Test of Hypothesis

Statistical hypothesis

In statistics, a hypothesis is a statement or claim about population.

Example

You think that the average amount of cola dispensed by a machine is less than 250 ml. Then, your research question and the related hypotheses are as follows.

Question: Is μ less than 250?

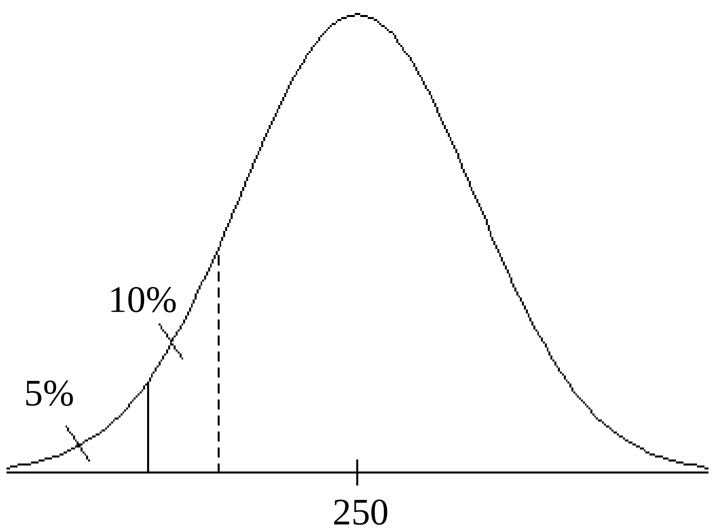
Null hypothesis (H_0): $\mu = 250$

Alternative hypothesis (H_1): $\mu < 250$

Test of hypothesis

We collect data to test whether the claim about the parameter is valid. To answer the above question, we may collect a sample of size 25 (say) and calculate $\bar{x}$. (We should check whether $\bar{x}$ is much less than 250.)

If the null hypothesis is true, sample mean $\bar{X}$ has the following distribution:



Level of significance shows how strictly we are judging the hypotheses. For 10% level, null hypothesis is rejected more quickly compared to 5% level. Usually, tests are done at 5% level of significance.

When data follow normal and population variance σ^2 is unknown:

We use the following "Test statistic" (statistic used for the test)

$$T = \frac{\bar{X} - \mu_0}{S/\sqrt{n}}$$

which follows t distribution with $n - 1$ d.f.

Suppose, we obtained the following results after collecting data.

$n = 25, \ \bar{x} = 247, \ s = 7.5$

Then, the value of the Test statistic:

$$T = \frac{\bar{X} - \mu_0}{S/\sqrt{n}} = \frac{247 - 250}{7.5/\sqrt{25}} = -2.0$$

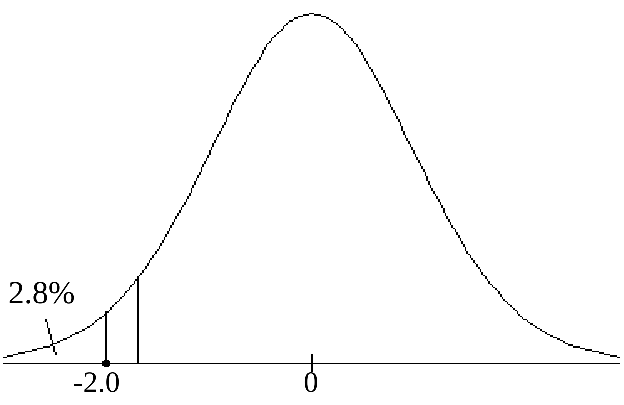
which follows t distribution with $n - 1 = 24$ d.f. if the null hypothesis is true. Here, μ_0 is the value of μ under the null hypothesis.

(We do not know σ. So, we used S in the calculation above. Therefore, the distribution is t, not normal.)

p-value (probability of observed or more extreme values under null hypothesis):

Here, p-value $= P(T \leq -2.0) = 0.028$

(That is, the area in the corner is 2.8%.)



About p value:

1. when p value<.05 NULL is rejected.
2.When p value>.05 NULL is accepted.

Here, p-value $<$ level of significance. Therefore, we reject the null hypothesis. We conclude that the average amount of cola dispensed by the machine is less than 250 ml.

Another way of performing the test is to determine $t_{0.05,24} = -1.71$, and then rejecting the null hypothesis because the observed (calculated) value of test statistic: $T_{\text{obs}} = -2$ falls in the "**critical region**" $(-\infty, -1.71)$.

When data follow normal and population variance σ^2 is "known"

We use the following Test statistic:

$$Z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}}$$

which follows standard normal distribution.

Simple and composite hypotheses

If a hypothesis completely specifies the distribution, it called a simple hypothesis. Otherwise, it is called a composite hypothesis. Suppose, the data follow normal distribution with known variance 4. Then, "$\mu = 250$" is a simple hypothesis, while "$\mu < 250$" is a composite hypothesis.

One-tailed and two-tailed tests

The following test is a one-tailed (upper-tailed) test:

H_0: $\mu = 400$

H_1: $\mu > 400$

The following test is a one-tailed (lower-tailed) test:

H_0: $\mu = 400$

H_1: $\mu < 400$

The following test is a two-tailed test:

H_0: $\mu = 400$

H_1: $\mu \neq 400$

Type I error and Type II error

If the null hypothesis is actually true but we reject it based on our sample, the error that has occurred is called Type I error. The probability of Type I error is usually denoted by α which is actually the level of significance.

$P(\text{rejecting } H_0 \mid H_0 \text{ is true}) = \alpha$ (i.e., level of significance)

If the null hypothesis is actually false but we accept it based on our sample, the error that has occurred is called Type II error. The probability of Type II error is usually denoted by β.

$P(\text{accepting } H_0 \mid H_0 \text{ is false}) = \beta$

Power of the test

The probability of rejecting the null hypothesis when it is false is called power of the test. This is actually $1 - \beta$.

For example, your claim is that the moble phone battery you developed works continuously for more than 72 hours, on an average, without recharging (more than other batteries). Then,

H_0: $\mu = 72$

H_1: $\mu > 72$

Level of significance: $\alpha = 0.05$

For discussion purposes, let $\sigma = 4$ be known and the sample size is 64. Then, the test statistic is

$$Z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} = \frac{\bar{X} - 72}{4/\sqrt{64}}$$

Using normal table, our critical region is: $Z > 1.645$ or $\bar{X} > 72.82$. That is, we reject the null hypothesis if $\bar{X} > 72.82$. If the actual value of μ is 76, the power of the above test is:

$P(\bar{X} > 72.82 \mid \mu = 76) = 0.787$