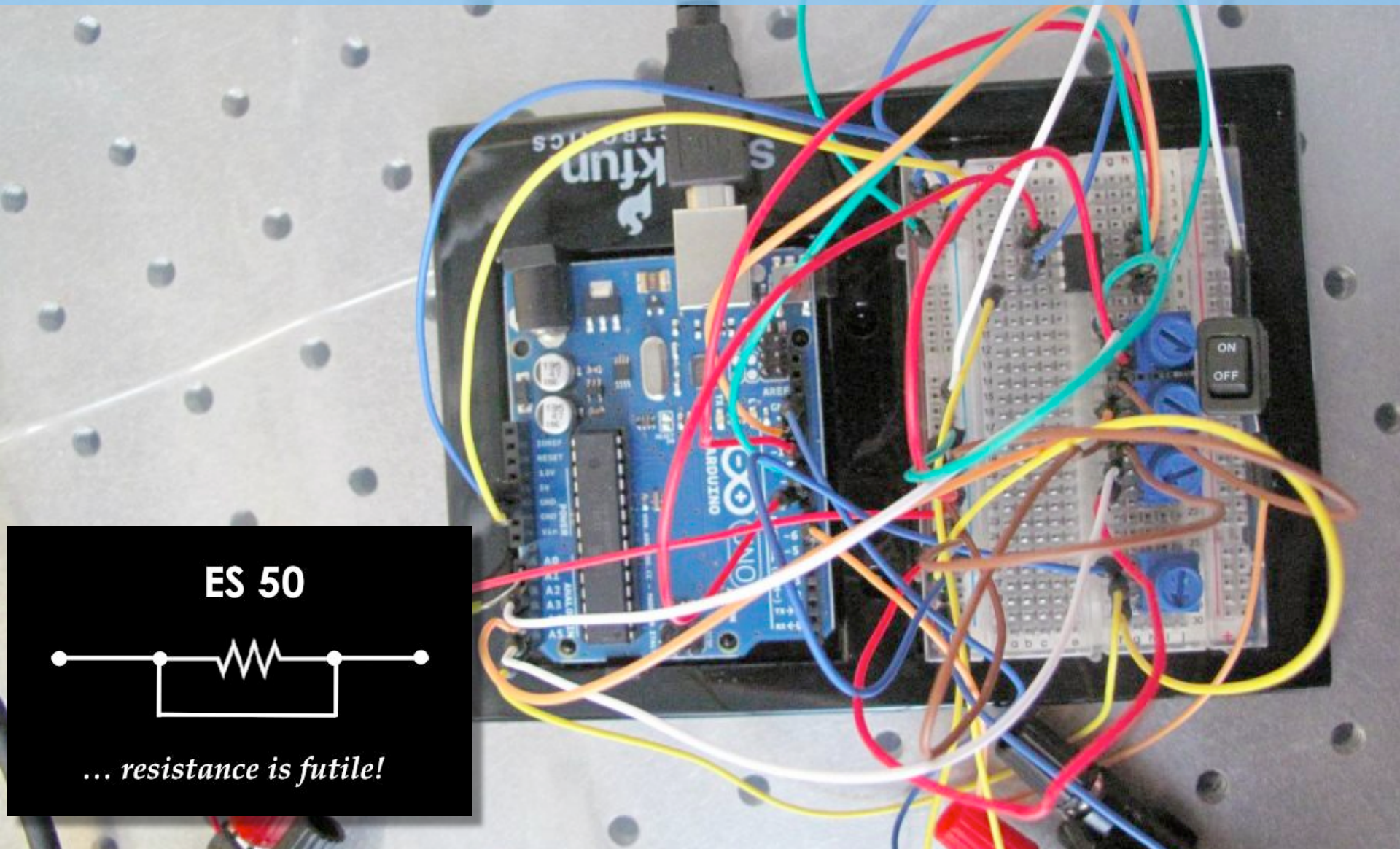
ES 50: Introduction to Electrical Eng.

Explore EE, satisfy a Gen Ed requirement & have fun



ES 50

… resistance is futile!

Lecture 9: Analog to Digital Conversion – Part 1

October 14, 2015

HARVARD
School of Engineering and Applied Sciences

Here is what you have learned so far….

L1 – L8: Analog world

Use electronics to sense the environment, amplify and communication the obtained information, and respond to it (e.g. moving something);

#	Date	Lecture Subject	Homework	Lab
L1	09/02	***Introduction to ES50 - Guide To Programming the Physical World*** *Course overview;* demos of prior years' final projects		
	09/07	*Labor Day*		
L2	09/9	***Analog Electronics: Electrical Circuits & their Elements*** Signals as electrical currents and voltages; voltage and current sources; resistors; Ohm's Law; electric power	**HW1** (due 09/12)	
L3	09/14	***Analog Electronics: Circuit Analysis:*** Kirchoff's laws	**HW2** (due 09/19)	**Lab 1:** Arduino µ-controller; generating & observing electrical signals;
L4	09/16	***Analog Electronics: Amplifiers***		
L5	09/21	***Analog Electronics: Circuits with Amplifiers*** Design and Analysis	**HW3** (due 09/26)	**Lab 2:** Testing simple circuits; soldering 101
L6	09/23	***Analog Electronics:*** *More examples & catch-up*		
L7	09/28	***Control Theory: Feedback vs Open-Loop Control*** *PID controller*	**HW4** (due 10/03)	**Lab 3:** Audio Amplifier
L8	09/30	***Analog Electronics: Inductors, Capacitors & Applications*** Filtering, sensing, actuation (motors); generation, transmission and storage of electrical energy		

And here is what you will learn in next 4 weeks or so…

L9 – L15: Digital world & digital signal processing

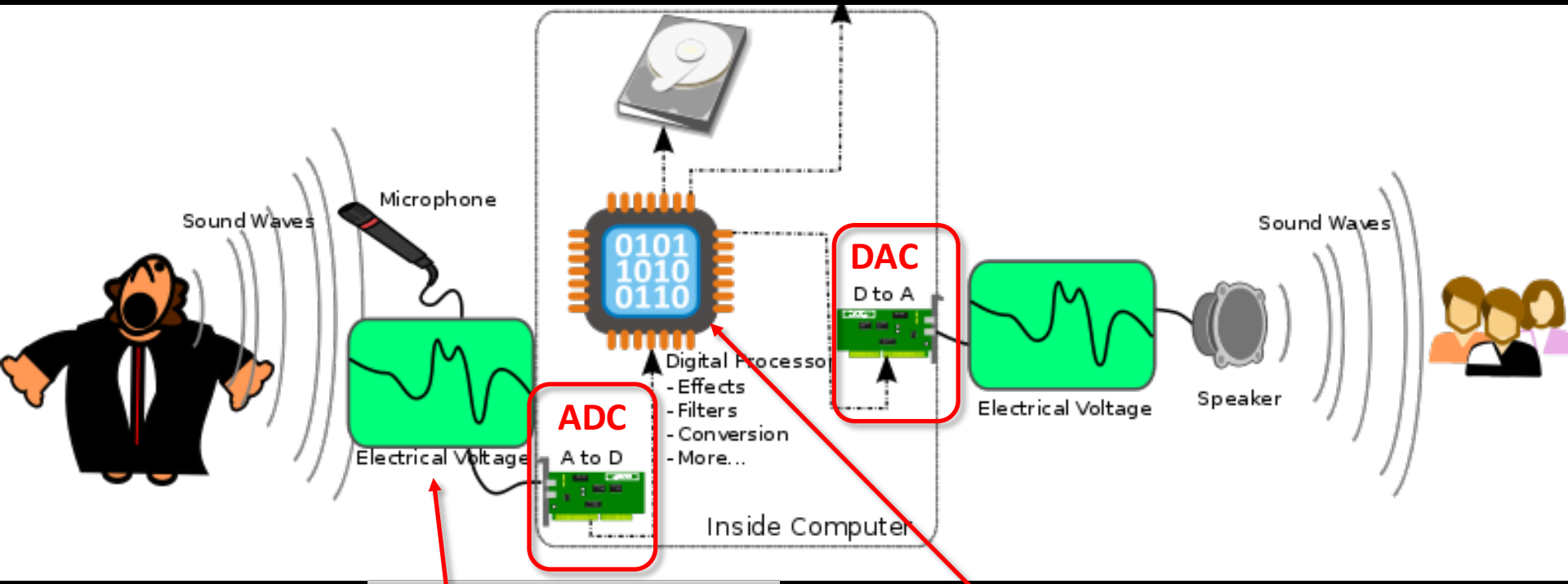
Why going digital? How do we obtain digital information and how do we manipulate, communicate, and respond to it? Examples: digital music, images, movies. How do computers work?

	10/12	*Columbus Day*		
L9	10/14	***Signals:* Analog-to-Digital Conversion (ADC)** Sampling and quantization; Nyquist theorem	**HW6** (due 10/17)	**Lab 5:** AD/DA conversion
L10	10/19	***Digital Electronics:* Logic Gates & Combinatory Circuits** How a computer adds, makes decisions, processes information, etc.	**HW7** (due 10/24)	**Lab 6:** Digital electronics: (shift registers, multiplexers, counters, etc.)
L11	10/21	***Digital Electronics:* Flip-Flops & Sequential Circuits** How does computer memory work?		
L12	10/26	***Digital Electronics:* Finite-State Machines; Timers;** Now you too can build vending & ATM machines, elevator brain, etc.	**Final project proposal** (due 10/31)	**Lab 7:** Final project: Brainstorming & proposal writing
L13	10/28	***Signals:* Images as 2-D Digital Signals** Color, gray scale and bit-map images		
L14	11/02	***Signals:* Image Processing and Machine Vision**	**HW8 & Final project part list** (due 11/07)	**Lab 8:** Arduino smart car (BoeBot)
	11/04	**Final Project Workshop:** 1 min & 1 slide presentation by each team		
L15	11/09	***Signals:* Lossless vs. Lossy Compression** Huffman code, JPEG and MPEG	**HW9** (due 11/14)	**Lab 9:** Machine vision
L16	11/11	***Communications:* Analog Information Transmission** How do we transmit information? Why modulate? AM & FM radio		
L17	11/16	***Communications:* Digital information Transmission** Shift keying (FSK, ASK, PSK); noise and codes to overcome it	**HW10** (due 11/21)	**Work on Final Project**
	11/18	***Review***		

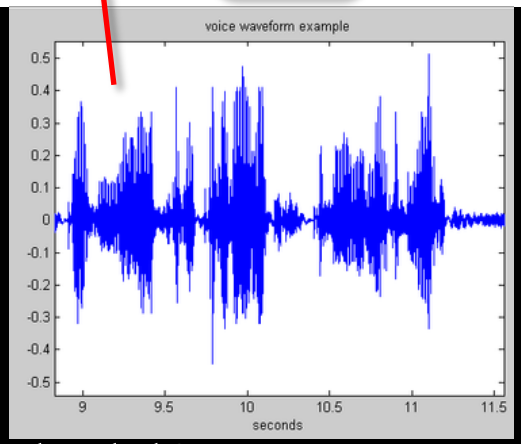
Analog vs Digital

ADC – analog to digital converter

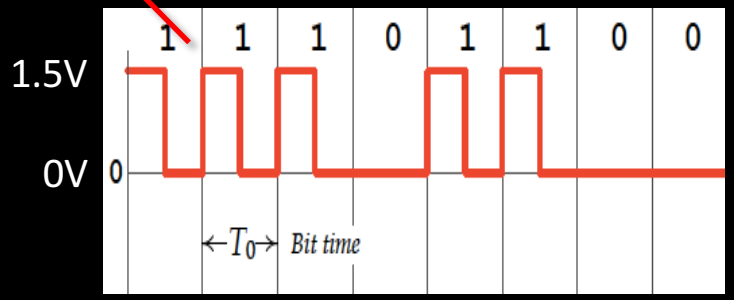
DAC – digital to analog converter



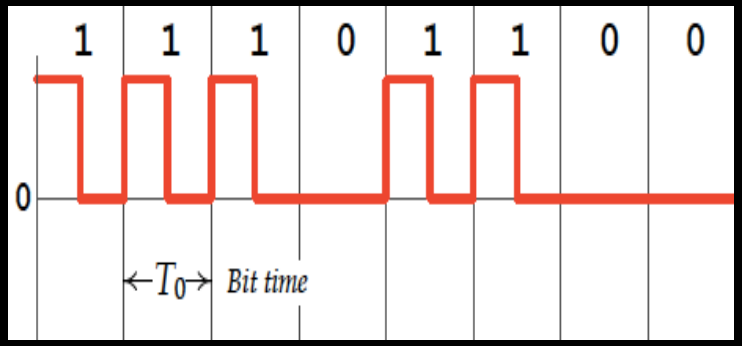
Analog: microphone signal in time domain

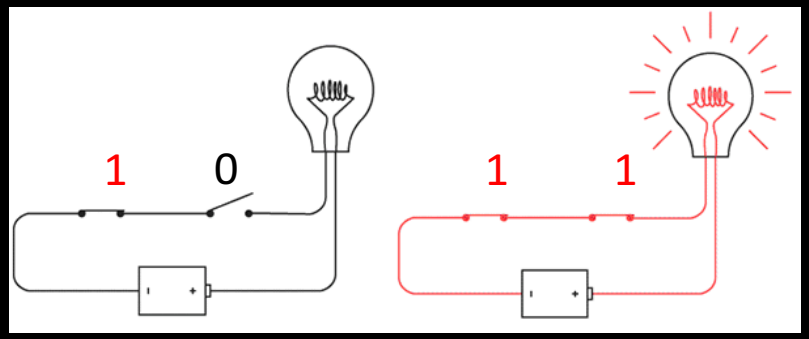


Digital signal: Inside computer



Why digital signals? Well, they are easy to represent, store, and manipulate!





At any point of time, digital signal can assume only one of two allowed vales: 0 or 1

- "0" ⇔ no signal, low voltage, switch off, "false"
- "1" ⇔ there is a signal, high voltage, switch on, "true"

And switches are really easy to make and scale up!

Music | Songs | Albums | Artists | Genres | Playlists

Name	Time	Artist	Album	Genre
beatles	0:15			

beatles

Summary | Info | Video | Sorting | Options | Lyrics | Artwork

No Artwork Available

beatles (0:15)

Also, how do we get the file size from this info?

Kind: WAV audio file
Size: 2.5 MB
Bit Rate: 1411 kbps
Sample Rate: 44.100 kHz
Date Modified: 4/23/11 9:00 PM
Plays: 1
Last Played: 1/28/12 9:09 PM

Channels: Stereo
Sample Size: 16 bit

What does this mean?

Sample Rate: 44.100 kHz
Sample Size: 16 bit
Channels: Stereo
Sample Rate: 1411kbps

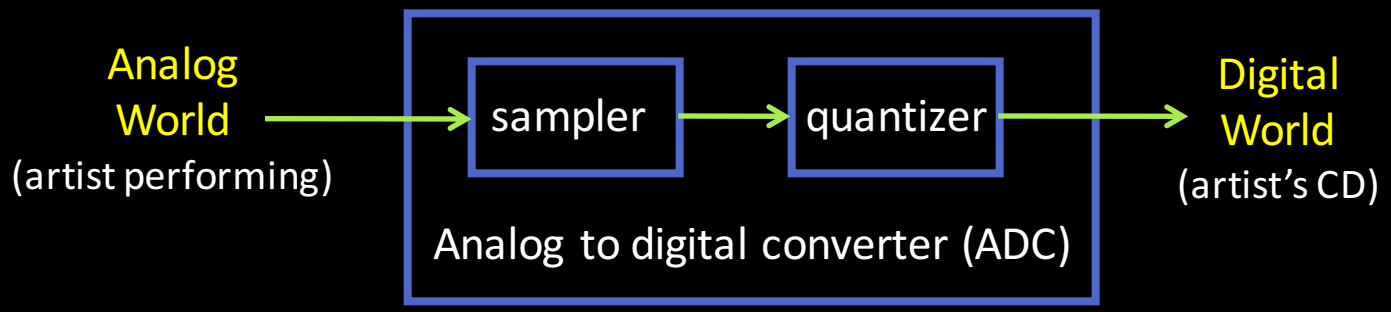
Where: /Users/markoloncar/Music/iTunes/iTunes Media/Music/Unknown Artist/Unknown Album/beatles.wav

Why sample rate not 10kHz? 100kHz?
What if sample size was 4 bit? What if it was 64bit?

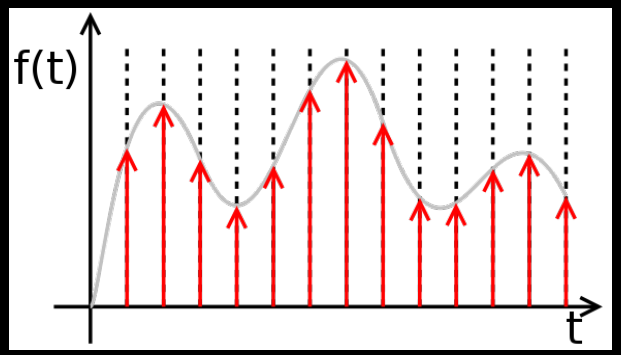
Previous | Next | Cancel | OK

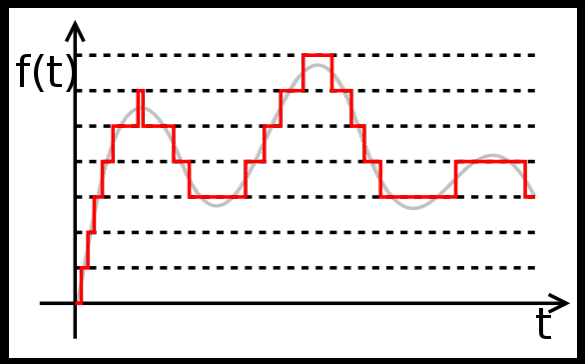
HA School

Analog to Digital Conversion (ADC) components: Sampling and Quantization



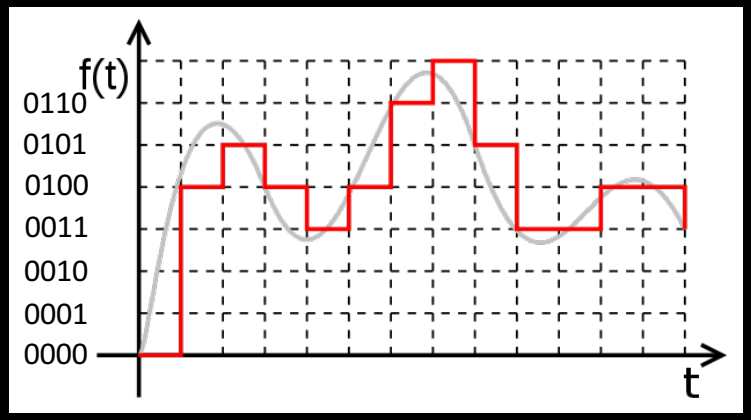
Sampled signal
Discrete in time, continuous in amplitude (can assume any value)



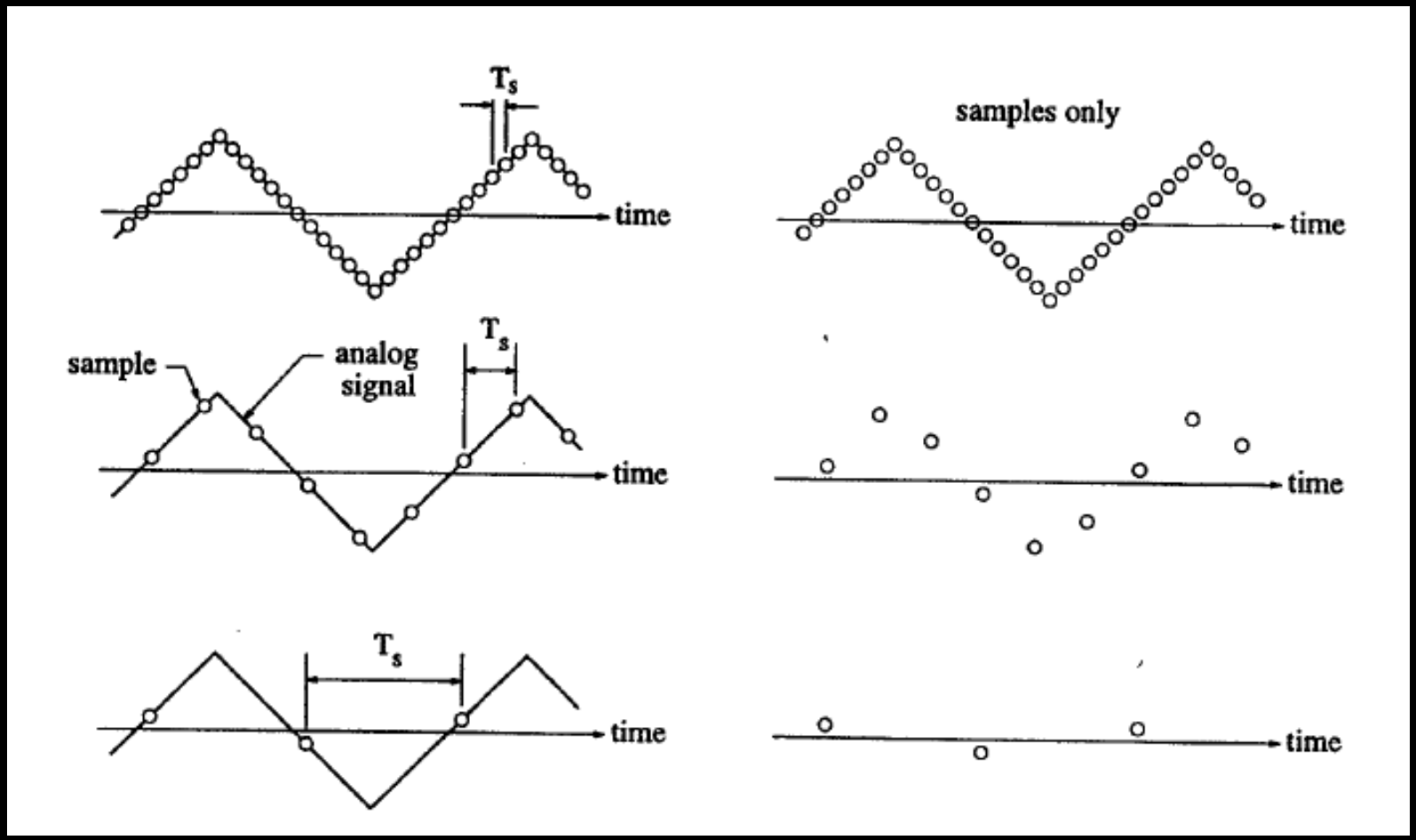


Quantized signal
Continuous in time, discrete in amplitude (can assume only certain values)

Sampled & Quantized signal
Discrete both in time and amplitude, & converted to stream of binary digits (bits)



How often do we have to take samples?



- More samples give us the signal that resembles original.
- However, more samples means that more memory is needed to store the signal.

How often do we have to sample?

Can an analog signal be represented by a sampled signal so that there is no difference between the two after reconstruction (that after going back from digital to analog)?

- YES… but…. Only under certain conditions!

Sampled signal can be a perfect representation of an analog original only if the spectrum of the original signal is bandwidth-limited!

In addition, sampling frequency needs to satisfy:

$$f_s > 2 \cdot f_{max}$$

That is, the sampling frequency f_S needs to be at least two times larger than the frequency of the fastest component of the signal! This condition is known as a Nyquist Sampling Criterion – it is EXTREMELY important for the digital age!

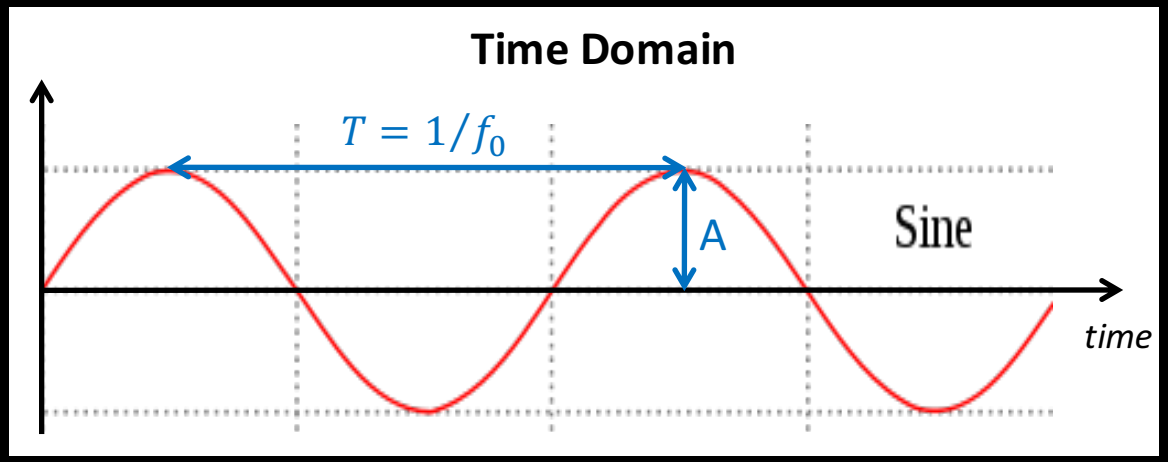
Note: sampling period $T_s = {}^{1}/_{f_s}$;

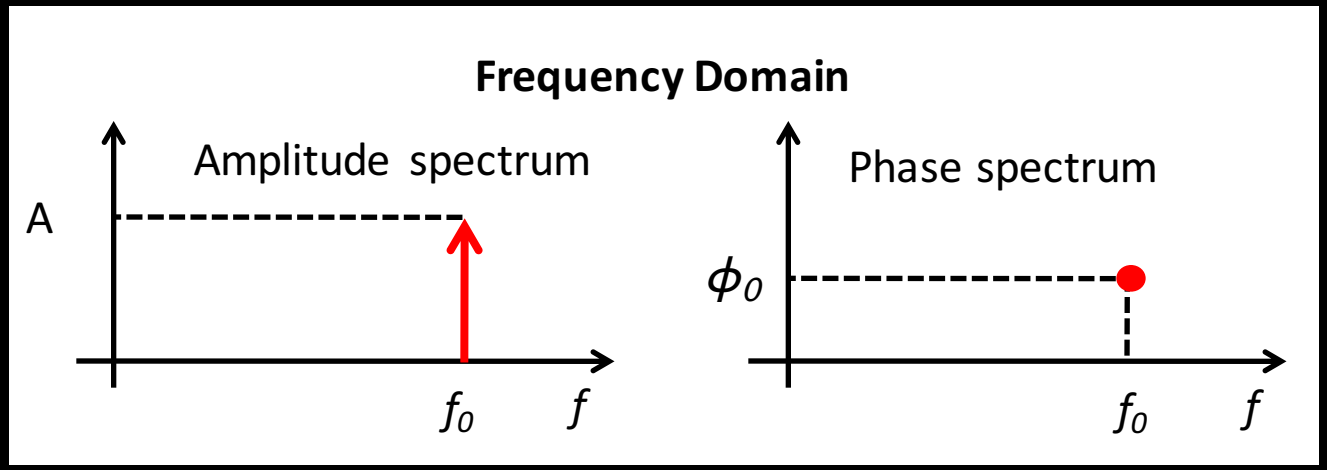
But what does it mean "bandwidth-limited signal" ?

Analog Signals: Time and Frequency Domain

$$f(t) = A\sin(2\pi f_0 t + \phi_0)$$

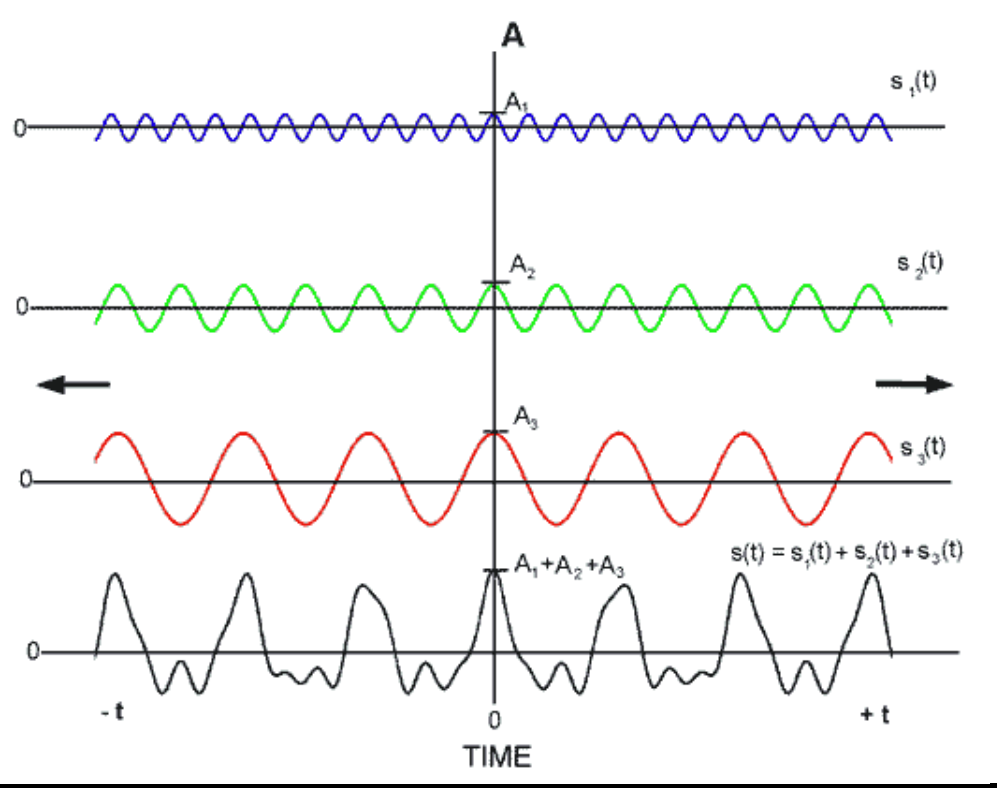
A – amplitude ($A > 0$)
f_0 – frequency
$T = 1/f_0$ = period
ϕ_0 – phase

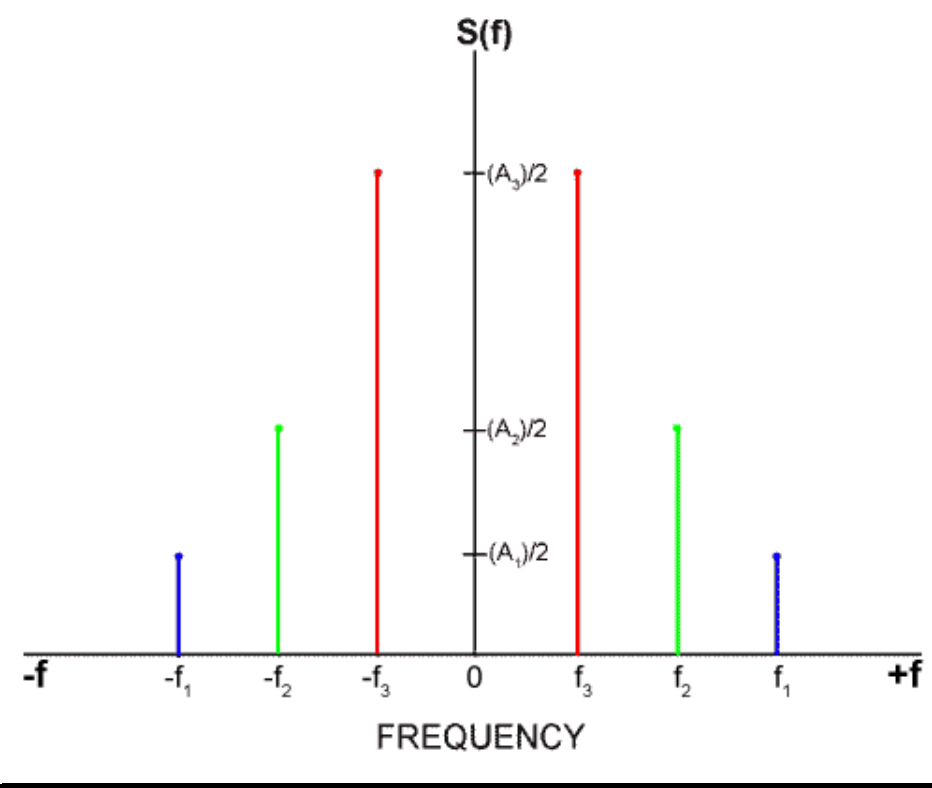




Both phase and amplitude are important!
But in ES 50, we will often "worry" about amplitude only.

Analog Signals: Time and Frequency Domain

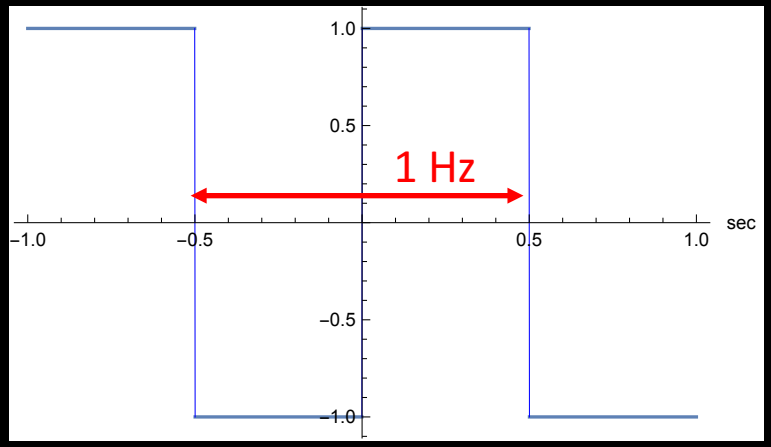




http://dagsaw.sdsu.edu/3.4.html

Analog Signals: Time and Frequency Domain

Frequency components of a square wave



$$f(t) = A\,sin(2\pi f_0 t + \phi_0)$$

$$f(t) = C_1\,sin(2\pi f_1 t + \phi_1) + C_2\,sin(2\pi f_2 t + \phi_2) + C_3\,sin(2\pi f_3 t + \phi_3) + C_4\,sin(2\pi f_4 t + \phi_4) + \cdots$$

Every time domain signal can be represented as a sum of sine and cosine waves

- Mathematical formalism used to achieve this is called Fourier transform
- Fourier transform and frequency domain representation are VERY powerful and important!

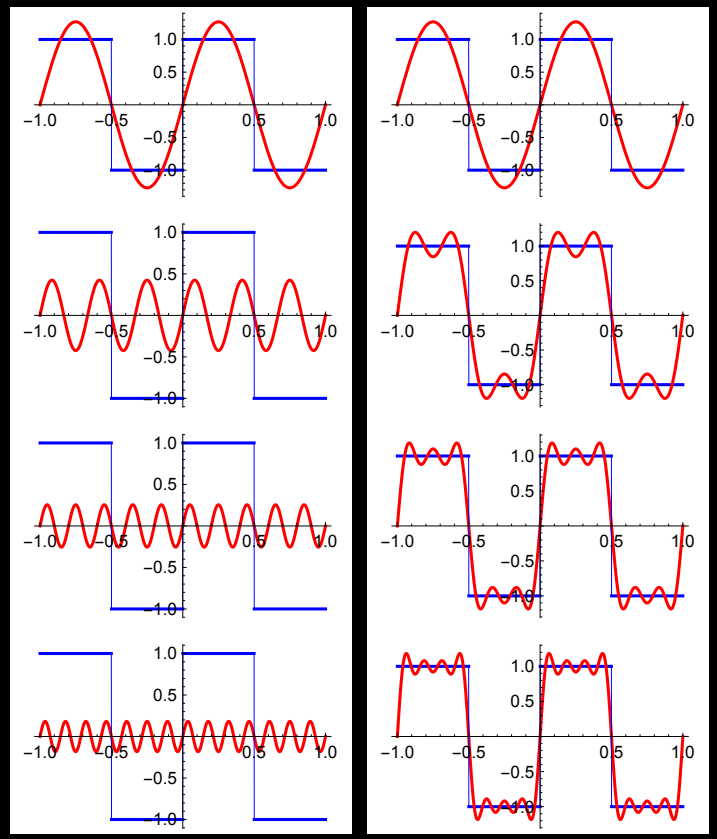
$$f(t) = \sum_{n=0}^{N} C_n\,sin(2\pi n f t + \phi_n)$$

C_1 = 1 Hz

C_3 = 3 Hz

C_5 = 5 Hz

C_7 = 7 Hz



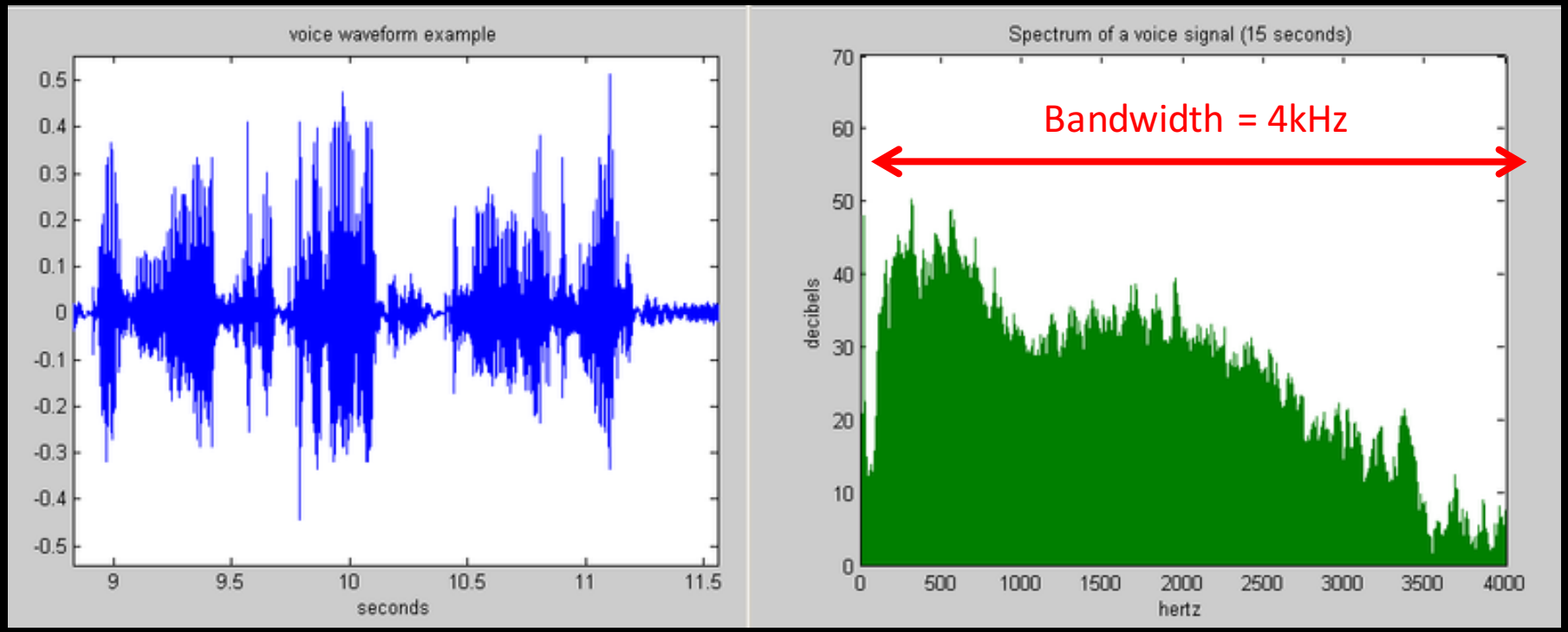
C_1

$C_1 + C_3$

$C_1 + C_3 + C_5$

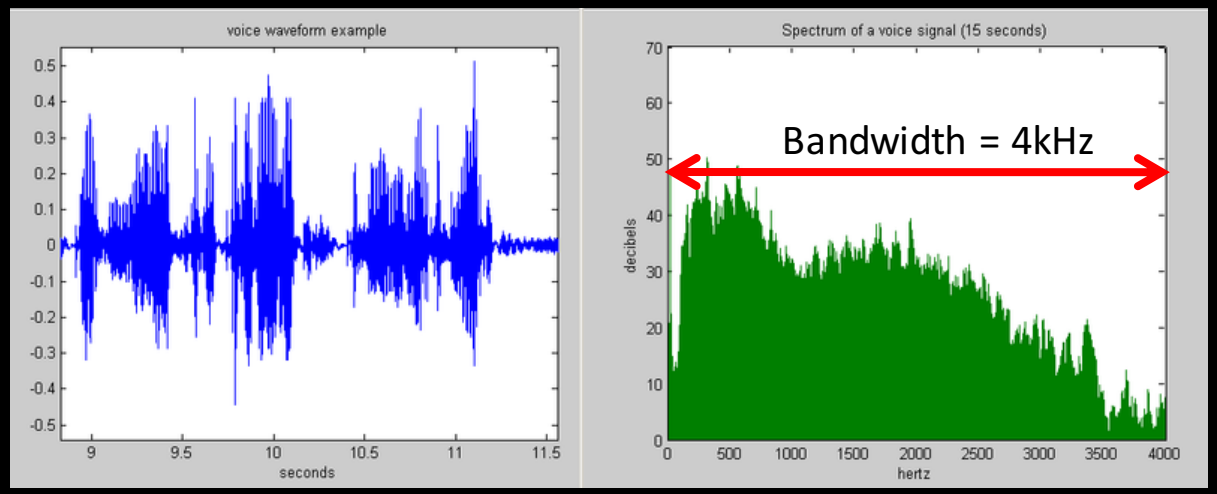
$C_1 + C_3 + C_5 + C_7$

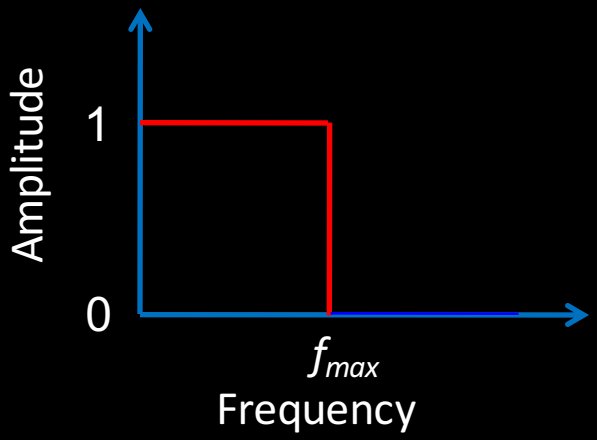
Frequency content of “real-life” speech signal obtained using a Fourier transform



Arduino spectrum analyzer: https://www.youtube.com/watch?v=X35HbE7k3DA

Bandwidth limited signals





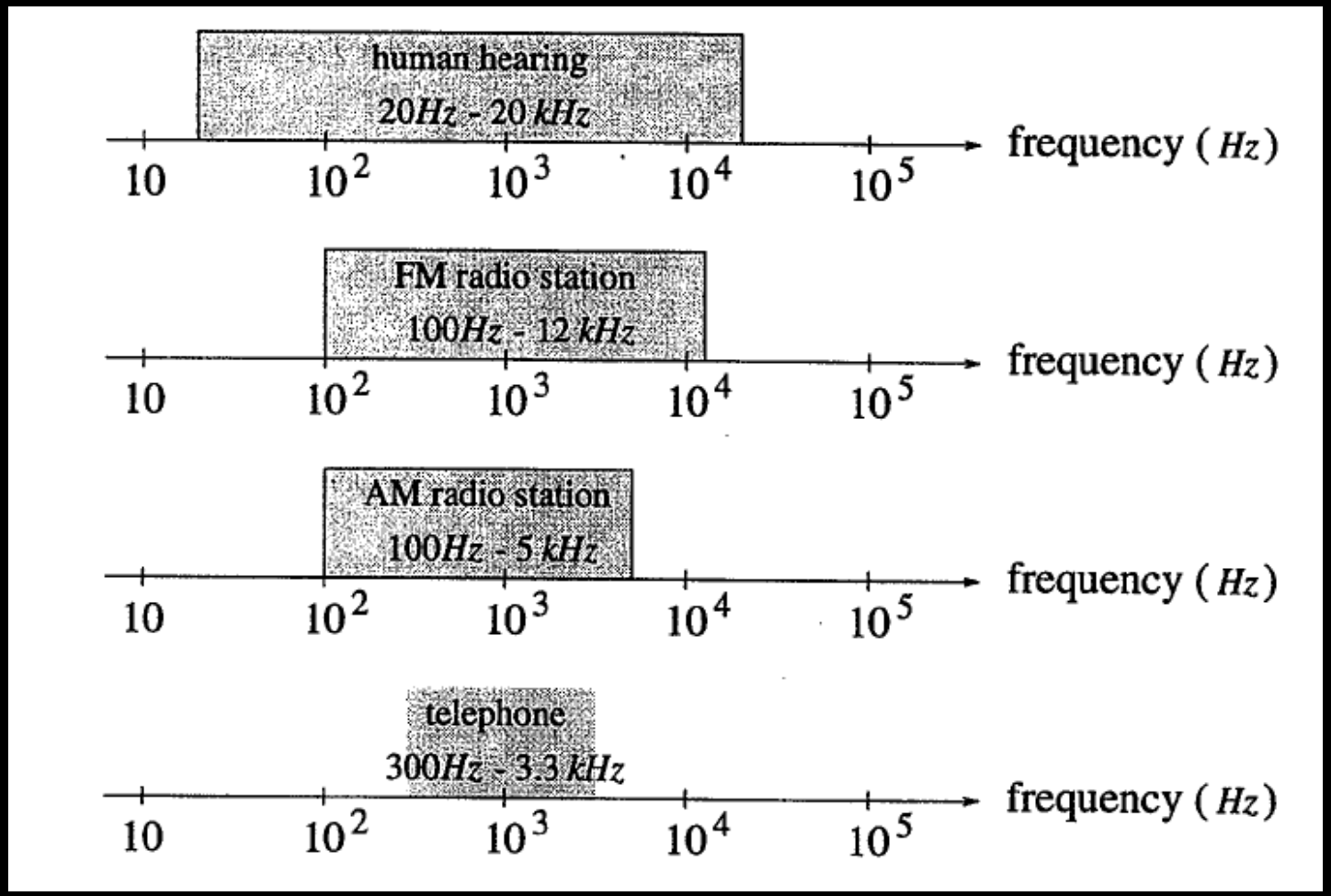
Bandwidth-limited signal is a signal that has maximum frequency, f_{max}. That is the amplitude of frequency components with $f > f_{max}$ is zero.

- Most natural signals are not bandwidth limited, but do come very close to it! For example, human voice spectrum has $f_{max} \approx 4\ kHz$. Signal can be made bandwidth-limited using filters.

Filter is a component that removes certain parts of signal's spectrum

- A human ear cannot hear anything above 20 kHz so there is no need to transmit or store frequency components of music signal with $f > 20\ kHz$. A *low-pass filter*, with cut-off frequency of 20 kHz, can be used to remove high frequency components from music signal and make it bandwidth limited.

Frequency bandwidth of common audio systems

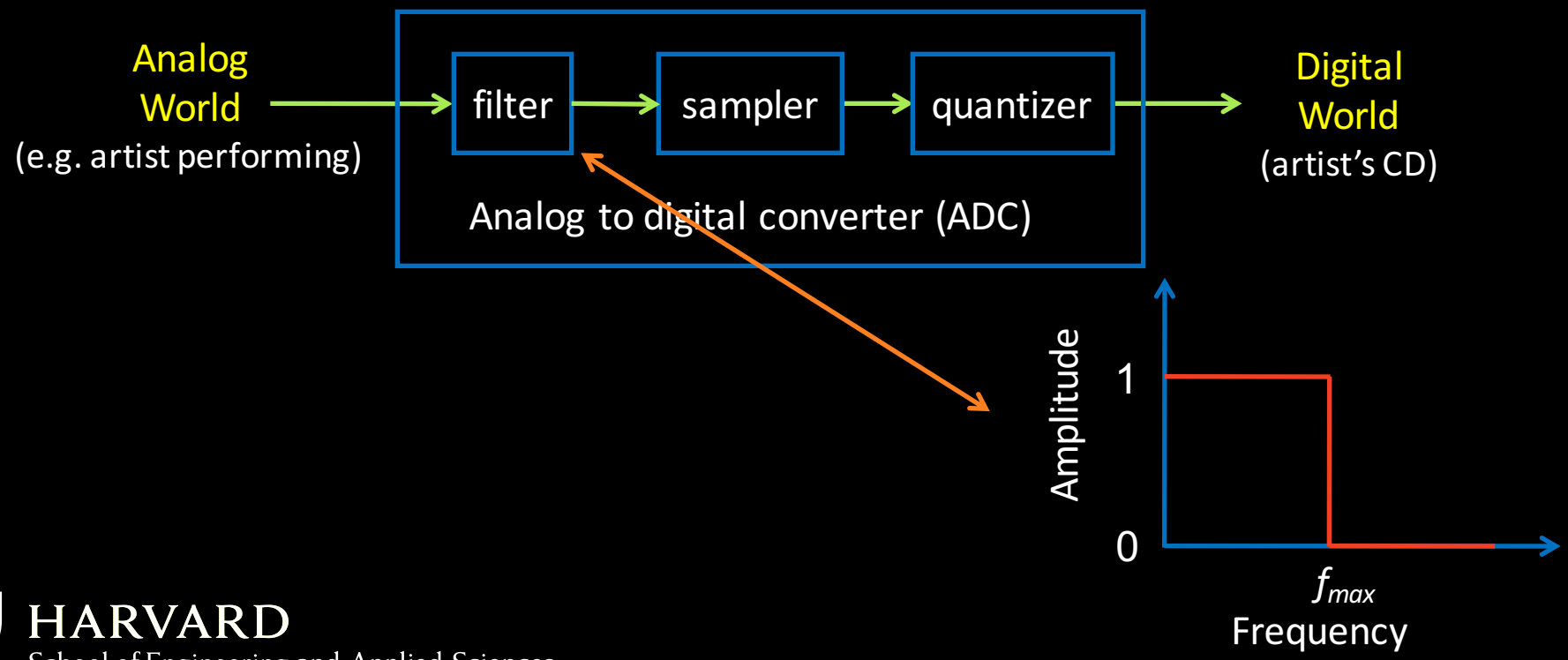


Example

A human ear cannot hear anything above 20kHz so is there a need to store information with frequency above 20kHz?

- A low-pass filter with $f_{max} = 20\ kHz$ can be used to remove high frequency components and make it bandwidth limited!

low-pass: lets low frequencies ($f < f_{max}$) through, and rejects high frequencies ($f > f_{max}$).



How often do we have to sample?

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- YES… but…. Only under certain conditions!

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In addition, sampling frequency needs to satisfy:

$$f_s > 2\, f_{max}$$

That is, the sampling frequency f_s needs to be at least two times larger than the frequency of the fastest component of the signal! This condition is known as a **Nyquist Sampling Criterion** – it is EXTREMELY important for the digital age!

Note: sampling period $T_s = 1/f_s$;

What happens if sampling frequency does not satisfy Nyquist Criterion?

If Nyquist criterion is not satisfied, aliasing effect occurs. So if

$$f_s < 2\, f_{max}$$

We have a problem!

- That is, recovered signal will be different from the original and will sound as if it has different frequency.
- The frequency of recovered signal is $f_{alias} = \left| f_{sample} - f_{original} \right|$

Original Recording

Sampling rate: 10 kHz

Sampling rate: 8 kHz

http://www.coe.montana.edu/ee/rmaher/aes_cd/1.1.htm

What happens if sampling frequency does not satisfy Nyquist Criterion? → Aliasing Error!

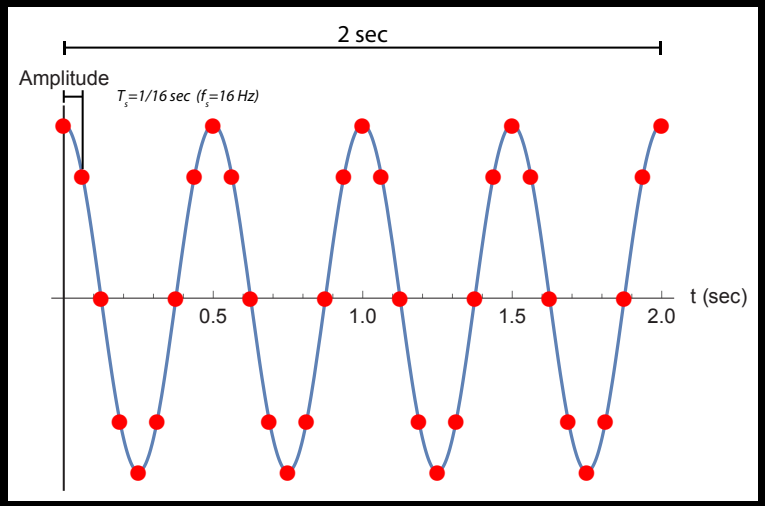
Example: Consider cosine signal with $f_0 = 2\ Hz$ & $f_{max} = 2\ Hz$

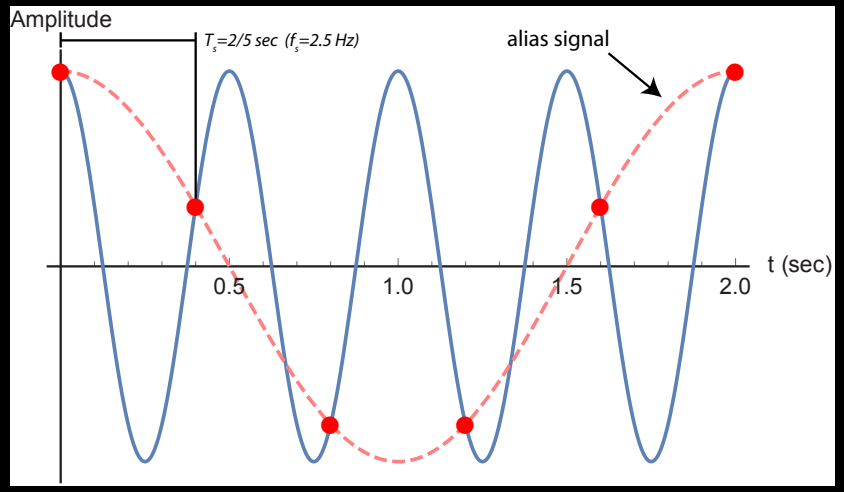
Let's see what happens with this signal after is sampled with different f_{sample} and then recovered:

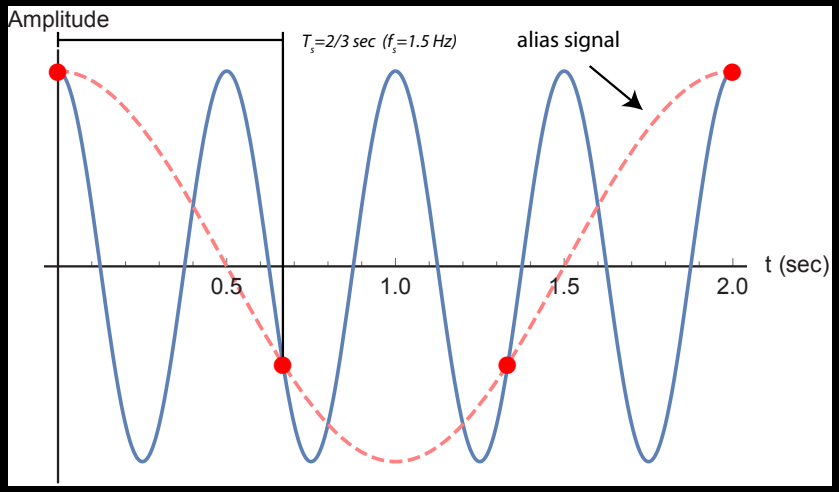
a) $f_{sample} = 16\ Hz \rightarrow f_{recovered} = f_{org} = 2\ Hz$

b) $f_{sample} = 2.5\ Hz \rightarrow f_{rec} = |f_{sample} - f_{org}| = 0.5\ Hz$

c) $f_{sample} = 1.5\ Hz \rightarrow f_{rec} = |f_{sample} - f_{org}| = 0.5\ Hz$



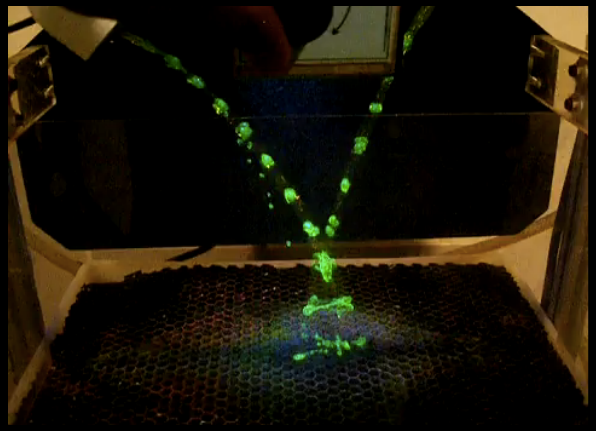




Examples

https://www.youtube.com/watch?v=R-IVw8OKjvQ

https://www.youtube.com/watch?v=jHS9JGkEOmA



https://www.youtube.com/watch?v=ZRlNOyxWWf8

What is going on in these three clips?!?!

"Strange" Aliasing

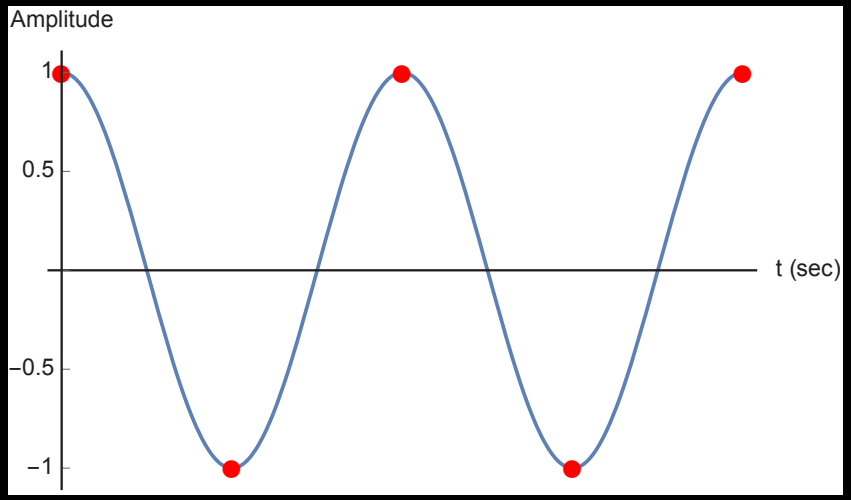
What happens when $f_s = 2f_{max}$?

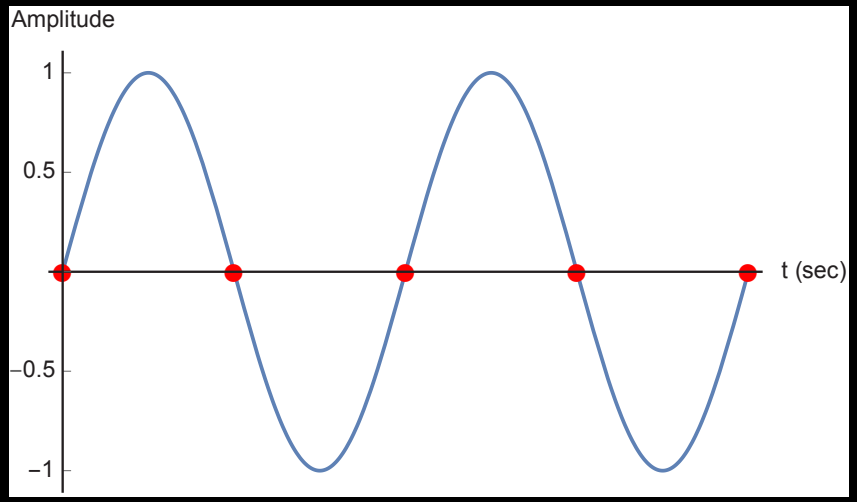
If waveform is $\cos(2\pi f_0 t)$...

- Samples occur with $f_s = 2f_0$ which are located at maxima & minima

If waveform is $\sin(2\pi f_0 t)$...

- Samples occur with $f_s = 2f_0$ which are located at zeros!





Severe Under-Sampling

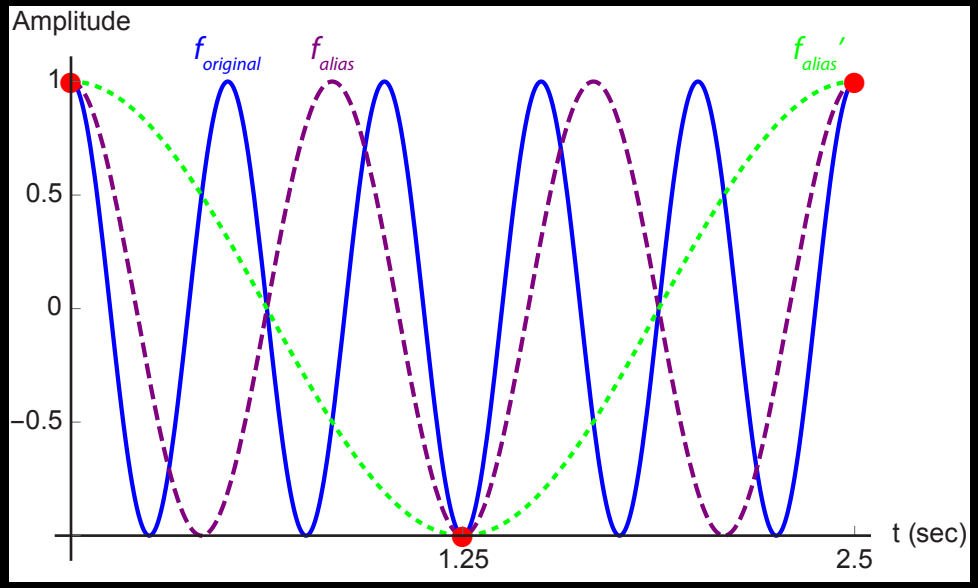
It can be shown, that when aliasing occurs the perceived signal always needs to satisfy

$$f_{alias} \leq f_{sample}/2$$

- If $|f_{sample} - f_{original}| > f_{sample}/2$, then we need to continue computing alias frequency by substituting the computed value of f_{alias} for the original frequency $f_{original}$

Example: let's assume that $f_{original} = 2\ Hz$ and $f_{sample} = 0.8\ Hz$

- $f_{alias} = |f_{sample} - f_{original}| = 1.2\ Hz$, which is bigger than $f_{sample}/2$
- Therefore, we need to continue the process, $f_{alias}{}' = |f_{sample} - f_{alias}| = 0.4\ Hz$
- Therefore, the perceived signal, after this under-sampling, will have frequency of $f'_{alias} = 0.4\ Hz$



Music | Songs | Albums | Artists | Genres | Playlists

Name	Time	Artist	Album	Genre
beatles	0:15			

beatles

Summary | Info | Video | Sorting | Options | Lyrics | Artwork

No Artwork Available

beatles (0:15)

Also, how do we get the file size from this info?

Kind: WAV audio file
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Channels: Stereo
Sample Size: 16 bit

What does this mean?

Sample Rate: 44.100 kHz
Sample Size: 16 bit
Channels: Stereo
Sample Rate: 1411kbps

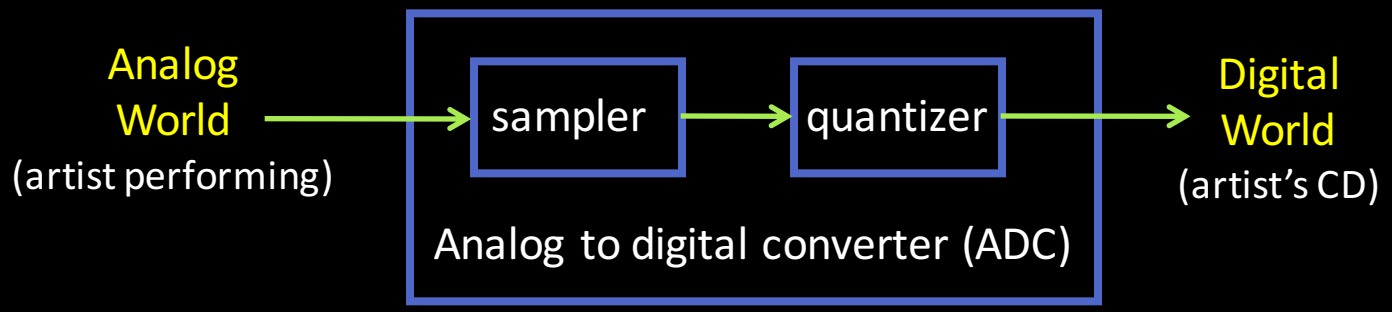
Where: /Users/markoloncar/Music/iTunes/iTunes Media/Music/Unknown Artist/Unknown Album/beatles.wav

Why sample rate not 10kHz? 100kHz?
What if sample size was 4 bit? What if it was 64bit?

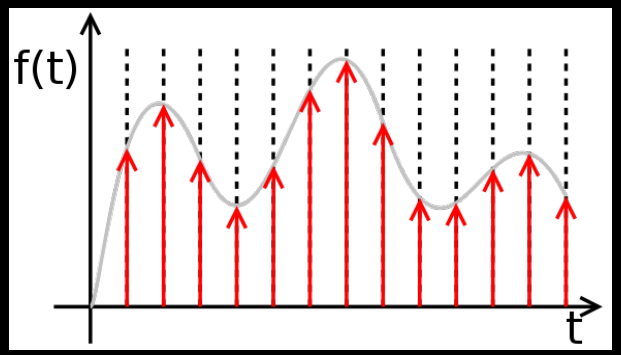
Previous | Next | Cancel | OK

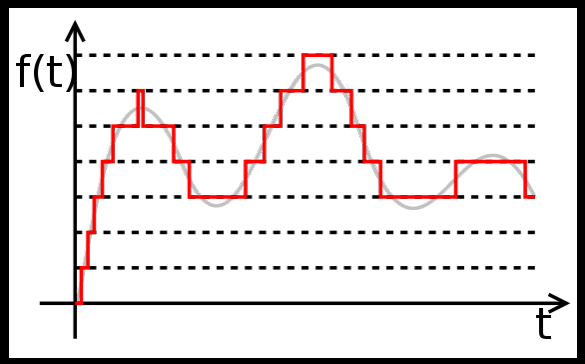
HA School

Analog to Digital Conversion (ADC) components: Sampling and Quantization



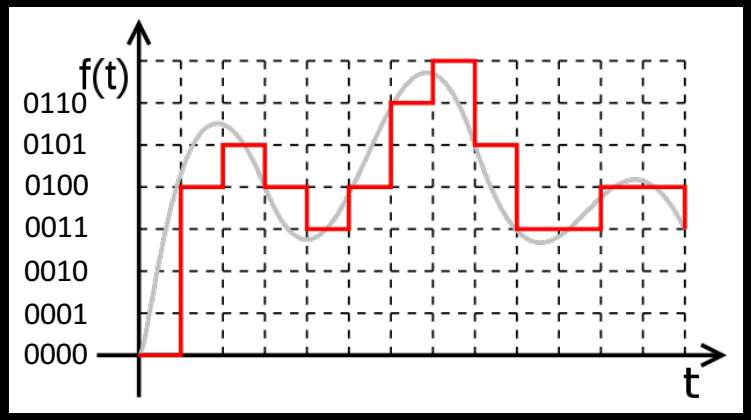
Sampled signal
Discrete in time, continuous in amplitude (can assume any value)





Quantized signal
Continuous in time, discrete in amplitude (can assume only certain values)

Sampled & Quantized signal
Discrete both in time and amplitude, & converted to stream of binary digits (bits)



How do we convert (decimal) signals to binary system?

How do we go from decimal system that we are used to, to binary (or any other system)?

Let's take a look at decimal system first, and representation of number **2413**
Note: to indicate that this is decimal system, that is *system with base 10*, we should write 2413_{10}

Most significant digit (place value)

$$2413_{10} = (2 * 10^3) + (4 * 10^2) + (1 * 10^1) + (3 * 10^0)$$

base

Least significant digit (place value)

$$= (2 * 1000) + (4 * 100) + (1 * 10) + (3 * 1) = 2000 + 400 + 10 + 3 = 2413_{10}$$

Ok, now, let's see assume that this was number in base 5, that is 2413_5. What is the value of this number in decimal system?
Well, we follow the same procedure:

$$2413_5 = (2 * 5^3) + (4 * 5^2) + (1 * 5^1) + (3 * 5^0) = (2 * 125) + (4 * 25) + (1 * 5) + (3 * 1)$$
$$= 250_{10} + 100_{10} + 5_{10} + 3_{10} = 358_{10}$$

Neat! Now we are ready for binary!

What is the value of binary number 2413_2 in decimal system? (careful, this is the trick question! ☺)

- Well 2413_2 does not make sense since digits 2, 4 and 3 DO NOT EXIST in binary system!

How do we represent decimal numbers in the binary system?

Now let's try a binary number, how about $1\ 1010\ 1110_2$?

$$1\ 1010\ 1110_2$$
$$= (0 * 2^0) + (1 * 2^1) + (1 * 2^2) + (1 * 2^3) + (0 * 2^4) + (1 * 2^5) + (0 * 2^6) + (1 * 2^7) + (1 * 2^8)$$
$$= (0 * 1) + (1 * 2) + (1 * 4) + (1 * 8) + (0 * 16) + (1 * 32) + (0 * 64) + (1 * 128) + (1 * 256)$$
$$= 2_{10} + 4_{10} + 8_{10} + 32_{10} + 128_{10} + 256_{10} = 430_{10}$$

Isn't that cool? Now you try ☺

What are $1111\ 1111_2$ & $1011\ 0100_2$ in decimal system?

$$1111\ 1111_2 = 1 + 2 + 4 + 8 + 16 + 32 + 64 + 128 = 255_{10}$$
$$1011\ 0100_2 = 4 + 16 + 32 + 128 = 180_{10}$$

How do we go from decimal to binary?

Let's convert number 53_{10} to binary system.

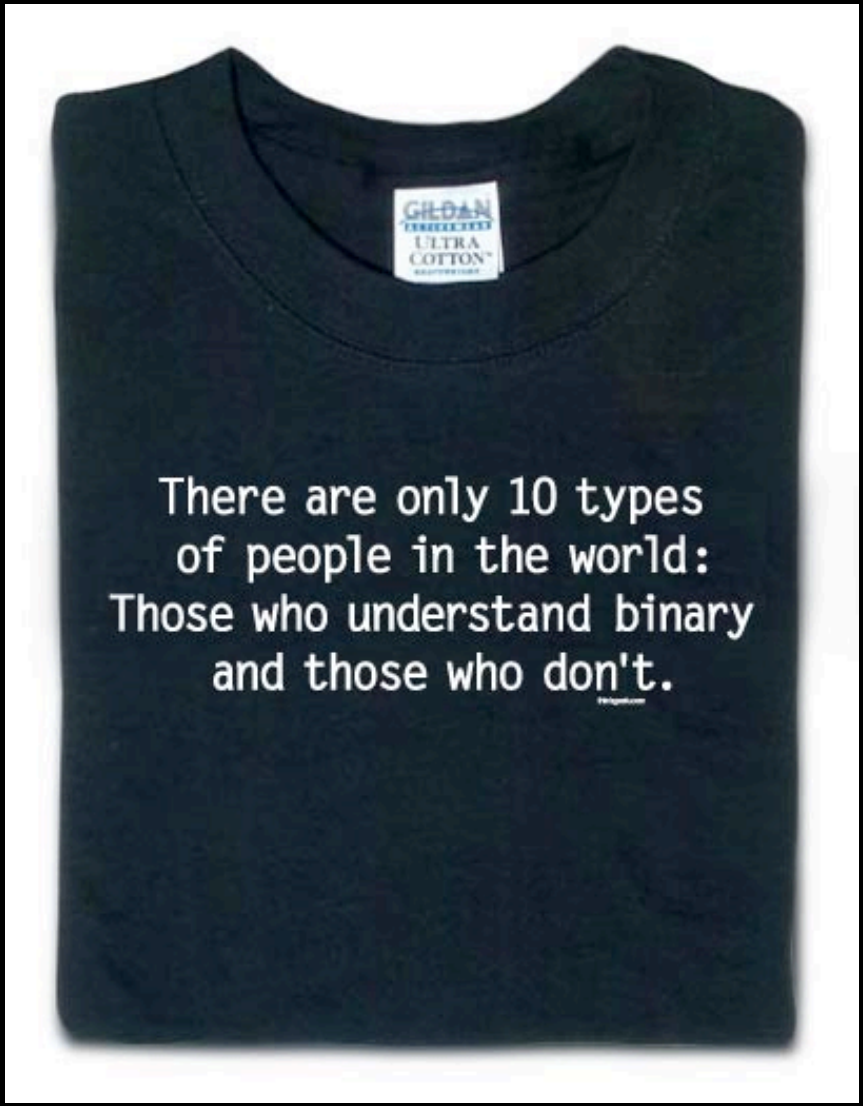
division	Integer quotient	remainder	bit	
$53/2$	26	1	b_0	least significant bit
$26/2$	13	0	b_1	
$13/2$	6	1	b_2	
$6/2$	3	0	b_3	
$3/2$	1	1	b_4	
$1/2$	0	1	b_5	most significant bit

So $53_{10} = b_5 b_4 b_3 b_2 b_1 b_0 = 11\ 0101_2$

Now you try: Represent 231_{10} as a binary number?

And $231_{10} = b_7 b_6 b_5 b_4 b_3 b_2 b_1 b_0 = 1110\ 0111_2$

Take home message…



Interesting facts – Maya Numerals

Did you know that Maya used system with base 20?

- They must have used toes and fingers to count! ☺
- They also wrote numbers vertically, with MSB on top and LSB at the bottom.

MSB

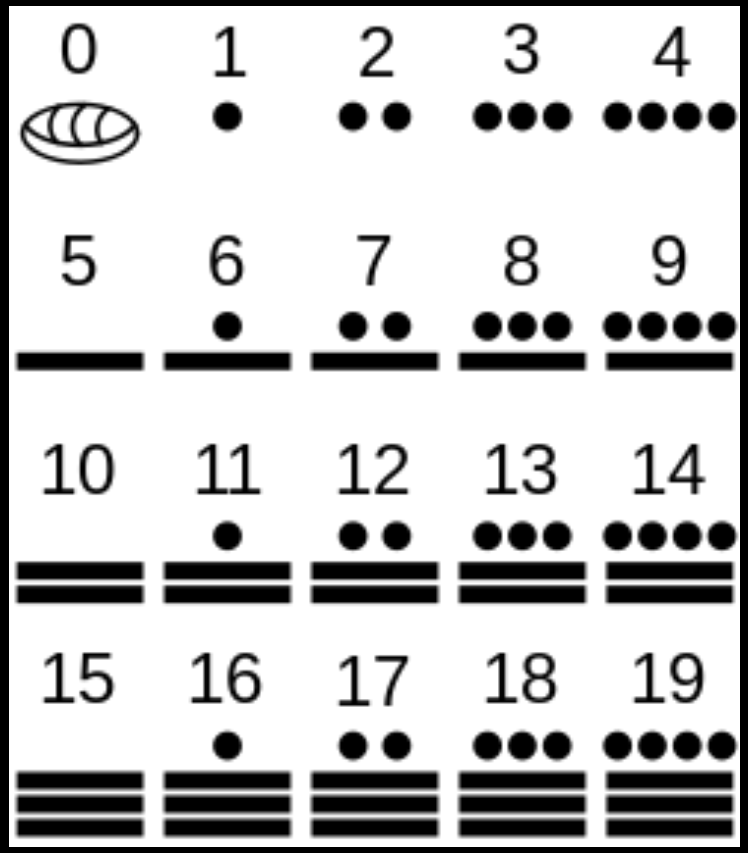
LSB

$1 * 20^3 = 8000$

$3 * 20^2 = 1200$

$12 * 20^1 = 240$

$9 * 20^0 = 9$



The mystery Maya number is 9449_{10}

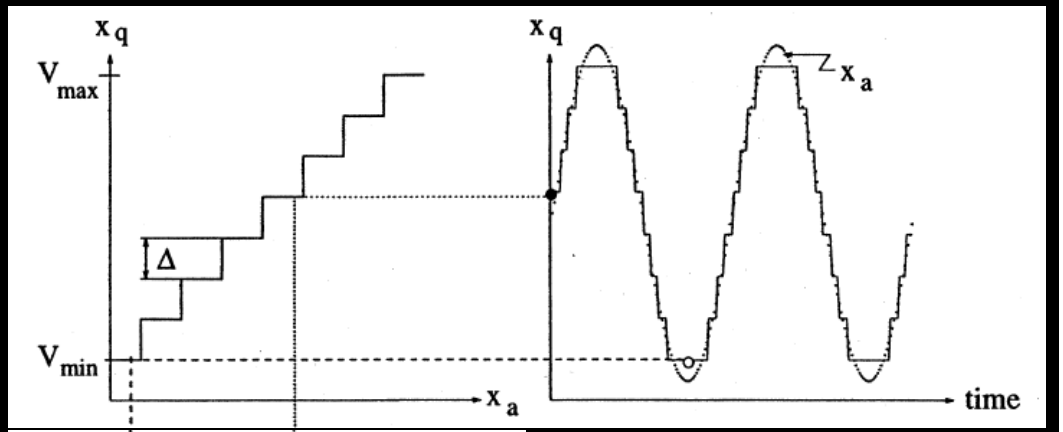
Cheat Sheet

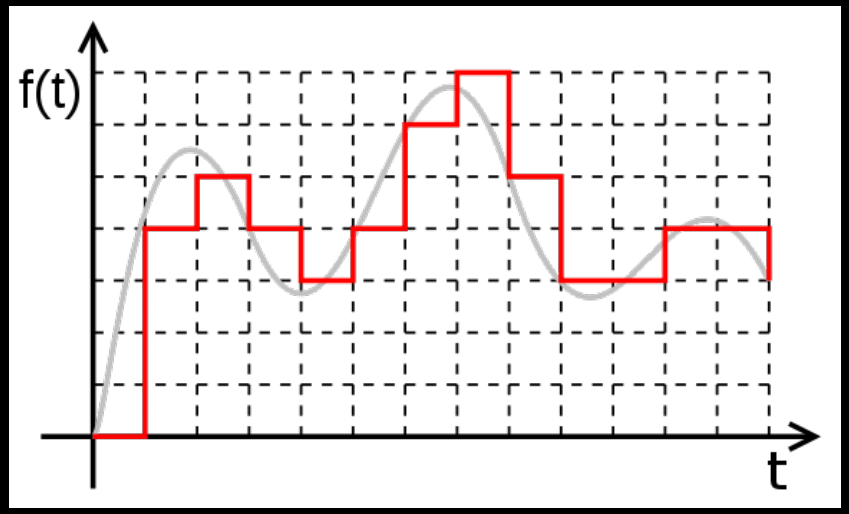
Decimal – Base 10	Binary – Base 2	Hex – Base 16	Base 4	Base 8
0	0	0	0	0
1	1	1	1	1
2	10	2	2	2
3	11	3	3	3
4	100	4	10	4
5	101	5	11	5
6	110	6	12	6
7	111	7	13	7
8	1000	8	20	10
9	1001	9	21	11
10	1010	A	22	12
11	1011	B	23	13
12	1100	C	30	14
13	1101	D	31	15
14	1110	E	32	16
15	1111	F	33	17

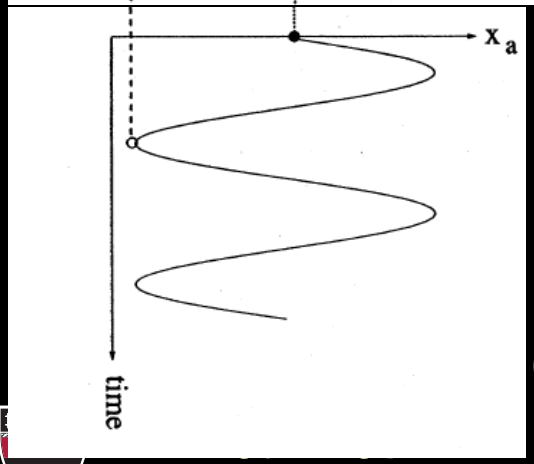
Quantizer

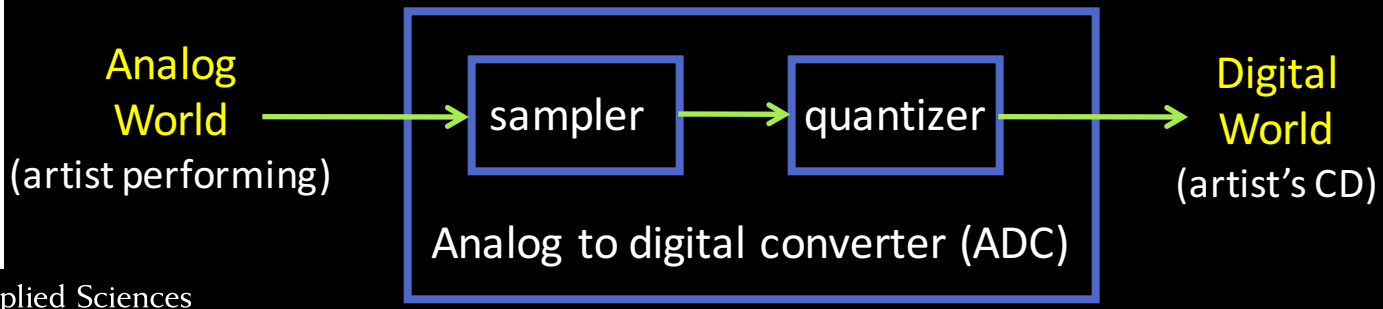
Why do we have to quantize in the first place?

- A sample of an analog signal can have any value. However, digital signals, represented with ***b*** bits can take only 2^b values so we need to round "analog" numbers to set of discrete values









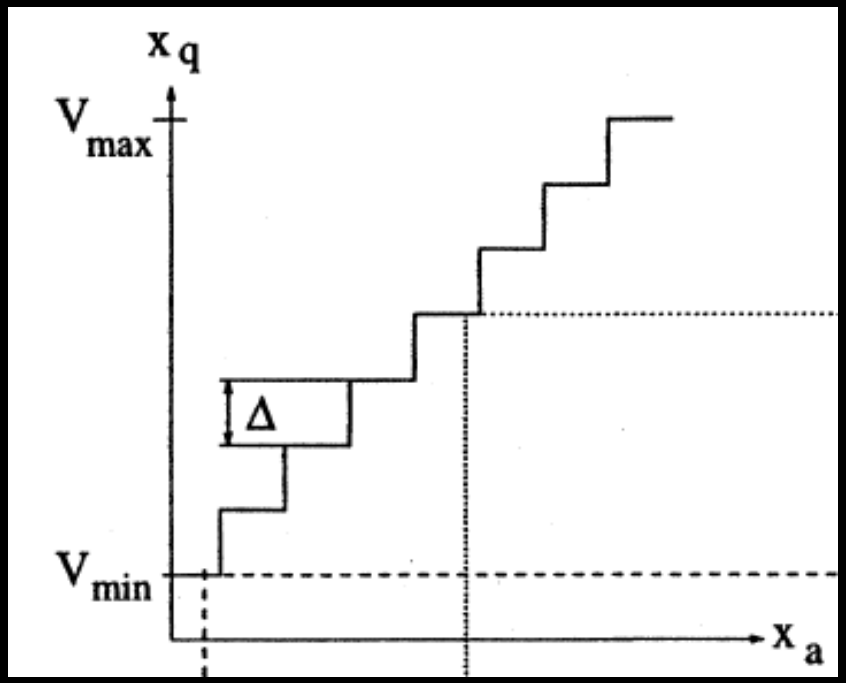
Quantizer Example

Let's say that we have a signal with $V_{min} = 0\ V$, $V_{max} = 7\ V$, and want to quantize with 3 bits.

- There are either possible binary values as $N_{steps} = 2^3 = 8$.
- Step Size = 1V

Step Size $\Delta = \frac{V_{max} - V_{min}}{N_{steps} - 1}$

Analog Value	Binary Number
0 V	000
1 V	001
2 V	010
3 V	011
4 V	100
5 V	101
6 V	110
7 V	111

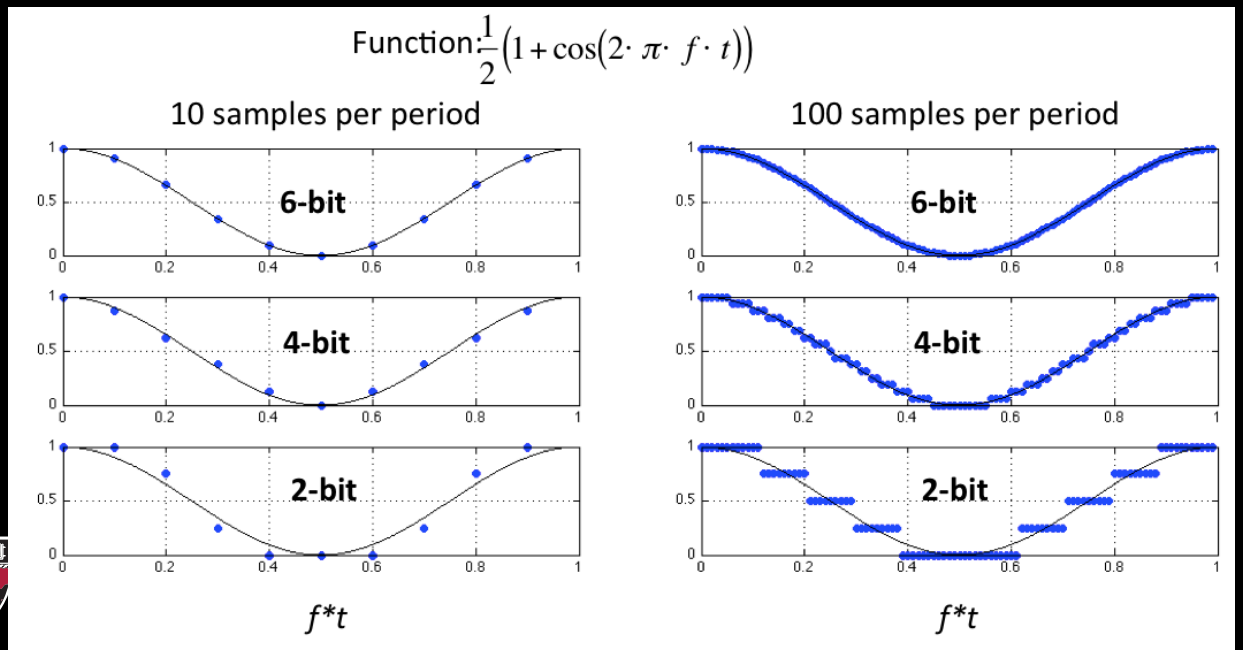


What about 4.3V input?

- It will be rounded up to 5V with error! (in this case, error is 0.7V)

ADC: Sampling & Quantization

- **Sampler:** Repeatedly **measures (samples) the amplitude** (volume) of an incoming music signal (an electrical voltage)
- **Quantizer:** rounds the sample amplitude to the nearest "allowed" level. Number of level is determined by number of bits used in quantizer. It also converts the quantized levels into digital data, stream of 0s and 1s.
  - 4 bits ⇔ 16 levels: 0000, 0001, 0010, 0011, 0100, 0101, 0110, 0111, 1000, 1001, 1010, 1011, 1100, 1101, 1110, 1111
- **Higher sample rates produce better quality recordings but also bigger file sizes** which demand greater space on storage devices (such as hard drives), and faster processors (CPUs) to manipulate.
- **Sample length in bits determines the accuracy of each sample measurement**. Better accuracy means less distortion which results in better sound!



Sampled & Quantized signal
Discrete both in time and amplitude, & converted to stream of binary digits (bits)