Problem Set 2

Due Date: Sat Sept 26 2015
by 9 PM

Name:________________________________

Lab Section & TF:_______________________

Collaborators:__________________________

For Grading Purposes Only:

Q1: _____ / 10
Q2: _____ / 22
Q3: _____ / 10
Q4: _____ / 8
Q5: _____ / 8
Q6: _____ / 12
Q7: _____ / 12

Total:_____ / 82

Instructions:

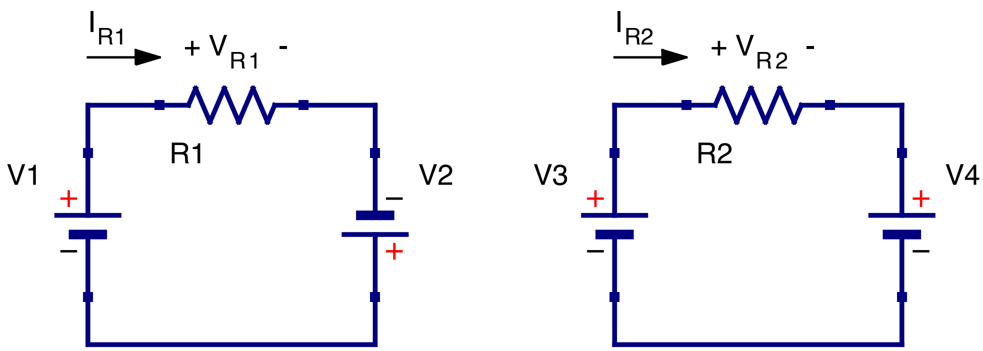
Please place your homework in the appropriate Dropbox in the basement of Maxwell Dworkin.

Please staple your homework. If your homework is not stapled, please hand it in with your name written on every page, all the pages numbered front & back, and the total number of pages in the homework written on the first page (so we don't lose any of your work).

Show all your work. If we can't figure out how you reached the answer, you won't get credit. More importantly, if you show your work but messed up the figures, you can get partial credit for your thought process.

MENTION ALL YOUR COLLABORATORS and which problems you collaborated on. There is no negative marking for collaborating on problems.

Problem 1: More Circuits (10 points)



a. For the circuits shown above, find I_R as a function of other circuit elements. Also, find the power *generated* by each source, in each circuit. (6 points)

$$I_{R1} = \frac{V_{R1}}{R_1} = \frac{V_1 + V_2}{R_1}$$

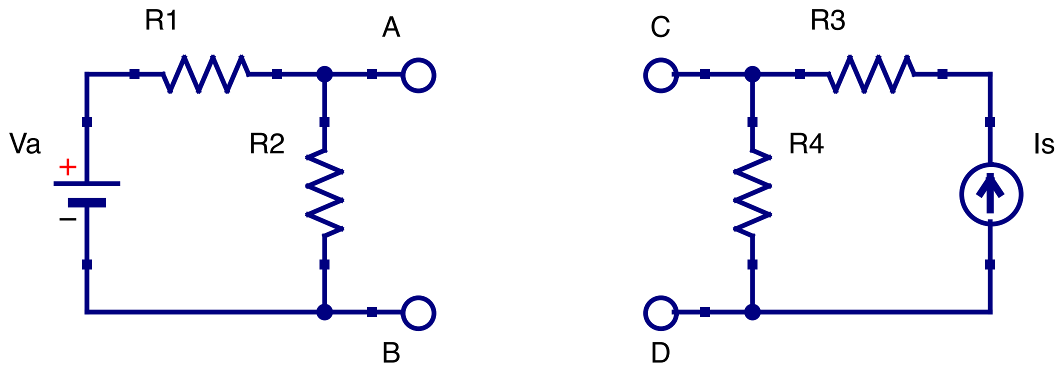
$$P_1 = V_1 I_1 = V_1 I_{R1} = V_1 \frac{V_1 + V_2}{R_1}$$

$$P_2 = V_2 I_2 = V_2 I_{R1} = V_2 \frac{V_1 + V_2}{R_1}$$

$$I_{R2} = \frac{V_{R2}}{R_2} = \frac{V_3 - V_4}{R_2}$$

$$P_3 = V_3 I_3 = V_3 I_{R2} = V_3 \frac{V_3 - V_4}{R_2}$$

$$P_4 = V_4 I_4 = V_4(-I_{R2}) = -V_4 \frac{V_3 - V_4}{R_2} = V_4 \frac{V_4 - V_3}{R_2}$$



b. Find voltages V_{AB} and V_{CD} in circuits below where $V_A = 5\ V, I_s = 2\ A, R_1 = R_3 = 40\ \Omega, R_2 = R_4 = 20\ \Omega$. (4 points)

$$V_{AB} = \frac{R_2}{R_1 + R_2} V_a = \frac{20\ \Omega}{40\ \Omega + 20\ \Omega} 5\ V = \frac{1}{3}(5\ V) = 1.67\ V$$

$$V_{CD} = I_4 R_4 = I_s R_4 = 2\ A * 20\ \Omega = 40\ V$$

Problem 2: Circuit Comprehension (22 points)

For the following questions, you may use calculations to illustrate your choices and circuits OR explain your logic in detail, using ideas about current/voltages/resistances in a circuit.

Leaving ES 50 class one day, you notice a little medallion lying on the floor with the words, ‘Resistance is futile!’ As you pick up the medallion, your hand accidently rub it and lo and behold! 5 genii appear in front of you and announce that they had been imprisoned by an evil ES professor for sleeping during class. They are so thankful to you, they become your slaves.

‘Great!’ you think, ‘No more psets!’

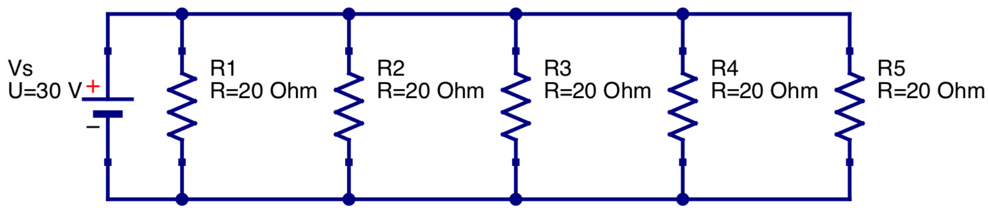
‘Er...’ says the first genie: there’s a small problem. The professor linked our powers forever to these light bulbs’.

Each genie quickly produces a light bulb and one also supplies a *30 V* battery. ‘The brighter the light bulb,’ explains the first genie, ‘the more powerful we are’.

‘Oh’, you say, desperately trying to remember your ES 50 lecture, ‘ Er...how much resistance does each bulb have?’

The genii look blankly at each other. They have no clue (living in a medallion is not an exciting intellectual environment). ’*20 Ω*?’ guesses one.

a. You connect the *30 V* battery to a circuit. Assuming that the genie's guess is correct, draw the circuit that will give each bulb its maximum brightness. Label the components with the correct voltage and resistance values. How much power will each bulb get? (4 points)



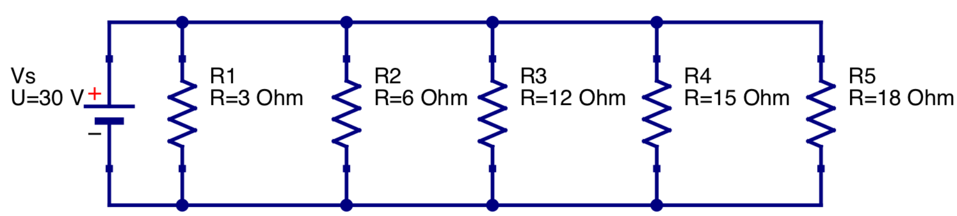
Voltage drop over each resistor is 30 V.

$$P_R = \frac{{V_R}^2}{R} = \frac{(30\ V)^2}{20\ \Omega} = 45\ W\ per\ bulb$$

'You donkey!' says another genie to the first. 'Don't you remember that the professor gave each of us different light bulbs? Those of us who begged for mercy the most got the lowest resistance, and those who showed the greatest defiance got bulbs with the highest resistance'.

Further investigation on your part reveals that the resistances are *3, 6, 12, 15, and 18 Ω* for the different light bulbs.

b. Thoroughly annoyed that the genie waited for you to finish the circuit before sharing this piece of information, you disconnect the entire circuit and then proceed to build the new circuit. What configuration of the bulbs will you use to give each bulb its maximum brightness? Now how much power will each bulb get? (4 points)



Voltage drop over each resistor is still 30 V.

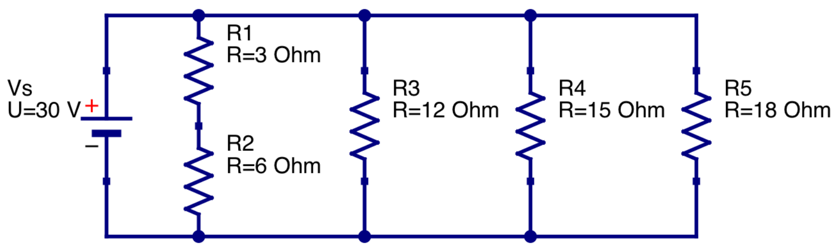
$$P_1 = \frac{{V_1}^2}{R_1} = \frac{(30\ V)^2}{3\ \Omega} = 300\ W, P_2 = \frac{{V_2}^2}{R_2} = \frac{(30\ V)^2}{6\ \Omega} = 150\ W$$

$$P_3 = \frac{{V_3}^2}{R_3} = \frac{(30\ V)^2}{12\ \Omega} = 75\ W, P_4 = \frac{{V_4}^2}{R_4} = \frac{(30\ V)^2}{15\ \Omega} = 60\ W$$

$$P_5 = \frac{{V_5}^2}{R_5} = \frac{(30\ V)^2}{18\ \Omega} = 50$$

Now that the genii are fully awake and working at maximum power, life has become much easier. However, you quickly discover that the most powerful genii aren't doing any work at all, but staying nice and warm, drinking hot coffee and bullying the less power genies into doing your work.

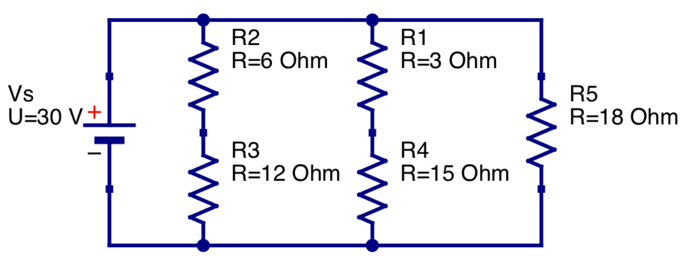
c. Thoroughly displeased at this state of affairs, you decide to reduce the power of the two most powerful genii from part b. How will you reconnect the circuit? Which is the most powerful genie now? (Answer should be of the form, 'the genie with the light bulb with R = ...') (4 points)



The 12 Ω genie is the most powerful, $P_3 = \frac{{V_3}^2}{R_3} = \frac{(30\ V)^2}{12\ \Omega} = 75\ W$

In order to distribute the load more fairly, you divide up the genies into three groups: 2 groups of 2 genii and a single genie. No group should be more powerful than the other.

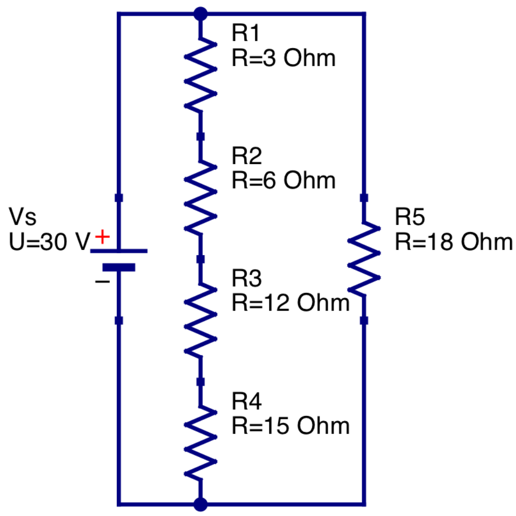
d. Which genii will you pair with each other, and how will you connect the circuit? Draw the new circuit. (4 points)



The ungrateful genii soon grow tired of you getting them to do your ES 50 homework for you, bringing you pizza and tampering with fire alarms so you can illegally smoke in your room. Reneging on the deal, they grab you one night and drop you in the middle of the Gobi desert.

However, the least powerful genie (who is that?) is still loyal to you and secretly leaves you the bulbs, batteries and rest of the circuit.

e. How will you connect the bulbs now so that your friend is the most powerful? Draw this circuit. (4 points)



Unfortunately, your little pal is letting things go to his head and thinks he's too grand to do work. By now, you've realized that things aren't exactly working out as they did in Aladdin and you're completely fed up. You decide to put an end to the genies once and for all. You grab the circuit and...

f. Complete the sentence. (2 points)

Problem 3: Equivalent Resistance (10 points)

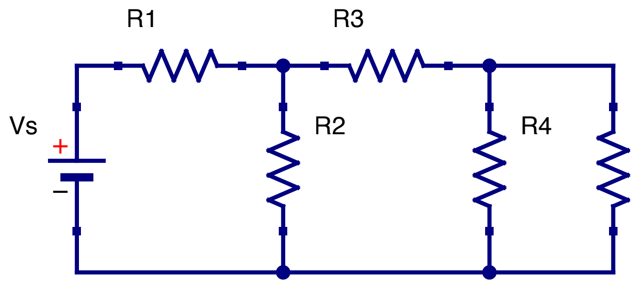


Figure A

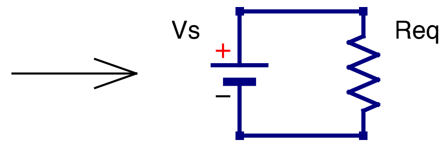


Figure B

a. For the circuit shown in *Figure A*, find the expression for the equivalent resistance R_{eq} seen by the voltage source V_s so that the circuit can be simplified as shown in *Figure B*. (6 points)

$$R_{eq} = R_1 + \{R_2||[R_3 + (R_4||R_5)]\}$$

b. Using the information from part a, find the power generated by the voltage source V_s. You can assume that $R_1 = R_2 = R_3 = R_4 = R_5 = 10\ \Omega, and\ V_s = 32\ V$. (2 points)

$$R_{eq} = R_1 + \{R_2||[R_3 + (R_4||R_5)]\}$$

$$R_{45} = R_4||R_5 = \frac{R_4 R_5}{R_4 + R_5} = \frac{(10\ \Omega)(10\ \Omega)}{10\ \Omega + 10\ \Omega} = 5\ \Omega$$

$$R_{345} = R_3 + R_{45} = 10\ \Omega + 5\ \Omega = 15\ \Omega$$

$$R_{2345} = R_2||R_{345} = \frac{R_2 R_{345}}{R_2 + R_{345}} = \frac{(10\ \Omega)(15\ \Omega)}{10\ \Omega + 15\ \Omega} = 6\ \Omega$$

$$R_{eq} = R_1 + R_{2345} = 10\ \Omega + 6\ \Omega = 16\ \Omega$$

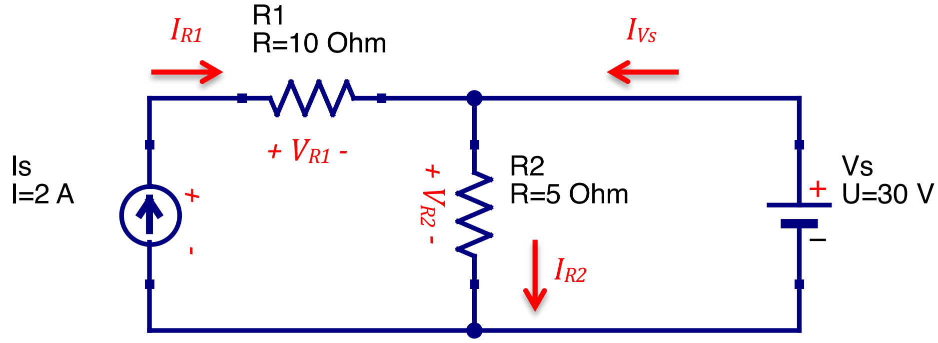
$$P_{generated} = P_{dissapated} = \frac{{V_s}^2}{R_{eq}} = \frac{(32\ V)^2}{16\ \Omega} = 64\ W$$

c. What is the total power dissipated by ALL resistors together? (Hint: this can be done without finding all currents and voltages for all resistors) (2 points)

$$P_{generated} = P_{dissapated} = \frac{{V_s}^2}{R_{eq}} = \frac{(32\ V)^2}{16\ \Omega} = 64\ W$$

Problem 4: KVL, KCL, and Ohm's Law (8 points)

For the circuit below:



a. Label all the currents and voltages across the resistors, consistent with references used in Ohm's Law. (2 points)

b. Solve for all the values of resistor currents. (2 points)

$$I_{R_1} = I_s = 2\ A$$
$$I_{R_2} = \frac{V_2}{R_2} = \frac{30\ V}{5\ \Omega} = 6\ A$$

c. Also solve for the values of the current going through the voltage source and the value of voltage across the current source. Indicate the direction of the voltage across the current source (which side is + , which side is -). (2 points)

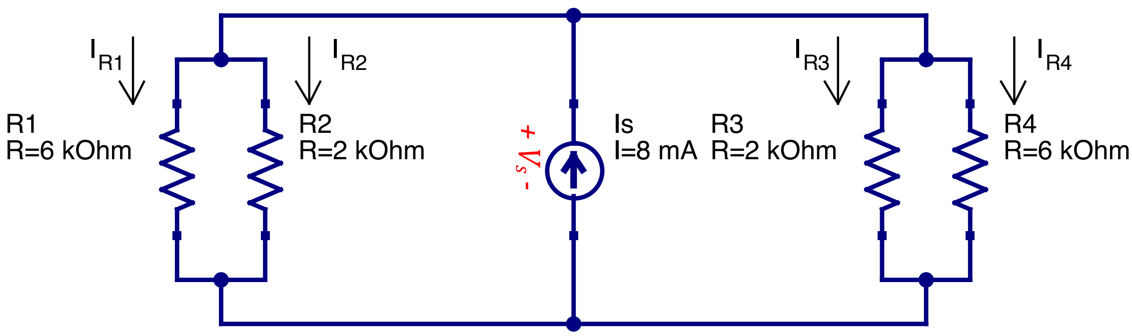
$$I_{V_s} + I_{R_1} = I_{R_2}$$
$$I_{V_s} = I_{R_2} - I_{R_1} = 6\ A - 2\ A = 4\ A$$
$$V_{R_2} = V_s = V_{I_s} - V_{R_1}$$
$$V_{I_s} = V_s + V_{R_1} = V_s + I_{R_1}R_1 = V_s + I_sR_1 = 30\ V + 2\ A * 10\ \Omega = 50\ V$$

d. What is the power generated by both sources? (2 points)

$$P_{I_s} = V_{I_s} I_s = 50\ V * 2\ A = 100\ W$$
$$P_{V_s} = V_s I_{I_s} = 30\ V * 4\ A = 120\ W$$
$$P_{total} = P_{V_s} + P_{I_s} = 120\ W + 100\ W = 220\ W$$

Problem 5: KVL, KCL, and Ohm's Law (8 points)

For the circuit below:



a. Solve for the values of I1, I2, I3, and I4 in the circuit above. (4 points)

$$R_{12} = R_{34} \rightarrow I_{12} = I_{34} = 4\ mA$$
$$I_1 = I_{12}\frac{R_2}{R_1 + R_2} = 4\ mA\frac{2\ k\Omega}{6\ k\Omega + 2\ k\Omega} = 1\ mA$$
$$I_2 = I_{12}\frac{R_1}{R_1 + R_2} = 4\ mA\frac{6\ k\Omega}{6\ k\Omega + 2\ k\Omega} = 3\ mA$$
$$I_3 = I_2 = 3\ mA$$
$$I_4 = I_1 = 1\ mA$$

b. What is the voltage across the current source (direction and magnitude)? (2 points)

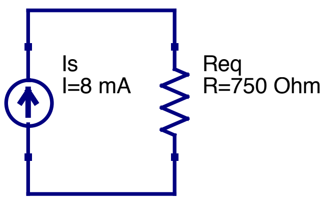
$$V_{I_s} = V_{R_{12}} = I_{12}R_{12} = 4\ mA \frac{6\ k\Omega * 2\ \text{k}\Omega}{6\ k\Omega + 2\ k\Omega} = 4\ mA * 1.5\ k\Omega = 6\ V$$

The positive end in the end where the current exits the source (upper).

The negative end in the end where the current enters the source (lower).

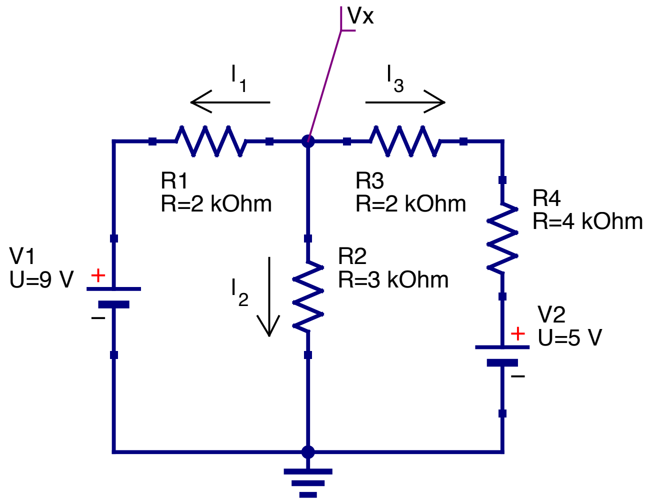
c. You can simplify the circuit above and represent it by a current source and a single *equivalent resistor*, R_{eq}. What is the value of R_{eq}? What is the value of the power generated by the current source? (2 points)

$$R_{eq} = R_{12}||R_{34} = \frac{1.5\ k\Omega * 1.5\ \text{k}\Omega}{1.5\ k\Omega + 1.5\ k\Omega} = 0.75\ k\Omega = 750\ \Omega$$



$$P_{generated} = V_{I_s}I_s = 6\ V * 8\ mA = 48\ mW$$

Problem 6: Nodal Analysis of Circuits (12 points)



a. For the circuit shown above, solve for I_1, I_2, I_3, and V_x. Here the ground is shown (triangular symbol). In the other parts of the problem, you should indicate where the ground is (on the circuit). Also note that we have indicated directions for the currents, to help you start solving the problem. (4 points)

$$I_1 + I_2 + I_3 = 0$$

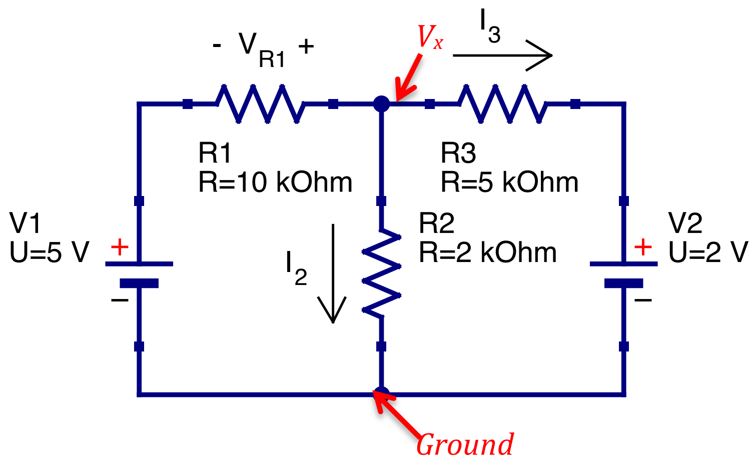
$$\frac{V_x - V_1}{R_1} + \frac{V_x}{R_2} + \frac{V_x - V_2}{R_3 + R_4} = \frac{V_x - 9\ V}{2\ k\Omega} + \frac{V_x}{3\ k\Omega} + \frac{V_x - 5\ V}{6\ k\Omega} = 0$$

$$V_x = \frac{32}{6}\ V = 5.33\ V$$

$$I_1 = \frac{V_x - V_1}{R_1} = \frac{5.33\ V - 9\ V}{2\ k\Omega} = -1.84\ mA$$

$$I_2 = \frac{V_x}{R_2} = \frac{5.33\ V}{3\ k\Omega} = 1.78\ mA$$

$$I_3 = \frac{V_x - V_2}{R_3 + R_4} = \frac{5.33\ V - 5\ V}{6\ k\Omega} = 0.055\ mA$$



b. For the circuit shown above, find V_{R_1}, I_1, and I_2. Identify where your ground is. (4 points)

$$I_1 + I_2 + I_3 = 0$$

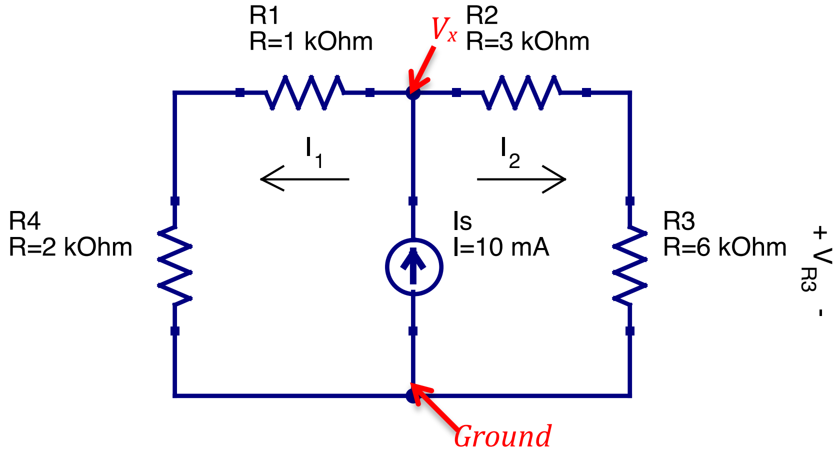
$$\frac{V_x - V_1}{R_1} + \frac{V_x}{R_2} + \frac{V_x - V_2}{R_3 + R_4} = \frac{V_x - 5\ V}{10\ k\Omega} + \frac{V_x}{2\ k\Omega} + \frac{V_x - 2\ V}{5\ k\Omega} = 0$$

$$V_x = \frac{9}{8}\ V = 1.125\ V$$

$$V_{R_1} = I_1 R_1 = \frac{V_x - V_1}{R_1} R_1 = V_x - V_1 = 1.125\ V - 5\ V = -3.875\ V$$

$$I_2 = \frac{V_x}{R_2} = \frac{1.125\ V}{2\ k\Omega} = 0.563\ mA$$

$$I_3 = \frac{V_x - V_2}{R_3} = \frac{1.125\ V - 2\ V}{5\ k\Omega} = -0.175\ mA$$



c. For the circuit shown above, find I_1, I_2, and V_{R_3}. Identify where your ground is. (4 points)

$$I_s = I_1 + I_2$$

$$I_s = \frac{V_x}{R_1 + R_4} + \frac{V_x}{R_2 + R_3} = \frac{V_x}{1\ k\Omega + 2\ k\Omega} + \frac{V_x}{3\ k\Omega + 6\ k\Omega} = 10\ mA$$

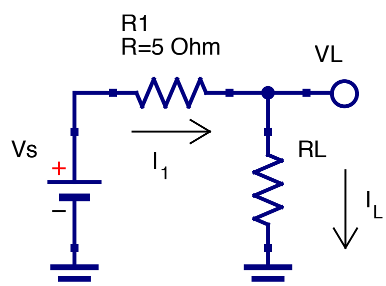
$$V_x = \frac{90}{4}\ V = 22.5\ V$$

$$I_1 = \frac{V_x}{R_1 + R_4} = \frac{22.5\ V}{3\ k\Omega} = 7.5\ mA$$

$$I_2 = \frac{V_x}{R_2 + R_3} = \frac{22.5\ V}{9\ k\Omega} = 2.5\ mA$$

$$V_{R_3} = I_2 R_3 = 2.5\ mA * 6\ k\Omega\ = 15\ V$$

Problem 7: Difference Between Ordinary Circuits and Op-Amps (12 points)



a. For the circuit above, $V_s = 5\ V$. Find I_L and V_L when $R_L = 10\ \Omega$ and when $R_L = 10\ k\Omega$. (4 points)

For $R_L = 10\ \Omega$:

$$V_L = V_s \frac{R_L}{R_1 + R_L} = 5\ V \frac{10\ \Omega}{5\ \Omega + 10\ \Omega} = 3.33\ V$$

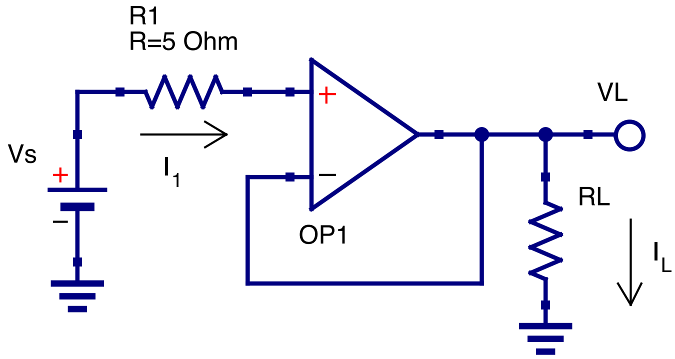
$$I_L = \frac{V_s}{R_1 + R_L} = \frac{5\ V}{5\ \Omega + 10\ \Omega} = 0.33\ A$$

For $R_L = 10\ k\Omega$:

$$V_L = V_s \frac{R_L}{R_1 + R_L} = 5\ V \frac{10\ k\Omega}{5\ \Omega + 10\ k\Omega} = 4.9975\ V \approx 5\ V$$

$$I_L = \frac{V_s}{R_1 + R_L} = \frac{5\ V}{5\ \Omega + 10\ k\Omega} = 0.500\ mA$$

Now let's take a look at an op-amp circuit:



b. For the circuit above, $V_s = 5\ V$. Find I_L and V_L when $R_L = 10\ \Omega$ and when $R_L = 10\ k\Omega$. (4 points)

$I_1 = 0$, so $V_+ = V_s$ and $V_- = V_+$ so $V_L = V_s = 5\ V$ regardless of R_L

For $R_L = 10\ \Omega$:

$$I_L = \frac{V_L}{R_L} = \frac{V_s}{R_L} = \frac{5\ V}{10\ \Omega} = 0.5\ A$$

For $R_L = 10\ k\Omega$:

$$I_L = \frac{V_L}{R_L} = \frac{V_s}{R_L} = \frac{5\ V}{10\ k\Omega} = 0.5\ mA$$

c. In a couple of sentences, write the main difference(s) you observed, if any, between how an op-amp and an ordinary circuit functions. (2 points)

The op-amp acts as a buffer between the input resistor and the load resistor. Without the buffer, the value of R_L influences the behavior of the output voltage. With the op-amp in place, V_L is set without being effected by the load. Additionally, in the second circuit, V_s provides no power because the op-amp draws no current on the input (V_+ and V_-) terminals. Therefore the entire load is driven by the op-amp, rather than V_s.

d. What is the bias of an op-amp so that the output voltage can follow input voltage, V_s without any distortion, clipping etc. using the circuit in part b? You can assume $V_s = 5\,V * sin(2\pi * 1000\,Hz * t)$. (2 points)

We know that $V_L = V_s$ and $max(V_s) = 5\,V$, we know that we need to bias the op-amp somewhat greater than *5 V* (and less than -5 V on the negative rail). For the sake of simplicity, let's use two *9 V* batteries: