Statistical Quality Control

Control charts are used to monitor how the production process is going, so that we can take measures when the process is out of control. To employ such a chart, the data generated by the manufacturing process are divided into subgroups. Subgroup statistics — such as the subgroup average and subgroup standard deviation — are computed and plotted in the control chart. If the values within the upper and lower control limits, then the process is in control.

$\bar{X}$ control chart

Suppose that, when in control, the items produced have a measurable characteristic that is normally distributed with mean μ and variance σ^2. We take k small samples (subgroups) of size n each, where k should be chosen so that $k \geq 20$ and $nk \geq 100$. We calculate $\bar{X}_i, i = 1, 2, \cdots, k$. We estimate μ by $\bar{\bar{X}}$ which is calculated as

$$\bar{\bar{X}} = \frac{1}{k}(\bar{X}_1 + \bar{X}_2 + \cdots + \bar{X}_k).$$

To estimate σ, let S_i denote the sample SD of the ith subgroup. Let

$$\bar{S} = \frac{1}{k}(S_1 + S_2 + \cdots + S_k).$$

The statistic $\bar{S}$ is not an unbiased estimate of σ, that is, $E(\bar{S}) \neq \sigma$. To build an unbiased estimate of σ, let us recall that

$$\frac{(n-1)S_i^2}{\sigma^2} \sim \chi_{n-1}^2$$

It can be shown that

$$E(\bar{S}) = E(S_i) = c(n)\,\sigma$$

where

$$c(n) = \frac{\sqrt{2}\ \Gamma(n/2)}{\sqrt{n-1}\ \Gamma((n-1)/2)}$$

Thus, $\bar{S}/c(n)$ is an unbiased estimate of σ.

We compute the control limits as follows:

$$\mathrm{LCL} = \bar{\bar{X}} - \frac{3\,\bar{S}}{\sqrt{n}\,c(n)}$$

$$\mathrm{UCL} = \bar{\bar{X}} + \frac{3\,\bar{S}}{\sqrt{n}\,c(n)}$$

We now check whether each of the subgroup averages falls within these lower and upper limits. Any subgroup whose average value does not fall within the limits is removed (assuming that the process was temporarily out of control) and the estimates are recomputed. We then again check that all the remaining subgroup averages fall within the control limits. If not, then they are removed, and so on. If too many of the subgroup averages fall outside the control limits, then the process is out of control.

Example (do it yourself)

Let the subgroup size is 4 and the subgroup means and subgroup standard deviations are as given in Example 13.2b on page 554 of textbook. Check whether the process is in control.

Example (do it yourself)

In the above example, suppose that a customer's specifications are 3 ± 0.1. Assuming that the process is in control, what percent of the items will meet the desired specifications?

S-control chart

Suppose that, when in control, the items produced have a measurable characteristic that is normally distributed with mean μ and variance σ^2. We take k small samples (subgroups) of size n each, where k should be chosen so that $k \geq 20$ and $nk \geq 100$. Let S_i denote the sample SD of the ith subgroup. We have

$$E(S_i) = c(n)\,\sigma$$

and

$$V(S_i) = E\left(S_i^2\right) - (E(S_i))^2 = \sigma^2 - c^2(n)\,\sigma^2 = \sigma^2(1 - c^2(n))$$

Let

$$\bar{S} = \frac{1}{k}(S_1 + S_2 + \cdots + S_k).$$

$\bar{S}/c(n)$ is an unbiased estimate of σ. We compute the control limits as follows:

$$\mathrm{LCL} = \bar{S}\left(1 - 3\sqrt{\frac{1}{c^2(n)} - 1}\right)$$

$$\mathrm{UCL} = \bar{S}\left(1 + 3\sqrt{\frac{1}{c^2(n)} - 1}\right)$$

Exercise (do it yourself)

Example 13.3a on page 557 of textbook.