6.18 The following prime implicant table (chart) is for a four-variable function $f(A, B, C, D)$.

(a) Give the decimal representation for each of the prime implicants.

(b) List the maxterms of f.

(c) List the don't-cares of f, if any.

(d) Give the algebraic expression for each of the essential prime implicants.

	2	3	7	9	11	13
–0–1		×		×	×	
–01–	×	×			×	
--11		×	×		×	
1--1				×	×	×

Solution

Step 1

List all binary representation of 4-input Variables from 0-15 and then group them in Increasing number of 1's.

	ABCD	GROUPS ABCD	
0	0000	0 0000	GROUP 1
1	0001	1 0001	GROUP 2
2	0010	2 0010	
3	0011	4 0100	
4	0100	8 1000	
5	0101	3 0011	GROUP 3
6	0110	5 0101	
7	0111	6 0110	
8	1000	9 1001	
9	1001	10 1010	
10	1010	12 1100	
11	1011	7 0111	GROUP 4
12	1100	11 1011	
13	1101	13 1101	
14	1110	14 1110	
15	1111	15 1111	GROUP 5

Step 2.

Considering each of the prime Implicants Individually

1) PI $-0-1$

Thus from the question, PI $-0-1$ is covered by minterms 3, 9, and 11.

Thus taking minterm 9 and 11 and combining,

$\Rightarrow 9 \rightarrow 1001 \Rightarrow 9,11 = 10-1$
$11 \rightarrow 1011$

Thus from step 1, minterm 3 is in group 3 and
Thus by checking group 2 one, by-one to see which will
Combine with minterm 3 and then will combine with 9, 11
to give $-0-1$

Thus in group 2, minterm 3 can combine with minterm 1 and 2, but from the given question, minterm 2 does not cover the PI $-0-1$

Thus combining 3 and 1 $\Rightarrow$

$$\begin{array}{ll} 1 & 0001 \\ 3 & 0011 \end{array} \Rightarrow 1,3 \rightarrow 00-1$$

and hence combining 1,3, 9,11 $\Rightarrow$

$$\begin{array}{ll} 1,3 \cdot & 00-1 \\ 9,11 \cdot & 10-1 \end{array} = -0-1$$

which is PI $-0-1$ and thus from the given PI chart, minterm $\underline{1}$ is not on the chart thereby signifying that minterm $\underline{1}$ is a don't care since don't care columns are not included in PI charts but are used during groupings,.

Hence the decimal representation of PI $-0-1$ is; $\underline{1, 3, 9, 11}$

ii) <u>PI $-01-$</u>

From the given question, PI $-01-$ is covered by minterms 2, 3 and 11

Thus taking minterm 2 and 3 & combining.

$$\begin{array}{ll} 2 \rightarrow & 0010 \\ 3 \rightarrow & 0011 \end{array} \Rightarrow 2,3 \quad 001-$$

Also Checking the table in step 1, minterm 11 is in group 4, thus it can combine with minterms 9 and 10 of group 3. But from the question minterm 9 doesn't cover PI -01- (PI chart given).
(no asterik under minterm 9 for PI -01-)

Thus combining with 10 → 10 1010 ⇒ 10,11 101-
11 1011

and hence combining 2,3 and 10,11 ⇒ 2,3 001- ⇒ 2,3,10,11 -01-
10,11 101-

which is PI -01- and since 10 is not on the PI chart, minterm 10 is also a don't care.

Thus the decimal representation of PI -01- is 2, 3, 10, 11

iii) PI --11

From the given question, PI --11 is covered by minterms 3, 7 and 11

Thus combining minterm 3 and 7

→ 3 → 0011 ⇒ 3,7 ⇒ 0-11
7 → 0111

Also checking the table in step 1, minterm 11 is in group 4 and can combine with 15 in group 5 because from the given chart minterm 9 doesnot cover

--11 even though it can combine with minterm 11

thus Combining 11 & 15 ⇒ 11 1011 ⇒ 11, 15 1-11

15 1111

Thus Combining 3, 7 and 11, 15 ⇒ 3, 7 0-11 ⇒ 3, 7, 11, 15 --11

11, 15 1-11

Which is PI --11 and since minterm 15 is not on the PI Chart it is also a don't care.

thus the decimal representation of PI --11 is <u>3, 7, 11, 15</u>

iv) <u>PI 1--1</u>

From the given question, PI 1--1 is covered by minterms 9, 11 and 13.

thus by combining 9 and 11

⇒ 9 → 1001 ⇒ 9, 11 10-1

11 → 1011

thus Checking the table in step 1, minterm 13 is in group 4 and can combine with minterm 9 and 12 in group 3 but from the given question 12 is not on the PI Chart and thus 9 & 13 can combine also but checking group 5, it can combine with 15 so that the desired PI 1--1 can be obtained.

Thus

combining 13 & 15

$\Rightarrow$

13 → 1101 ⇒ 13, 15 11–1

15 → 1111

Thus by grouping 9, 11 and 13, 15

⇒ 9, 11 10–1 ⇒ 9, 11, 13, 15 ⇒ 1––1

13, 15 11–1

which is PI 1––1 and since minterm 15 is not on the PI chart it is also a don't care.

Thus the decimal representation of PI 1––1 is <u>9, 11, 13, 15</u>

b) <u>List the maxterms of F</u>

The maxterms of F are the remaining excluding the minterms and don't cares;

$\therefore F = \pi M(0, 4, 5, 6, 8, 12, 14)$

c) <u>List the don't cares of F.</u>

From the first part of the question

$F\{\Sigma d(1, 10, 15).\}$ → minterms 1, 10 & 15 are don't cares.

d) From the given PI chart)

	2	3	7	9	11	13
–0–1		X		X	X	
–01–	X (EPI)	X			X	
––11		X	X (EPI)		X	
1––1				6	X	X (EPI)

Thus PIs $-01-$, $--11$ and $1--1$ are essential PI, and their algebraic expression is

	A	B	C	D		
(i)	$-$	0	1	$-$	=	$\bar{B}C$
ii)	$-$	$-$	1	1	=	CD
iii)	1	$-$	$-$	1	=	AD.

6.19 Packages arrive at the stockroom and are delivered on carts to offices and laboratories by student employees. The carts and packages are various sizes and shapes. The students are paid according to the carts used. There are five carts and the pay for their use is

Cart C1: $2
Cart C2: $1
Cart C3: $4
Cart C4: $2
Cart C5: $2

On a particular day, seven packages arrive, and they can be delivered using the five carts as follows:

C1 can be used for packages P1, P3, and P4.
C2 can be used for packages P2, P5, and P6.
C3 can be used for packages P1, P2, P5, P6, and P7.
C4 can be used for packages P3, P6, and P7.
C5 can be used for packages P2 and P4.

The stockroom manager wants the packages delivered at minimum cost. Using minimization techniques described in this unit, present a systematic procedure for finding the minimum cost solution.

Solution

Thus by representing the given information on a PI chart, let the minterm columns be the packages and the carts should be the prime implicants.

PI	P_1	P_2	P_3	P_4	P_5	P_6	P_7
C1	X		X	X			
C2		X			X	X	
C3	X	X			X	X	X
C4			X			X	X
C5		X		X			

note cart 1 (C1) can be used for packages p1, p3 and p4 that is why on the PI chart under row C1, P_1, P_3 & P_4 have asterisks. similarly that is how the whole chart was obtained.

Thus by using petrick method.

$P_1 = (C_1 + C_3)$

$P_2 = (C_2 + C_3 + C_5)$

$P_3 = (C_1 + C_4)$

$P_4 = (C_1 + C_5)$

$P_5 = (C_2 + C_3)$

$P_6 = (C_2 + C_3 + C_4)$

$P_7 = (C_3 + C_4)$.

5

$\Rightarrow P = (C_1+C_3)(C_2+C_3+C_5)(C_1+C_4)(C_1+C_5)(C_2+C_3)(C_2+C_3+C_4)(C_3+C_4).$

Thus let $C_2+C_3 = X$

$\Rightarrow P = (C_1+C_3)(X+C_5)(C_1+C_4)(C_1+C_5)(C_2+C_3)(X+C_4)(C_3+C_4).$

Thus by using absorbitive law $[(a+b)(a+c) = (a+bc)]$

$\Rightarrow P = (C_1+C_3)(C_1+C_4)(X+C_5)(X+C_4)(C_2+C_3)(C_3+C_4)(C_1+C_5)$

$\Rightarrow P = (C_1+C_3C_4)(X+C_4C_5)(C_3+C_2C_4)(C_1+C_5)$

$P = (C_1+C_3C_4C_5)(X+C_4C_5)(C_3+C_2C_4)$

Thus by expanding

$P = (C_1X + C_1C_4C_5 + XC_3C_4C_5 + C_3C_4C_5)(C_3+C_2C_4)$

$P = C_1C_3X + XC_1C_2C_4 + C_1C_3C_4C_5 + C_1C_2C_4C_5 + XC_3C_4C_5 + XC_2C_3C_4C_5$

$+ C_3C_4C_5 + C_2C_3C_4C_5$

Thus by factorizing C_1C_3, $C_3C_4C_5$ and $C_1C_2C_4$

$\Rightarrow P = C_1C_3(X+C_4C_5) + C_3C_4C_5(\overbrace{1 + \cancel{C_2} + \cancel{XC_2} + X + C_1}^{1}) + C_1C_2C_4(X+C_5)$

$\Rightarrow P = C_3C_4C_5 + C_1C_3X + C_1C_3C_4C_5 + C_1C_2C_4X + C_1C_2C_4C_5$

Thus by eliminating those with 4 number of variables.

$\Rightarrow P = C_3C_4C_5 + C_1C_3X + C_1C_2C_4X$

recalling $X = C_2+C_3$

$P = C_3C_4C_5 + C_1C_3(C_2+C_3) + C_1C_2C_4(C_2+C_3)$

$= C_3C_4C_5 + C_1C_2C_3 + C_1C_3 + C_1C_2C_4 + C_1C_2C_3$

$= C_3C_4C_5 + C_1C_3(\overbrace{1+\cancel{C_2}+C_2}^{1}) + C_1C_2C_4$

Hence the minimum solutions are $C_3C_4C_5$, C_1C_3 & $C_1C_2C_4$

Thus to find the minimal cost;

i) $C_3 C_4 C_5 =$ $C_3 \rightarrow \$4$

$C_4 \rightarrow \$2$

$C_5 \rightarrow \$2$

$+$ ———

$\$8$

ii) $C_1 C_3 =$ $C_1 \rightarrow \$2$

$C_3 \rightarrow \$4$

———

$\$6$

iii) $C_1 C_2 C_4 =$ $C_1 \rightarrow \$2$

$C_2 \rightarrow \$1$

$C_4 \rightarrow \$2$

———

$\$5$

Hence the carts C_1 C_2 & C_4 should be used for minimal cost, and all packages will be delivered

because $C_1 \rightarrow P_1$ P_3 & P_4

$C_2 \rightarrow P_2$ P_5 & P_6

$C_4 -$ P_3 P_6 & P_7

$\therefore$ $C_1 C_2 C_4 \rightarrow P_1, P_2, P_3, P_4, P_5, P_6$ & P_7

6.21 Shown below is the prime implicant chart for a completely specified four-variable combinational logic function $r(w, x, y, z)$.

(a) Algebraically express r as a product of maxterms.

(b) Give algebraic expressions for the prime implicants labeled A, C, and D in the table.

(c) Find all minimal sum-of-product expressions for r. You do **not** have to give algebraic expressions; instead just list the prime implicants (A, B, C, etc.) required in the sum(s).

	0	4	5	6	7	8	9	10	11	13	14	15
A	×	×										
B			×		×					×		×
C				×	×						×	×
D						×	×	×	×			
E								×	×		×	×
F							×		×	×		×
G	×					×						
H		×	×	×	×							

a) Solution

From the given prime Implicant Chat, the Columns are minterms and thus since it is a completely specified function thus it has no don't cares thus the minterms that are not present on the PI chat are the maxterms.

Thus the maxterms of r are;

$$r = \pi m(1, 2, 3, 12)$$

Thus Also $r(w, x, y, z)$

and $1 = 0001 = (w+x+y+\bar{z})$

$2 = 0010 = (w+x+\bar{y}+z)$

$3 = 0011 = (w+x+\bar{y}+\bar{z})$

$12 = 1100 = (\bar{w}+\bar{x}+y+z$

Thus algebraic expression as a product of maxterms.

$$r = (w+x+y+\bar{z})(w+x+\bar{y}+z)(w+x+\bar{y}+\bar{z})(\bar{w}+\bar{x}+y+z)$$

b). The Algebraic expression are;

(i) PI A

From the given PI Chart PI A is covered by minterms 0 and 4, thus by Combining 0 & 4

$\Rightarrow$ $0 \to 0000$ ∴ $0,4 = \overset{wxyz}{0-00} = \bar{A}\bar{C}\bar{D}$

$4 \to 0100$

~~PI A = $\bar{A}\bar{C}\bar{D}$~~

$\boxed{\text{PI A} = \bar{w}\bar{y}\bar{z}}$

ii) PI C

From the given PI Chart PI C is covered by minterms 6, 7 and 14, 15

Thus by combining 6 & 7, 14 & 15 and combining again

$\Rightarrow$ 6 $\rightarrow$ 0110 $\Rightarrow$ 6,7 $\Rightarrow$ 011_
7 $\rightarrow$ 0111

14 $\rightarrow$ 1110 $\Rightarrow$ 14,15 = 111_
15 $\rightarrow$ 1111

Thus 6,7 and 14,15 $\Rightarrow$ 6,7 $\rightarrow$ 011_
14,15 $\rightarrow$ 111_ $\Rightarrow$ 6,7,14,15 = _11_ (W X Y Z)

∴ ~~PI C = BC~~

answer to be in W, X, Y & Z

$\boxed{\text{PI C} = XY}$

iii) PI D

From the given PI chart, PI D is covered by minterms 8, 9, 10 & 11.

Thus by combining 8 & 9, 10 & 11 and combining again

$\Rightarrow$ 8 $\rightarrow$ 1000 8,9 $\Rightarrow$ 100_
9 $\rightarrow$ 1001

10 $\rightarrow$ 1010 $\Rightarrow$ 10,11 $\Rightarrow$ 101_
11 $\rightarrow$ 1011

$\Rightarrow$ 8, 9, 10, 11 = 8,9 100_
10,11 101_ 8, 9, 10, 11 10_ _ (W X Y Z)

$\boxed{\therefore \text{PI D} = W\bar{X}}$

(c) Thus by employing petricks method;

$P = (A+G)(A+H)(B+H)(C+H)(B+C+H)(D+G)(D+F)(D+E)(D+E+F)$
$(B+F)(C+E)(B+C+E+F)$

Thus let $B+H = X$, $D+E = Y$, $B+F = Z$ and $C+E = W$

$\Rightarrow P = (A+G)(A+H)(X)(C+H)(X+C)(D+G)(D+F)(Y)(Y+F)$
$(Z)(W)(W+Z)$

$\Rightarrow$ by rearranging

$P = (A+G)(A+H)(C+H)(X)(X+C)(D+G)(D+F)(Y)(Y+F)(Z)(W)(W+Z)$

recall that $A(A+B) = AA+AB = A+AB = A(\overset{1}{\cancel{1+B}}) = A$.

thus $X(X+C) = X$, $W(W+Z) = W$, $Y(Y+F) = Y$.

$\Rightarrow P = (A+G)(A+H)(C+H)(X)(D+G)(D+F)(Y)(Z)(W)$.

Thus by using absorption rule.

$\Rightarrow P = (A+GH)(C+H)(X)(D+GF)(Y)(Z)(W)$.

$\Rightarrow P = (A+GH)(C+H)(D+GF)(WXYZ)$

$\Rightarrow P = (AC+AH+CGH+HG)(D+GF)(WXYZ)$

$\Rightarrow P = (ACD+ACGF+AHD+AHGF+CDGH+CGFH+HGD+HGF)$
$(WXYZ)$.

by Factoring out HGF

$P = (ACD+ACGF+AHD+HGF(\overset{1}{\cancel{A+C+1}})+CDGH+HGD)(WXYZ)$

$P = (ACD+ACGF+AHD+HGF+\underbrace{CDGH+HGD}_{HGD(\overset{1}{\cancel{1+C}})})(WXYZ)$.

$P = (ACD + ACGF + AHD + HGF + HGD)(WXYZ).$

Thus by resolving $WXYZ$ $W = C+E$, $X = B+H$, $Y = D+E$, $Z = B+F$

$\Rightarrow (C+E)(B+H)(D+E)(B+F)$

$\Rightarrow (C+E)(D+E)(B+H)(B+F)$ and thus by using absorption law

$\Rightarrow (E+CD)(B+FH) \Rightarrow BE + EFH + BCD + CDFH.$

$\Rightarrow P = (ACD + ACGF + AHD + HGF + HGD)(BE + EFH + BCD + CDFH)$

$\Rightarrow P = (ABCDE + ACDEFH + ABCD + ACDFH + ABCEGF + ACGFEH$
$+ ACGFBD + ACGFDH + AHDBE + AHDEF + AHDBC + AHDCF$
$+ HGFBE + HGFE + HGFBCD + HGFCD + HGDBE + HGDEF +$
$HGDBC + HGDCF)$

Thus by factoring out $ABCD$ and $EFGH$.

~~$\Rightarrow P = \{ABCD(1+E) + EFGH($~~

$P = \{ABCD(1 + E + \cancel{GF} + H\} + EFGH(1 + \cancel{AC} + D)$ and neglecting other products having 5 number of literals.

$\Rightarrow P = ABCD + EFGH$

Thus First solution $= A+B+C+D$

Second solution $= E+F+G+H.$