Homework #6
Due Monday, Oct. 25

#1. Show the following languages are not regular

a) $L = \{a^n b^n a^n \mid n \geq 0\}$
b) $L = \{a^i \mid i \text{ is prime}\}$

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Proof by contradiction using Pumping Lemma

L is infinite. Assume that L is regular.
Because L is infinite, we can pick
Pick $w = a^m b^m a^m$ where m id the constant in the PL
Then

$W = xyz$ with $|xy| \leq m$, $|y| \geq 1$;
this makes xy all "a"s

Suppose $|xy| = s$ and $|y| = k \geq 0$

Then $z = a^{m-s} b^m a^m$

Pumping y twice gives

$a^{m+k} b^m a^m$ which is not of the form of $a^n b^n a^n$

This is a contradiction and thus L is not a regular language.

#2. Exercise 6.1.1 a), b) and c), Page 228. Also describe L(M)

Given the PDA $P = (\{q,p\}, \{0,1\}, \{Z_0, X\}, \delta, q, Z_0, \{p\})$
with the following transition functions:

1. $\delta(q, 0, Z_0) = \{(q, XZ_0)\}$
2. $\delta(q, 0, X) = \{(q, XX)\}$
3. $\delta(q, 1, X) = \{(q, X)\}$
4. $\delta(q, \varepsilon, X) = \{(p, \varepsilon)\}$
5. $\delta(p, \varepsilon, X) = \{(p, \varepsilon)\}$
6. $\delta(p, 1, X) = \{(p, XX)\}$
7. $\delta(p, 1, Z_0) = \{(p, \varepsilon)\}$

Show all reachable ID's when

a) w = 01

$(q, 01, Z_0) \rightarrow^{1} (q, 1, XZ_0) \rightarrow^{3} (q, \varepsilon, XZ_0) \rightarrow^{4} (p, \varepsilon, Z_0)$

$\rightarrow^{1} (q, 1, XZ_0) \rightarrow^{4} (p, 1, Z_0) \rightarrow^{7} (p, \varepsilon, \varepsilon)$

b) w = 0011

$(q, 0011, Z_0) \rightarrow^{1} (q, 011, XZ_0) \rightarrow^{2} (q, 11, XXZ_0) \rightarrow^{3} (p, 1, XZ_0) \rightarrow^{4} (p, 1, Z_0)$
$\rightarrow^{7} (p, \varepsilon, \varepsilon)$

$\rightarrow^{1} (q, 011, XZ_0) \rightarrow^{2} (q, 11, XXZ_0) \rightarrow^{4} (p, 11, XZ_0) \rightarrow^{6} (p, 1, XXZ_0)$
$\rightarrow^{6} (p, \varepsilon, XXZ_0)$

$\rightarrow^{1} (q, 011, XZ_0) \rightarrow^{2} (q, 11, XXZ_0) \rightarrow^{3} (q, 1, XZ_0) \rightarrow^{3} (q, \varepsilon, Z_0)$

$\rightarrow^{1} (q, 011, XZ_0) \rightarrow^{4} (p, 011, XZ_0)$

$\rightarrow^{1} (q, 011, XZ_0) \rightarrow^{2} (q, 11, XXZ_0) \rightarrow^{4} (p, 11, XZ_0) \rightarrow^{6} (p, 1, XXZ_0)$
$\rightarrow^{5} (p, 1, XZ_0) \rightarrow^{5} (p, 1, Z_0) \rightarrow^{7} (p, \varepsilon, \varepsilon)$

$\rightarrow^{1} (q, 011, XZ_0) \rightarrow^{2} (q, 11, XXZ_0) \rightarrow^{6} (p, 1, XXXZ_0) \rightarrow^{6} (p, \varepsilon, XXXXZ_0)$

b) w = 010

$(q, 010, Z_0) \rightarrow^{1} (q, 10, XZ_0) \rightarrow^{3} (q, 0, XZ_0) \rightarrow^{2} (q, \varepsilon, XXZ_0)$

$\rightarrow^{1} (q, 10, XZ_0) \rightarrow^{4} (p, 10, Z_0) \rightarrow^{7} (p, 0, \varepsilon)$

$\rightarrow^{1} (q, 10, XZ_0) \rightarrow^{3} (q, 0, XZ_0) \rightarrow^{4} (p, 0, Z_0)$

#3. Exercise 6.2.1 c)

Design a PDA to accept the set of all strings of 0's and 1's with an equal number of 0's and 1's.

1. $\delta(q, 0, Z_0) = \{(q, 0Z_0)\}$ recording 0's
2. $\delta(q, 1, Z_0) = \{(q, 1Z_0)\}$ recording 1's
3. $\delta(q, 0, 0) = \{(q, 00)\}$ recording 0's
4. $\delta(q, 0, 1) = \{(q, \varepsilon)\}$ matching
5. $\delta(q, 1, 1) = \{(q, 11)\}$ recording 1's
6. $\delta(q, 1, 0) = \{(q, \varepsilon)\}$ matching
7. $\delta(q, \varepsilon, Z_0) = \{(p, \varepsilon)\}$ accepting by empty stack (or final state!)

The PDA $P = (\{q\}, \{0, 1\}, \{0, 1, Z_0\}, T, q, Z_0, \{p\})$, where T consists of the transitions 1-7 defined above.

#4. a) Create a PDA which accepts the same language as that generated by:

$S \rightarrow a\,A\,B \mid a\,B$
$A \rightarrow a\,A\,B \mid a\,B$
$B \rightarrow b$

c) Show a derivation of and a computation with *aaabbb*

1 $\delta(q, \varepsilon, S) = \{(q, aAB)\}$
2 $\delta(q, \varepsilon, S) = \{(q, aB)\}$
3 $\delta(q, \varepsilon, A) = \{(q, aAB)\}$
4 $\delta(q, \varepsilon, A) = \{(q, aB)\}$
5 $\delta(q, \varepsilon, B) = \{(q, b)\}$
6 $\delta(q, a, a) = \{(q, \varepsilon)\}$
7 $\delta(q, b, b) = \{(q, \varepsilon)\}$

Derivation (left-most):
S ➔ aAB ➔ aaABB ➔ aaaBBB ➔ aaabBB ➔ aaabbB ➔ aaabbb

Computation:
$(q, aaabbb, S) \vdash^{1} (q, aaabbb, aAB) \vdash^{6} (q, aabbb, AB) \vdash^{3} (q, aabbb, aABB)$
$\vdash^{6} (q, abbb, ABB) \vdash^{4} (q, abbb, aBBB) \vdash^{6} (q, bbb, BBB) \vdash^{5} (q, bbb, bBB)$
$\vdash^{7} (q, bb, BB) \vdash^{5} (q, bb, bB) \vdash^{7} (q, b, B) \vdash^{5} (q, b, b) \vdash^{7} (q, \varepsilon, \varepsilon)$

#5. (extra) Draw a set diagram that shows the relationship between regular languages, context-free languages, ambiguous context-free languages, unambiguous context-free languages, DPDA languages, and non-context-free languages: