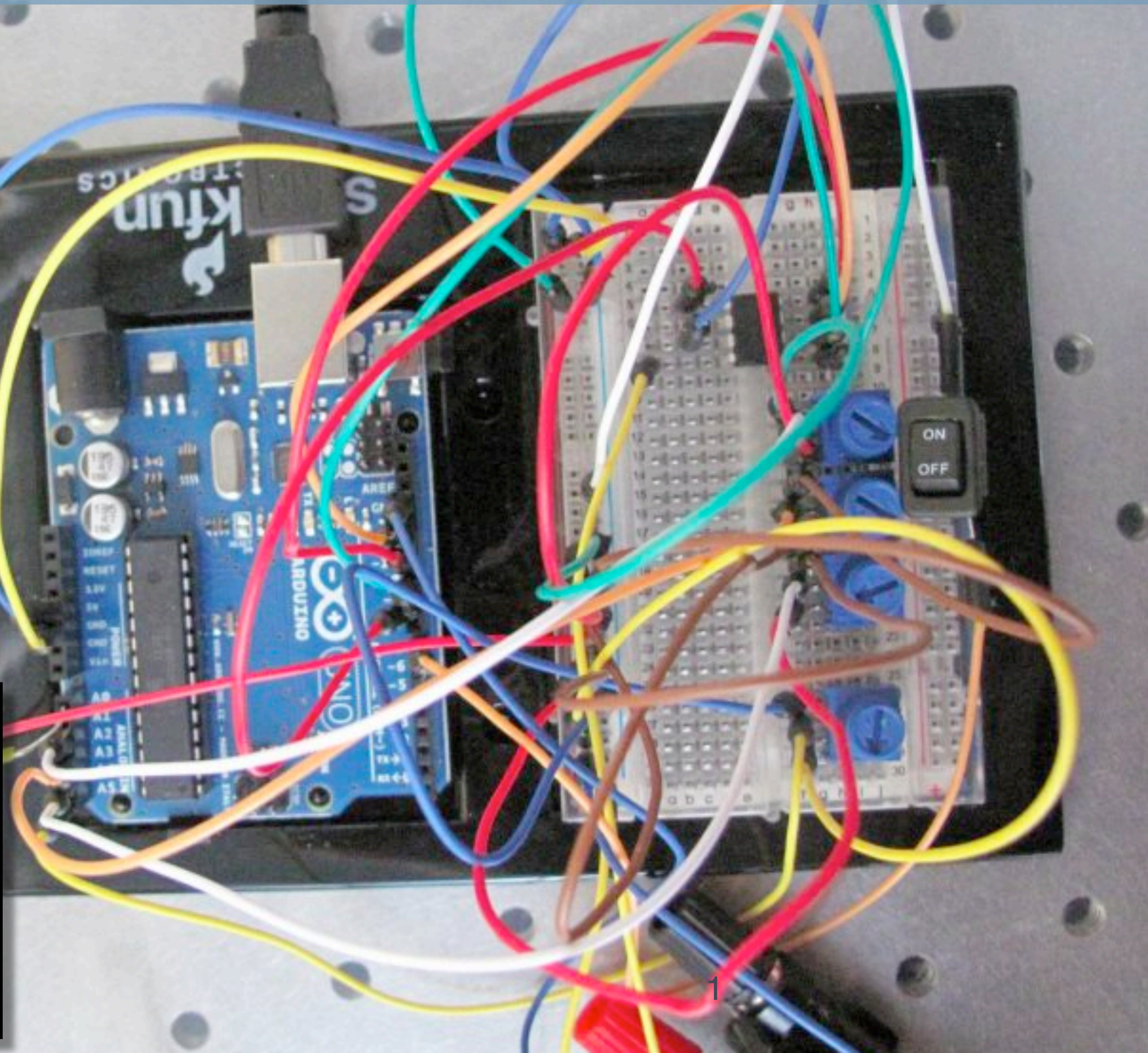
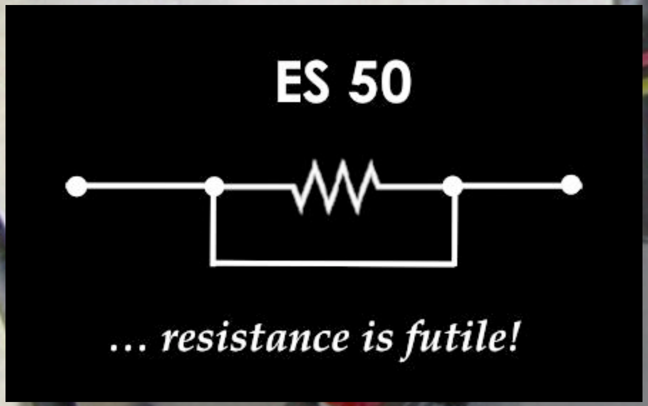
ES 50: Introduction to Electrical Eng.

Explore EE, satisfy a Gen Ed requirement & have fun





Lecture 5: Amplifiers (Part 2)

ES 50: Introduction to Electrical Engineering

Announcements

- **Review / problem sessions Wed & Thur 7-9:30PM moved to *MD119 lobby***

Last time…

- Dependent (voltage and current) sources
- Thevenin and Norton equivalent circuits
- Operational Amplifiers
  - Ideal characteristics
  - Realistic example
- Op-amp circuits

Today,

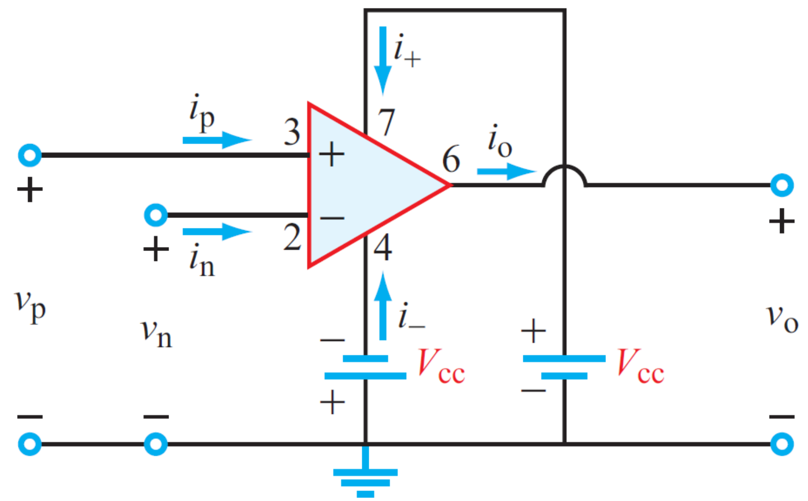
- More op-amp circuit topologies
- Several op-amp examples

Operational Amplifier “op-amp” summary

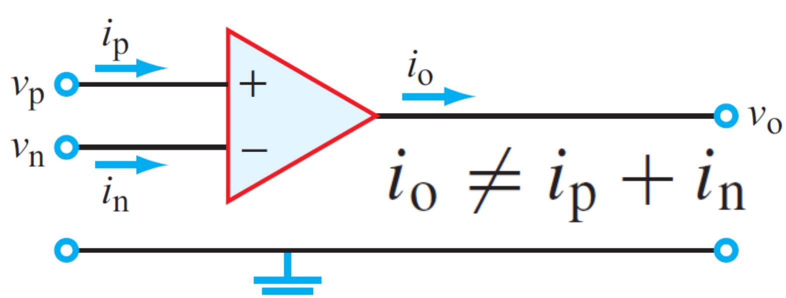
Important Characteristics:

- **One output terminal (V_o)**
  - Behaves as an ideal voltage source (i.e., no output resistance)
- **Two input terminals:**
  - positive (non-inverting): if V_+ increases V_o **increases**
  - negative (inverting): if V_- increases V_o **decreases**
  - $V_{in}=V_+-V_-$
  - Input resistance between + and – terminal is extremely large!
- **Needs power supply**
  - Often not shown in ES 50 circuits

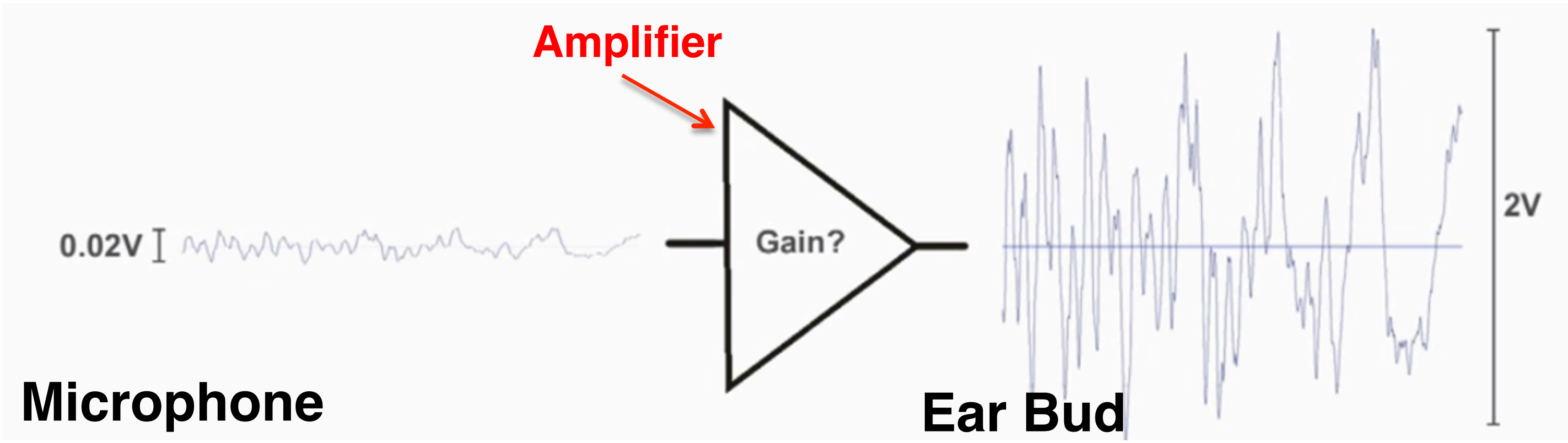
Op-amp showing power supply rails



Op Amp with power supply rails **not** shown (how we usually show op-amps in ES 50)



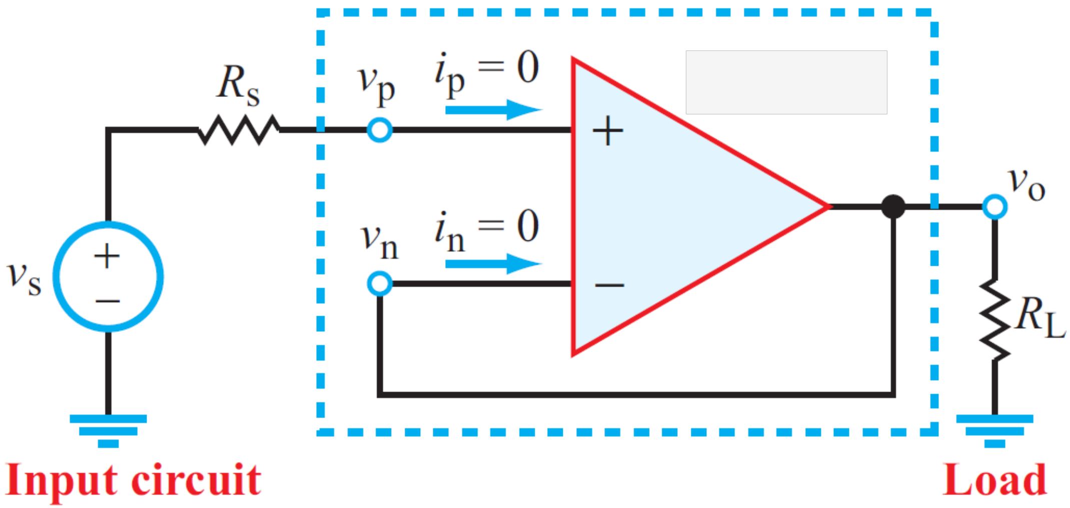
Example: Let's build an amplifier that picks up a heart beat and turns it into sound or flashing light



We need a gain of ~100, but the op-amp has a gain of $A=10^6$.
What do we do?

Negative Feedback is used to control the gain

- Feedback: Return some of the output to the input



- $v_n = v_o$ (shorted)

Op-Amp Golden rules:

- $i_p = 0$ ➔ No voltage drop across R_S ➔ $v_p = v_S$
- $v_o = A * (v_p - v_n)$

then…

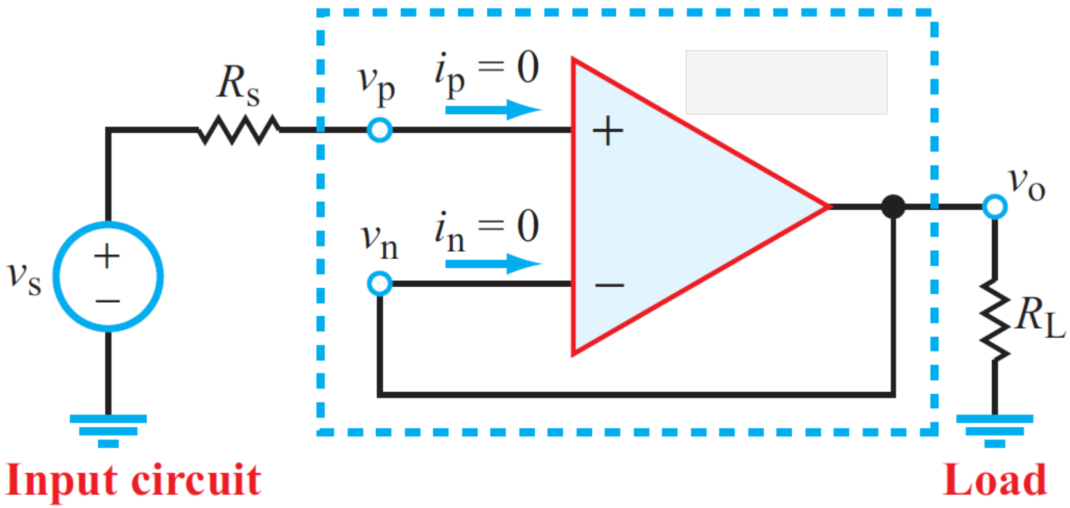
$$v_o = A(v_s - v_o)$$
$$v_o(1 + A) = Av_s$$
$$\boxed{v_o = v_s \frac{A}{1 + A}}$$

As A → ∞ we get the following approximation:

$$\boxed{v_o \approx v_s}$$

Negative Feedback is used to control the gain

- Feedback: Return some of the output to the input



As A → ∞ we get the following approximation:

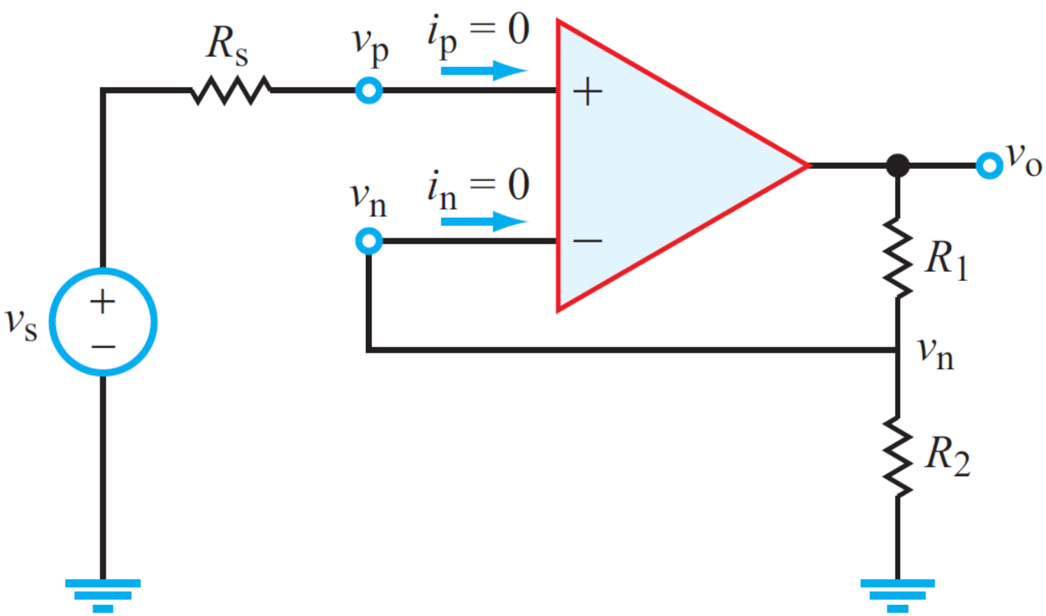
$$v_o = v_s$$

$$v_p = v_n$$

Also called a unity-gain buffer or voltage follower

Important property when solving op-amp circuits

How can we get gain > 1?



$$v_p = v_s$$

$$v_n = v_o \frac{R_2}{R_1 + R_2}$$

$$v_p = v_n$$

$$v_s = v_o \frac{R_2}{R_1 + R_2}$$

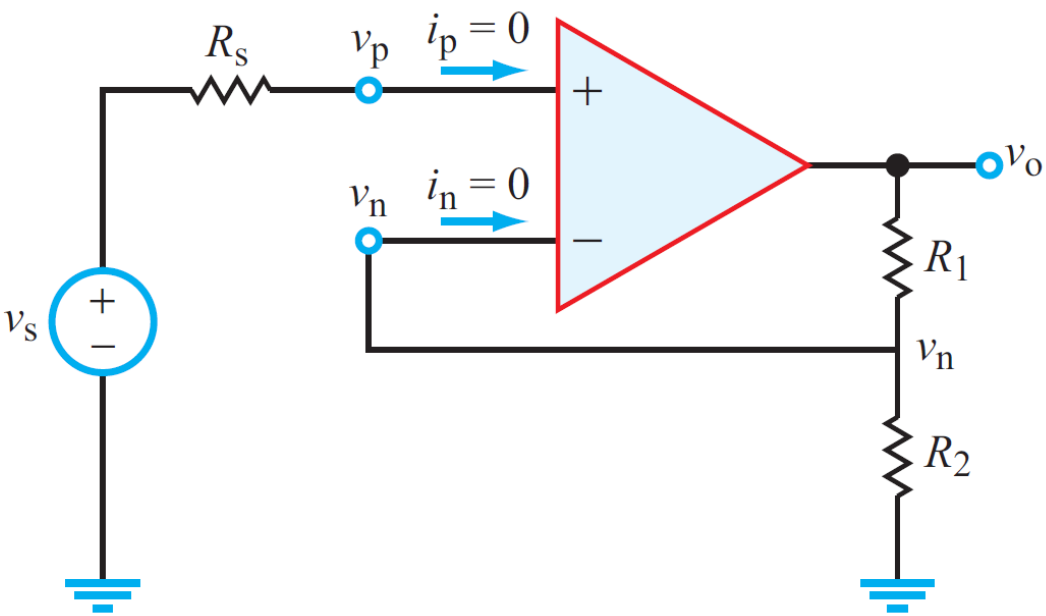
$$v_o = v_s \frac{R_1 + R_2}{R_2}$$

$$\boxed{G = \frac{v_o}{v_s} = \frac{R_1 + R_2}{R_2}}$$

To get **closed-loop gain** G = 100:

- $R_1 = 99 * R_2$
- Let $R_1 = 100\text{k}\Omega \rightarrow R_2 = 1\text{k}\Omega$
  (sensitive to accurate resistor values)

Can find G using the node voltage method



Recall node voltage method:
Pick nodes w/ unknown voltage and compute KCL

At node v_n:

$$\frac{v_o - v_n}{R_1} = \frac{v_n}{R_2} + i_n$$

$$\frac{v_o}{R_1} = v_n\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$v_o = v_n\frac{R_1 + R_2}{R_2}$$

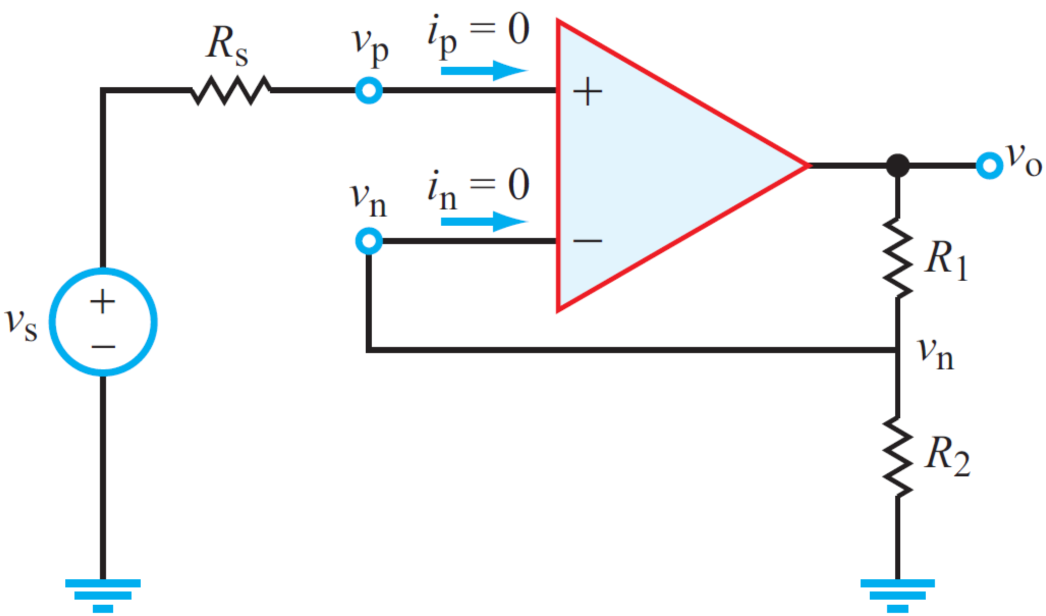
At node v_p:

$$v_p = v_s$$ **b/c $i_p = 0$**

$$v_p = v_n$$ **b/c $A = \infty$**

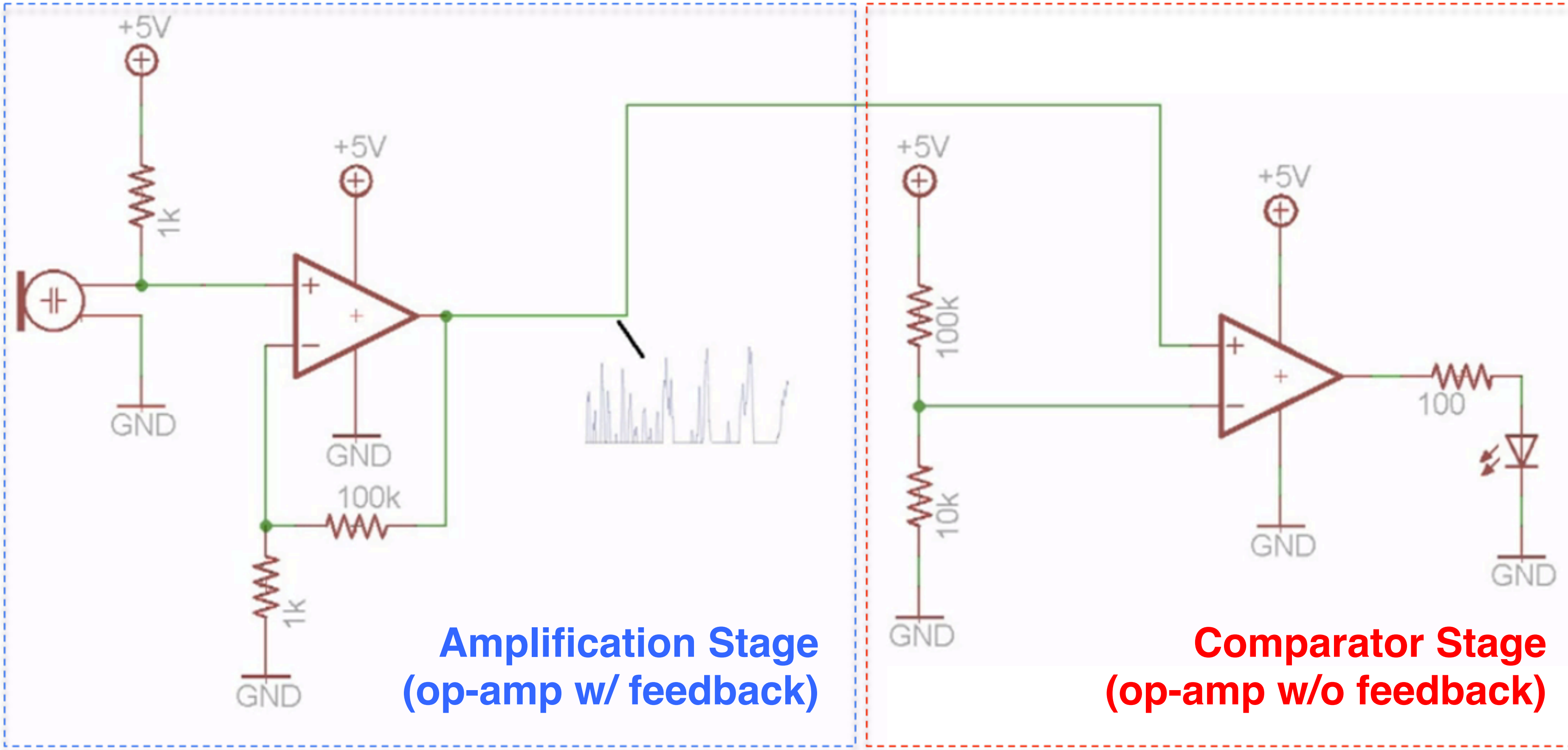
$$G = \frac{v_o}{v_s} = \frac{R_1 + R_2}{R_2}$$

Again, what's going on intuitively?



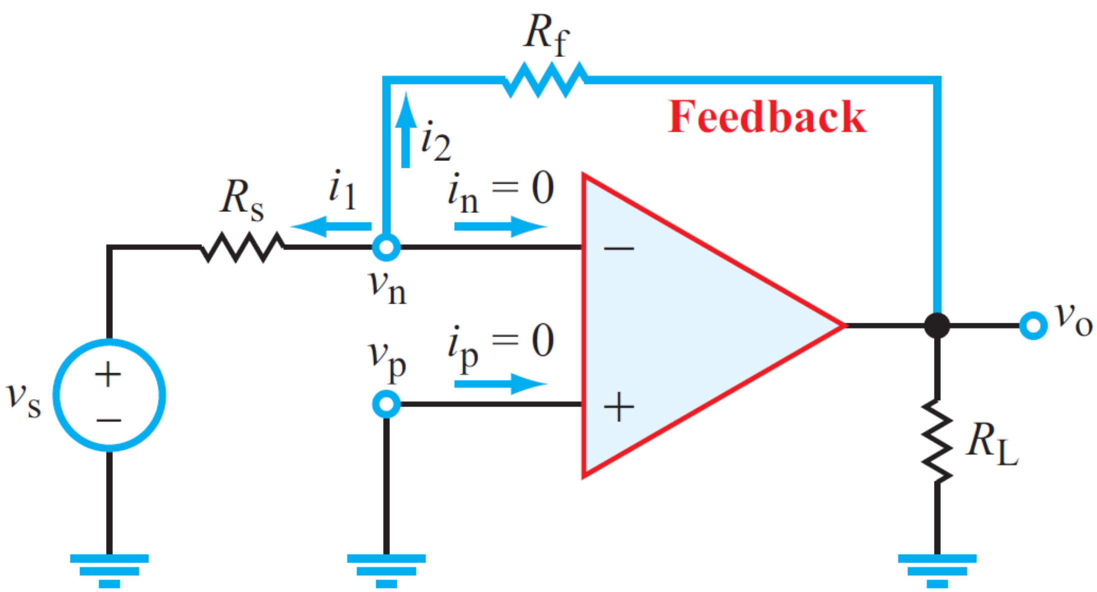
- Let's go around the loop one more time.
  - Assume v_s starts at 0V (v_o also at 0V) and increases by ΔV.
  - v_p-v_n will increase by ΔV and cause the amplifier to drive v_o towards A*ΔV and v_n towards A*ΔV*(R_2/ (R_1+R_2)).
  - But with negative feedback, we get $v_o/v_s = (R_1+R_2) / R_2$.

And here is our heart-rate indicator ☺



The same circuit can work as a clap-detector: http://www.youtube.com/watch?v=y0Q0ERSP24A
(also a very nice tutorial on op-amps!)
Music beat sensor: http://www.youtube.com/watch?v=OTwg2Eal8x4
Or, turn it into an Arduino-based heart rate monitor: http://www.youtube.com/watch?v=5ekHuCT2a1M

Another very common topology: *Inverting* Amplifier



KCL at node v_n:

$$\frac{v_n - v_s}{R_s} + \frac{v_n - v_o}{R_f} + i_n = 0$$

$$v_n = v_p = 0$$

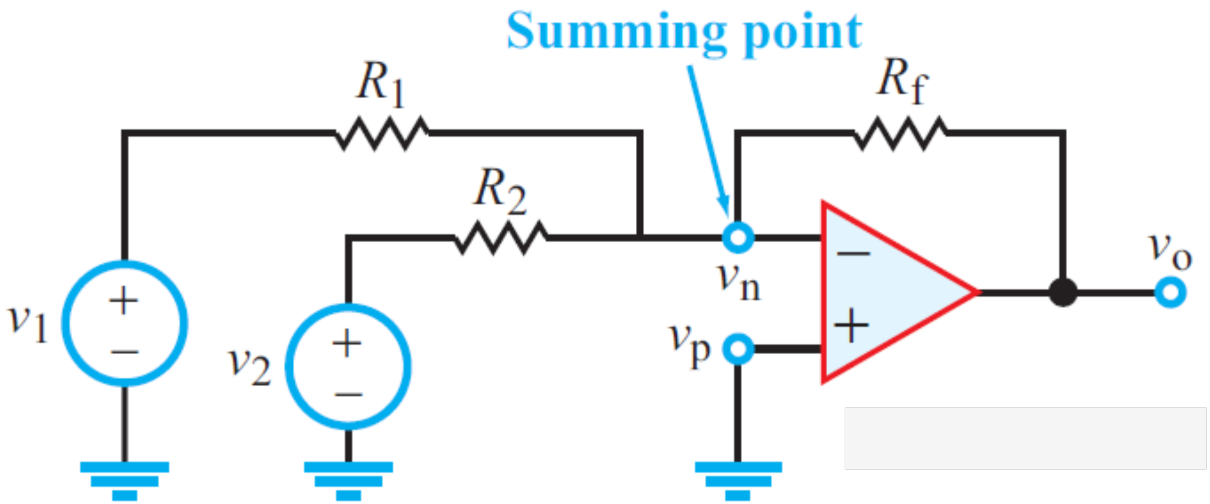
$$\frac{v_s}{R_s} + \frac{v_o}{R_f} = 0$$

$$\boxed{G = \frac{v_o}{v_s} = -\frac{R_f}{R_s}}$$

Notice: Input, $\boldsymbol{v_s}$, connected to the ***inverting*** input of the op-amp

Inverting amplifier topology:
Negative closed-loop gain

Summing Amplifier



KCL at node v_n:

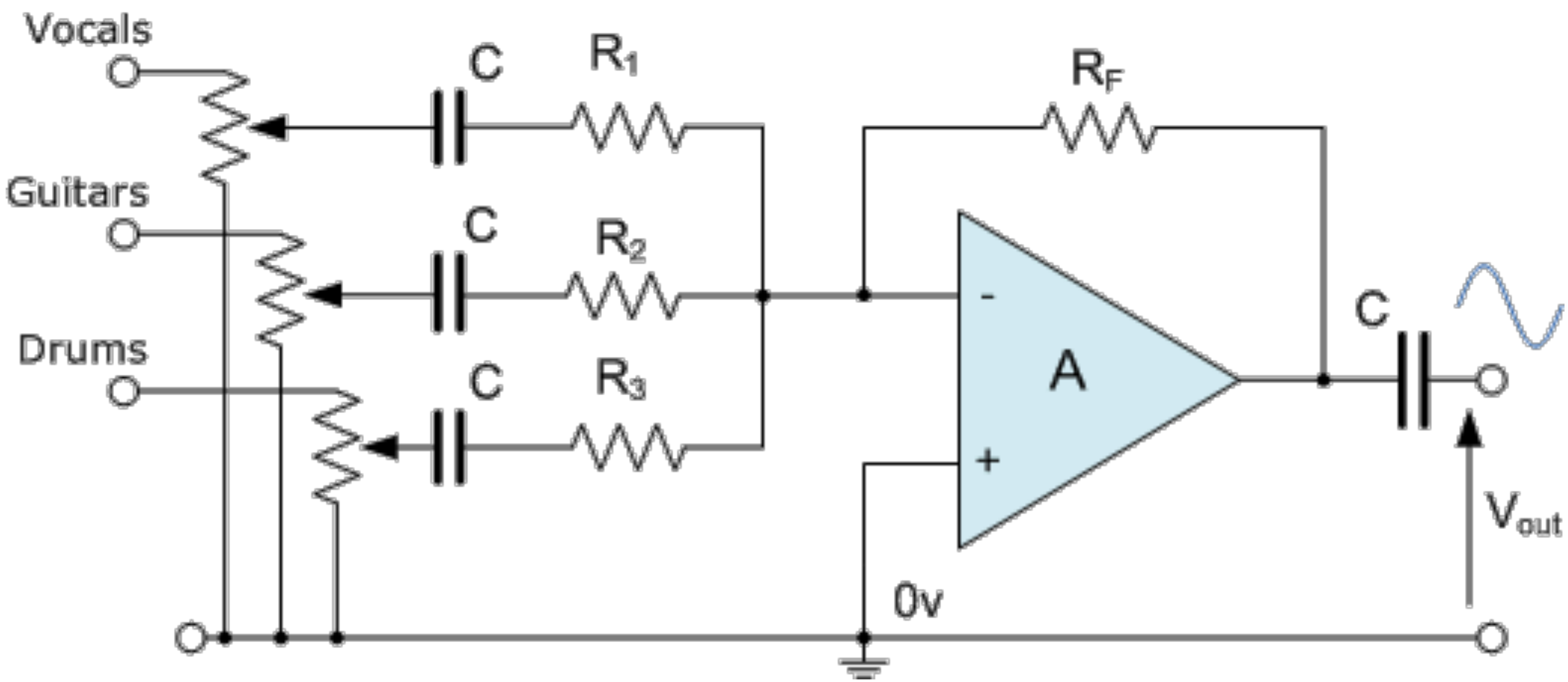
$$-\frac{v_o - v_n}{R_f} = \frac{v_1 - v_n}{R_1} + \frac{v_2 - v_n}{R_2}$$

$$-\frac{v_o}{R_f} = \frac{v_1}{R_1} + \frac{v_2}{R_2}$$

$$\boxed{v_o = -\left(\frac{R_f}{R_1}v_1 + \frac{R_f}{R_2}v_2\right)}$$

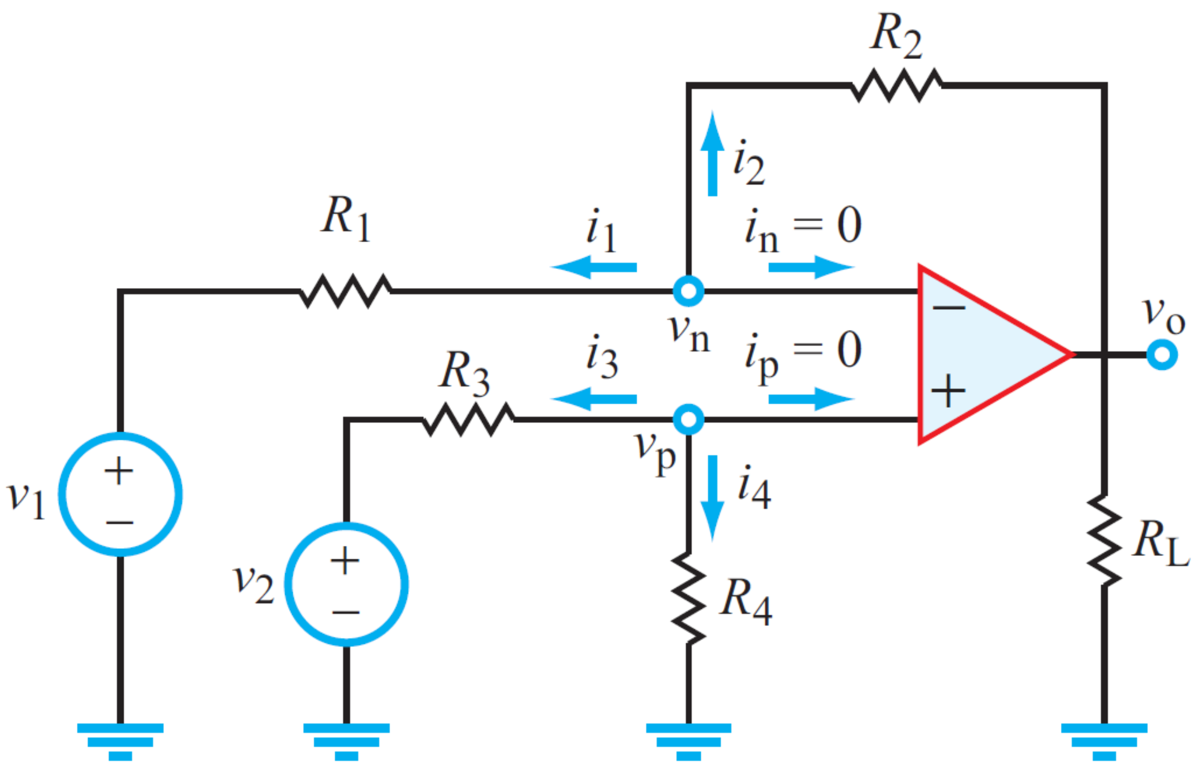
We could have also used superposition

Application of summing amplifier: **Music Mixer** (any DJs around?)



RC circuits are often used for filtering. Here, amplifier A amplifies a combination of the three incoming signals. Relative amplitudes set via potentiometers (voltage dividers).

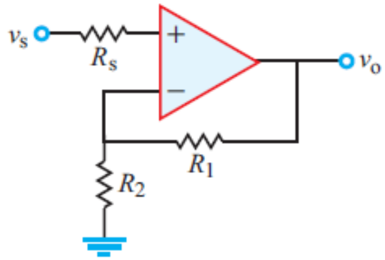
Difference Amplifier – Details in HW



$$v_o = \left[\left(\frac{R_4}{R_3 + R_4}\right)\left(\frac{R_1 + R_2}{R_1}\right)\right] v_2 - \left(\frac{R_2}{R_1}\right) v_1,$$

Table 4-3: Summary of op-amp circuits.

<table>
<tr><th>Op-Amp Circuit</th><th>Block Diagram</th></tr>
<tr><td>



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$$v_s \rightarrow \boxed{G = \frac{R_1 + R_2}{R_2}} \rightarrow v_o = Gv_s$$

Noninverting Amp
(v_o independent of R_s)

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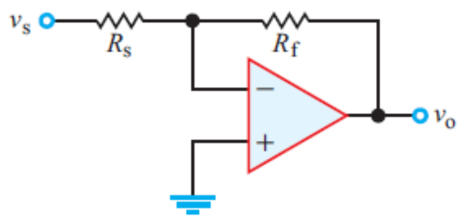

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$$v_s \rightarrow \boxed{G = -\frac{R_f}{R_s}} \rightarrow v_o = Gv_s$$

Inverting Amp

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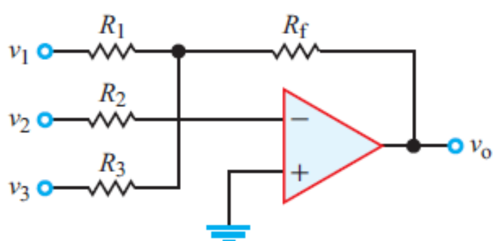

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$$v_1 \rightarrow \boxed{G_1 = -R_f/R_1}$$
$$v_2 \rightarrow \boxed{G_2 = -R_f/R_2} \rightarrow \oplus \rightarrow v_o = G_1 v_1 + G_2 v_2 + G_3 v_3$$
$$v_3 \rightarrow \boxed{G_3 = -R_f/R_3}$$

Inverting Summer

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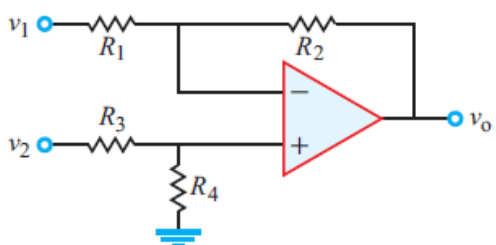

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$$v_1 \rightarrow \boxed{G_1 = -\frac{R_2}{R_1}}$$
$$\oplus \rightarrow v_o = G_1 v_1 + G_2 v_2$$
$$v_2 \rightarrow \boxed{G_2 = \left(\frac{R_1 + R_2}{R_1}\right)\left(\frac{R_4}{R_3 + R_4}\right)}$$

Subtracting Amp

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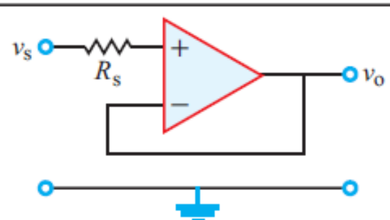

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$$v_s \rightarrow \boxed{G = 1} \rightarrow v_o = v_s$$

Voltage Follower
(v_o independent of R_s)

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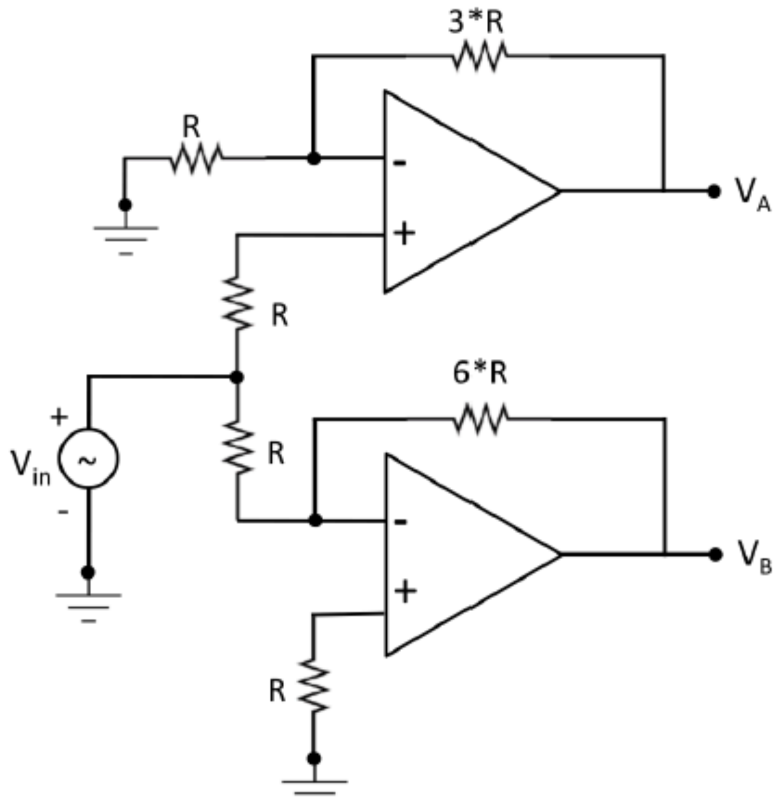
Example 1



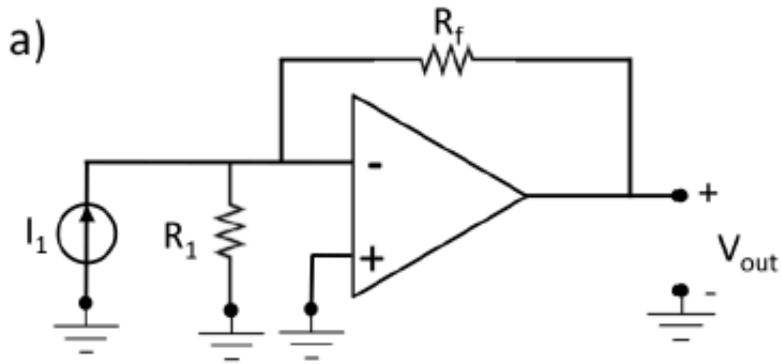
$$\frac{V_A}{V_{in}} = \frac{R+3R}{R} = 4$$

$$\frac{V_B}{V_{in}} = -\frac{6R}{R} = -6$$

$$\frac{Vout}{V_{in}} = \frac{V_A - V_B}{V_{in}}$$
$$= 4-(-6) = 10$$

1. Find V_A as a function of V_{in} and all resistor values.
2. Repeat for V_B.
3. If we define $V_{out} = V_A - V_B$, what is the gain, V_{out}/V_{in}?

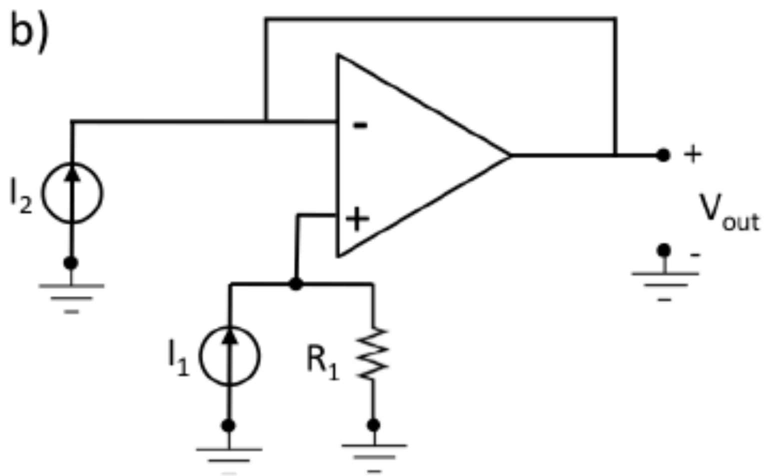
Example 2



$$\frac{V_{out}}{I_1} = -R_f$$

- For the circuit (a) above, find V_{out} in terms of R_1, R_f, I_1, and I_2. Hint: Can we use Thevenin and Norton equivalent circuits?

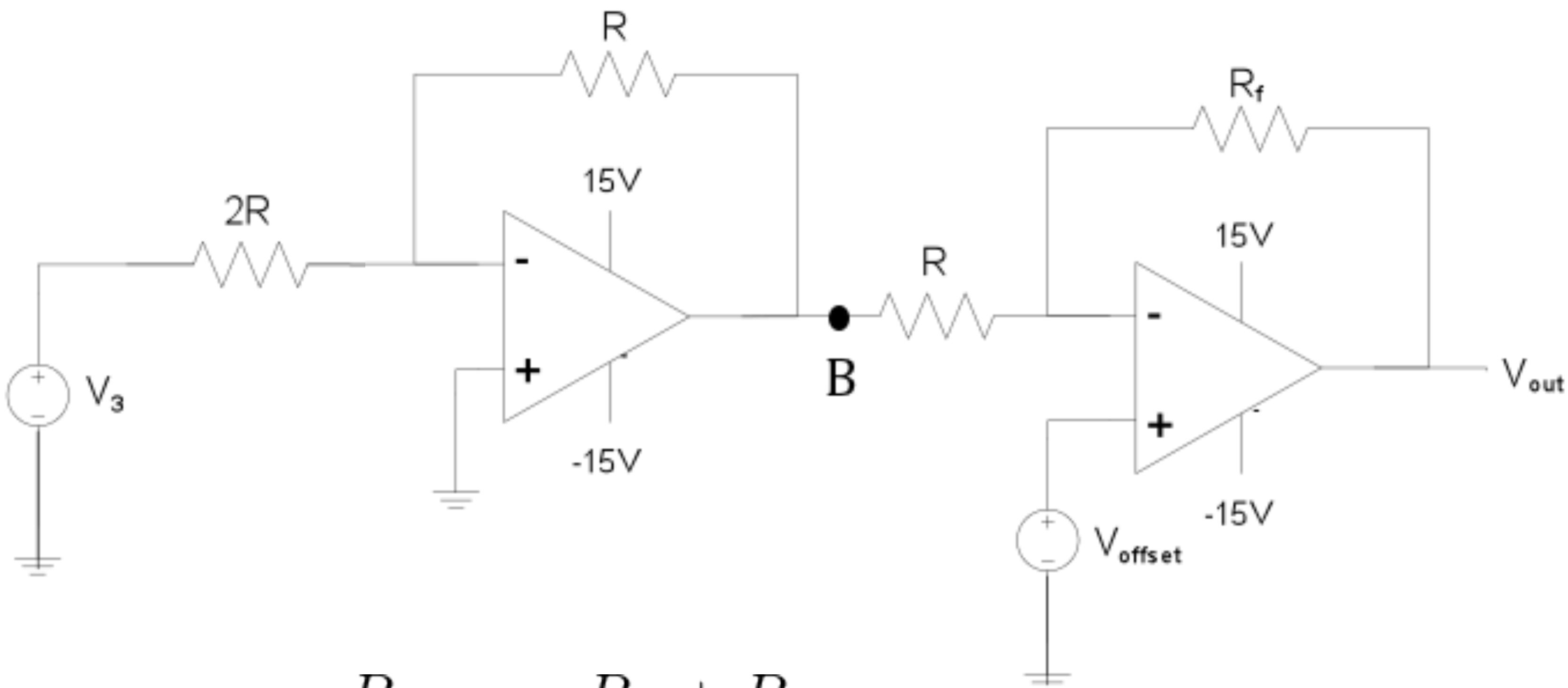
Example 3



$$\frac{V_{out}}{I_1} = R_1$$

- For the circuit (b) above, find V_{out} in terms of R_1, R_f, I_1, and I_2. Hint: Can we use Thevenin and Norton equivalent circuits?

Example 4

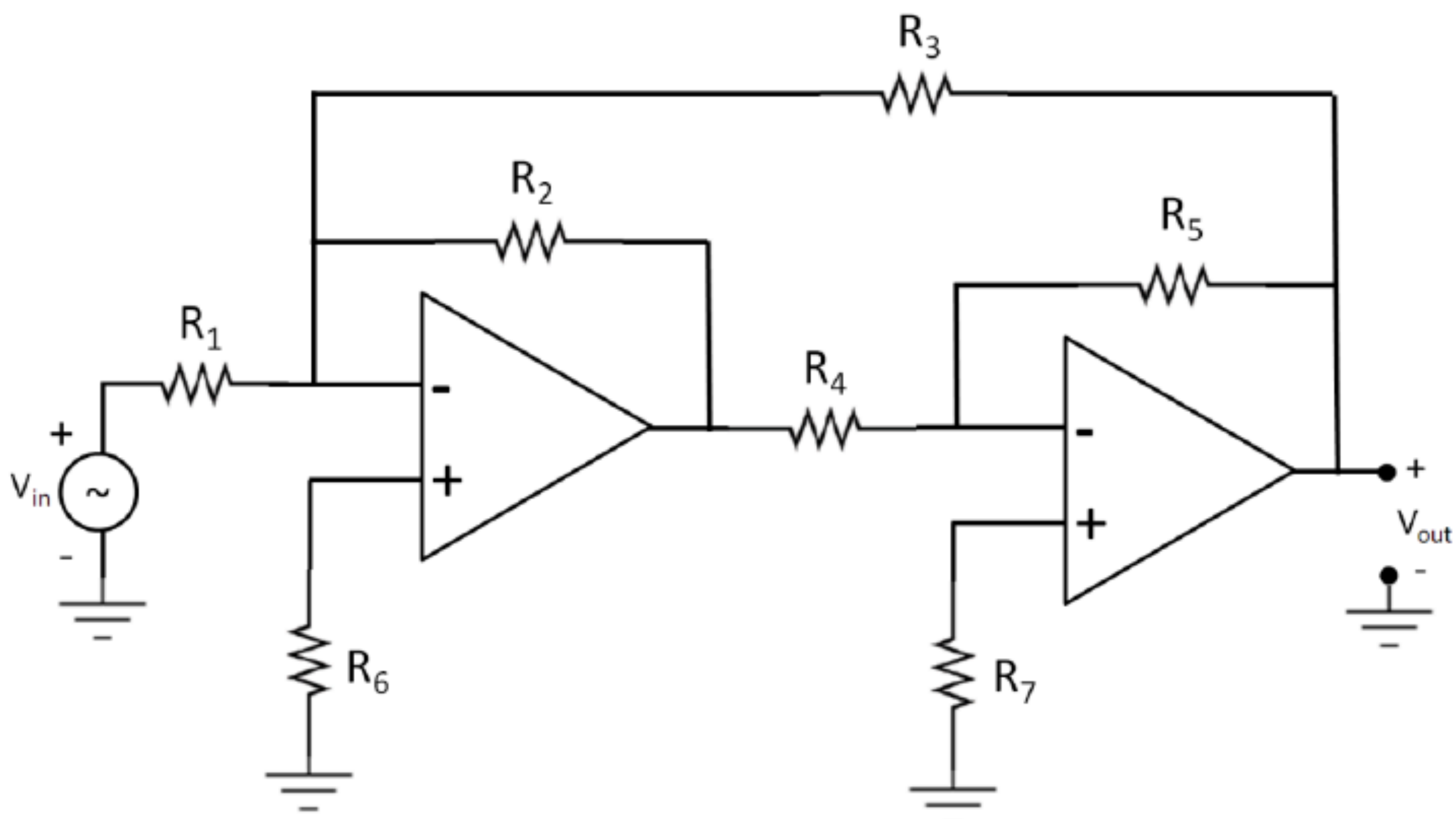


$$V_{out} = \frac{R_f}{2R} V_3 + \frac{R_f + R}{R} V_{\text{offset}}$$

- Find V_{out} in terms of V_3, R, R_f, and V_{offset}.

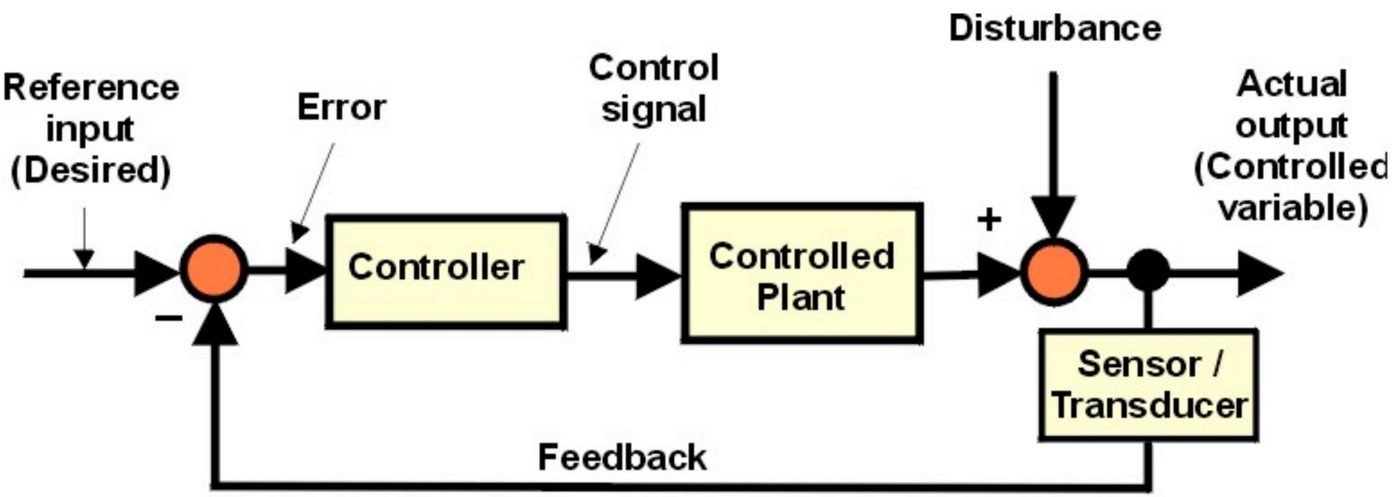
Last example

$$\frac{V_{out}}{V_{in}} = \frac{R_2 R_3 R_5}{R_1 R_3 R_4 - R_1 R_2 R_5}$$



- Find V_{out}/V_{in} in terms of R_1~R_7.

Feedback: A general discussion (basics of control theory)



- **<u>Feedback:</u>** Used to provide a desired output in spite of fluctuations, disturbances, nonlinearity, etc. in the system
- **<u>The main idea:</u>** The result of an action is fed back to the input and compared to the desired outcome.
- The principles of feedback are widespread, and can be found in many disciplines:
  - Robotics, electrical and mechanical engineering, manufacturing plants, navigation systems, ecology (wolves and rabbits), business & economics, social systems (health care), and many many more