Exercise-I

I.

$H_0 : \mu = 200$

$H_1 : \mu < 200$

level of significane 0.05

$n = 8$

$\sigma = 5$

$$\bar{x} = \frac{\Sigma x_i}{n} = \frac{1592.9}{8} = 199.1125$$

$$\therefore z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

$$= \frac{199.1125 - 200}{5/\sqrt{8}}$$

$$= -0.502$$

at 5% level of significanse-

$z = -1.645$

As $z_{obs} > z_{table}$ so null accepted.

P value:- $P(z < -0.502)$

$= \Phi(-.502) = 0.3015$

II. $H_0 : \mu = 200$

$H_1 : \mu < 200$

$n = 8$

$\bar{x} = 199.1125$

SD :-

$\Sigma(x_i - \bar{x})^2 = 170.7085$

$\therefore S_D = \sqrt{\frac{(\Sigma x_i - \bar{x})^2}{n-1}} = 4.939$

$\therefore T = \frac{\bar{x} - \mu}{S_D / \sqrt{n}} = -5.083$

$t_{0.05,7} = -1.895$ [from t_table]

As, $t_{obs} > t_{0.05,7}$

So, null accepted.

iii. $H_0: \mu = 75$

$H_1: \mu > 75$

$n = 50, \quad \sigma = 5$

$$\therefore z = \frac{\bar{x} - 75}{5/\sqrt{50}}$$

at 5% level of significance

$z = 1.645$

$$\therefore 1.645 = \frac{\bar{x} - 75}{5/\sqrt{50}}$$

$$\therefore \bar{x} = 76.162$$

So, the rejection region is $(76.16, \infty)$

if $\bar{x} > 76.162$, we reject the null hypothesis.

now the true mean is 77.

$$\therefore z = \frac{76.162 - 77}{5/\sqrt{50}} = -1.1834$$

$$\therefore P(z > -1.1834) = 1 - \Phi(-1.1834)$$
$$= 0.881$$ Ans

4. $H_0: \mu_1 = \mu_2$

$H_1: \mu_1 > \mu_2$

$\bar{x} = 11.17, \ \bar{y} = 11.9875$

variance, $\sigma^2 = 0.09$

$$\therefore z = \frac{|\bar{x} - \bar{y}|}{\sigma \sqrt{\frac{1}{n} + \frac{1}{m}}}$$

$$= \frac{11.9875 - 11.17}{0.\cancel{0}3 \sqrt{\frac{1}{10} + \frac{1}{8}}}$$

$$= 5.744$$

$z_{5\%} = 1.645$

As, $z_{obs} > z_{5\%}$, so, we reject the null hypothesis. So, two lakes are equally contaminated.

(5). Let,

$\mu_1 = 59700$ | $sp_1 = 2400$

$\mu_2 = 58400$ | $sp_2 = 2200$

$H_0 : \mu_1 = \mu_2$

$H_1 : \mu_1 > \mu_2$

$$\therefore sp^2 = \frac{(n-1)\, sp_1^2 + (m-1)\, sp_2^2}{n+m-2}$$

$$\therefore sp = 2302.172$$

$$\therefore T = \frac{\mu_1 - \mu_2}{sp\sqrt{\frac{1}{n} + \frac{1}{m}}}$$

$$\therefore T = 1.5971$$

$$t_{30,0.05} = 1.697$$

Since, $T_{obs} < t_{30,0.05}$, so, we can't reject the mult. professors claim is not correct.

(6). $\bar{D} = \dfrac{(74-70) + (86-85) + (98-90) + (-8) + 7 + 4 + 10 - 4}{8}$

$= 2.75$

$D_i = X_i - Y_i$

$\bar{D} = \Sigma D_i$

$s_D^2 = \dfrac{\Sigma (D_i - \bar{D})^2}{n-1}$

$= \dfrac{(4-2.75)^2 + (1-2.75)^2 + (8-2.75)^2 + (-8-2.75)^2 + (7-2.75)^2 + (4-2.75)^2 + (10-2.75)^2 + (-4-2.75)^2}{8-1}$

$\therefore s_D = 6.1586$

$\therefore T = \dfrac{\bar{D}}{s_D / \sqrt{n}} = \dfrac{2.75}{6.1586/\sqrt{8}}$

$= 1.2629$

$t_{0.05,7} = 1.895$

since, $t_{obs} < t_{0.05,7}$, we conclude that jogging has not an effect on the pulse rates.

Scanned with CamScanner

(7). $H_0 : \sigma^2 = (0.10)^2 = 0.01$

$H_1 : \sigma^2 < 0.01$

the test statistic is —

$$= \frac{(n-1) s^2}{\sigma^2}$$

$$= \frac{(50-1) \cdot (0.08)^2}{0.01}$$

$= 31.36$.

from χ^2 table,

$\chi^2_{0.1, 49} = \chi^2_{0.9, 49} = 37.69$

Since, $\chi^2_{obs} < \chi^2_{0.1, 49}$

so, we reject the null hypothesis.

(8). $H_0 : P = 0.72$

$H_1 : P_1 > 0.72$

we know that —

$$z = \frac{\Sigma x_i - nP_0}{\sqrt{nP_0(1-P_0)}}$$

$$= \frac{42 - 50 \times 0.72}{\sqrt{50 \times 0.72(1-0.72)}}$$

$$= 1.889$$

$z_{0.95} = 1.645$.

As, $z_{obs} > z_{0.95}$, so, null rejected.

p-value –

$$= P(z > 1.889)$$

$$= \cancel{\Phi ++} \; 1 - \Phi(1.889)$$

$$= 1 - 0.9706$$

$$= 0.0294$$ [vary করতে পারে]

(9). a. We know –

$$\bar{y} = \hat{\alpha} + \hat{\beta} \bar{x}$$

Here, $\bar{x} = 2.67$

$\bar{y} = 0.0385$

$$\sum(x_i-\bar{x}) = -1.67 - 0.67 - 0.17 + 0.33 + 0.83$$
$$+1.33 = -0.02$$

$$\sum(x_i-\bar{x})^2 = 5.8334.$$

$$\sum(x_i-\bar{x})\cdot(y_i-\bar{y}) = 0.0685$$

$$\therefore \hat{\beta} = \frac{\sum(x_i-\bar{x})\cdot(y_i-\bar{y})}{\sum(x_i-\bar{x})^2}$$

$$= \frac{0.0685}{5.8334}$$

$$= 0.01\cancel{7}4$$

$$\hat{\alpha} = \bar{y} - \hat{\beta}\bar{x}$$
$$= 0.007$$

$\therefore$ fitted linear regration line —

Avarage percent gain = 0.00714 + (0.01174) × periods of exposure.

(b). from a —

$\bar{Y} = 0.007147 + (0.01174 \times x)$

$x = 3.2$

$\therefore \bar{Y} = 0.0447$ Ans

(c). error variance is —

$= \frac{SSe}{(n-2)}$ — (1)

$SSe = \Sigma (Y_i - \alpha - \beta x_i)^2$

$= (0.02 - 0.00714 - (0.0117 \times 1))^2 + \ldots$

$\ldots + (0.054 - 0.007147 - (0.0117 \times 4))^2$

$= 7.124 \times 10^{-6}$

$\therefore (1) \Rightarrow \frac{SSe}{n-2} = 1.781 \times 10^{-6}$ Ans (d). $H_0: \beta_1 = 0$

$H_1: \beta \neq 0$

test statistic —

$$T = \frac{\hat{\beta} - \beta}{\sqrt{SSe/((n-2)(\Sigma x_i - \bar{x})^2)}}$$

$= 42.5$

~~t_{obs}~~ $t_{0.025,4} = 2.776$

$t_{obs} > t_{0.025,4}$.

so, null can be rejected.

(e). you have to find the value —

$$R^2 = 1 - \frac{SSe}{SSy}$$

$SSy = \Sigma(y_i - \bar{y})^2 = 8.115 \times 10^{-4}$

$$\therefore R^2 = 1 - \frac{1.781 \times 10^{-6}}{8.115 \times 10^{-4}}$$

$$= 99.78\%$$

(16). sample co-relation co-efficient

is —

$$r = \frac{\Sigma (x_i - \bar{x}) \cdot (y_i - \bar{y})}{\sqrt{(\Sigma x_i - \bar{x})^2} \cdot \sqrt{(\Sigma y_i - \bar{y})^2}}$$

$$= 99.56\%$$