Problem Set 3

Due Date: Sat Oct 3 2015
by 9 PM

Name:________________________________

Lab Section & TF:______________________

Collaborators:_________________________

For Grading Purposes Only:

Q1: _____ / 10
Q2: _____ / 11
Q3: _____ / 5
Q4: _____ / 5
Q5: _____ / 32
Q6: _____ / 8
Q7: _____ / 11

Total:_____ / 82

Instructions:

Please place your homework in the appropriate Dropbox in the basement of Maxwell Dworkin.

Please staple your homework. If your homework is not stapled, please hand it in with your name written on every page, all the pages numbered front & back, and the total number of pages in the homework written on the first page (so we don't lose any of your work).

Show all your work. If we can't figure out how you reached the answer, you won't get credit. More importantly, if you show your work but messed up the figures, you can get partial credit for your thought process.

MENTION ALL YOUR COLLABORATORS and which problems you collaborated on. There is no negative marking for collaborating on problems.

Problem 1: Engineering Course Survey (10 points)

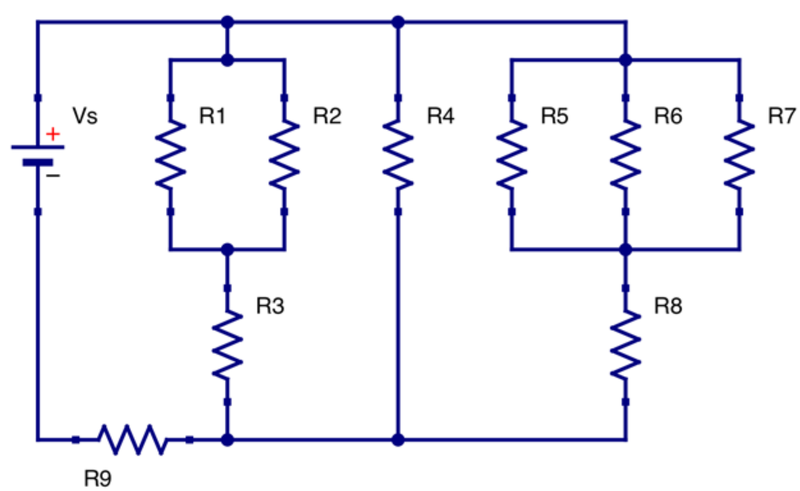
a. Please follow the link below and fill out the course survey for introductory engineering courses. You will be graded on completing the survey. While we ask for your names and HUIDs for purposes of grading the assignment, your responses to the survey will not be associated with your name and HUID. If you have any questions or concerns about completing this survey please contact Dr. Lombardo.

   Please access the survey using one of the following methods:
   - https://harvard.az1.qualtrics.com/SE/?SID=SV_3L60LwrXJFFbQb3
   - https://goo.gl/Y6HPqf

Problem 2: Circuit Drawing Method (11 points)

a. Show the schematic of the resistive network that consists of resistors R_1 through R_9 according to the following description:
   - R_1 and R_2 are connected in parallel
   - R_3 is connected in series with the combination of R_1 and R_2
   - R_5, R_6, and R_7 are connected in parallel
   - R_8 is connected in series with combo of R_5 through R_7
   - The combo of R_5 through R_8 is in parallel with the combo of R_1 through R_3
   - R_4 is connected in parallel with the combo of R_5 through R_8 and R_1 through R_3
   - R_9 is in series with the network synthesized above.

   The whole network is powered by voltage source V_s. Include your voltage source in the schematic too. (3 points)



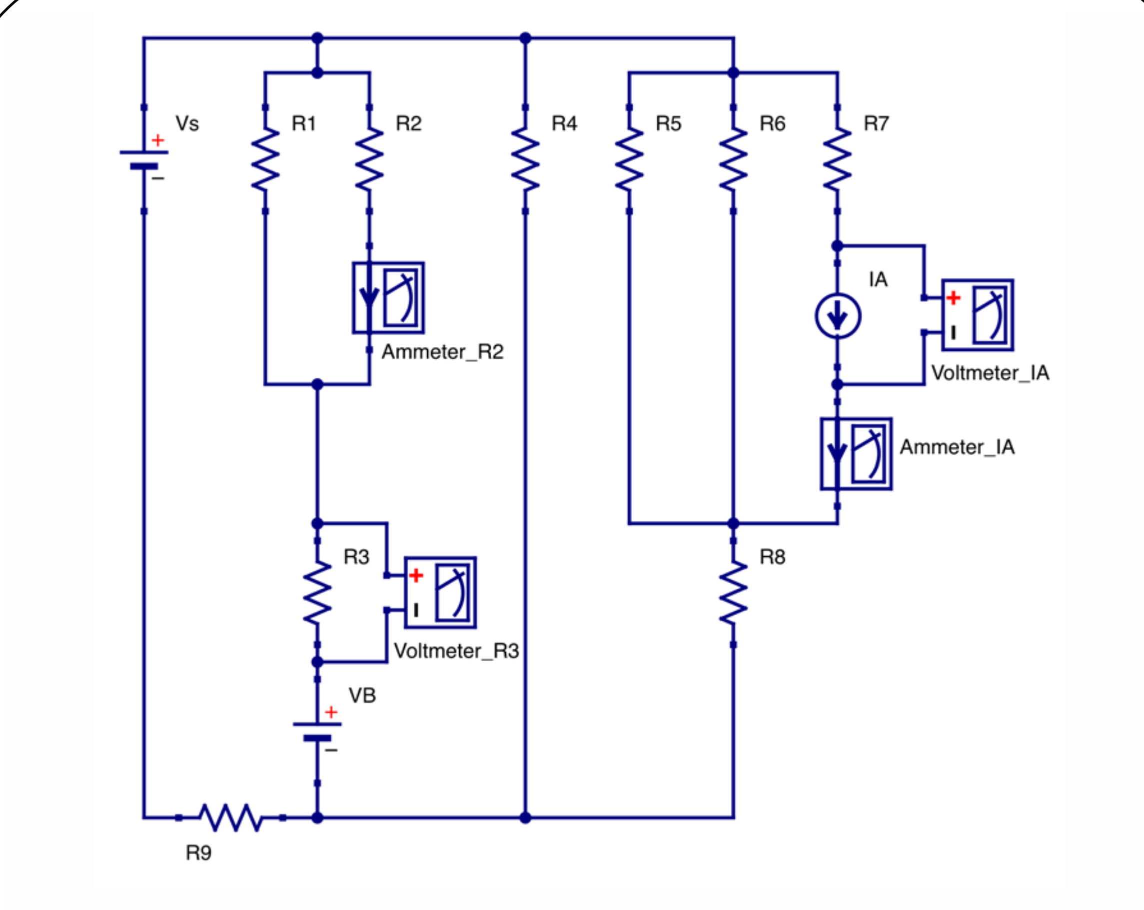
b. Find the expression for the overall resistance of the resistive network "seen" by the voltage source V_s. (2 points)

Note: expression means if you have, for e.g. 2 resistors in series, in parallel with the third resistor, the total resistance expression would be $(R_1 + R_2)||R_3$. Use parentheses to clearly delineate your combinations.)

$$R_{eq} = \{[(R_1||R_2) + R_3] \; || \; R_4 \; || \; [(R_5 \; || \; R_6 \; || \; R_7) + R_8]\} + R_9$$

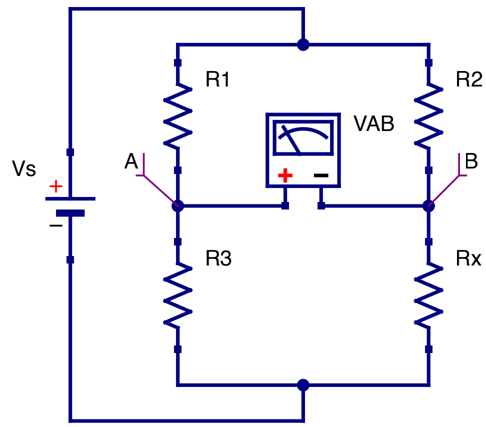
c. Redraw the circuit and add the following: (6 points)

- A current source I_A in series with R_7, and voltage source V_B in series with R_3.
- An ammeter to measure current through R_2.
- A voltmeter to measure voltage across R_3.
- Include voltmeter and ammeter to measure the current provided by the current source I_A and voltage on its terminals.



Problem 3: Useful Circuits – Wheatstone Bridge (5 points)

A Wheatstone bridge, shown below, can be used to measure the resistance of an unknown resistor R_x.



a. What is the value of R_x when the Wheatstone bridge is balanced, that is when voltmeter measures $V_{AB} = 0\ V$? (Please find R_x in terms of R_1, R_2, and R_3) (5 points)

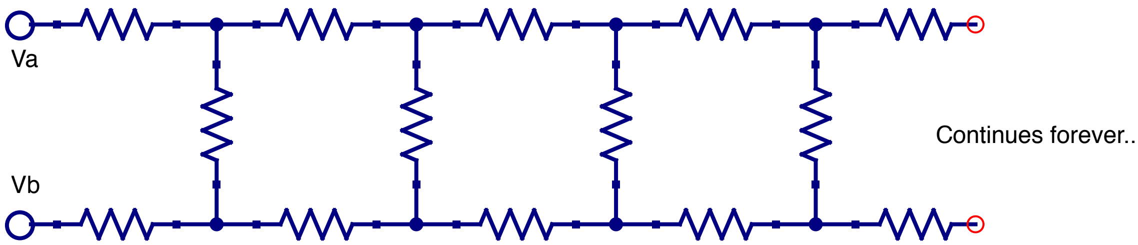
If $V_{AB} = 0$, then we know that current through the voltmeter is zero so we know that $V_A = V_B$. Since there is no current flowing from A to B, we can treat the voltage dividers containing $R_1 + R_3$ and $R_2 + R_x$ separately. On the left side, $V_A = V_s \frac{R_3}{R_1+R_3}$ and on the right side, $V_A = V_s \frac{R_x}{R_2+R_x}$. Since $V_A = V_B$:

$$V_s \frac{R_3}{R_1 + R_3} = V_s \frac{R_x}{R_2 + R_x}$$

$$R_x = \frac{R_2 R_3}{R_1}$$

Problem 4: Equivalent Resistance (5 points)

We have an infinite network of resistors, with each resistor having a resistance R (shown below)



a. Find the resistance between V_a and V_b in terms of R. In other words, what resistance would you measure if you attach a digital multimeter between points V_a and V_b?

$$R_{eq} = 2R + (R||R_{eq}) = 2R + \frac{RR_{eq}}{R + R_{eq}}$$

$$R_{eq}(R + R_{eq}) = 2R(R + R_{eq}) + RR_{eq}$$

$$R_{eq}{}^2 - 2RR_{eq} - 2R^2 = 0$$

Use quadratic formula:

$$R_{eq} = \frac{2R \pm \sqrt{4R^2 - 4(-2R^2)}}{2} = R \pm \sqrt{3}R$$

Must choose "+" because negative resistance is not possible.

$$R_{eq} = R(1 + \sqrt{3})$$

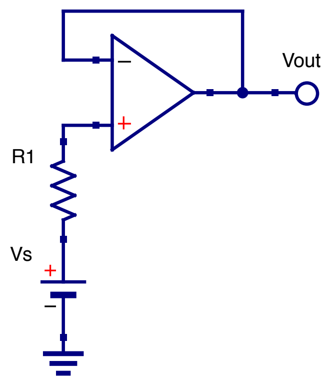
Problem 5: Understanding Op-Amp Connections (32 points)

This question is meant to help you understand what the effect of connecting different components to different parts of the op-amp does. So as you work through each part, note the effect of each component on the final answer.

KNOW THE IDEAL OP-AMP RULES LIKE THE BACK OF YOUR HAND. IF YOU DON'T KNOW THEM, YOU WILL STRUGGLE IN THIS QUESTION – NOT TO MENTION IT WILL TAKE TWICE AS LONG.

Note: Remember, we always want to express our output voltages using ONLY our known component values: voltages for voltage sources, resistances for resistors, etc. Throughout this question, you should express output voltages using known component values.

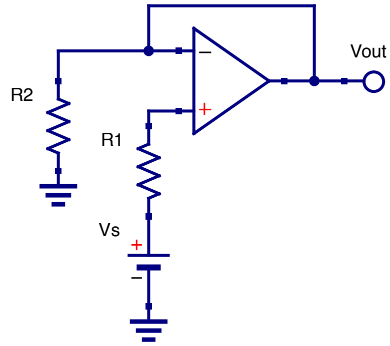
a. What is the current through R_1? What is V_{out} in following circuit? (2 points)



$$I_{V_+} = 0 \rightarrow I_{R_1} = 0$$

$V_{out} = V_-$ and $V_- = V_+$ due to the "virtual short between" the inputs of the op-amp so, $V_{out} = V_+$. Now since $I_{R_1} = 0$, there will be no voltage drop across R_1, so $V_- = V_s$. Therefore, $V_{out} = V_s$.

b. Let us modify the circuit by adding a resistor to the inverting-input terminal, V_-, of the op-amp. What is the current through R_1 and R_2? What is V_{out} now? (3 points)



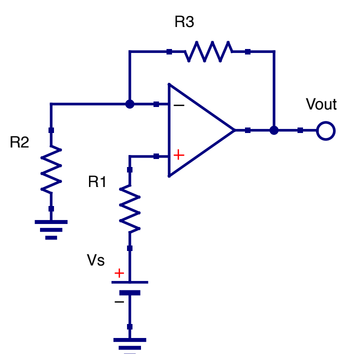
$$I_{V_+} = 0 \rightarrow I_{R_1} = 0$$

$$I_{V_-} = 0 \rightarrow I_{R_2} = \frac{V_{out}}{R_2} = \frac{V_-}{R_2} = \frac{V_+}{R_2}$$

Just as before since $I_{R_1} = 0$, there will be no voltage drop across R_1, so $V_+ = V_s$ and $I_{R_2} = \frac{V_s}{R_2}$.

Also just as before, $V_{out} = V_s$.

c. Now let us add a resistor to the feedback from the output (shown below). What is the current through R_1, R_2, and R_3? (3 points)



$I_{V_+} = 0 \rightarrow I_{R_1} = 0$, same as before.

Also as before, $V_- = V_+ = V_s$ so $I_{R_2} = \frac{V_s}{R_2}$.

$$I_{R_3} = I_{R_2} = \frac{V_s}{R_2}$$

d. What is the output voltage? (3 points)

$$V_{out} = V_{-} + V_{R_3} = V_s + I_{R_3} R_3 = V_s + \frac{V_s}{R_2} R_3$$

$$V_{out} = V_s \left(1 + \frac{R_3}{R_2}\right)$$

e. What happens with the output voltage if we increase each resistor, one by one, by a factor of 10? First R_1, then R_2, and then R_3? (3 points)

Note: you do not necessarily have to do a lengthy calculation if it is evident to you – just state the answer.

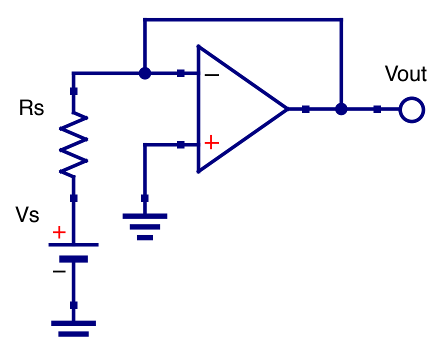
Increasing R_1 by a factor of 10 does nothing.

Increasing R_2 by a factor of 10 decreases the gain.

Increasing R_3 by a factor of 10 increases the gain.

The amount that the gain changes is based on the relative values of R_2 and R_3.

Let us go off on a different track. Suppose we reverse the diagram entirely:

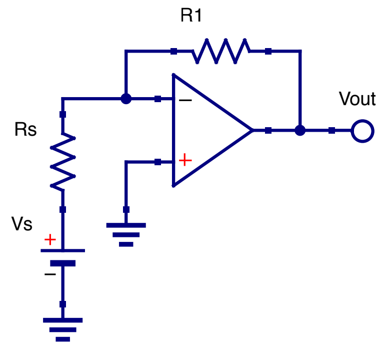


f. What is the current through R_s and the output voltage? (2 points)

$$V_{out} = V_{-} = V_{+} = 0$$

$$I_s = \frac{V_s}{R_s}$$

Let us do the same thing as we did above and add a resistor to the feedback from as shown below:



g. What is the output voltage now? (2 points)

$V_{out} = V_s - V_{R_s} - V_{R_1}$, but $V_- = 0$ so $V_{R_s} = V_s$.

$$V_{out} = -V_{R_1} = -I_{R_1}R_1 = -I_s R_1 = -\frac{V_s}{R_s}R_1$$

$$V_{out} = -V_s\frac{R_1}{R_s}$$

h. What is the relationship between V_s and V_{out} if $R_1 > R_s$? What about when $R_1 < R_s$? (1point)

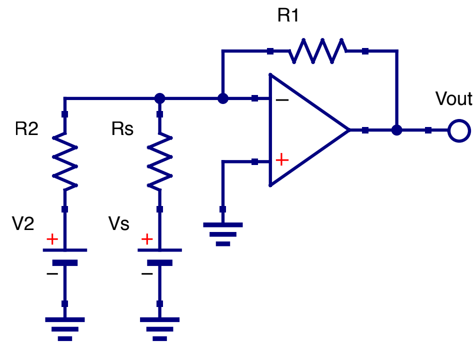
If $R_1 > R_s$ then $V_{out} > V_s$.

If $R_1 < R_s$ then $V_{out} < V_s$.

i. Thus, what is similar and what is different to the circuit shown in part c? (2 points)
Note: Also reference magnitude, sign, etc. if relevant for your answer.

This circuit inverts the input signal at the output. Additionally, the circuit is able to provide a gain of less than 1. Both, signals are able to amplify a signal.

j. What will the output be if we add another resistor and voltage as shown in circuit below? Without doing further calculations, guess what would be the effect if you had N such voltage and resistor combinations instead of just two as shown in the diagram. (4 points)



From part g:

$$V_{out,1} = -V_s \frac{R_1}{R_s}$$

From superposition considering only the V_2 branch:

$$V_{out,2} = -V_2 \frac{R_1}{R_2}$$

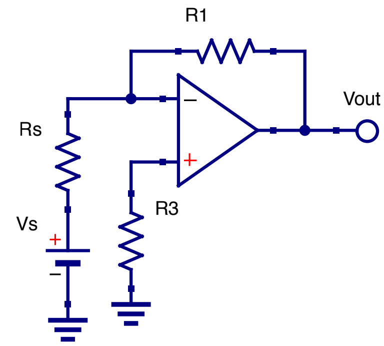
Summing:

$$V_{out} = V_{out,1} + V_{out,2} = -V_s \frac{R_1}{R_s} - V_2 \frac{R_1}{R_2} = -R_1 \left(\frac{V_s}{R_s} + \frac{V_2}{R_2} \right)$$

For N resistor/source combinations:

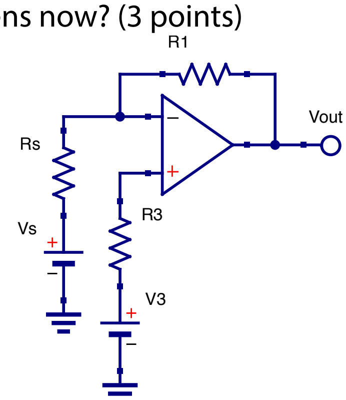
$$V_{out} = -R_1 \sum_{n=s}^{N} \frac{V_n}{R_n}$$

k. Let's go back to part g. Suppose we add a resistor to the non-inverting terminal as in the circuit below. What is the output voltage now? (1 point)



The result is unchanged, $V_{out} = -V_s \frac{R_1}{R_s}$, because $V_- = V_+ = 0$ since both inputs draw no current.

l. Suppose we add a voltage source to the resistor attached to the positive combination as in the circuit below. What happens now? (3 points)



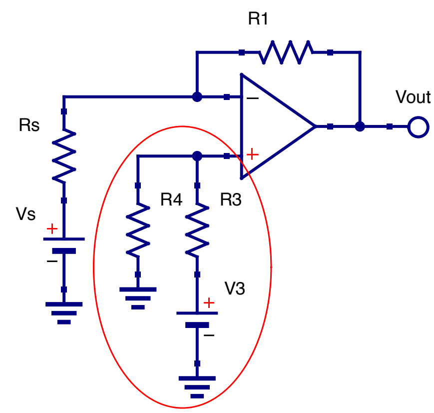
$I_3 = 0$, so $V_+ = V_3$ therefore $I_s = \frac{V_s - V_3}{R_s}$.

$$V_{out} = V_3 - V_1 = V_3 - I_1 R_1 = V_3 - \frac{V_s - V_3}{R_s} R_1$$

$$V_{out} = V_3 - \frac{R_1}{R_s}(V_s - V_3)$$

m. Let's make it a bit more complicated. Suppose I add another resistor as shown below, what is the effect now? In terms of the input voltages, V_s and V_3 what does this circuit do (i.e. does it amplify, diminish, add, subtract, etc.)? (4 points)

Note: When calculating, note that the circled part of the diagram forms a closed loop.



$$V_+ = V_3 \frac{R_4}{R_3 + R_4}$$

From part l:

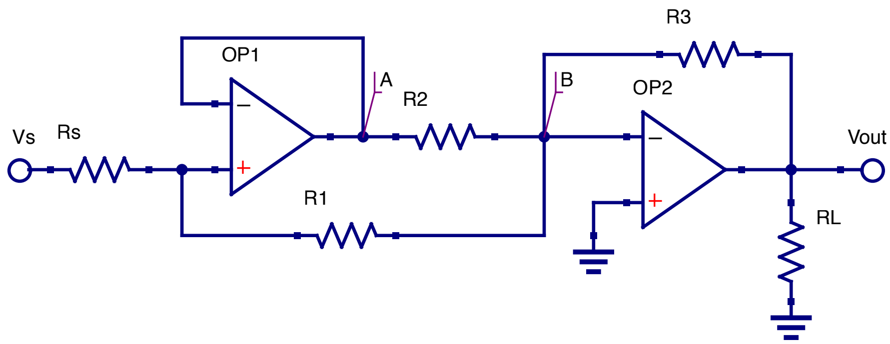
$$V_{out} = V_+ - \frac{R_1}{R_s}(V_s - V_+)$$

$$V_{out} = V_3 \frac{R_4}{R_3 + R_4} - \frac{R_1}{R_s}\left(V_s - V_3 \frac{R_4}{R_3 + R_4}\right)$$

Problem 6: Multiple Op-Amps (8 points)

In the circuit below, we have two op-amps connected to each other. Find an expression for the output, V_{out}, relative to the input voltage, V_s, in terms of all resistors in the circuit.

Hint: Method of nodal voltages may come in handy!



$V_B = 0$ because $V_{+,2} = 0$.

$$V_{+,1} = V_s \frac{R_1}{R_s + R_1}$$

Since $V_{-,1} = V_{+,1}$

$$V_A = V_s \frac{R_1}{R_s + R_1}$$

For node B:

$$I_1 + I_2 = I_3$$

$$I_1 = \frac{V_s}{R_s + R_1}$$

$$I_2 = \frac{V_A}{R_2} = \frac{V_s}{R_2} \frac{R_1}{R_s + R_1}$$

$$I_3 = I_1 + I_2 = \frac{V_s}{R_s + R_1} + \frac{V_s}{R_2} \frac{R_1}{R_s + R_1} = \frac{(R_1 + R_2)}{R_2(R_s + R_1)} V_s$$

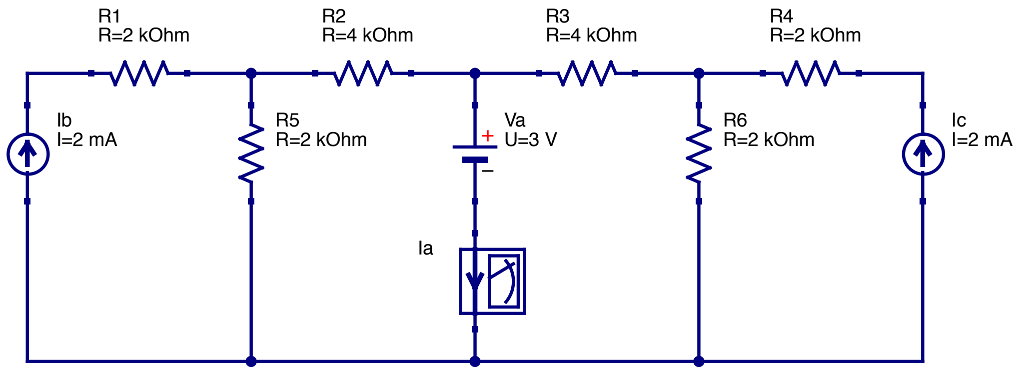
Now for V_{out}:

$$V_{out} = V_B - I_3 R_3 = 0 - \frac{(R_1 + R_2)}{R_2(R_s + R_1)} V_s * R_3$$

$$V_{out} = -\frac{R_3(R_1 + R_2)}{R_2(R_s + R_1)} V_s$$

Problem 7: Superposition (11 points)

Superposition is an important method to simplify circuit and make easier to solve for voltages and currents.

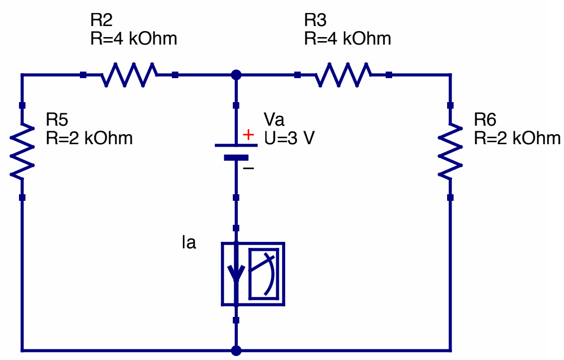


a. Redraw the circuit above and solve for the current flowing through V_a solely due to the voltage source, V_a. (3 points)

$$R_{eq} = (R_2 + R_4)||(R_6 + R_6) = (4\ k\Omega + 2\ k\Omega)||(4\ k\Omega + 2\ k\Omega)$$

$$R_{eq} = 6\ k\Omega||6\ k\Omega = 3\ k\Omega$$

$$I_{a,V_a} = -\frac{V_a}{R_{eq}} = -\frac{3\ V}{3\ k\Omega} = -1\ mA$$

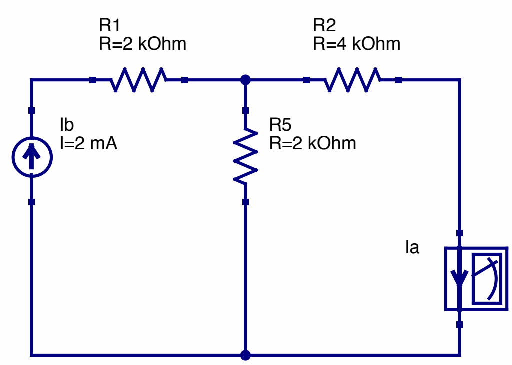


b. Redraw the circuit above and solve for the current flowing through V_a solely due to the current source I_b. (3 points)

$$I_{a,I_b} = I_{R_2} = I_{R_1}\frac{R_5}{R_2 + R_5} = I_b\frac{R_5}{R_2 + R_5}$$

$$I_{a,I_b} = 2\ mA\frac{2\ k\Omega}{4\ k\Omega + 2\ k\Omega} = 2\ mA * \frac{1}{3}$$

$$I_{a,I_b} = \frac{2}{3}\ mA$$

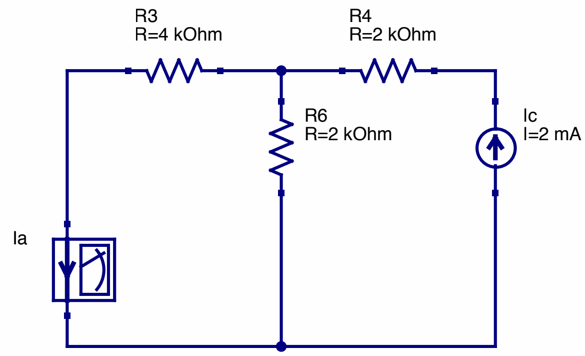


c. Redraw the circuit above and solve for the current flowing through V_a solely due to the current source I_c. (3 points)

$$I_{a,I_c} = I_{R_3} = I_{R_4}\frac{R_6}{R_3 + R_6} = I_c\frac{R_6}{R_3 + R_6}$$

$$I_{a,I_c} = 2\ mA\frac{2\ k\Omega}{4\ k\Omega + 2\ k\Omega} = 2\ mA * \frac{1}{3}$$

$$I_{a,I_c} = \frac{2}{3}\ mA$$



d. What is the total current flowing through V_a? (2 points)

$$I_a = I_{a,V_a} + I_{a,I_b} + I_{a,I_c}$$

$$I_a = -1\ mA + \frac{2}{3} mA + \frac{2}{3} mA$$

$$I_a = \frac{1}{3}\ mA$$