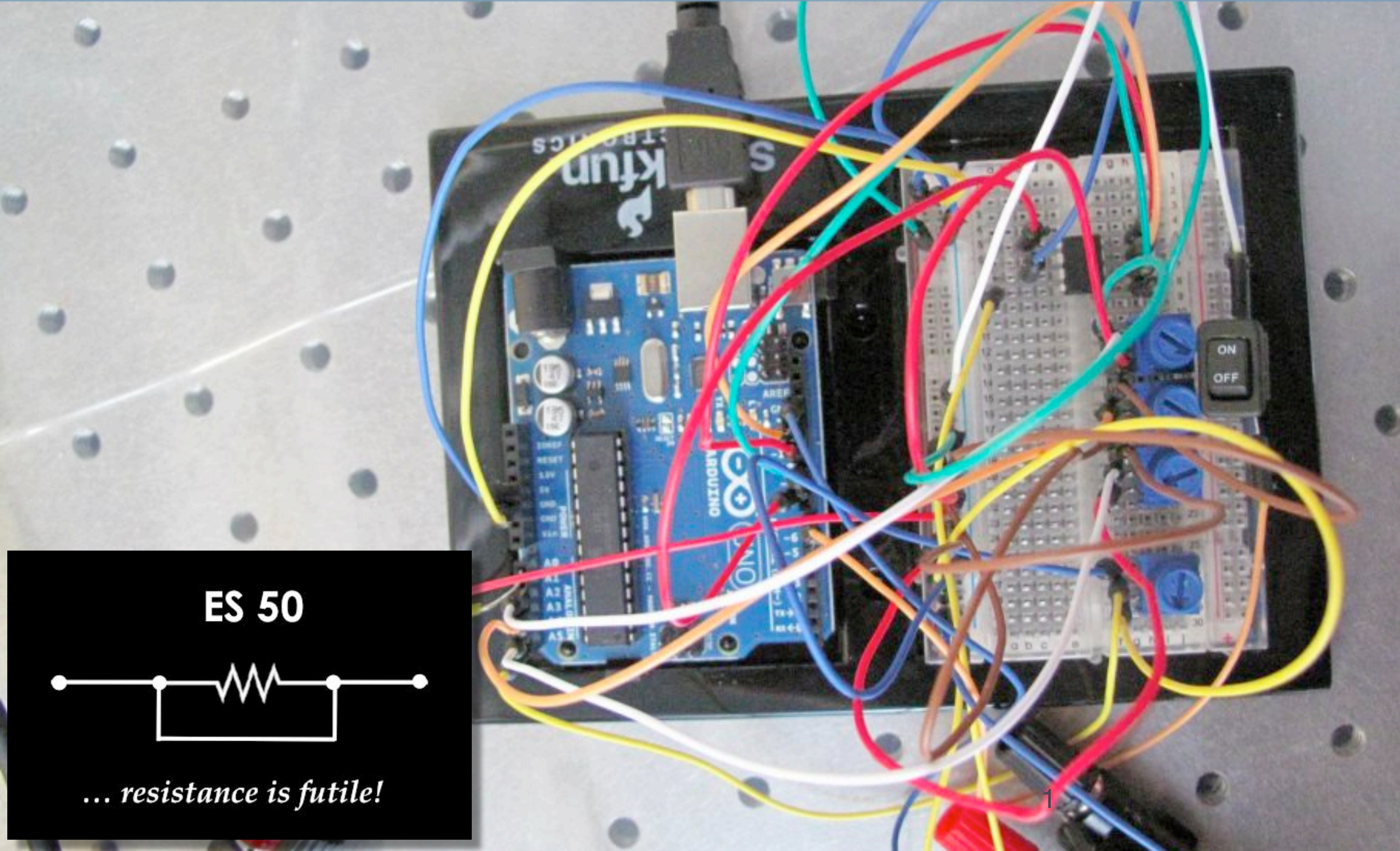
ES 50: Introduction to Electrical Eng.

Explore EE, satisfy a Gen Ed requirement & have fun



Lecture 7: Feedback

ES 50: Introduction to Electrical Engineering

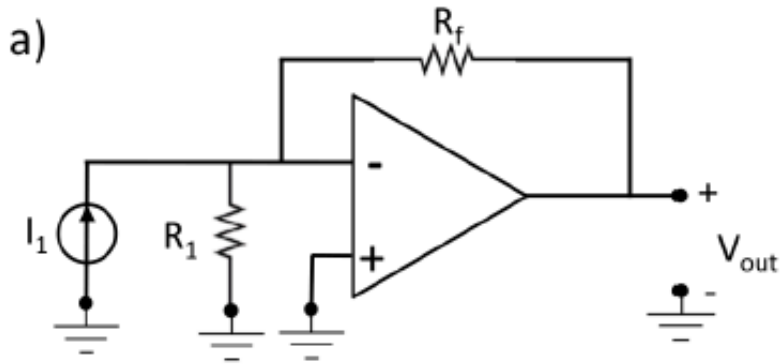
Announcements

- In-class Quiz #1 on Wed, Oct 7… It's next week!
  - Everything up to and including inductors and capacitors
  - Allowed one 8.5x11 inch sheet of notes (front and back)
  - Calculators allowed, computers not allowed
- No pset due Oct 10 (week of quiz #1), but labs **will** continue next week
- We are moving next week's review sessions to Mon (10/5) & Tues (10/66) evenings… Location TBD
- No review sessions next week Wed (10/7) or Thurs (10/8)
- If you need special accommodations for quiz #1, please email Gu and Chris ASAP.

Today,

- Finish up op-amp examples
- Feedback: A more formal treatment

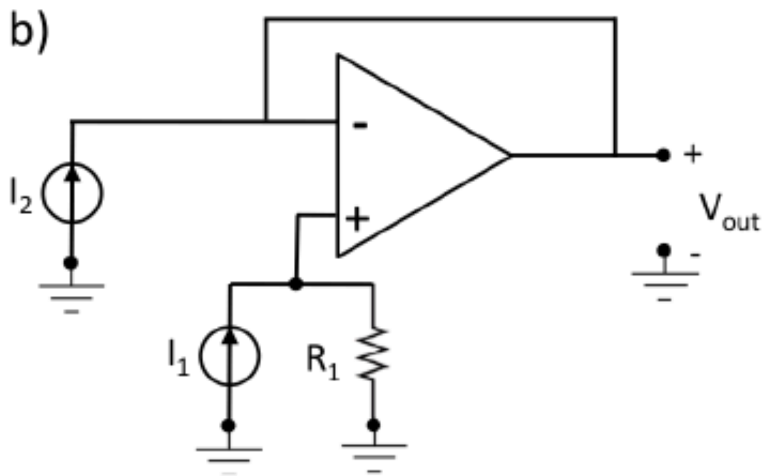
Example 2



$$\frac{V_{out}}{I_1} = -R_f$$

- For the circuit (a) above, find V_{out} in terms of R_1, R_f, I_1, and I_2. Hint: Can we use Thevenin and Norton equivalent circuits?

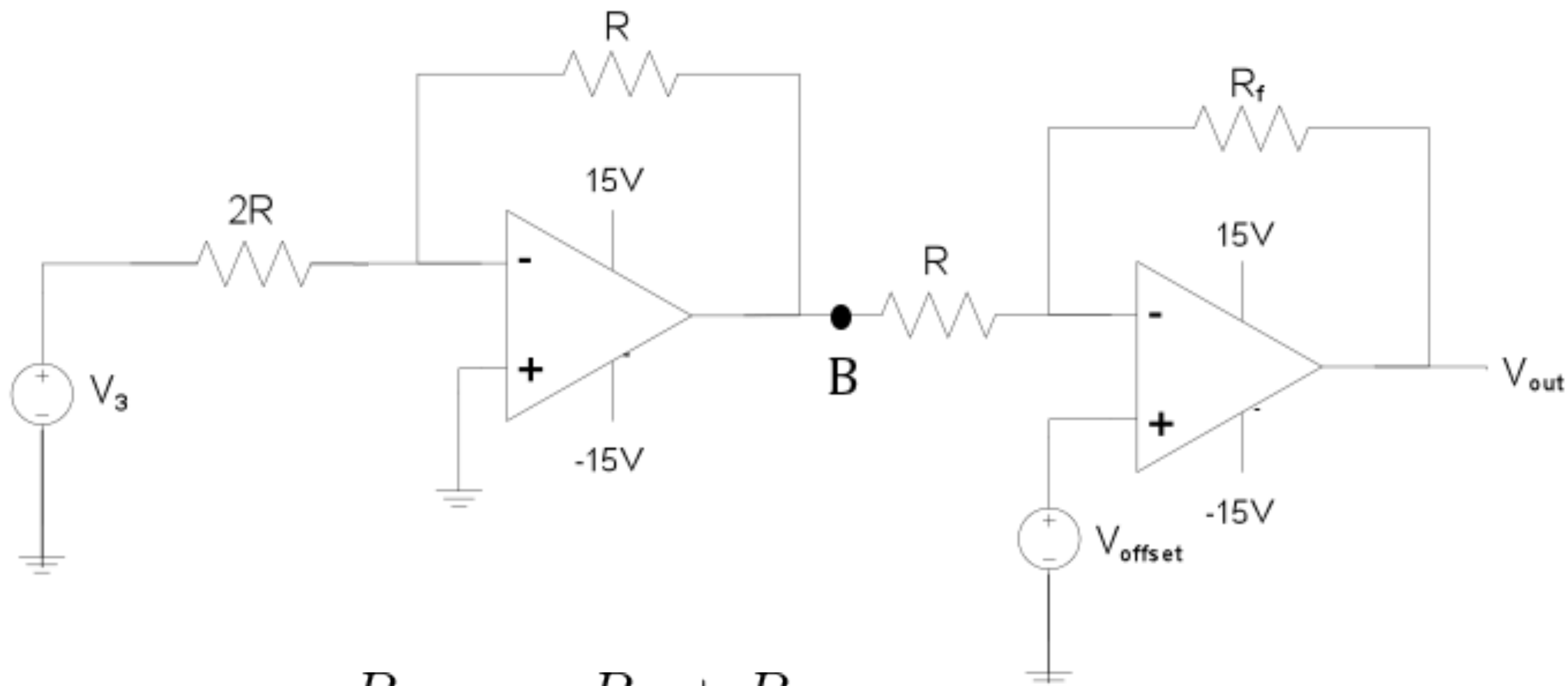
Example 3



$$\frac{V_{out}}{I_1} = R_1$$

- For the circuit (b) above, find V_{out} in terms of R_1, R_f, I_1, and I_2. Hint: Can we use Thevenin and Norton equivalent circuits?

Example 4

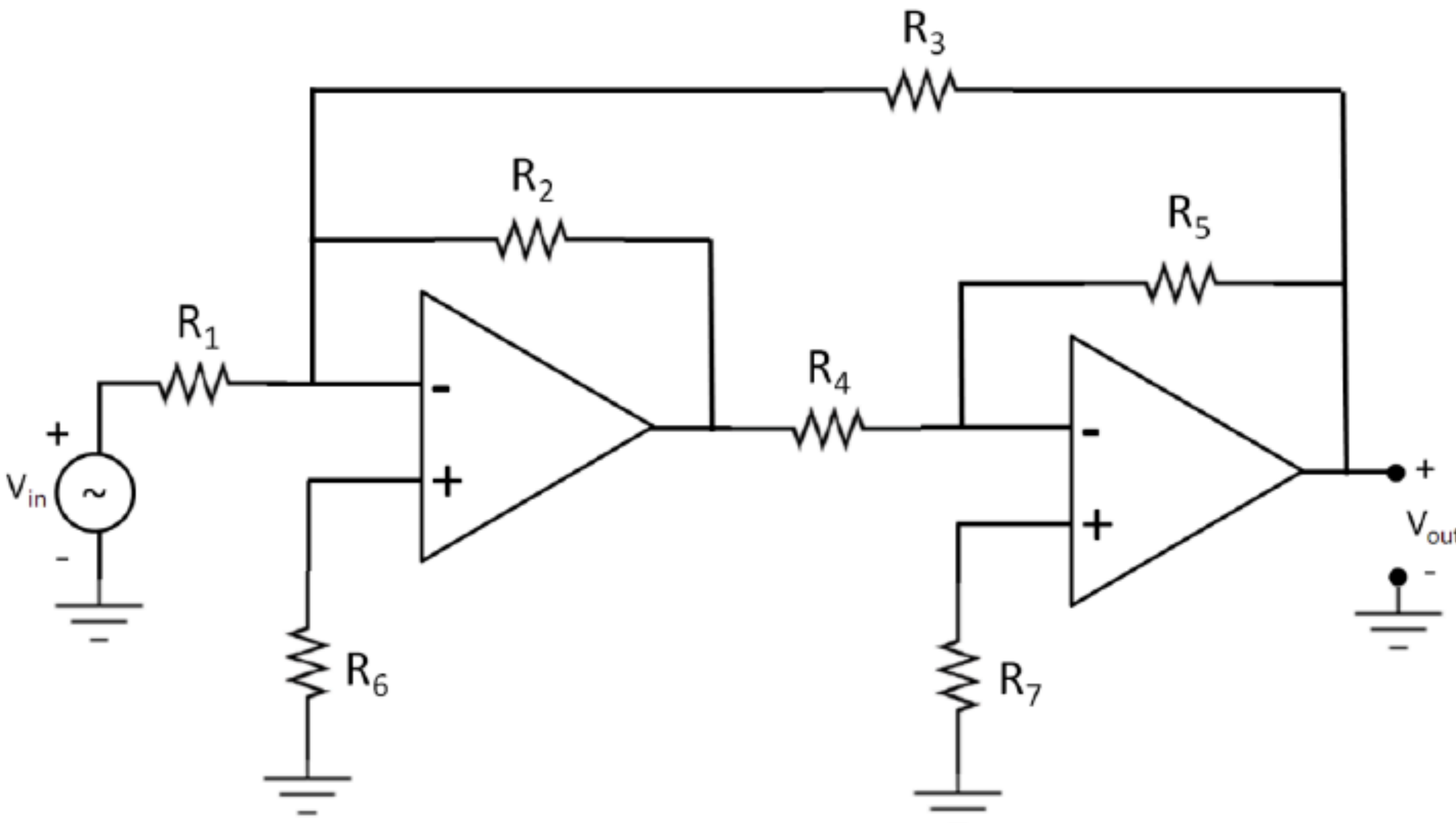


$$V_{out} = \frac{R_f}{2R} V_3 + \frac{R_f + R}{R} V_{\text{offset}}$$

- Find V_{out} in terms of V_3, R, R_f, and V_{offset}.

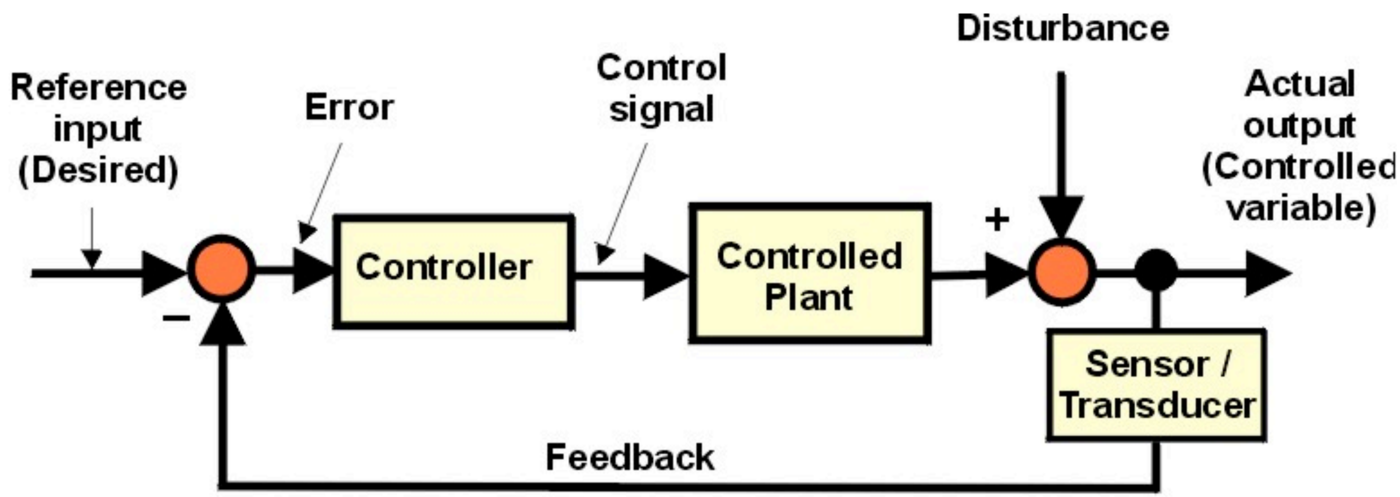
Last example

$$\frac{V_{out}}{V_{in}} = \frac{R_2 R_3 R_5}{R_1 R_3 R_4 - R_1 R_2 R_5}$$



- Find V_{out}/V_{in} in terms of R_1~R_7.

Feedback: A general discussion (basics of control theory)

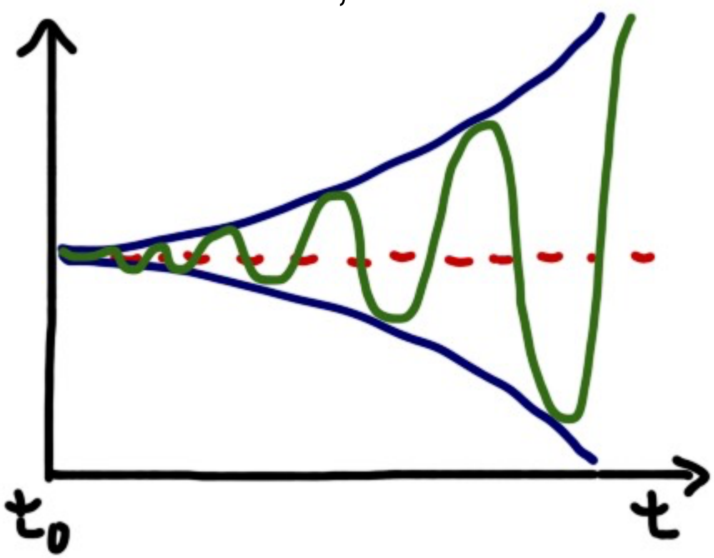


- **<u>Feedback:</u>** Used to provide a desired output in spite of fluctuations, disturbances, nonlinearity, etc. in the system
- **<u>The main idea:</u>** The result of an action is fed back to the input and compared to the desired outcome.
- The principles of feedback are widespread, and can be found in many disciplines:
  - Robotics, electrical and mechanical engineering, manufacturing plants, navigation systems, ecology (wolves and rabbits), business & economics, social systems (health care), and many many more

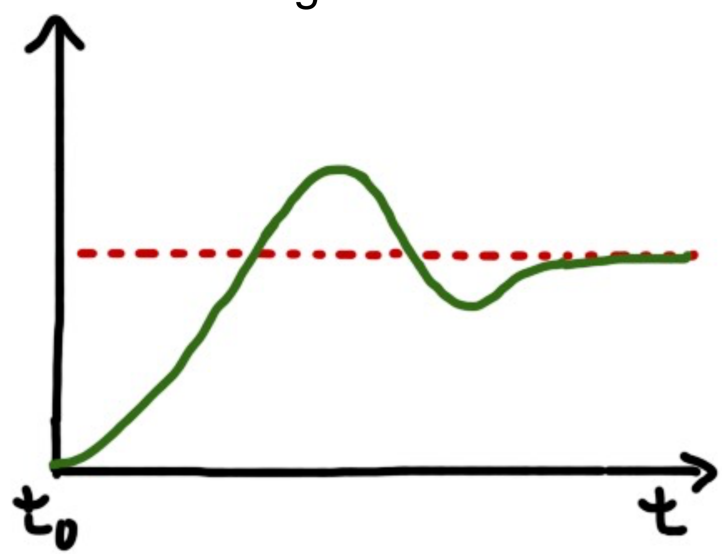
Positive vs. Negative Feedback

- **Positive feedback:** Fed-back signal alternates action in the same direction as preceding result
- **Negative feedback**: Fed-back signal provides action in the opposite direction

Positive feedback:
cumulative, unstable



Negative feedback:
stabilizing



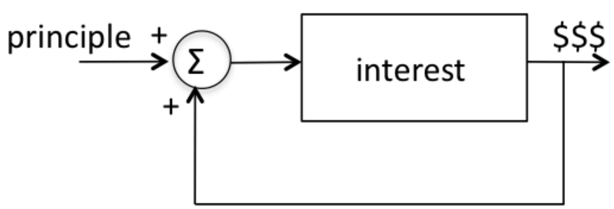
Positive feedback: Can make the system unstable

Negative feedback: Can stabilize the system

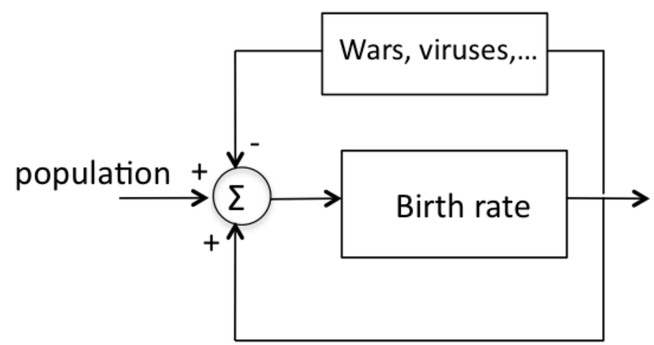
Examples of positive/negative feedback

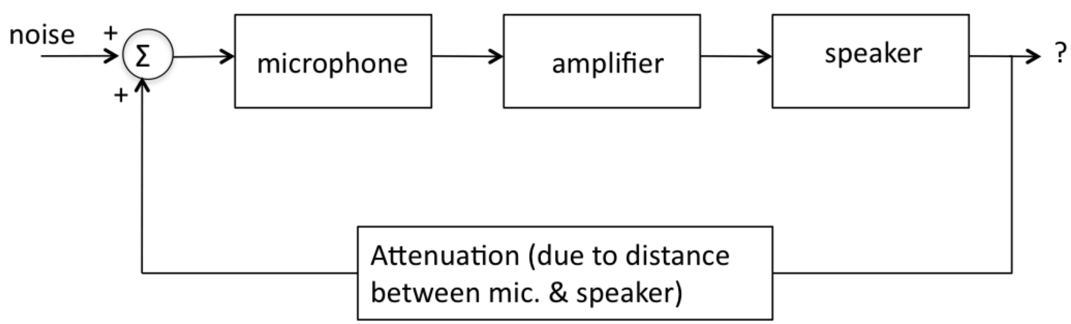
- **Q:** What kind of feedback is this?

Compound Interest



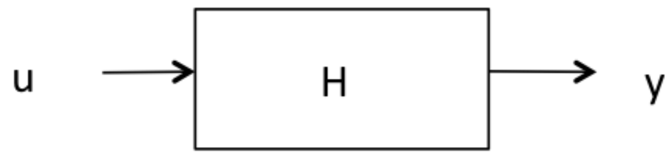
Population





Block Diagram Representation

- We will use block diagrams to study *systems* and compute their output as a function of their inputs.

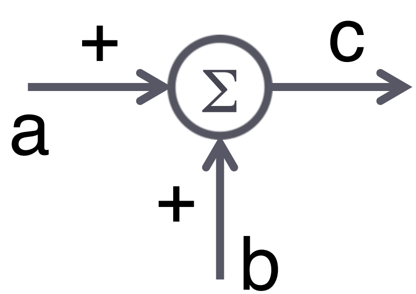


- This is a block-diagram representation for: $\boldsymbol{y = H * u}$
where $\boldsymbol{u}$ is an input**,** $\boldsymbol{y}$ is the output**,** and $\boldsymbol{H}$ (= y/u) is called the *transfer function* of the system.

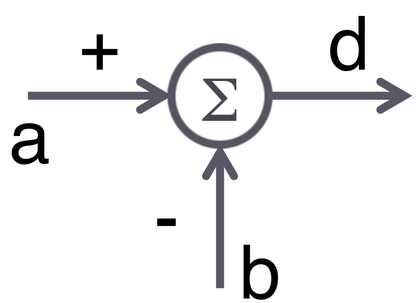
- <u>Example:</u> If H is an amplifier with gain K, then $y = K * u$. In this case, $\boldsymbol{u}$ and $\boldsymbol{y}$ are voltage potentials.
- BTW, H does not have to be linear! It can be a nonlinear function.
- Also, H can have different response for different frequencies – for example, filters – low-pass filter, high-pass filter, band-pass filter

Important notations

$$y = H \cdot u$$

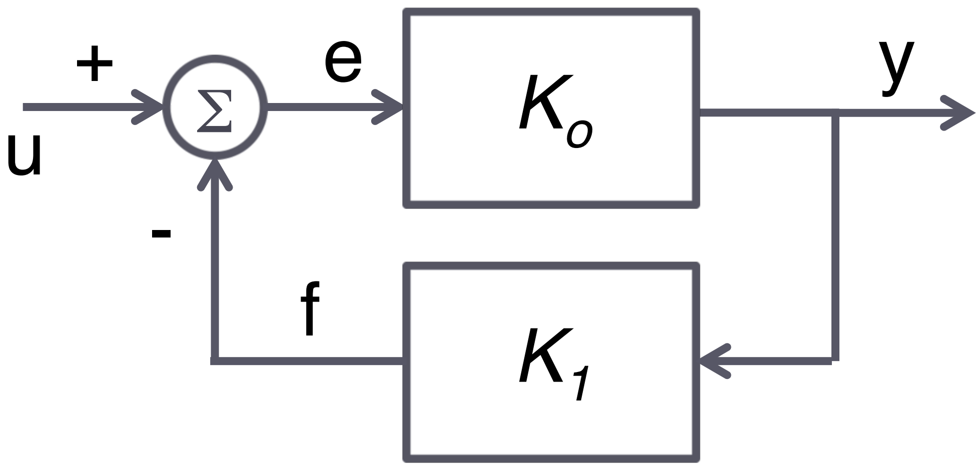


$$c = a + b$$



$$d = a - b$$

Negative Feedback Example

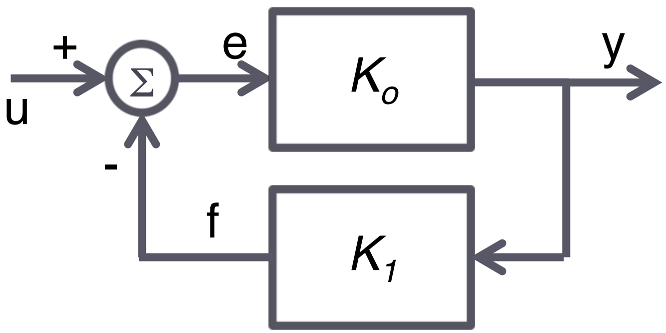


Let's find the closed-loop gain

$$
\begin{aligned}
y &= K_o e \\
f &= K_1 y = K_o K_1 e \\
e &= u - f = u - K_o K_1 e \\
u &= e\left(1 + K_o K_1\right) \\
e &= u \frac{1}{1 + K_o K_1} \\
y &= K_o e = u \frac{K_o}{1 + K_o K_1} \\
H_{\text{closed-loop}} &= \frac{y}{u} = \frac{K_o}{1 + K_o K_1}
\end{aligned}
$$

- Lingo:
  - K_o : feed-forward or open-loop gain
  - K_1 : feed-back gain
  - K_o*K_1 : loop gain
  - H : overall closed-loop gain

Where have we seen this before?



$$y = u \frac{K_o}{1 + K_o K_1}$$

$$H_{\text{closed-loop}} = \frac{y}{u} = \frac{K_o}{1 + K_o K_1}$$

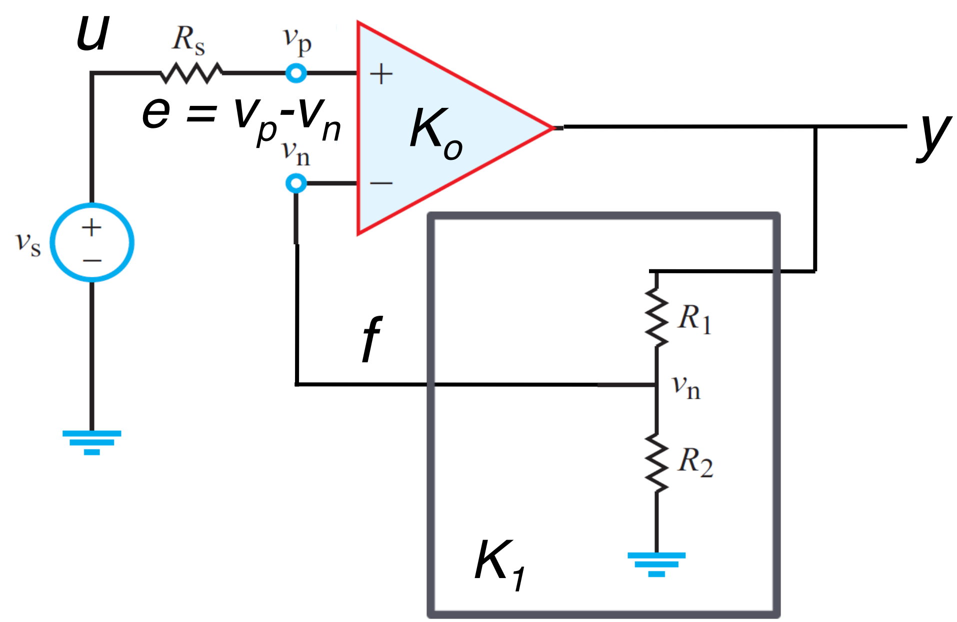
$$K_o = A$$

$$K_1 = \frac{R_2}{R_1 + R_2}$$

$$H_{\text{closed-loop}} = \frac{A}{1 + A\frac{R_2}{R_1+R_2}}$$

$$A \rightarrow \infty$$

$$H_{\text{closed-loop}} = \frac{R_1 + R_2}{R_2} = G$$

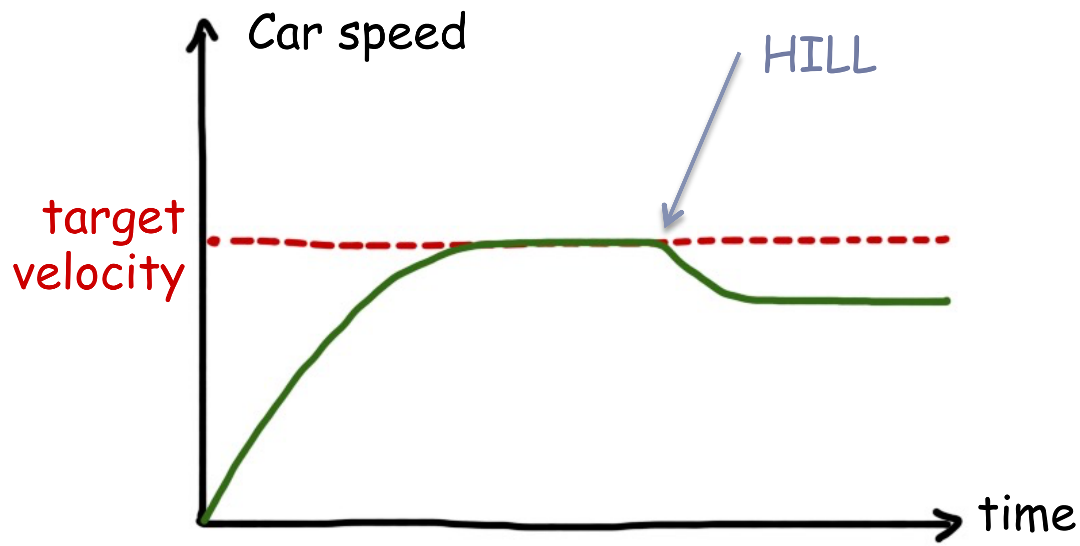


What is the benefit of a closed loop system?

Tracking: The most important application of feedback and control theory in general

Example: *Cruise control in your car is a closed loop system.* The goal is to have car cruise at a constant speed (e.g., 60 mph) regardless of external disturbances (e.g., hills, bumps, etc.).

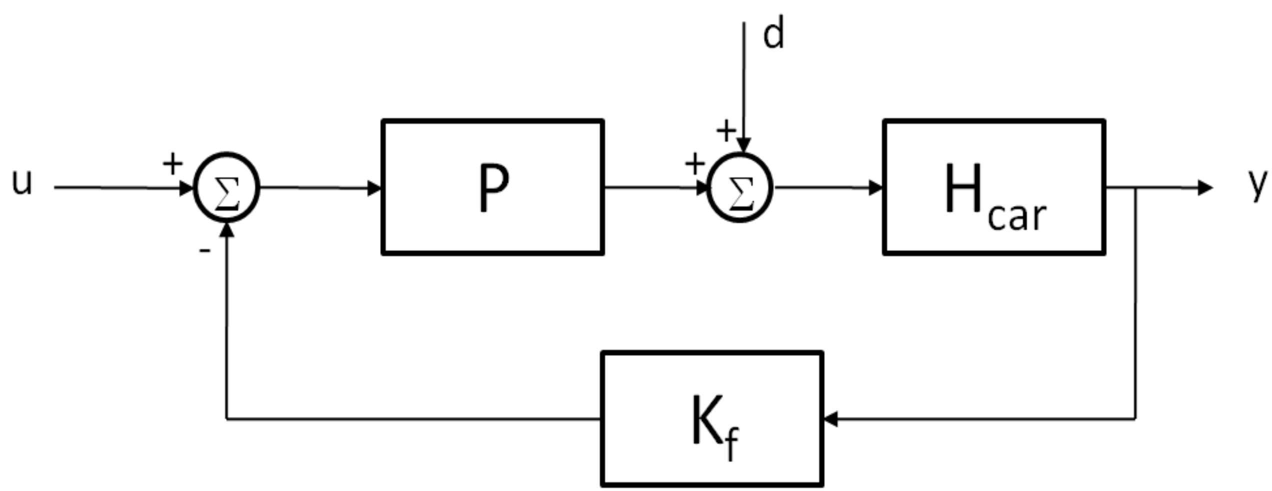
- Without feedback, the speed of your car as a function of time could look like the following…



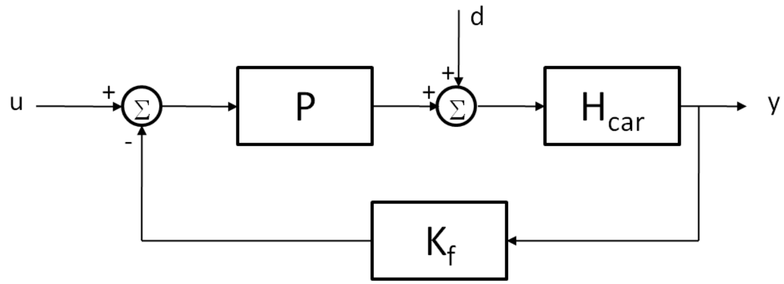
where the drop in the speed is due to climbing up a steep hill.

Example of feedback: Cruise control

- In order to maintain the speed we can add feedback to the system (car)
  1. Measure the speed (y) of car (block K_f) (e.g., speedometer)
  2. Compare it to the desired speed setting (u)
  3. Take some action (more/less gas pedal) based on these parameters (block P)
  4. Gas pedal adjusts speed of the car (system H_{car})
- *d* is a disturbance that affects the car (e.g., hill)



Example of feedback: Cruise control



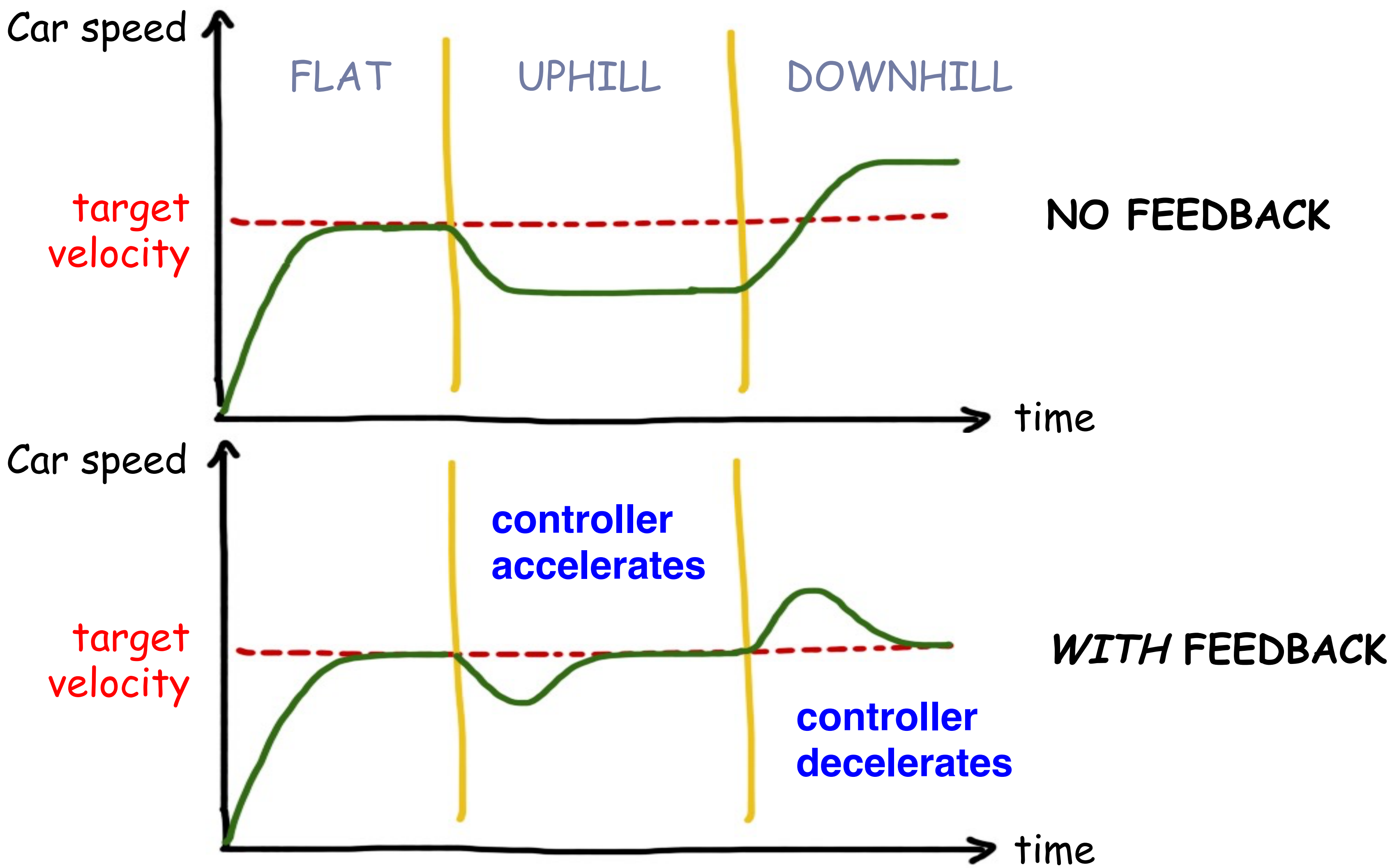
$$y = (P \cdot e + d)\, H_{\mathrm{car}}$$
$$e = u - K_f \cdot y$$

$$\begin{aligned} y &= (P\,(u - K_f \cdot y) + d)\, H_{\mathrm{car}} \\ y &= P H_{\mathrm{car}} u - P K_f H_{\mathrm{car}} y + H_{\mathrm{car}} d \\ y\,(1 + P K_f H_{\mathrm{car}}) &= P H_{\mathrm{car}} u + H_{\mathrm{car}} d \end{aligned}$$

$$y = \frac{P H_{\mathrm{car}}}{1 + P K_f H_{\mathrm{car}}} u + \frac{H_{\mathrm{car}}}{1 + P K_f H_{\mathrm{car}}} d$$

Result of feedback: As *P* increases, discrepancy between y and u is reduced! So, it appears that it is beneficial to add a lot of gain (P) in the system to reduce the influence of disturbances.

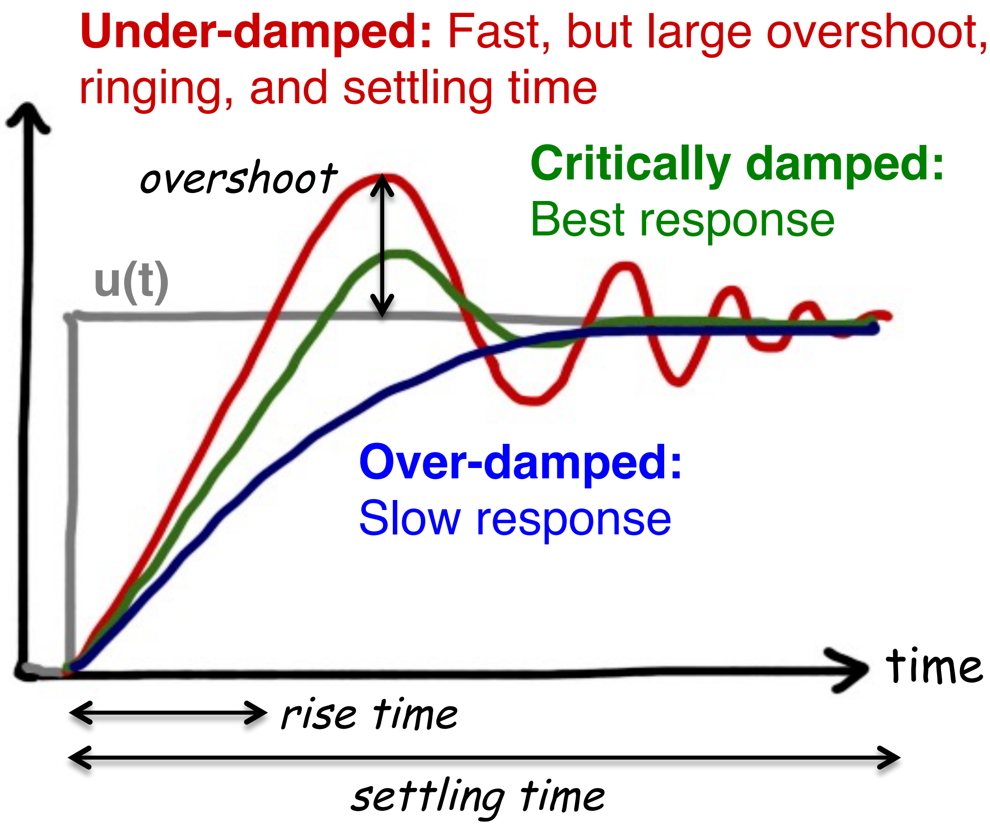
Example of feedback: Cruise control



- Q: What would you expect from the controller in terms of response time, errors, etc?

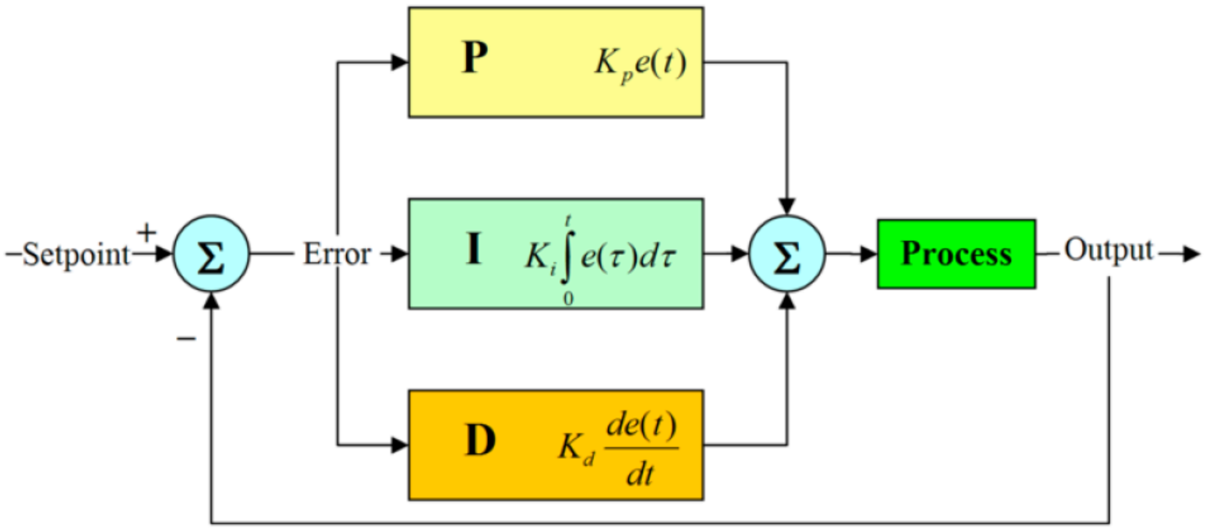
Controller

- <u>Control circuit needs to:</u>
  - Respond quickly
  - Have minimal oscillations
  - Have small overshoot

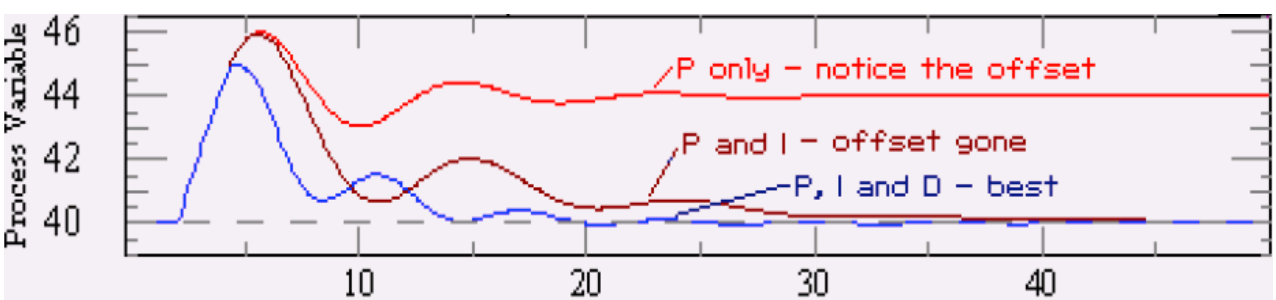


Therefore, the control circuit needs to:
<u>Cancel present error:</u> Use **proportional control**, which can be achieved by adding *a lot of forward gain* to the system. However, in practice, this can result in oscillations due to presence of noise at the input
<u>Average out past errors:</u> Use **integral control**, which makes sure that error goes to zero. This cannot be done using *proportional* control only!
<u>Anticipate future errors:</u> Use **derivative control**, which predicts the future a bit to significantly speed up the response of the system.

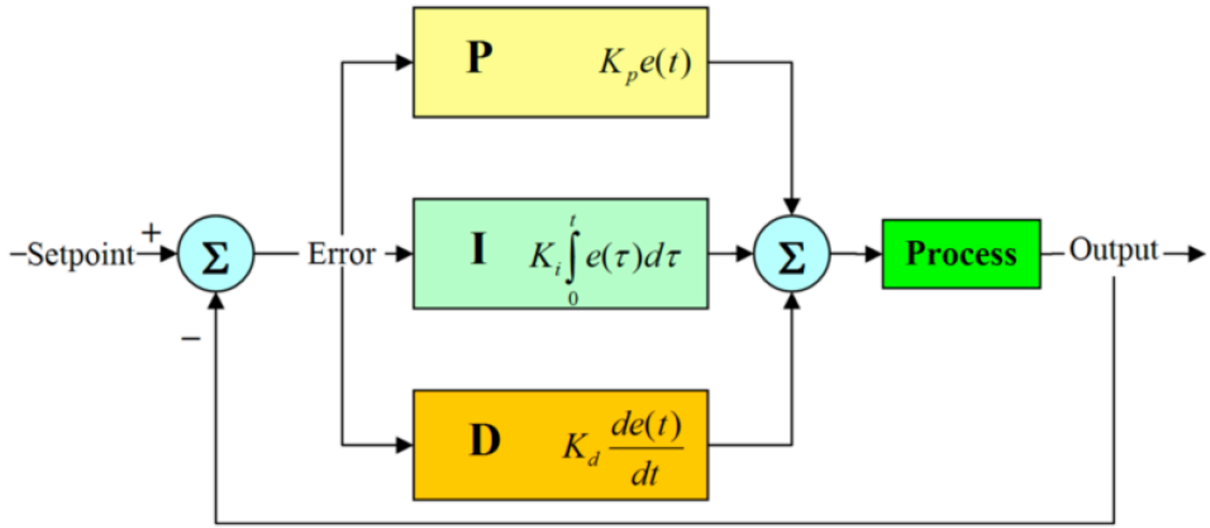
PID Controller



What is the influence of each of these parameters P, I, and D? Consider following example that shows change of car speed, as a function of time, in response to disturbance (e.g., going downhill) at t=2.



PID Controller

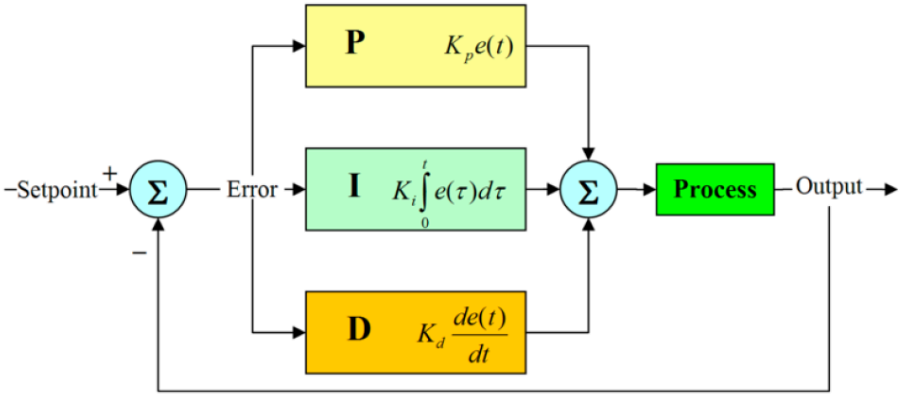


Effects of different PID parameters:

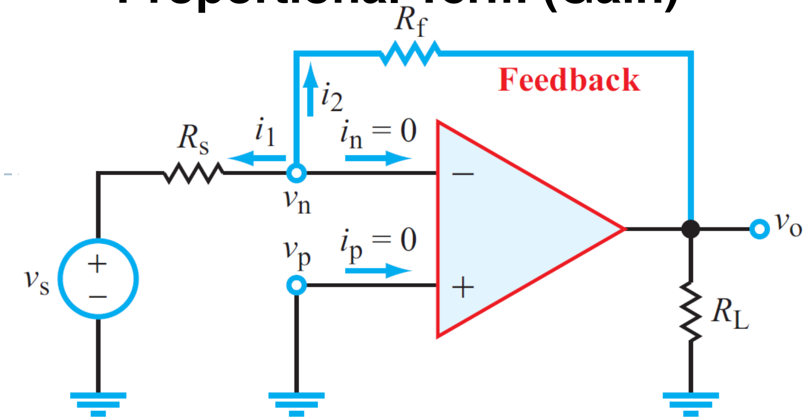
Parameter	Rise time	Overshoot	Settling Time	Steady State Error
P (K_P)	Decreases	Increases	Small effect	Decreases
I (K_I)	Decreases	Increases	Increase	Eliminates it
D (K_D)	Small effect	Decreases	Decrease	No effect

How do we make these integrators and differentiators?

PID Controller

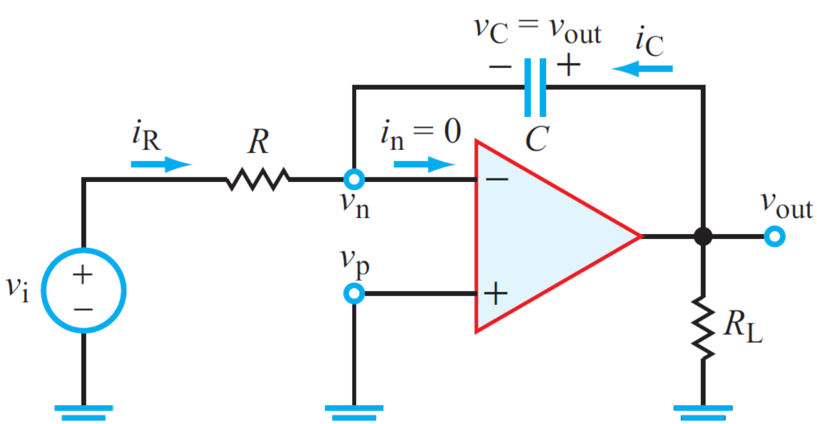


Proportional Term (Gain)



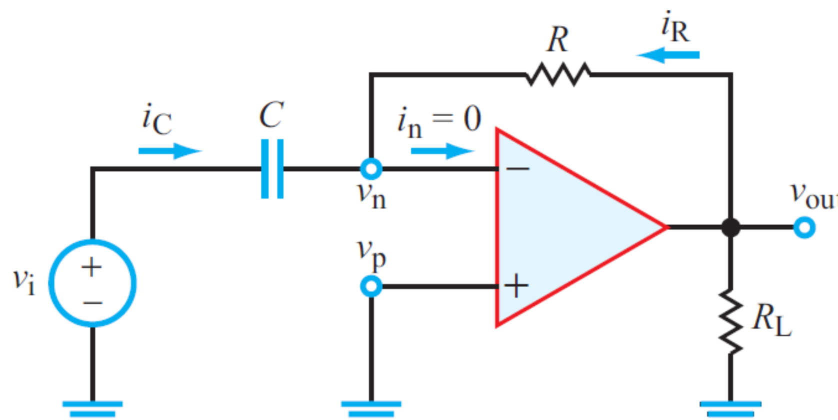
$$G = \frac{v_o}{v_s} = -\left(\frac{R_f}{R_s}\right).$$

Integral Term



$$v_{out}(t) = -\frac{1}{RC}\int_{t_0}^{t} v_i \, dt + v_{out}(t_0).$$

Derivative Term



$$v_{out} = -RC\frac{dv_i}{dt},$$