

1. The joint pmf of X and Y is given below. Determine (a) the value of k (b) the marginal pmf of X and (c) $P(Y = 2 | X = 0)$. Are X and Y independent?

$x \backslash y$	1	2	3
-1	$\frac{1}{17}$	$\frac{3}{17}$	$\frac{2}{17}$
0	$\frac{3}{17}$	k	$\frac{1}{17}$
1	$\frac{1}{17}$	0	$\frac{1}{17}$

2. The joint probability density function of X and Y is given below. Determine (a) the value of k (b) $P(0.2 < X < 0.7, 0.6 < Y < 0.9)$ (c) the marginal density of X and (d) the condition density $f_{Y|X}(y | 0.25)$. Are X and Y independent?

$$f(x, y) = \begin{cases} k(2x + y) & 0 < x < 1, 0 < y < 1 \\ 0 & \text{otherwise.} \end{cases}$$

3. The probability function of X is given below. Determine the mean and variance of X .

x	1	2	3	4
$p(x)$	0.1	0.3	0.4	0.2

4. The density of X is given below. Determine the mean and variance of X .

$$f(x) = \frac{3}{8}x^2, \quad 0 < x < 2.$$

5. The joint density of X and Y is given below. Calculate $E(X^2Y)$.

$$f(x, y) = \begin{cases} x + y & 0 < x < 1, 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

6. In an MCQ test, there are 10 questions each with 5 choices. If Smith chooses all the answers at random, what is the probability that the number of correct answers will be less than 2?
7. Refer to Question 6. If Smith and Jones independently choose all the answers at random, what is the probability that (a) the number of correct answers will be 2 and 3, respectively? (b) total number of correct answers will be 5?
8. Mean and variance of a binomial random variable are 4 and 2, respectively. Calculate $P(X = 0)$.

9. A box contains 20 mobile phones that are in good condition and 10 that are defective. You randomly selected 5 phones from the box without replacement. What is the probability that at least 2 of them will be in good condition?
10. Refer to Question 9. If the box contains 2000 good phones and 1000 defective ones, find an approximate probability that at least 2 out of 5 randomly selected phones will be good.
11. Let $X \sim \text{Poisson}(\lambda)$. If $V(X) = 4$, calculate $P(X = 1)$.
12. The average number of misprints in one page of a book is 3.2. What is the probability that there will be 2 and 4 misprints, respectively, in the next two pages?
13. The average number of people entering a bank between 9:00-10:00 A.M. and 10:00-11:00 A.M. are 3 and 2, respectively. Find the probability that at most 2 people will enter the bank between 9:00-11:00 A.M. tomorrow.
14. A certain genetic characteristic will express itself in a person with probability 0.001. Use two different distributions to calculate the probability that at least one out of 1000 people will display the characteristic.
15. The mean and variance of a uniformly distributed random variable X are 4 and 3, respectively. Calculate $P(4 < X < 6 \mid X > 5)$.
16. The weight of a particular type of fish (in lbs) is normally distributed with mean 20 and standard deviation 4. What is the probability that the next fish you catch will weigh more than 24 lbs?
17. Let $X_1 \sim N(20, 9)$ and $X_2 \sim N(40, 16)$, and they are independent. Find $P(X_1 + X_2 > 70)$.
18. The price of an egg (BDT) is normally distributed with mean 10 and standard deviation 1. What is the probability that the total price of four eggs will be more than 44?
19. The weight of a particular type of fish (in lbs) is normally distributed with mean 10 and standard deviation 1. What is the probability that the total weight of next 4 fish you catch will be more than 44 lbs?
20. $P(-k < Z < k) = 0.90$. Determine the value of k .
21. Lifetime of a particular type of radio follows exponential distribution with an average lifetime of 2 years. If you buy a 3-year-old radio, what is the probability that it will survive at least 4 more years?
22. Refer to Question 21. Specify the distribution of the total lifetime of 5 randomly chosen radios.

23. Let X follow normal distribution with mean 40 and standard deviation 4. Let a sample of size 16 be taken from the distribution. What is the probability that \bar{X} will be less than 39.5?
24. Survival time (in years) after a certain operation has mean 5 and SD 1. What is the approximate probability that the average survival time of 100 patients will be more than 5.2?
25. Let X_1, X_2, \dots, X_n be a random sample from $\text{Poisson}(\lambda)$ distribution. Obtain the MLE of λ .
26. An automatic machine is calibrated to fill cups with 250 ml of cola. You want to check whether the machine is calibrated correctly. A sample of 25 cups yielded a mean of 248 ml and a standard deviation of 10 ml. Construct a 95% confidence interval for the true mean and comment on machine calibration.
27. Average income of 64 males is 33.7 (thousand), while average income of 100 females is 35.5 (thousand). The two populations are independently normal with known variances 8 and 12.5, respectively. Construct a 90% confidence interval for the difference in means of the two populations.
28. Out of a random sample of 100 students at a university, 18 stated that they were smokers. Based on this, construct a 90% CI for the proportion of all students at the university who are smokers.
29. Let X_1, X_2, \dots, X_n be a random sample from $\text{exponential}(\lambda)$ distribution. Explain how we can obtain a 95% CI for λ .
30. Let $X \sim \text{binomial}(n, \theta)$, where the prior distribution of θ is $\text{uniform}(0, 1)$. Obtain the Bayes' estimator of θ . In a particular experiment, if there are 13 successes out of 20 Bernoulli trials, what is the Bayes' estimate of θ ?