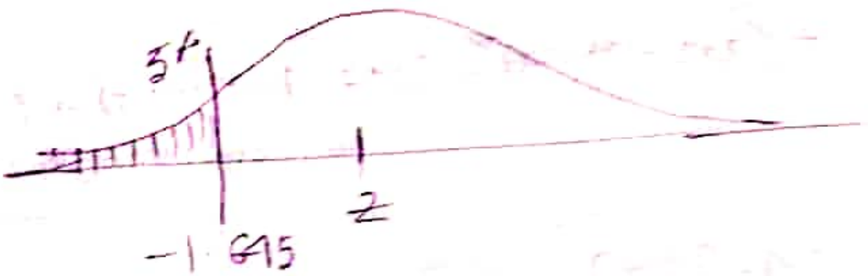
1)

$\mu = 200$ $\bar{x} = 199.1125$

$\mu < 200$



$$Z_{obs} = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

$$= \frac{199.1125 - 200}{5/\sqrt{8}}$$

$$= -0.5020$$

$Z_{obs} > -Z_{0.05}$

Null accepted : The mean breaking strength of the fibre is not less than the target

p value : $P(Z < -0.5020)$

$= \Phi(-0.502)$

$= 1 - \Phi(0.502)$ $= 0.3015$

$= 1 - 0.6985$

$\mu = 200$

$\mu < 200$

2) $\bar{x} = 199.1125$

$$S_D^2 = \frac{(210-199.1125)^2 + (198-199.1125)^2 + (195-199.1125)^2 + (202-199.1125)^2 + (197.4-199.1125)^2 + (196-199.1125)^2 + (199-199.1125)^2 + (195.5-199.1125)^2}{8-1}$$

$= 24.3869$

$S_D = 4.9383$

$t_{0.05,7} = -1.895$

$$t_{obs} = \frac{199.1125 - 200}{4.9383/\sqrt{8}}$$

$= -0.5083$

$t_{0.05,7} < t_{obs}$ null accepted

3) $\mu = 75$

$\mu > 75$

$z_{0.95} = 1.695$

$\Rightarrow \dfrac{\bar{x} - 75}{5/\sqrt{50}} = 1.695$

$\Rightarrow \bar{x} = 76.1631$

Rejection area/region $(76.1631, \infty)$

$P(\bar{x} > 76.1631 \mid \mu = 77)$

$= P\left(Z > \dfrac{76.163 - 77}{5/\sqrt{50}}\right)$

$= P(Z > -1.1836)$

$= 1 - \phi(-1.1836) = 1 - 1 + \phi(1.1836)$.

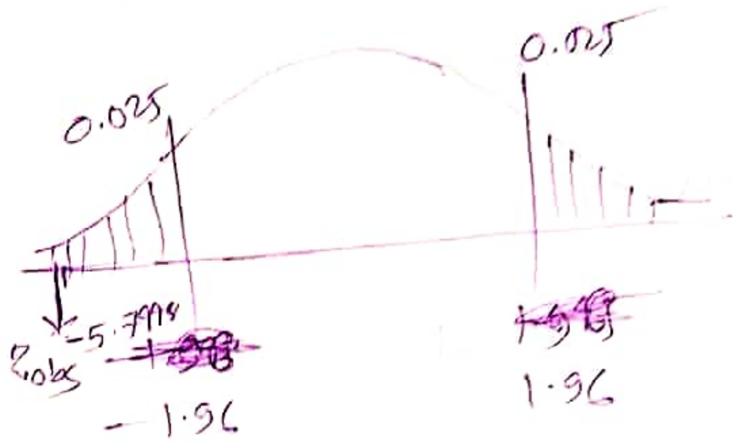
$= 0.8810$

4) $\bar{x} = 11.17$ $\bar{Y} = 11.9875$ $\sigma^2 = 0.09$

$\sigma = 0.3$

$\mu_1 = \mu_2$

$\mu_1 \neq \mu_2$



$$Z_{obs} = \frac{\bar{x} - \bar{Y}}{\sigma/\sqrt{n}} = \frac{11.17 - 11.9875}{0.3\sqrt{\frac{1}{11} + \frac{1}{8}}}$$

$$= -5.7498$$

$Z_{obs} < Z_{0.025}$

$\therefore$ Null rejected.

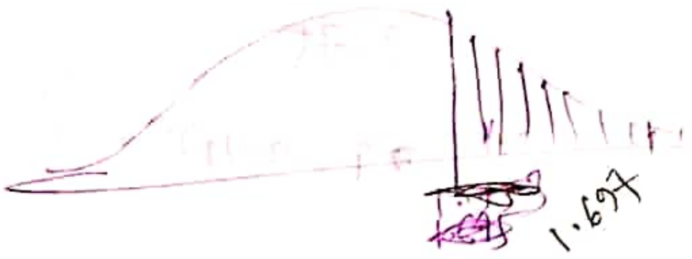
$\therefore$ Yes I can reject the claim that two lakes are equally contaminated.

5) $\mu_1 = \mu_2$ $\qquad$ $\bar{X} = 59700$

$\mu_1 > \mu_2$ $\qquad$ $\bar{Y} = 58400$

$s_1 = 2400$

$s_2 = 2200$



$t_{0.05,30} = \cancel{1.753}\ 1.697$

$$S_p^2 = \frac{(n-1)S_1^2 + (m-1)S_2^2}{n+m-2} \Rightarrow S_p = 2302.1728$$

$$t_{obs} = \frac{\bar{X} - \bar{Y}}{S_p\sqrt{\frac{1}{n} + \frac{1}{m}}} = \frac{59700 - 58400}{2302.1728\sqrt{\frac{1}{18} + \frac{1}{16}}}$$

$$= 1.5971$$

$\cancel{t_{obs}}\ t_{0.05,30} < t_{obs}$

$\cancel{t_{obs} < t_{0.05,30} > t_{obs}}$ $\quad$ $t_{0.05,30} < t_{obs}$

∴ Null can't be rejected.

∴ Prof. claim is not correct

$\mu_d = 0$

$\mu_d \neq 0$

6)

$\bar{D} = 0$

$\bar{D} > 0$

$$\bar{d} = \frac{(79-70)+(86-85)+(98-90)+(102-110)+(78-71)+(89-80)+(79-69)+(70-74)}{8}$$

$$= 2.75$$

$$s_D^2 = \frac{(9-2.75)^2+(1-2.75)^2+(8-2.75)^2+(-8-2.75)^2+(7-2.75)^2+(9-2.75)^2+(10-2.75)^2+(-4-2.75)^2}{8-1}$$

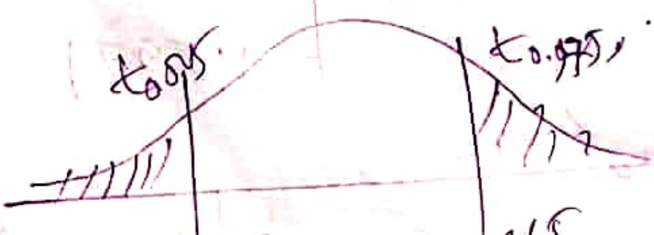
$$= 37.9285$$

$$s_D = 6.1586$$

$$t_{obs} = \frac{\bar{d}}{s_d/\sqrt{n}}$$

$$= \frac{2.75}{6.1586/\sqrt{8}}$$

$$= 1.262974$$



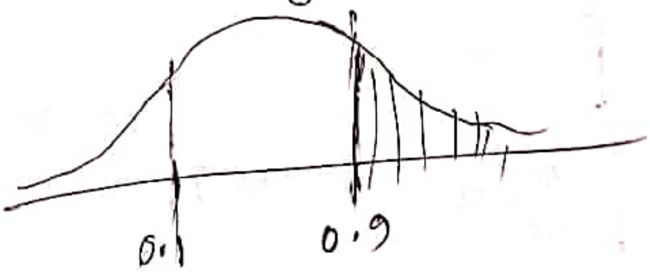
$t_{0.025} < t_{obs} < t_{0.975}$ ∴ Null accepted

$T = \frac{D}{S_D/\sqrt{n}}$ $T = \frac{D}{S_D/\sqrt{n}}$

We cannot conclude that Jogging has had an effect on the pulse rate.

$\sigma^2 = 0.01$

$\sigma^2 > 0.01$



7) $\sigma^2 = 0.01$

$\sigma^2 > 0.01$

Now, the statistic

$$\chi^2_{obs} = \frac{(n-1)s^2}{\sigma^2} = \frac{49\times(0.08)^2}{(0.1)^2} = \cancel{36.2}\ 31.36$$

$$\chi^2_{0.9,49} = 36.89$$

$$\chi^2_{obs} < \chi^2_{0.1,49}$$

Null Accepted, shouldn't discontinued.

8)

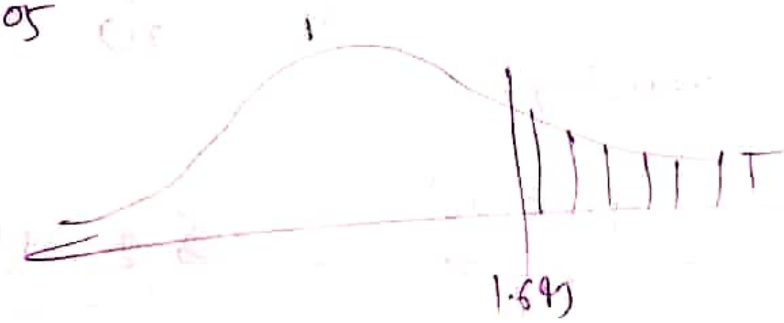
$p = 0.72$

$p > 0.72$

$$Z_{obs} = \frac{\sum x_i - np_0}{\sqrt{np(1-p)}} = \frac{42 - 50 \times 0.72}{\sqrt{50 \times 0.72 (1 - 0.72)}}$$

$$= 1.8898$$

$Z_{0.05} = 1.695$



$Z_{obs} > Z_{0.05}$

null rejected

$\therefore$ The drug is more effective.

P-value $= P(Z > 1.14898)$

$= 1 - 0.9627$

$= 0.0373$

9)

x	Y	$\bar{x}$	$\bar{Y}$	$x-\bar{x}$	$Y-\bar{Y}$	$(x-\bar{x})^2$	$(Y-\bar{Y})^2$
1	0.02			-1.67	-0.0185	2.7889	3.422×10^{-4}
2	0.03	8/3	0.0385	-0.67	-0.0085	0.4489	7.22×10^{-5}
2.5	0.035	2.67		-0.17	-0.0035	0.0289	1.22×10^{-5}
3	0.042			0.33	0.0035	0.1089	1.22×10^{-5}
3.5	0.05			0.83	0.0115	0.6889	1.322×10^{-4}
4	0.059			1.33	0.0155	1.7685	2.4×10^{-4}
total				-0.02	0	5.8334	0.008112

$\Sigma(x-\bar{x})(Y-\bar{Y}) =$

0.030895
0.005695
0.000595
0.001155
0.005545
0.020615

$\Sigma(x-\bar{x})(Y-\bar{Y}) = 0.0685$

(a)

$$\hat{\beta} = \frac{0.0685}{5.8334} = 0.011742$$

$$\hat{\alpha} = \bar{Y} - \hat{\beta}\bar{x} = 0.0385 - \frac{8}{3} \times 0.011742$$

$$= 0.007186$$

$$\bar{Y} = \hat{\alpha} + \hat{\beta}\bar{x}$$

$$\bar{Y} = \cancel{0.011742}(0.007186) + (0.011742)\bar{x}$$

(b) $\bar{Y} = 0.007186 + 0.011742 \times 3.2$

$$= 0.04976$$

(c) $$\sigma^2 = \frac{SS_e}{\cancel{\Sigma(x_i-\bar{x})^2}\ n-2}$$

$$SS_e = \Sigma(Y_i - \hat{\alpha} - \hat{\beta}x_i)^2$$

$$= (0.02 - 0.007186 - \cancel{0.01174x} - 0.02398)^2$$
$$+ (0.03 - 0.007186 - 0.02935)^2$$
$$+ (0.035 - 0.007186 - 0.03522)^2$$
$$+ (0.042 - 0.007186 - 0.04109)^2$$
$$+ (0.05 - 0.007187 - 0.04109)^2$$
$$+ (0.054 - 0.007187 - 0.04696)^2$$
$$= 7.1146 \times 10^{-6}$$

$\hat{\sigma}^2 = 1.778\times10^{-6}$

(d) $\hat{\beta} = 0.011742$

$\beta = 0$

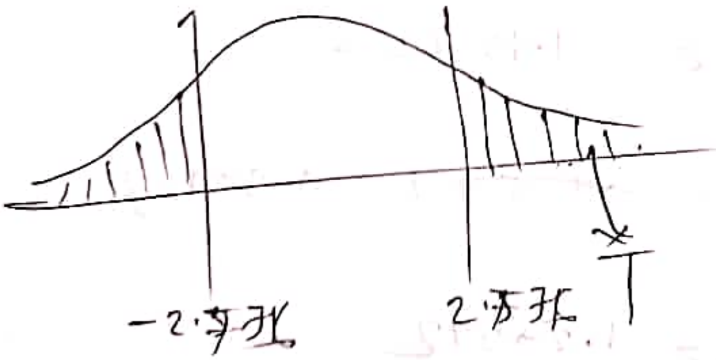
$\beta \neq 0$

$$T_{obs} = \frac{\hat{\beta}-\beta}{\sqrt{\hat{\sigma}^2/4\times\sum(x-\bar{x})^2}} = \frac{0.011742}{\sqrt{1.778\times10^{-6}/4\times5.8334}} =$$

$\therefore T_{obs} = 21.2645\times 2 = 42.529$

$t_{0.025,5} = -2.571$

$t_{0.975,5} = 2.576$



$T_{obs} > t_{0.025,5}$

$\therefore$ Null rejected

$\therefore \beta > 0$.

(e) $R^2 = 1 - \dfrac{SS_e}{SS_y}$

$= 1 - \dfrac{7.1146\times10^{-6}}{0.0008175}$ $\qquad SS_y = \Sigma(Y-\bar{Y})^2$

$= 0.99123$ $\qquad = 0.001032$

$R^2 = 99.12\%$

$\hat{\alpha} = 0.007182$

$\hat{\beta} = 0.011742$

(f) $e_1 = \dfrac{Y_i - \hat{\alpha} - \hat{\beta} x_i}{\sqrt{SS_e/(n-2)}} = \dfrac{0.02 - 0.007182 - 0.01174}{0.001333}$

$= 0.8057$

$e_2 = -0.499624$

$e_{2.5} = -1.15228$

$e_3 = \cancel{-1.15228}\ -0.30957$

$e_{3.5} = 1.27332$

$e_4 = -0.10952$

$e_{3.5}$
e_1
e_4
e_3
e_2
$e_{2.5}$
(0,0)
1
2
3
4
1.3
1.2
1.1
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
-0.7
-0.8
-0.9
-1.0
-1.1
-1.2

10)

$$r = \frac{\Sigma(x-\bar{x}_i)(y-\bar{y}_i)}{\sqrt{\Sigma(x-\bar{x}_i)^2}\ \sqrt{\Sigma(y-\bar{y})^2}}$$

$$= \frac{0.0685}{\sqrt{5.8734}\ \sqrt{0.000815}}$$

$$= \cancel{0.6628}\ 0.9956$$

$$\cancel{r^2 = 0.7794}\ \ r^2 = 0.9912 = R^2$$

11)

$$T = t s^{-n}$$

$$\log T = \log t + (-n \log s)$$

$$\hat{\beta} = \frac{\Sigma(Y-\bar{Y})(n-\bar{n})}{\Sigma(n-\bar{n})^2}$$

$$Y = \log T \rightarrow \log_e T$$

$$\hat{\alpha} = \log t$$

$$\hat{\beta} = -\log s$$

$$\therefore Y = \hat{\alpha} + \hat{\beta} n$$

Y	n	$Y-\bar{Y}$	$n-\bar{n}$	$(Y-\bar{Y})(n-\bar{n})$	$(Y-\bar{Y})^2$	$(n-\bar{n})^2$
3.109	0	0.271	−3	−0.813		9
3.0587	1	0.221	−2	−0.442		4
2.9806	2	0.143	−1	−0.143		1
2.747	3	−0.09	0	0		0
2.721	4	−0.116	1	−0.116		1
2.6318	5	−0.205	2	−0.41		4
2.6173	6	−0.22	3	−0.66		9
Total		$\Sigma(Y-\bar{Y})$	$\Sigma(n-\bar{n})$	$\Sigma(Y-\bar{Y})(n-\bar{n})$ $= -2.598$	$\Sigma(Y-\bar{Y})^2$	$\Sigma(n-\bar{n})^2$ 28

$$\bar{Y} = 2.8379$$

$$\bar{n} = 3$$

$$\hat{\beta} = \frac{-2.598}{28}$$

$$\hat{\alpha} = \bar{Y} - \hat{\beta}\bar{n}$$

$$= 2.8379 + 0.091 \times 3$$

$$\log t = 3.1109$$

$$\therefore t = 22.441$$

$$-\log s = -0.091$$

$$\Rightarrow \log s = 0.091$$

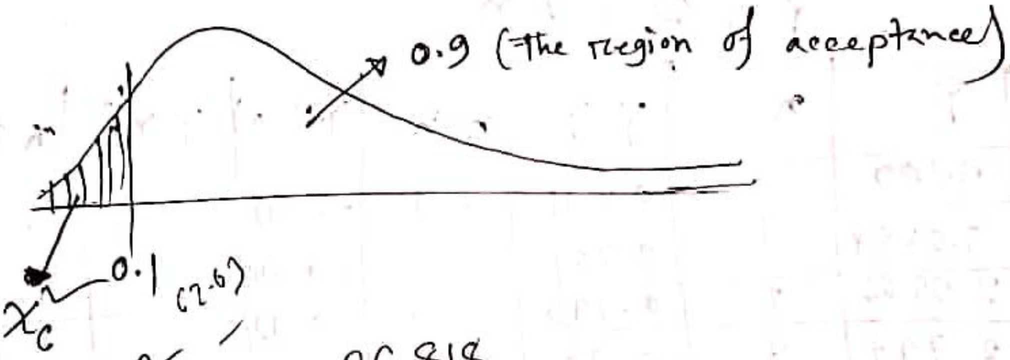
$$s = 1.0952$$

7) $H_0: \sigma^2 = 0.01$

$H_1: \sigma^2 < 0.01$

Now, the statistics

$$\chi_c^2 = \frac{(n-1)s^2}{\sigma^2} = 31.36$$



$$\chi^2_{0.9,49} = 36.818$$

$$\therefore \chi^2_c < \chi^2_{0.9,49}$$

χ_c is in rejection region.

∴ Null hypothesis is rejected

∴ The system should not be discontinued

2011

H_0: Two distributions are Identical
H_1: Two distributions are different

$T = 57$

$$E(T) = \mu = \frac{n(m+n+1)}{2} = \frac{7(6+7+1)}{2} = 49$$

$$V(T) = \sigma^2 = \frac{mn(m+n+1)}{12} = \frac{7 \times 6 \times 11}{12} = 49$$

$T > \mu$

$$p\text{-value} = 2\left(1 - \Phi\left((T-\mu)/\sigma\right)\right)$$
$$= 2\left(1 - \Phi(1.428)\right)$$
$$= 2\left(1 - 0.9222\right)$$
$$= 0.1556$$

$0.05 < p\text{-value}$

Null accepted.

→ Tortoises and hares are not different in racing.

4|| For factor A.

H_0: All α_i are equals ~~[illegible]~~ $\mu_1 = \mu_2 = \mu_3$

H_1: All α_i are not equal

For factor B.

$H_0 : \mu_1 = \mu_2 = \mu_3 = \mu_4$

H_1 : At least two mean are not equal

$$SS_A = n \sum_{i=1}^{m} (\bar{Y}_{i.} - \bar{Y})^2 \qquad \bar{Y}_{i.} = \frac{1.35 + 1.[illegible] + 1.0[illegible] + 0.95}{4} = \cancel{1.4133}\ 1.13$$

$$= 4 \times (1.4133 - 1.2775)^2 \qquad \bar{Y} = 1.2775$$

$$= 4\{(1.13 - 1.2775)^2 + (1.2775 - 1.2775)^2 + (1.425 - 1.2775)^2\}$$

$$= 0.17405$$

$$SS_B = m \sum_{j=1}^{n} (\bar{Y}_{.j} - \bar{Y})^2 \qquad \bar{Y} = 1.2775$$

$$= 3 \times \{(1.4133 - 1.2775)^2 + (1.2733 - 1.2775)^2 + (1.29 - 1.2775)^2 + (1.1833 - 1.2775)^2\}$$

$$= 0.08621$$

3)] [illegible]

	O_i		
x		$\frac{1}{4}$	141
white	141		232
pink	291	$\frac{1}{2}$	

$$SS_T = \sum_{i=1}^{m} \sum_{j=1}^{n} (y_{ij} - \bar{y})^2$$

$$= 0.282925$$

$$\cancel{0.2816}$$

$$SS_E = SS_T - SS_A - SS_B = 0.02165$$

$$F = \frac{SS_A/(m-1)}{SS_E/[(m-1)(n-1)]}$$

$$= \frac{0.17905/2}{0.02165/6}$$

$$= 23.5579$$

b)| H_0: The result is fair
H_1: The result is not fair

x	O_i	p	E_i
white	141	$\frac{1}{4}$	141
pink	291	$\frac{1}{2}$	282
red	132	$\frac{1}{4}$	141

$$\chi^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}$$

$$= \frac{(141-141)^2}{141} + \frac{(291-282)^2}{282} + \frac{(132-141)^2}{141}$$

$$= 0.8616$$

$$\chi^2_{0.95,\,2} = 5.991 \; \cancel{4.103} \; 5.991$$

obs → smaller
accepted

$$\chi^2_{0.95,\,2} > \chi^2_{obs}$$

∴ Null ~~rejected~~ accepted.

H_0: Smoking has no effect on hypertension

6 11 H_1: Smoking has effect on hypertension

Class	Hypertension	No hypertension	total
Non-smoker	20 (33.26)	50 (36.74)	70
Moderate Smoker	38 (30.88)	27 (34.21)	65
Heavy smoker	28 (21.86)	18 (24.19)	46
total	86	95	181

$$\chi^2 = \sum_{i=1}^{r} \sum_{j=1}^{c} \frac{(O_{ij} - E_{ij})^2}{E_{ij}} \quad \Sigma\, \text{all}$$

$$= 16.485$$

$$\chi^2_{0.05,2} = 5.991$$

$$\chi^2_{0.05,2} < \chi^2$$

$\therefore$ Null rejected.

$\therefore$ Smoking has effect on Hypertension.

$\therefore$ An Individual having hypertension is not independent of how much that person smokes.

7||

$H_0 : m = 0$

$H_1 : m < 0$

$$I_i = \begin{cases} 1 & x_i < 0 \\ 0 & x_i \geq 0 \end{cases}$$

$p(x) = {}^{n}C_x\, p^x (1-p)^{n-x}$

5 10

9

17

22

$I_i \Rightarrow 110010011111110111$

$n = 18$

$$V = \sum_{i=1}^{n} I_i = 13$$

$$\text{p-value} = P\left(\sum_{i=1}^{n} I_i \leq 13\right)$$

$$= 1 - p(14) - p(15) - p(16) - p(17) - p(18)$$

$$= 0.98455$$

$V \sim$ Binomial $(18, 0.5)$

$0.05 < 0.98455$

Null can't be rejected.

~~medicine does not have~~

We can't say.

$1 + e^{-(a+bt)}$

-17

-85.5

527.2

10 Q11

$H_0: m = 0$

$H_1: m < 0$

$Y_i = X_i - 0$

$$I_i = \begin{cases} 1 & x_i < 0 \\ 0 & x_i > 0 \end{cases}$$

$|Y_i| \Rightarrow$

1.5	1.5	3	4	6	6	6	8.5	8.5	10
1	1	2	4	5	5	5	8	8	9
11	12	13	14	15	16	17			
11	12	15	16	18	21	22			
18									
25									

$$T_- = \sum_{i=1}^{n} i I_i = 1.5 + 6 + 6 + 8.5 + 10 + 11 + 12 + 13 + 14 + 15 + 16 + 17 + 18$$

$$= 148$$

$$E(T_-) = \frac{n(n+1)}{4} = \frac{18 \times 19}{4} = 85.5$$

$$V(T_-) = \frac{n(n+1)(2n+1)}{24} = 527.25$$

$$\text{p-value} = P(T_- \geq t)$$

$$\cancel{= P\left(\frac{148 - 85.5}{\sqrt{527.25}} > Z\right)}$$

$$= P\left(\frac{T_- - \mu}{\sigma} \geq \frac{148 - 85.5}{\sqrt{527.25}}\right)$$

$$\cancel{= 0.9966}$$

$$= 1 - \phi(2.721)$$

$$= 1 - 0.9967 = 0.0033$$

$\therefore$ ~~0.05 < 0.9966~~ $0.05 >$ p-value

$\therefore$ ~~We can not reject null~~.

$\therefore$ Null rejected

9)

Treatment group rank:

1	2	3	4	5	6	7	8	9	10.5
19	29	31	36	39	44	45	47	49	52

10.5	12	13	14	15
52	60	66	74	81

H_0: Two distributions are identical

H_1: Two distributions are different.

$T = 1+3+5+7+8+13+14+15$
$= 66$

$$E(T) = \mu = \frac{n(m+n+1)}{2} = \frac{8(15+1)}{2} = 64$$

$$V(T) = \sigma^2 = \frac{mn(m+n+1)}{12} = \frac{224}{3}$$

$p = 2(1 - \phi(T-\mu/\sigma))$

$p = \cancel{\phi\left(\frac{T-\mu}{\sigma}\right)}$

$= 2 \times (1 - \phi(0.231))$ ~~$P_{0.05}$ 0.5199~~

~~$= \phi(0.231)$~~

$= \cancel{0.5910}\ 2 \times (1 - 0.5910)$

$= 0.818$

~~$P > P_{0.05}$~~ $0.05 < p$

$\therefore$ Null ~~Rejected~~. Accepted.

$\therefore$ The articles have not had any effect.

10|1|

$\mu = 128.6$

Let the value less than mean be replaced by 0 and greater values by 1.

Now, quality levels: 000100000111111000111110101

H_0: Data are a random sample

H_1: Data are not a random sample

$R = 10$ $\quad E(R) = \mu = \frac{2mn}{m+n} + 1$

$= 13.4$

$V(R) = \sigma^2 = \frac{(\mu-1)(\mu-2)}{m+n-1} = 5.9696$

$R < \mu$

$p = 2\varphi((R-\mu)/\sigma)$

$= 2\varphi(-1.4213)$

$= 2 \times 0.0778$

$= 0.1556$

~~$T_{0.05} = 0.5199$~~

~~$p > 0.05$~~ $0.05 < p$

$\therefore$ Null ~~rejected~~ Accepted

$\therefore$ Data are a random sample from a population.

377.7994

$= 0.6$

395.616

$= 1.3126$