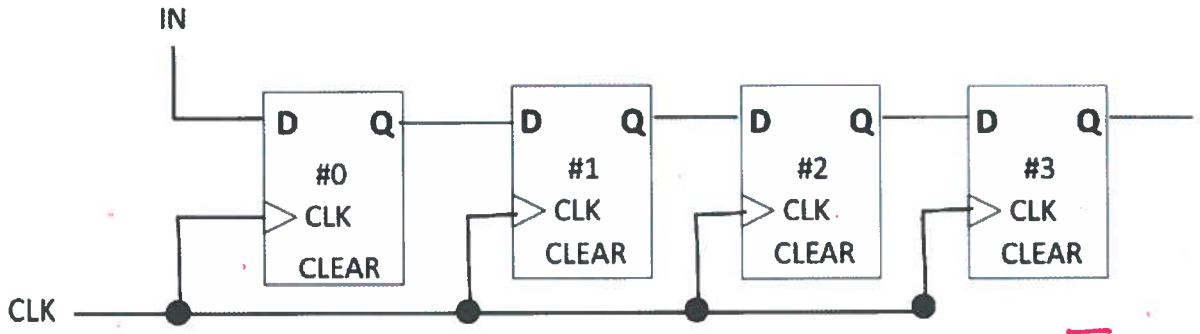


MANY CORRECT ANSWERS POSSIBLE

CAN USE 2-INPUT gates as well.

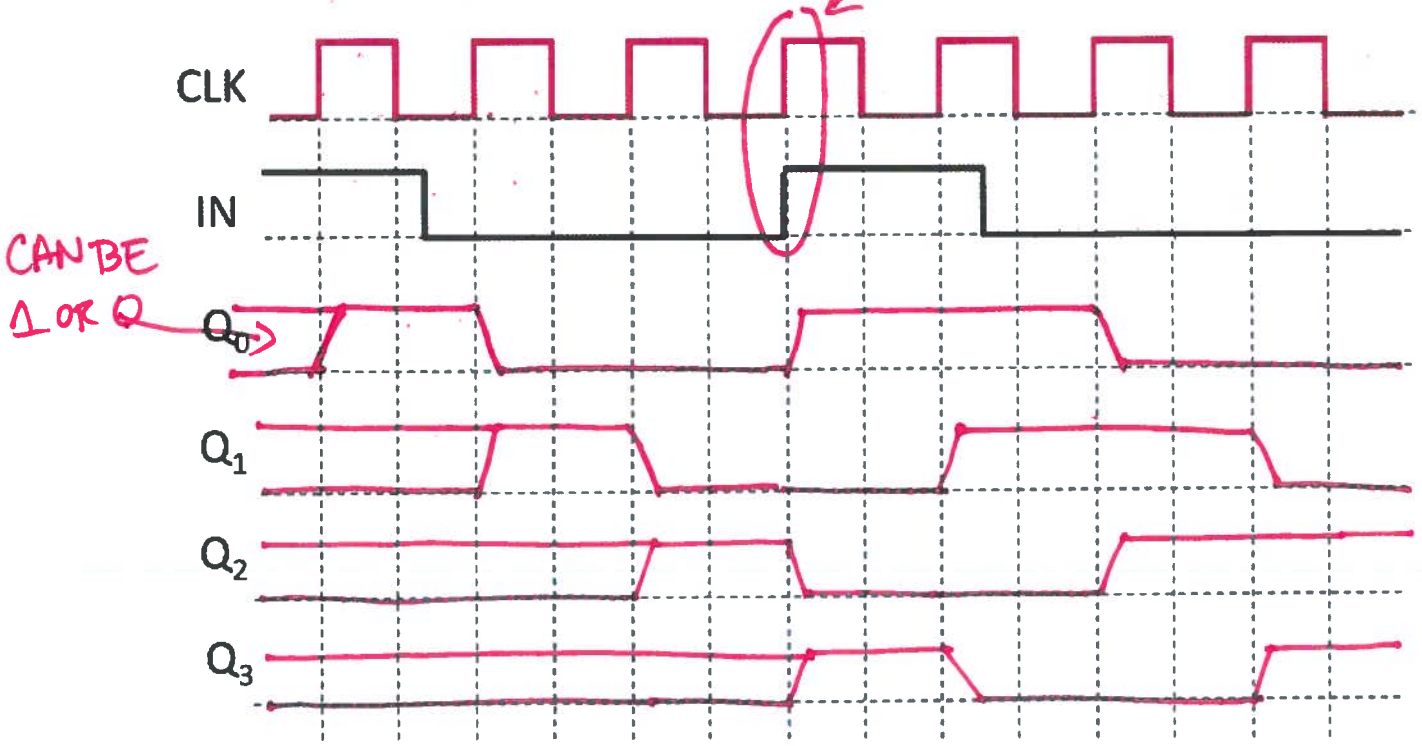
Problem 3: Sequential Logic with D Flip-Flops (10 points)

Consider the sequential logic circuit shown below made of four D flip-flops.



a. (8 points) Draw the waveforms for the outputs of each flip-flop (#0-3) below.

Assume IN ↑ BEFORE CLK ↑



b. (2 points) What does this circuit do? What is it called?

Circuit shifts data from IN through each flip-flop

Called a shift register

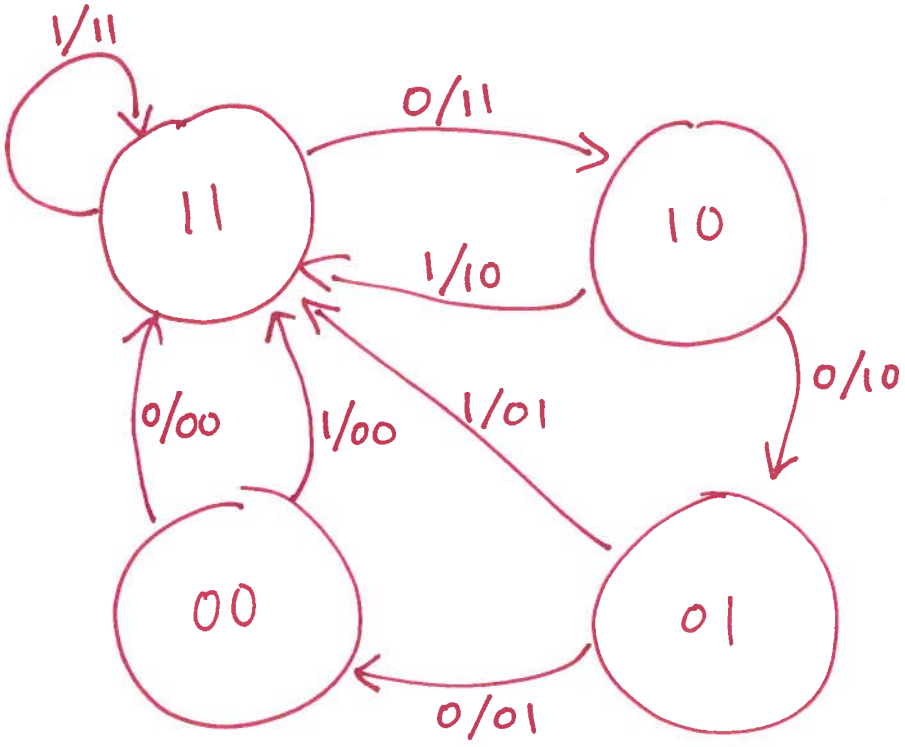
Problem 4: Counter (20 points)

Realize a count-down counter according to following rules:

- Counter has one input signal SET (in addition to the CLOCK signal)
- When SET=1, number 3 is written into counter
- When SET=0, counter counts down as follows 3, 2, 1, 0, 3, 2, 1, 0, ...
- Output of the counter is its present state

a. (7 points) Draw the finite state machine diagram for this counter.

IN/OUT = SET/COUNT



b. (3 points) How many input wires, output wires and flip-flops would you need to implement this FSM?

2 Flipflops (2 bits for state)

1 input (set)

2 output wires (count output)

c. (5 points) Draw the truth table for the counter (finite state machine) that describes the transitions from one state to another.

Set	Q_1	Q_0	$Q_1(t+1)$	$Q_0(t+1)$
0	0	0	1	1
0	0	1	0	0
0	1	0	0	1
0	1	1	1	0
1	0	0	1	1
1	0	1	1	1
1	1	0	1	1
1	1	1	1	1

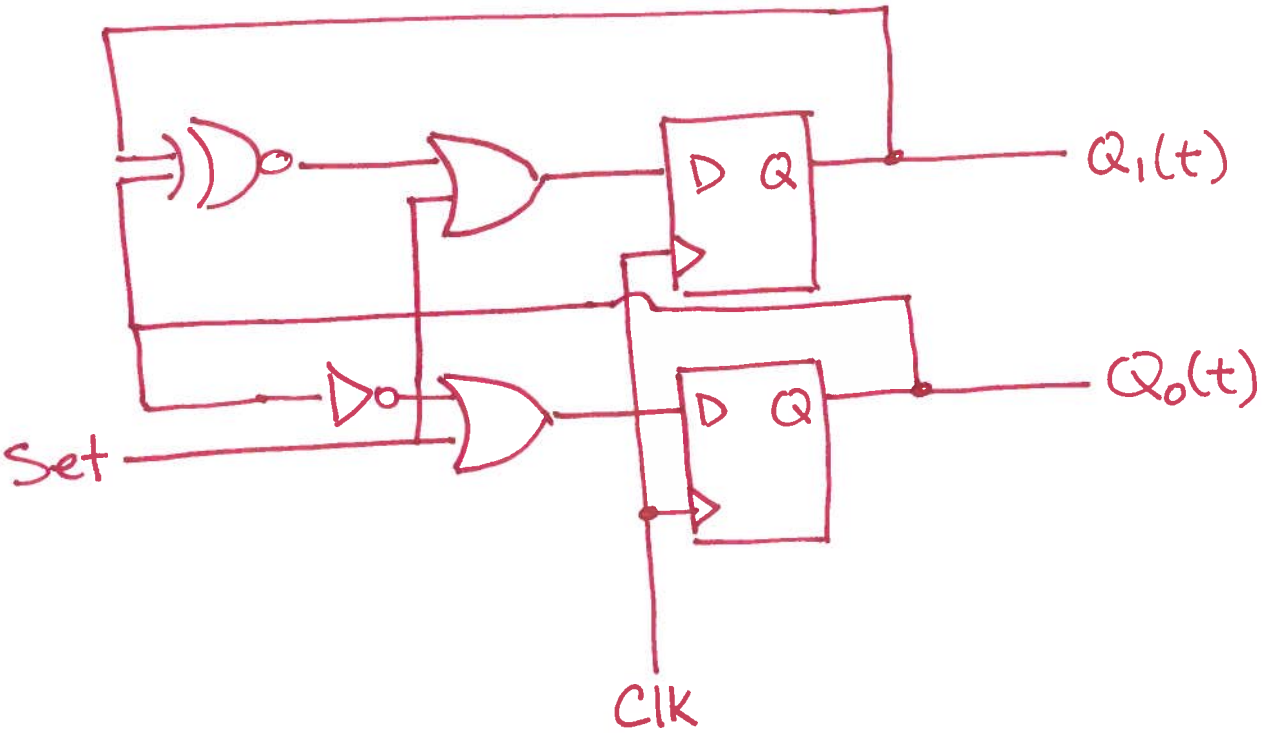
d. (5 points) Draw the circuit for this counter using synchronous D flip-flops sensitive to the rising edge of the clock signal and as many logic gates as needed. Logic gates can have as many inputs as you wish.

$$\overline{Q}_1(t+1) = \overline{S}(Q_1 \oplus Q_0)$$

$$Q_1(t+1) = S + \overline{(Q_1 \oplus Q_0)}$$

$$\overline{Q}_0(t+1) = \overline{S}Q_0$$

$$Q_0(t+1) = S + \overline{Q}_0$$



Problem 5: Finite State Machines (10 points)

One day while you were dozing off during ES50 lecture you were struck by stroke of genius!

"All of these electronic gizmos are so complicated nowadays," you think to yourself. "Who wants a phone that can do everything when you can have a phone that does next to nothing???"

As you start to rush off to the iLab, you get to thinking and realize that the only thing that people really do on their phones is call their parents once or twice a day, just to check in. You figure that you could design a phone that has three buttons, one to call each of your two parents, and one to end the current call.

Design an FSM for your invention according to the following rules:

- You should have states that correspond to each phone call and to an idle state while you're not in a call
- When you push button 1 or button 2 while the phone is idle, you should place a call
- If you push a button other than "End Call" while you're in the middle of a call, your phone should stay on the same call and ash an LED to remind you to keep talking
- Pressing "End Call" while your phone is idle should elicit a recording that enthusiastically reminds you to "Call your mom!"
- Also be sure that your phone doesn't hang up when you don't push any buttons!

a) (2 points) How many states, input, and outputs does this phone have?

3 states IDLE, CALL1, CALL2

4 inputs nothing, button1, button2, end call

3 outputs "Call Mom!", LED ON, NOTHING

b) (3 points) How many flip-flops would you need to represent your states? How many input and output wires do you need?

2 Flip flops for 3 states

2 input wires for 4 input possibilities

2 output wires for 3 possible outputs

c) (5 points) Draw your FSM. On your drawing, you should utilize (and identify somewhere) encoded inputs, outputs, and states.

STATES

Idle : 00
Call1: 01
Call2: 10

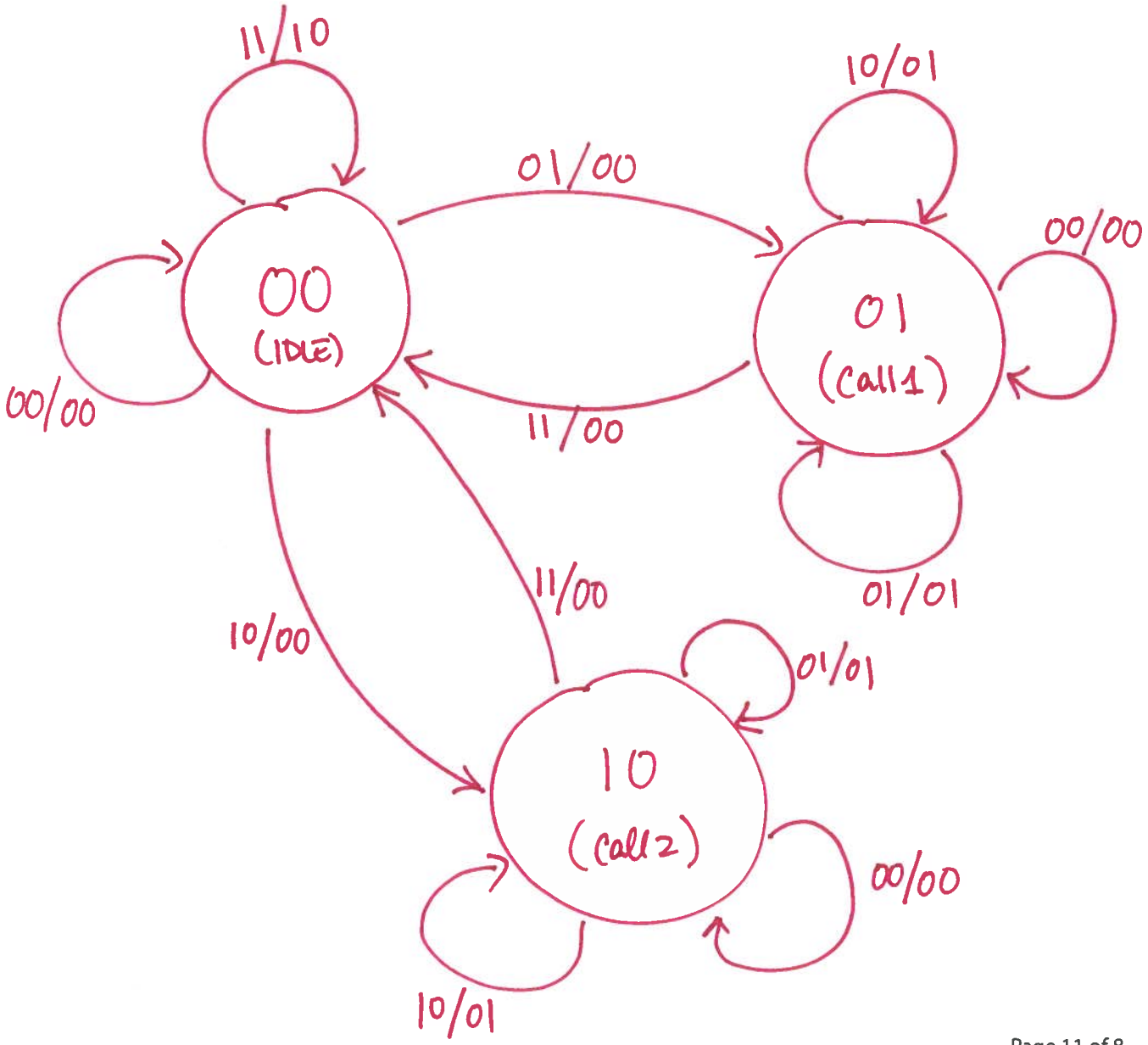
INPUTS

No Input : 00
Push Button 1 : 01
Push Button 2 : 10
End Call : 11

OUTPUTS

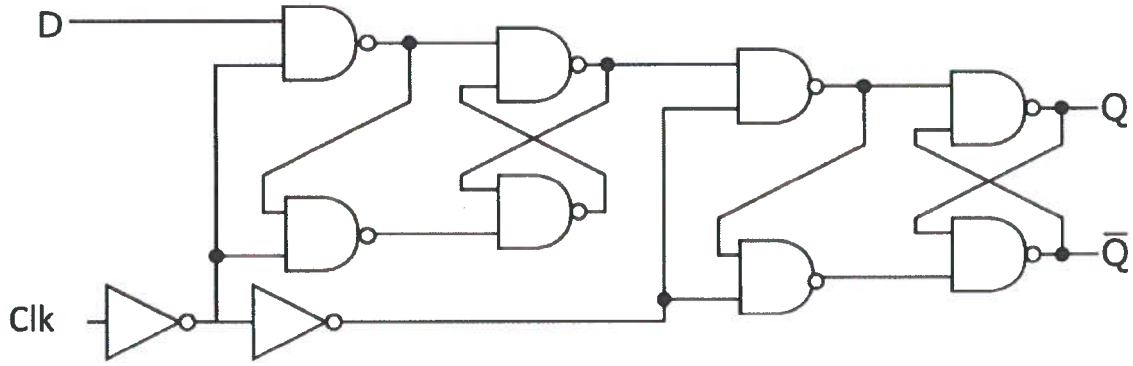
Nothing : 00
LED ON : 01
CALL MOM : 10

INPUT/OUTPUT.

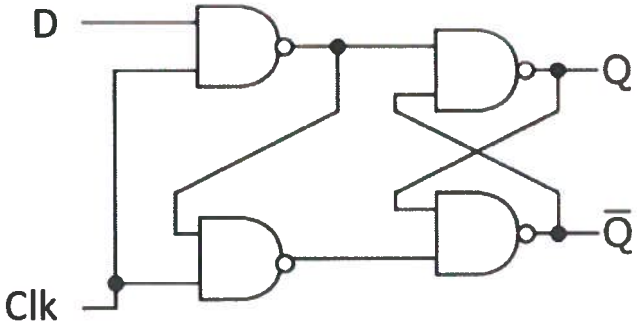


Problem 6: Sequential Logic (14 points)

Refer to the circuit below



a) (5 points) The circuit above has two distinct blocks of sequential logic. The second is replicated below. Fill out a truth table for this block by examining the circuit for all possible inputs.



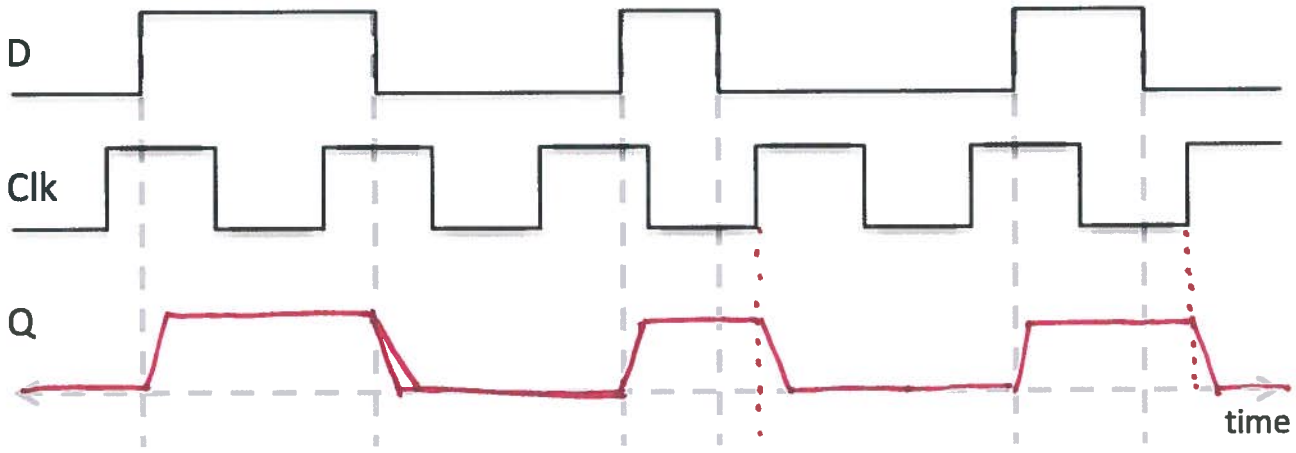
D	Clk	Q(t)	Q(t+1)
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

b) (2 points) Can you identify this block as a standard sequential logic gate?

Clock level sensitive D latch

(Not edge sensitive)

c) (3 points) Based on your truth table for part (a), draw the block's output in response to the arbitrary input signal and clock signal below



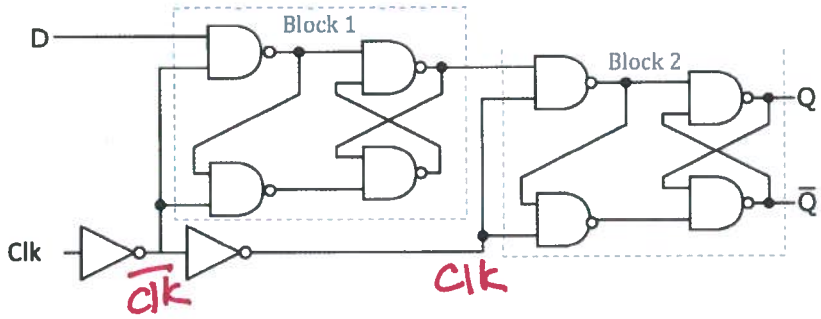
d) (2 points) From part (a) and part (b), it appears that the output Q can only change while the clock has a particular value. In other words, Q is "frozen" for one of the clock levels. For what clock level (high or low) is the gate "frozen" and for which level is it "active"? Explain the significance of a "frozen" output.

When clk is high (clk=1) Q=D "ACTIVE"

When clk is low (clk=0) Q holds last value of D "FROZEN" when clk was high

"Frozen" state holds data, implements memory

e) (2 points) Let's return to the entire circuit, shown below. Is it possible for both of the sequential logic blocks to be "active" simultaneously? (Hint: look at the two inverters. Is it possible for both of the clock signals to be "high" at the same time?)



No. By design, Block1 + Block2 get complementary clock signals $\overline{clk}$ + clk