Control chart for fraction defective

There are situations in which the items produced have quality characteristics that are classified as either being defective or non-defective. Let us suppose that when the process is in control, each item produced will independently be defective with probability p. If we let X denote the number of defective items in a subgroup of n items, then assuming control, X will be a binomial random variable with parameters (n, p). If $F = X/n$ is the fraction of the subgroup that is defective, then assuming the process is in control, its mean and standard deviation are given by

$$E(F) = \frac{E(X)}{n} = p$$

$$V(F) = \frac{V(X)}{n^2} = \frac{np(1-p)}{n^2} = \frac{p(1-p)}{n}$$

Thus,

$$\text{LCL} = p - 3\sqrt{\frac{p(1-p)}{n}}$$

$$\text{UCL} = p + 3\sqrt{\frac{p(1-p)}{n}}$$

Since p is unknown, we estimate it by the average of the subgroup proportions, denoted by $\bar{F}$, which is obtained as

$$\bar{F} = \frac{1}{k}(F_1 + F_2 + \cdots + F_k)$$

The upper and lower control limits are now given by

$$\text{LCL} = \bar{F} - 3\sqrt{\frac{\bar{F}(1-\bar{F})}{n}}$$

$$\text{UCL} = \bar{F} + 3\sqrt{\frac{\bar{F}(1-\bar{F})}{n}}$$

Exercise

Example 13.4a on page 560 of textbook.

Control chart for number of defects

There are situations in which we are concerned about the number of defects in an item or a group of items. Often it is reasonable to assume that, when the process is in control, the number of defects in a 'unit' (an item or a group of items) has a Poisson distribution with parameter λ.

Let X_i denote the number of defects in the ith unit produced. Then, $E(X_i) = \lambda$ and $V(X_i) = \lambda$. Thus,

$$\text{LCL} = \lambda - 3\sqrt{\lambda}$$

$$\text{UCL} = \lambda + 3\sqrt{\lambda}$$

Since λ is unknown, we estimate it by the average of the number of defects in k units, denoted by $\bar{X}$, which is obtained as

$$\bar{X} = \frac{1}{k}(X_1 + X_2 + \cdots + X_k)$$

The upper and lower control limits are now given by

$$\text{LCL} = \bar{X} - 3\sqrt{\bar{X}}$$

$$\text{UCL} = \bar{X} + 3\sqrt{\bar{X}}$$

- For large units, the value of λ is large, so that the distribution of X_i can be approximated by a normal distribution with mean λ and standard deviation $\sqrt{\lambda}$.

Exercise

Example 13.5a on page 563 of textbook.