Problem Set 4

Due Date: Sat Oct 17 2015
by 9 PM

Name: Solutions

Lab Section & TF: ____________________

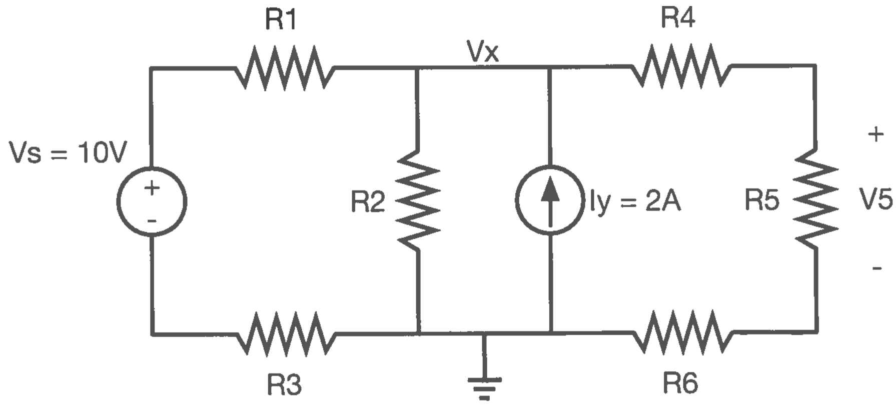
Collaborators: ____________________

For Grading Purposes Only:

Q1: _____ / 10
Q2: _____ / 10
Q3: _____ / 15
Q4: _____ / 10
Q5: _____ / 5

Total: _____ / 50

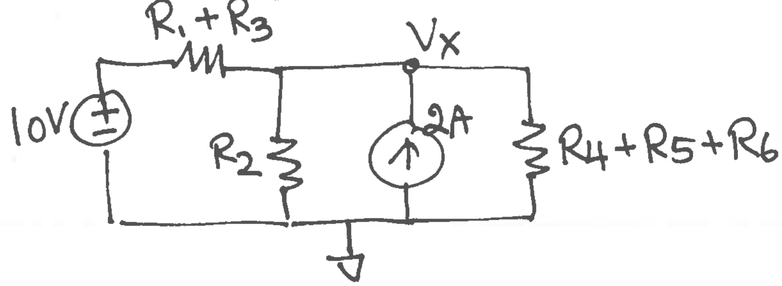
Problem 1: Circuit Analysis (10 points)



Assume the following resistances: R1=R3=1Ω, R2=2Ω, R4=R6=2Ω, and R5=1Ω.

a. (5 points) For the circuit above, solve for Vx and V5 using the **node voltage method**.

First, redraw to simplify.



KCL @ Vx gives

$$\frac{10-V_x}{R_1+R_3} - \frac{V_x}{R_2} + 2A - \frac{V_x}{R_4+R_5+R_6} = 0$$

$$V_x\left(\frac{1}{R_1+R_3} + \frac{1}{R_2} + \frac{1}{R_4+R_5+R_6}\right) = \frac{10}{R_1+R_3} + 2$$

$$V_x\left(\frac{1}{2} + \frac{1}{2} + \frac{1}{5}\right) = \frac{10}{2} + 2 = 7$$

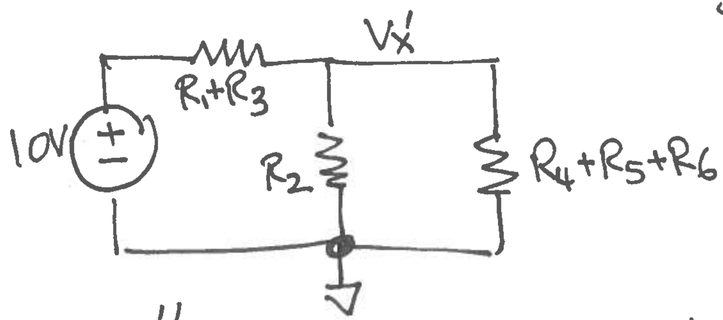
$$V_x \cdot \frac{12}{10} = 7 \Rightarrow \boxed{V_x = \frac{35}{6} V}$$

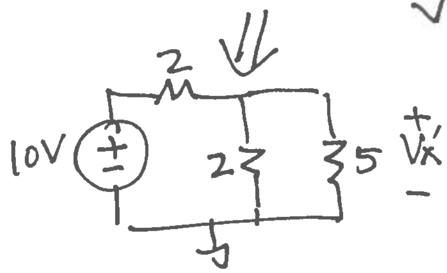
Voltage divider gives...

$$V_5 = \frac{R5}{R4+R5+R6} \cdot V_x = \frac{1}{5} \cdot \frac{35}{6} \Rightarrow \boxed{V_5 = \frac{7}{6} V}$$

b. (5 points) Now, solve for Vx and V5 using **superposition**.

First solve for Vs and assume $I_y = 0$



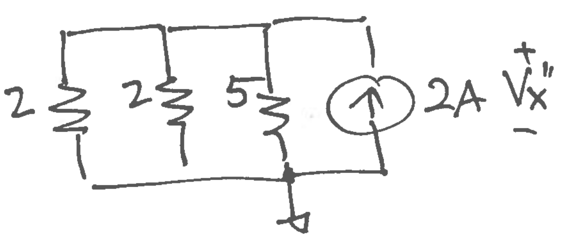


$$V_x' = \frac{\overset{10/7}{2 \| 5}}{2 + 2\|5} \cdot 10V$$

$$= \frac{10/7 \cdot 10}{2 + 10/7} = \frac{100}{24}$$

$$\boxed{V_x' = \frac{25}{6} V.}$$

Then solve for $I_y = 2$ + $V_s = 0$.



$$V_x'' = (2 \| 2 \| 5) \cdot 2A$$

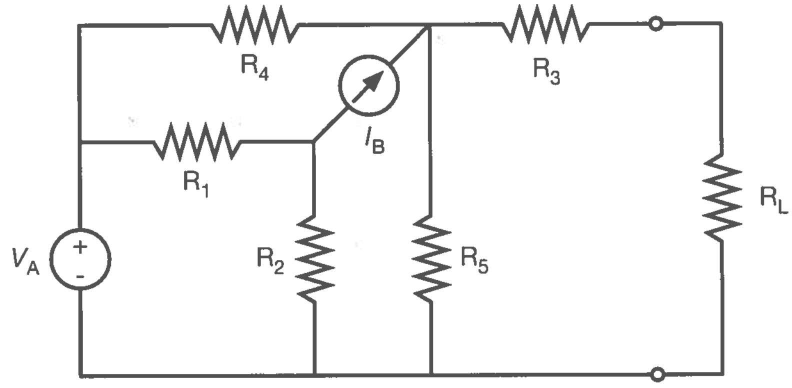
$$= \frac{2}{\frac{1}{2} + \frac{1}{2} + \frac{1}{5}} = \frac{20}{12}$$

$$V_x = V_x' + V_x'' = \boxed{35/6} \Leftarrow \boxed{V_x'' = 5/3}$$

$$\text{again, } V_5 = \frac{R_5 \quad V_x}{R_4 + R_5 + R_6}$$

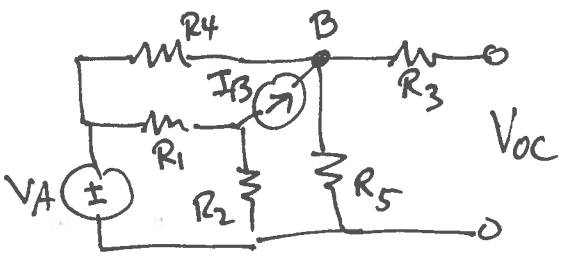
$$= \boxed{7/6 V}$$

Problem 2: Thevenin Equivalent Circuit (10 points)



(10 points) For the circuit shown above, find the Thevenin equivalent circuit from the perspective of R_L. Please find V_{THEV} and R_{THEV} in terms of V_A, I_B, and R_{1-5}.

To find V_{THEV}, solve for open-circuit voltage. (V_{OC})



KCL @ node B ...

$$\frac{V_A - V_B}{R_4} + I_B = \frac{V_B}{R_5}$$

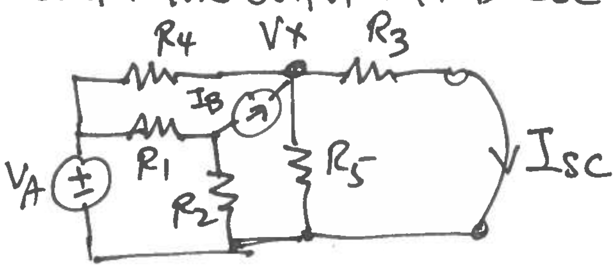
$$V_{OC} = V_B$$

$$\frac{V_A}{R_4} + I_B = \left(\frac{1}{R_4} + \frac{1}{R_5}\right) V_B$$

$$V_{OC} = V_B = \left(\frac{V_A}{R_4} + I_B\right)\left(\frac{1}{R_4} + \frac{1}{R_5}\right)^{-1}$$

$$\boxed{V_{THEV} = \left(\frac{V_A}{R_4} + I_B\right)\left(\frac{1}{R_4} + \frac{1}{R_5}\right)^{-1}}$$

SHORT THE OUTPUT & FIND I_{SC}



KCL @ Vx gives

$$\frac{V_A - V_X}{R_4} + I_B = \frac{V_X}{R_5} + \frac{V_X}{R_3}$$

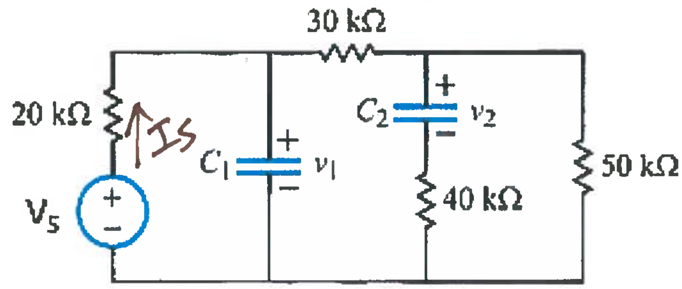
$$V_X = \left(\frac{V_A}{R_4} + I_B\right)\left(\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}\right)^{-1}$$

$$R_{THEV} = \frac{V_{THEV}}{I_{SC}}$$

$$R_{THEV} = \frac{V_{THEV}}{V_X / R_3} \leftarrow I_{SC} = V_X / R_3$$

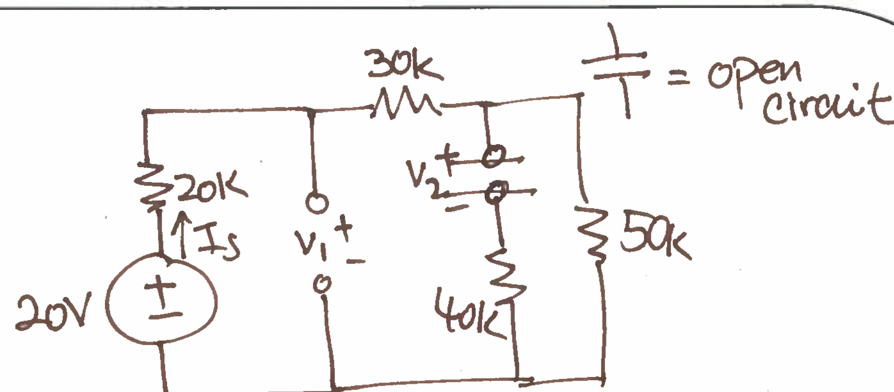
$$R_{THEV} = \frac{R_3\left(\frac{1}{R_4} + \frac{1}{R_5}\right)^{-1}}{\left(1/R_3 + 1/R_4 + 1/R_5\right)^{-1}}$$

Problem 3: RC Circuit (15 points)



For the circuit shown above, find v_1, v_2, and the *current* through the voltage source for two conditions below.

a. (5 points) The voltage source is a DC source (e.g., battery) with $V_s = 20V$.



$$V_1 = \frac{30K+50k}{20K+30K+50k} \cdot 20 = \boxed{16V}$$

$$V_2 = \frac{50}{100} \cdot 20 = \boxed{10V}$$

$$I_s = \frac{V_s}{100K} = \frac{20}{100K} = \boxed{0.2mA}$$

b. (5 points) The voltage source, V_s, is an AC source $V_s = 20\sin(2\pi f t)$, where f is very high frequency relative to the RC time constants of this circuit.

at high frequency, $C \rightarrow$ short circuit

therefore $\boxed{V_1 = V_2 = \phi V}$

$$\boxed{I_{sc} = \frac{V_s}{20K} = \frac{20}{20K} \cdot \sin(2\pi f t)}$$

c. (5 points) Calculate the average power delivered by the source for both (a) and (b) above.

(a) $P = I \cdot V = \frac{1}{5} \cdot 20 = \boxed{4W}$

(b) $P = \frac{V_{RMS}^2}{R}$ where $V_{RMS} = \frac{V_{amplitude}}{\sqrt{2}}$

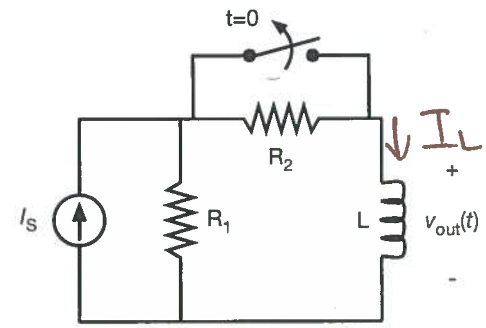
From Lecture 3

$$P = \frac{20^2}{2(20K)} = \frac{400}{40K} = \frac{1}{100}$$

$\boxed{\text{or } 10mW}$

Problem 4: Step Response of RL Circuit (10 points)

Please refer to the circuit shown (to the right) and answer the questions the below. The switch has been closed for a long time and opens at t = 0 and remains open. Please provide your answers in terms of I_S, R_1, R_2 and L.



a. (2 points) What is the current flowing through the inductor, L, at $t=0^-$ and $t=\infty$?

at $t=0^-$

$I_L = I_S$

at $t=\infty$

$I_L = \frac{R_1}{R_1+R_2} I_S$

Current divider

b. (3 points) What is the current flowing through R1 and R2 at $t=0^-$ and $t=\infty$?

at $t=0^-$

$I_{R1} = 0$

$I_{R2} = 0$

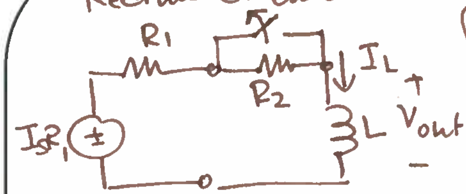
at $t=\infty$

$I_{R1} = \frac{R_2}{R_1+R_2} I_S$

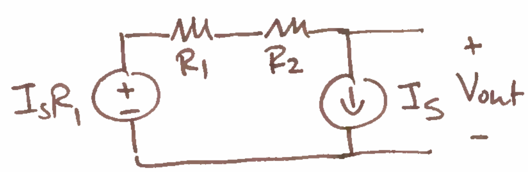
$I_{R2} = \frac{R_1}{R_1+R_2} I_S$

c. (5 points) Please *sketch* the voltage, $v_{out}(t)$, across the inductor vs. time.

Redraw circuit



at t=0 circuit looks like the following



At $t=0^-$, $V_{out}=0$

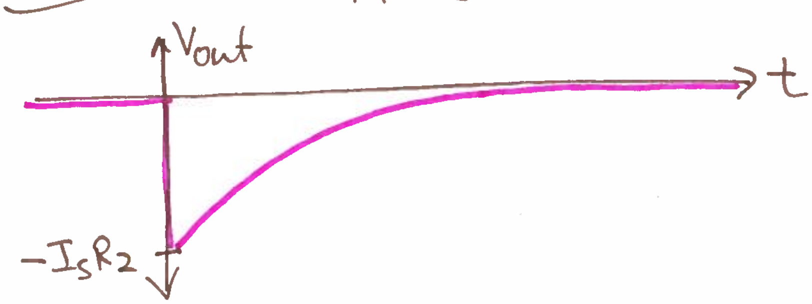
But $I_L = I_S$

$$\frac{I_SR_1 - V_{out}}{R_1+R_2} = I_S \Rightarrow V_{out}(t=0) = -I_SR_2$$

Also, at $t=\infty$

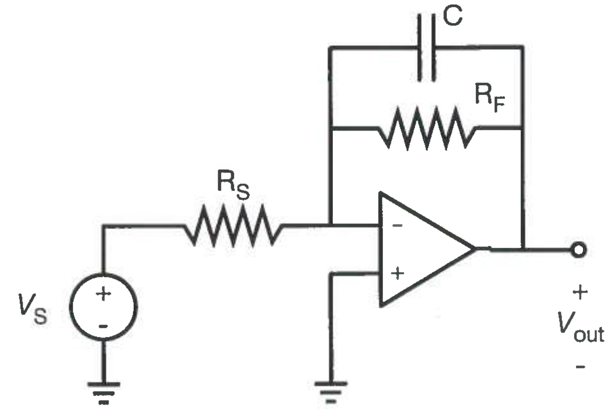
$V_{out}=0$

$I_L = \frac{I_SR_1}{R_1+R_2}$



Problem 5: Op-amp Circuit with C (5 points)

For this problem, assume V_s provides a unit voltage step, u(t), 0V for t<0 and transitions to 1V at t=0 and stays at 1V for t>0.



a. (5 points) For the op-amp circuit, sketch the resulting waveform for V_{out} versus time.

at $t=0^-$, $V_s=0$ + $V_{out}=0$

at $t=\infty$, $V_s=1$ + $V_{out} = -\frac{R_F}{R_s}\cdot V_s$

$= -\frac{R_F}{R_s}$