Problem Set 5

Due Date: Sat Oct 24 2015
by 9 PM

Name: Solutions

Lab Section & TF:____________

Collaborators:____________

For Grading Purposes Only:

Q1: _____ / 10
Q2: _____ / 10
Q3: _____ / 10
Q4: _____ / 15

Total:_____ / 45

Problem 1: Binary Numbers (10 points)

a. (5 points) Convert the following binary numbers to their decimal representations. **Please show your work, even if the answer seems obvious.**
   i. 101_2
   ii. 1011_2
   iii. 110010_2
   iv. 1101010_2
   v. 001111010_2

i) $1\times2^2 + 0\times2^1 + 1\times2^0 = 4+0+1 = \underline{5}$

ii) $1\times2^3 + 0\times2^2 + 1\times2^1 + 1\times2^0 = 8+0+2+1 = \underline{11}$

iii) $1\times2^5 + 1\times2^4 + 0\times2^3 + 0\times2^2 + 1\times2^1 + 0\times2^0$
$= 32 + 16 + 0 + 0 + 2 + 0 = \underline{50}$

iv) $1\times2^6 + 1\times2^5 + 0\times2^4 + 1\times2^3 + 0\times2^2 + 1\times2^1 + 0\times2^0$
$= 64 + 32 + 0 + 8 + 0 + 2 + 0 = \underline{106}$

v) $0\times2^8 + 0\times2^7 + 1\times2^6 + 1\times2^5 + 1\times2^4 + 1\times2^3 + 0\times2^2$
$+ 1\times2^1 + 0\times2^0$
$= 64 + 32 + 16 + 8 + 0 + 2 + 0 = \underline{142}$

b. (5 points) Convert the following decimal numbers to their binary representations

i. 42_{10}
ii. 50_{10}
iii. 101_{10}
iv. 1011_{10}
v. 2015_{10}

i) $42 \div 2 = 21$ $r = 0$ Least significant bit or LSB
$21 \div 2 = 10$ $r = 1$
$10 \div 2 = 5$ $r = 0$
$5 \div 2 = 2$ $r = 1$
$2 \div 2 = 1$ $r = 0$
$1 \div 2 = 0$ $r = 1$ most significant bit or MSB

$\Rightarrow$ $\underline{101010}$

ii) $50/2 = 25$ $r=0$ LSB
$25/2 = 12$ $r=1$
$12/2 = 6$ $r=0$
$6/2 = 3$ $r=0$
$3/2 = 1$ $r=1$
$1/2 = 0$ $r=1$ MSB

$\Rightarrow$ $\underline{110010}$

iii) $101/2 = 50$ $r=1$
see ii
$\Rightarrow$ $\underline{1100101}$

1(b) iv) 1011_{10}

$1011/2 = 505 \quad r = 1 \quad$ LSB

$505/2 = 252 \quad r = 1$

$252/2 = 126 \quad r = 0$

$126/2 = 63 \quad r = 0$

$63/2 = 31 \quad r = 1$

$31/2 = 15 \quad r = 1 \quad \Rightarrow \underline{1111110011}$

$15/2 = 7 \quad r = 1$

$7/2 = 3 \quad r = 1$

$3/2 = 1 \quad r = 1$

$1/2 = 0 \quad r = 1 \quad$ MSB

(v) 2015_{10}

$2015/2 = 1007 \quad r = 1 \quad$ LSB

$1007/2 = 503 \quad r = 1$

$503/2 = 251 \quad r = 1$

$251/2 = 125 \quad r = 1$

$125/2 = 62 \quad r = 1$

$62/2 = 31 \quad r = 0 \quad \Rightarrow \underline{111\ 1001\ 1111}$

$31/2 = 15 \quad r = 0$

$15/2 = 7 \quad r = 1$

$7/2 = 3 \quad r = 1$

$3/2 = 1 \quad r = 1$

$1/2 = 0 \quad r = 1 \quad$ MSB

Problem 2: Audio Signals (10 points)

Katy Perry is starting to work on her next album, and wants to make sure that it is the optimal length. Since it was her teenage dream to be an electrical engineer, she decides to calculate the maximum duration of stereo music a CD can store. Let's help her with this!

An audio CD can store 700 MB of data. The sampling rate of Katy's music is 44 kHz. In order to capture her perfect voice by avoiding significant clipping, Katy wants the quantizer to range from -8 V to + 8V and have a step size of (just over) 244 µV. Remember, Katy wants high quality stereo music:

a. (3 points) How many bits of quantization is she using?

$$\frac{16V}{244\mu V} = 65574$$

This is very close to $2^{16} = 65536$

So, 16 bits of quantization

b. (4 points) What is the duration of music that her CD can store?

Stereo requires two channels (left + right)

$$700\text{ MB} \cdot \frac{10^6 B}{MB} \cdot \frac{8\text{ bits}}{\text{Byte}} \cdot \frac{1\text{ sample}}{16\text{ bits}} \cdot \frac{1\text{ sec}}{44000\text{ samples}} \div 2\text{ channels}$$

$= 3977$ sec $= 66$ minutes $= 1.1$ hours

c. (3 points) What is the signal-to-noise ratio?

$SNR \approx \frac{3}{16} 4^b$ where $b = \#$ bits in quantizer

$SNR \approx 8 \times 10^8$

OR in decibels, $SNR_{dB} = 10 \log_{10}(SNR) = 89\text{dB}$

Problem 3: Sampling Signals (10 points)

a. (3 points) A 40 kHz sinusoidal is to be sampled. What is the minimum frequency that you can use to sample this signal so that it can be recovered without any distortions?

$f_s > 2 f_0$

so, $f_s > 80kHz$

b. (3 points) A 40 kHz sinusoidal is to be sampled at a frequency, $f_s = 60\ kHz$. What frequency will be observed when the signal is recovered?

$f_{alias} = |f_s - f_0| = 20\,kHz$

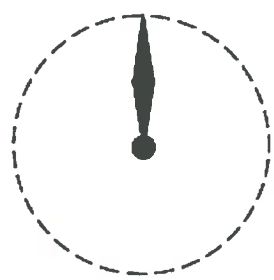
c. (4 points) What rate would you use to sample the following signal and why?
$y = \sin(2\pi\ 45\ t) + 2\ \sin(2\pi\ 125\ t)$

125Hz is the highest frequency in this signal so $f_s > 250Hz$ is sufficient. to satisfy Nyquist sampling criterion

Problem 4: (Non)-rotations (15 points)

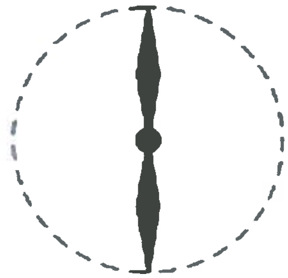
After an ES 50 lecture, one of your friends is really excited over the video that was shown of a moving helicopter with a non-rotating rotor. 'OMG, did you see that? How'd that happen?' After he repeats this at least 10 times, you're quite fed up. 'Look", you tell him; "the only reason you think that was so cool is because you were Facebooking your mother instead of paying attention to the unbelievably insightful lecture in ES 50!" You decide to totally impress him by giving him a math lesson with a camera and a toy helicopter. You know that reproducing the effect is merely a matter of calculating sampling frequency – in this case, the number of times you snap a picture in a given unit time, according to the correct *Nyquist criterion.*

a. (3 points) In grabbing the helicopter, your friend broke off part of the rotor on the top so it has only a single blade. If the rotor rotates with a frequency of f_0, write an expression for the time, T_0, taken for the rotor to look the same again once it starts moving. The rotor looks like this:



$T_0 = 1/f_0$

b. (3 points) Your friend somehow manages to fix the broken rotor. Write an expression for the new time, $T_{rotation}$, for the rotor to look the same again once it starts moving, in terms of the frequency f_0. The rotor now looks like this:



$$T_{rotation} = \frac{1}{2} T_0 = \frac{1}{2f_0}$$

b/c it takes half the time for the rotor to look the same. w/ two blades.

c. (3 points) While your friend debates how many more blades to add, you decide to one-up him by coming up with a formula relating T rotation, N (the number of blades) and the rotation frequency, f_0. Find the sampling frequency, which in this case is the number of camera frame-shots needed per second to avoid aliasing. (Note: the rotor will have blades arranged such that the angle between any given blade and the preceding/following blade in the circle will be equal).

$$T_{rotation} = \frac{1}{N} T_0 = \frac{1}{Nf_0}$$

$$f_{rotation} = \frac{1}{T_{rotation}} = Nf_0$$

$$f_s > 2 f_{rotation} \Rightarrow \underline{f_s > 2Nf_0}$$

d. (3 points) Your friend creates an 8-bladed rotor as shown below (the angles between all the blades are equal). If the 8-bladed rotor were rotating at 1.5 Hz (1.5 rotations per second), would aliasing be seen if you snapped pictures at 30 frames per second?

$f_s = 30\,Hz$

$2N f_{rotation} = 24\,Hz$

Since $f_s > 2N f_{rotation}$, NO ALIASING.

e. (3 points) If the rotor is rotating at 50 Hz and there are 8 spokes, find the expression for all sampling frequencies (frames/sec) for which the rotor appears stationary.

Given 8 blades, they look stationary if $T_s = \frac{T_0}{8}$

OR $f_s = 8 f_0 = 400\,Hz$

BUT, WHAT IF at $f_s = 200\,Hz$? The rotor moved by 2 blades. At $f_s = 400/3$, rotor moved by 3 blades and so forth.

So... $f_s = \frac{400\,Hz}{N}$ for any integer N