Αριθμιτικη αναλυση

Προαιρετικη εργασια εξαμηνου

Αθανασιος Καρρας 3ο Ετος
email: dit20082@go.uop.gr
ΑΜ: 202220200082
Πανεπιστημιο Πελοποννησου

Contents

1 Ασκηση 1

Η ασκηση υλοποιηθηκε με την χρηση του προγραμματος octave (7.3.0)
$f(x) = sin(x), g(x) = x^3 - 6x^2 + 4x + 12, h(x) = 2^x$

1.1 Κανόνας Τραπεζίου

```
function ftrap(x,y,fun)

%trapezoid formula
%error and relative  error calculations are inside the printf
resultTrapeziou=( x(2)-x(1) ) * ( (y(1)+y(2)) / 2 );

    printf("resultTrapeziou %d\n",resultTrapeziou);
    printf("True intergral result %d\n",integral(fun,1,3));
    printf("Error %d\n",error=integral(fun,1,3)-resultTrapeziou);
    printf("Relative error %d
        %%\n",errorRelative=abs(error/integral(fun,1,3))*100);

endfunction
```

Συγκριση προσεγγισης,Σφαλμα,Σχετικο σφαλμα

- $f(x) = sin(x)$
  resultTrapeziou 0.982591
  True intergral result 1.53029
  Error 0.547704
  Relative error 35.7907 %

- $g(x) = x^3 - 6x^2 + 4x + 12$
  resultTrapeziou 8
  True intergral result 8
  Error 3.55271e-15
  Relative error 4.44089e-14 %

- $h(x) = 2^x$
  resultTrapeziou 10
  True intergral result 8.65617
  Error -1.34383
  Relative error 15.5245 %

1.2 Κανόνας Simpson's 1/3

```
function simp1(x,y,fun)
h=(x(2)-x(1))/2; %  h=B-A/2
%simp1/3 formula
```

```
resultSimp=h/3*(y(1)+4*fun((x(2)+x(1))/2)+y(2));

    printf("resultSimp1/3 %d\n",resultSimp);
    printf("True intergral result %d\n",integral(fun,x(1),x(2)));
    printf("Error %d\n",error=integral(fun,x(1),x(2))-resultSimp);
    printf("Relative error %d
        %%\n",errorRelative=abs(error/integral(fun,x(1),x(2)))*100);

endfunction
```

Συγκριση προσεγγισης,Σφαλμα,Σχετικο σφαλμα

- $f(x) = sin(x)$
  resultSimp1/3 1.53993
  True intergral result 1.53029
  Error -0.0096321
  Relative error 0.629428 %
- $g(x) = x^3 - 6x^2 + 4x + 12$
  resultSimp1/3 8 True intergral result 8
  Error 3.55271e-15
  Relative error 4.44089e-14 %
- $h(x) = 2^x$
  resultSimp1/3 8.66667
  True intergral result 8.65617
  Error -0.0104964
  Relative error 0.121259 %

1.3 Κανόνας Simpson's 3/8

```
function simp3(x,y,fun)

h=(x(2)-x(1))/3; %  h=B-A/3

%simp3/8 formula apo to wikipedia
resultSimp=((3*h)/8)*(y(1)+3*(fun((2*x(1)+x(2))/3)+fun((x(1)+2*x(2))/3))+y(2));

    printf("resultSimp3/8 %d\n",resultSimp);
    printf("True intergral result %d\n",integral(fun,x(1),x(2)));
    printf("Error
        %d\n",error=abs(integral(fun,x(1),x(2))-resultSimp));
    printf("Relative error %d
        %%\n",errorRelative=abs(error/integral(fun,x(1),x(2)))*100);

endfunction
```

Συγκριση προσεγγισης,Σφαλμα,Σχετικο σφαλμα

- $f(x) = sin(x)$
  resultSimp3/8 1.53452

True intergral result 1.53029
Error 0.00422333
Relative error 0.275981 %

- $g(x) = x^3 - 6x^2 + 4x + 12$
  resultSimp3/8 8
  True intergral result 8
  Error 3.55271e-15
  Relative error 4.44089e-14 %

- $h(x) = 2^x$
  resultSimp3/8 8.66086
  True intergral result 8.65617
  Error 0.00469448
  Relative error 0.0542328 %

1.4 Σύνθετος κανόνας Τραπεζίου

1.5 Σύνθετος κανόνας Simpson's 1/3

1.6 Σύνθετος κανόνας Simpson's 3/8

2 Ασκηση 2

$f(x) = cos(x) + C, g(x) = 5x^5 + 12x^4 - x^3 + 9x^2 + 4x + 12, h(x) = 2^x + C$
$C = 2022202000082$
***Η σταθερα C=2022202000082 για λογους απλοτητας εχει την τιμη 82
Βεβαια μπορειτε να εξετασετε τις μεθοδους με την κανονικη της τιμη στο octave script(trapezi.m)***

2.1 Γραμμικη παρεμβολλη

```
function  linearInter(x,yi,fun)
        plotp=0; %init gia to plot
        plote=0; %init gia to plot tou error
        plotr=0; %init gia to plot tou Relative error
        for xx=0:9
                    %ousiastika oti einai mesa se auto to for einai h
                        pragmatikh parembolh
                    %0:9 giati trexei to function kai gia ta 5 gnwsta
                        (loop 0 mexri 5) kai ta 5 agnwsta (loop 5
                        mexri 9)
            sum=0;
            trueResult=fun(xx);
            for i=1:2               %apo ta slides dinotan gia n-th
                order,ara gia grammikh 1st order (2-1=1)
                product=yi(i);
                for j=1:2
                    if i!=j
                        product=product*( ( xx-x(j) )/( x(i)-x(j)) );
                    endif
                endfor
                sum=sum+product;
            endfor
                printf("Linear interplotation \nx=%d f(x)=%d
                    \nError=%d\nRelative error=%d
                    %%\n",xx,sum,trueResult-sum,abs(((trueResult-sum)/trueResult)*100));
            plotp(end+1)=sum;
            plote(end+1)=trueResult-sum;
            plotr(end+1)=abs(((trueResult-sum)/trueResult)*100);
        endfor
    yy=0:1:9
    plotp(1) = [];  %delete thn init timh tou plotp
    plote(1) = [];  %delete thn init timh tou plote
    plotr(1) = [];  %delete thn init timh tou plote
    subplot(3,1,1)
    plot(yy,plotp,'rs-');
    title('Linear interplotation'),xlabel('x'),ylabel('y'),grid on
    subplot(3,1,2)
    plot(yy,plote,'rs-');
    title('Error of Linear
        interplotation'),xlabel('x'),ylabel('Error'),grid on
```

```
    subplot(3,1,3)
    plot(yy,plotr,'rs-');
    title('Relative error of Linear
       interplotation'),xlabel('x'),ylabel('Relative Error'),grid on
    plotp
    plote
    plotr
endfunction
```

Συγκριση προσεγγισης,Σφαλμα,Σχετικο σφαλμα

- $f(x) = cos(x) + C$

```
    yy(diasthma stoixeiwn) =

   0   1   2   3   4   5   6   7   8   9

plotp (prediction)=

   83.000   82.540   82.081   81.621   81.161   80.702   80.242
         79.782   79.322   78.863

plote (error)=

        0         0  -0.4968  -0.6109   0.1851   1.5822   2.7184
             2.9718   2.5321   2.2261

plotr (relative error)=

        0         0   0.6089   0.7541   0.2276   1.9228   3.2767
             3.5911   3.0934   2.7453
```

- $g(x) = 5x^5 + 12x^4 - x^3 + 9x^2 + 4x + 12$

```
        yy =

   0   1   2   3   4   5   6   7   8   9

plotp =

     0    25    50    75   100   125   150   175   200   225

plote =

        0         0       330      2166      8172     23100     54390
             112770    212856    373752

plotr =

       NaN         0   86.8421   96.6533   98.7911   99.4618
          99.7250   99.8451   99.9061   99.9398
```

- $h(x) = 2^x + C$

```
yy =

   0   1   2   3   4   5   6   7   8   9

plotp =

   83   84   85   86   87   88   89   90   91   92

plote =

     0      0      1      4     11     26     57    120    247    502

plotr =

         0         0    1.1628    4.4444   11.2245   22.8070
           39.0411   57.1429   73.0769   84.5118
```

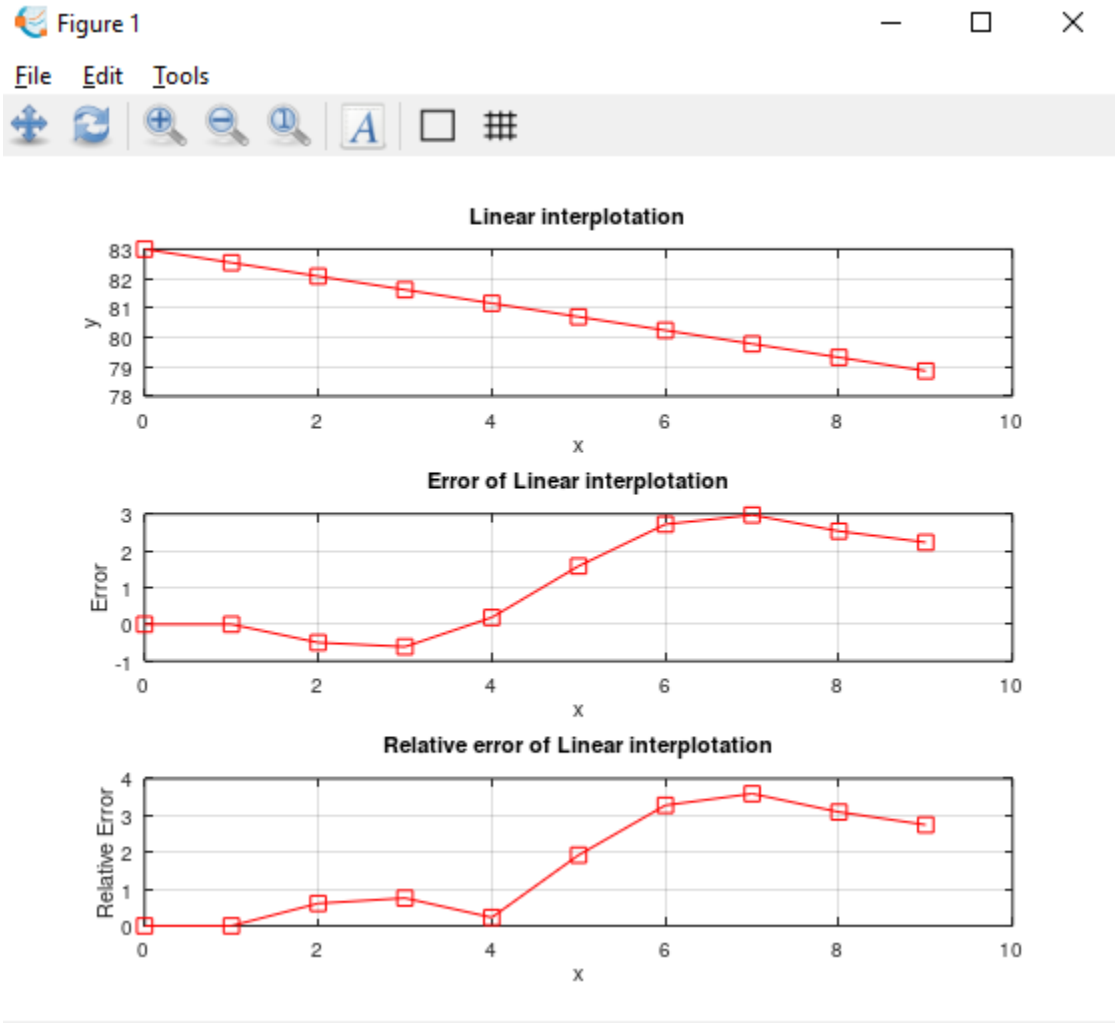

Figure 1: $f(x) = cos(x) + C$

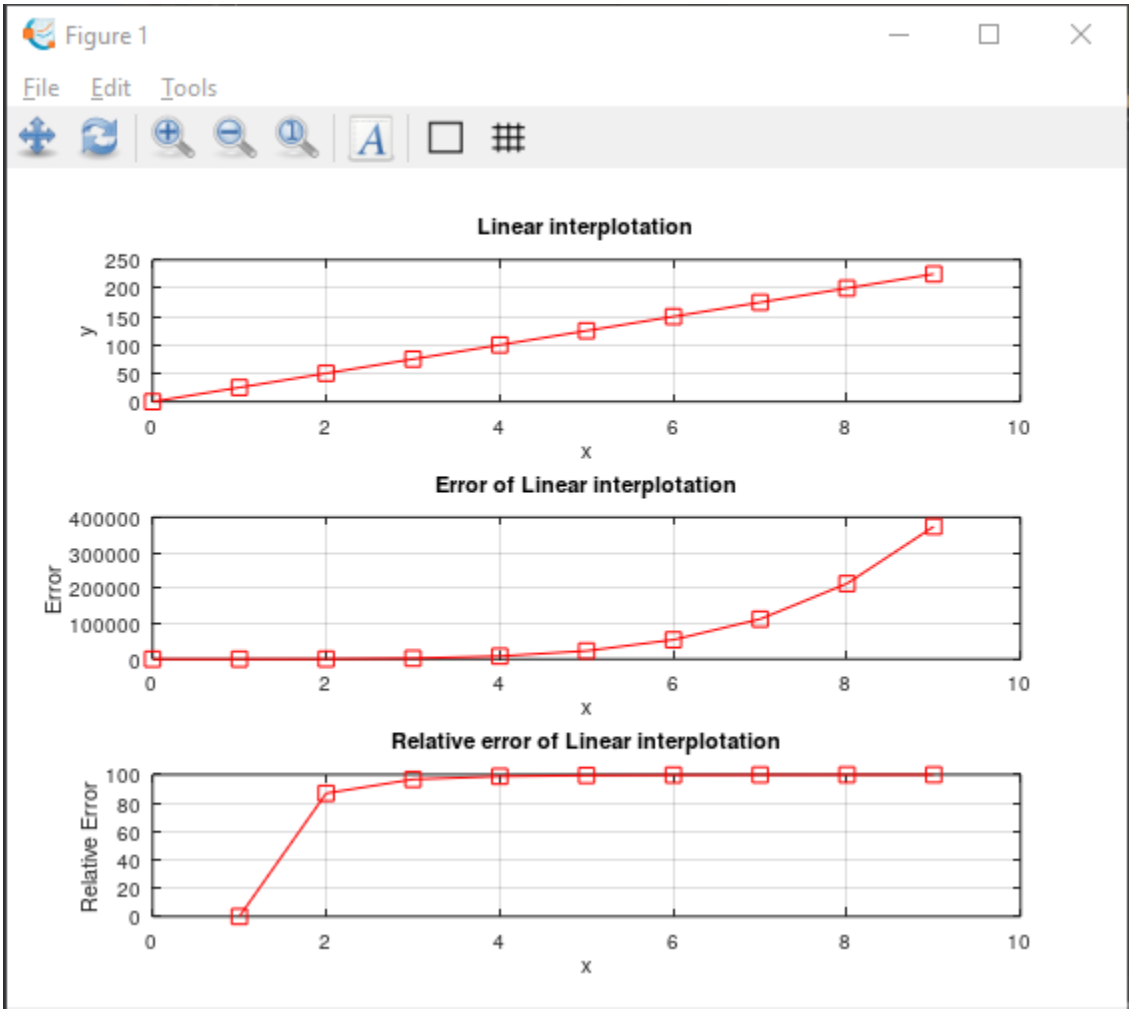

Figure 2: $g(x) = 5x^5 + 12x^4 - x^3 + 9x^2 + 4x + 12$

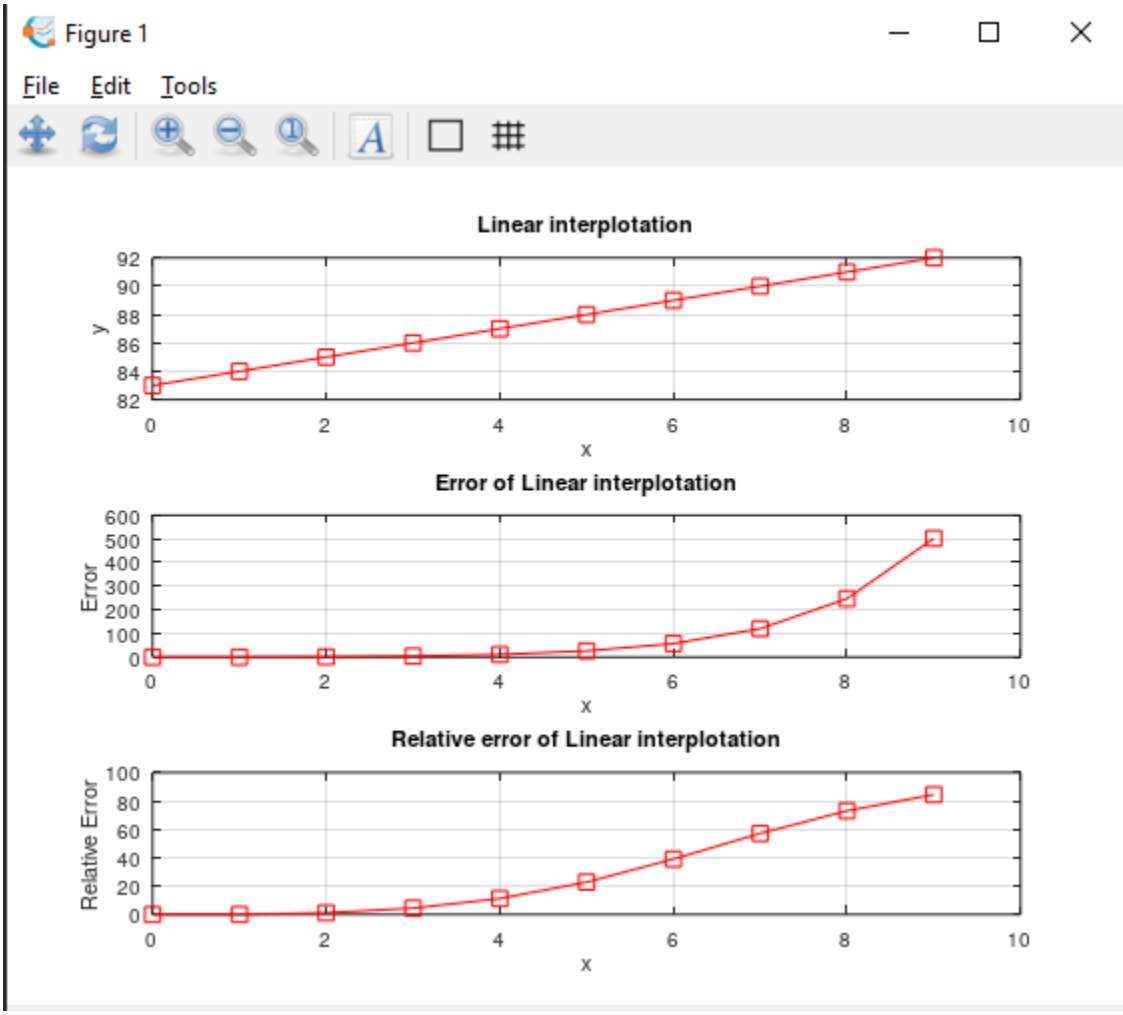

Figure 3: $h(x) = 2^x + C$

2.2 La grange παρεμβολλη

```
function  laGrange(x,yi,n,fun)
        plotp=0; %init gia to plot
        plote=0; %init gia to plot tou error
        plotr=0; %init gia to plot tou Relative error
        for xx=0:9
                          %ousiastika oti einai mesa se auto to for
                              einai h pragmatikh parembolh
                          %0:9 giati trexei to function kai gia ta
                              5 gnwsta (loop 0 mexri 5) kai ta 5
                              agnwsta (loop 5 mexri 9)
            sum=0;
            trueResult=fun(xx);
            for i=1:n           %apo ta slides
                product=yi(i);
                for j=1:n
                    if i!=j
                        product=product*( ( xx-x(j) )/( x(i)-x(j)) );
                    endif
                endfor
                sum=sum+product;
            endfor
            printf("Lagrange interplotation\nx=%d f(x)=%d
                \nError=%d\nRelative error=%d
                %%\n",xx,sum,trueResult-sum,abs(((trueResult-sum)/trueResult)*100));
            plotp(end+1)=sum;
            plote(end+1)=trueResult-sum;
            plotr(end+1)=abs(((trueResult-sum)/trueResult)*100);
        endfor
    yy=0:1:9;
    plotp(1) = [];  %delete thn init timh tou plotp
    plote(1) = [];  %delete thn init timh tou plote
    plotr(1) = [];  %delete thn init timh tou plote
    subplot(3,1,1)
    plot(yy,plotp,'rs-');
    title('Linear interplotation'),xlabel('x'),ylabel('y'),grid on
    subplot(3,1,2)
    plot(yy,plote,'rs-');
    title('Error of Linear
        interplotation'),xlabel('x'),ylabel('Error'),grid on
    subplot(3,1,3)
    plot(yy,plotr,'rs-');
    title('Relative error of Linear
        interplotation'),xlabel('x'),ylabel('Relative Error'),grid on
    plotp
    plote
    plotr
endfunction
```

Συγκριση προσεγγισης,Σφαλμα,Σχετικο σφαλμα

- $f(x) = cos(x) + C$

```
plotp =

   83.000   82.540   81.584   81.010   81.346   82.769   85.101
         87.816   90.034   90.523

plote =

        0         0         0         0         0  -0.4851  -2.1410
            -5.0621  -8.1793  -9.4344

plotr =

         0         0         0         0         0    0.5895
             2.5808    6.1171    9.9925   11.6346
```

- $g(x) = 5x^5 + 12x^4 - x^3 + 9x^2 + 4x + 12$

```
plotp =

        0       25      380     2241     8272    22625    50940
              100345   179456   298377

plote =

       0        0        0        0        0      600     3600    12600
             33600    75600

plotr =

       NaN         0         0         0         0    2.5834
           6.6007   11.1559   15.7705   20.2151
```

- $h(x) = 2^x + C$

```
plotp =

    83    84    86    90    98   113   139   181   245   338

plote =

     0     0     0     0     0     1     7    29    93   256

plotr =

         0         0         0         0         0    0.8772
             4.7945   13.8095   27.5148   43.0976
```

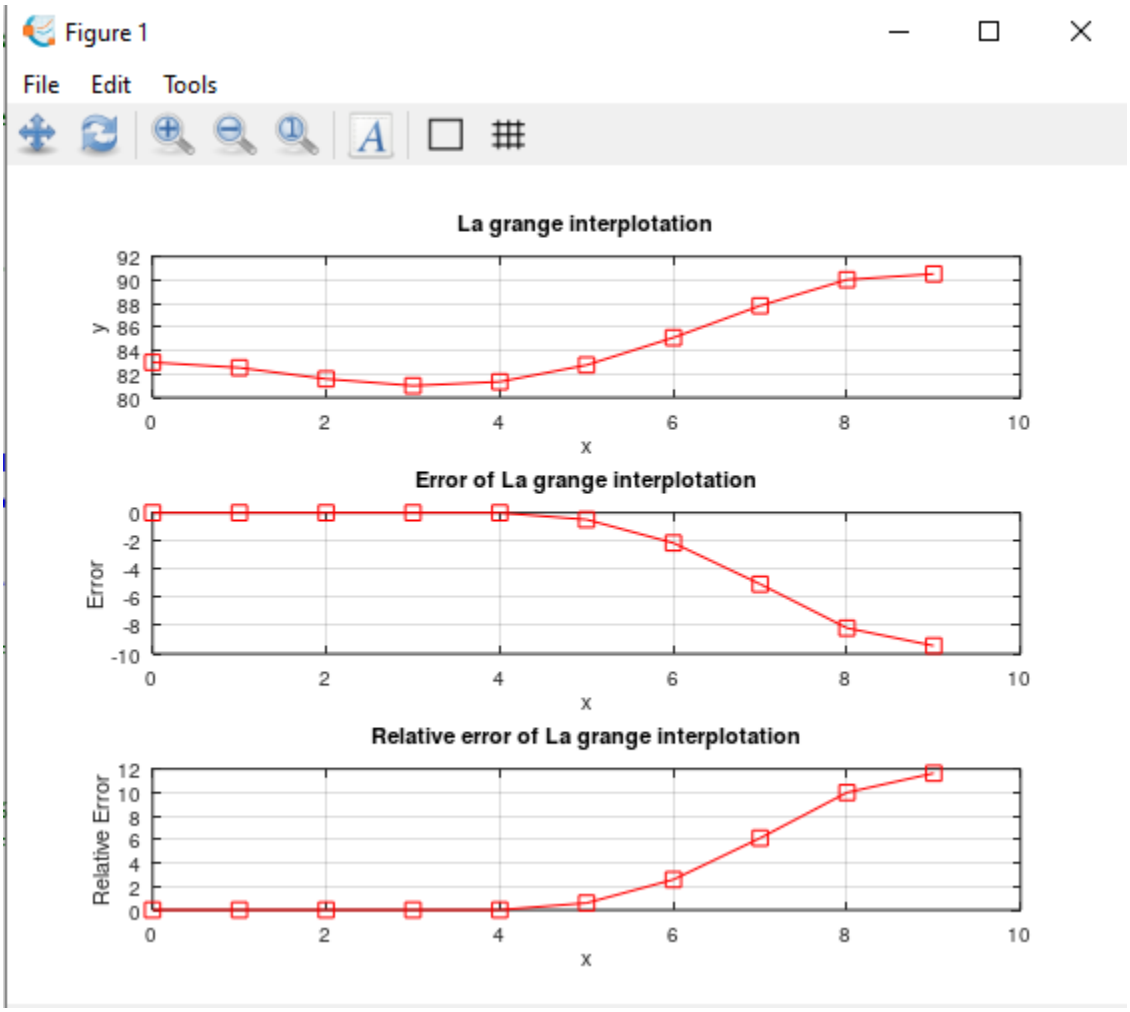

Figure 4: $f(x) = cos(x) + C$

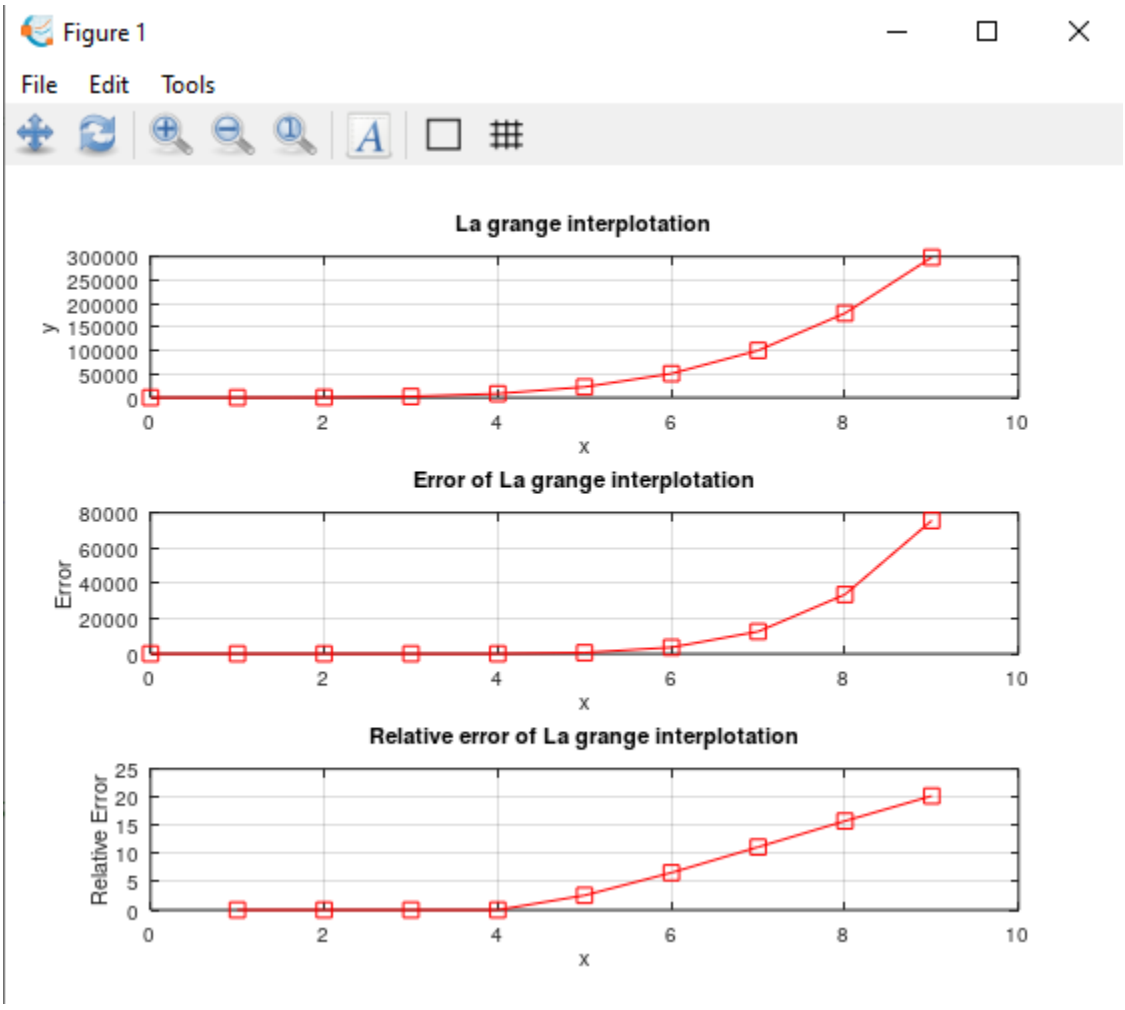

Figure 5: $g(x) = 5x^5 + 12x^4 - x^3 + 9x^2 + 4x + 12$

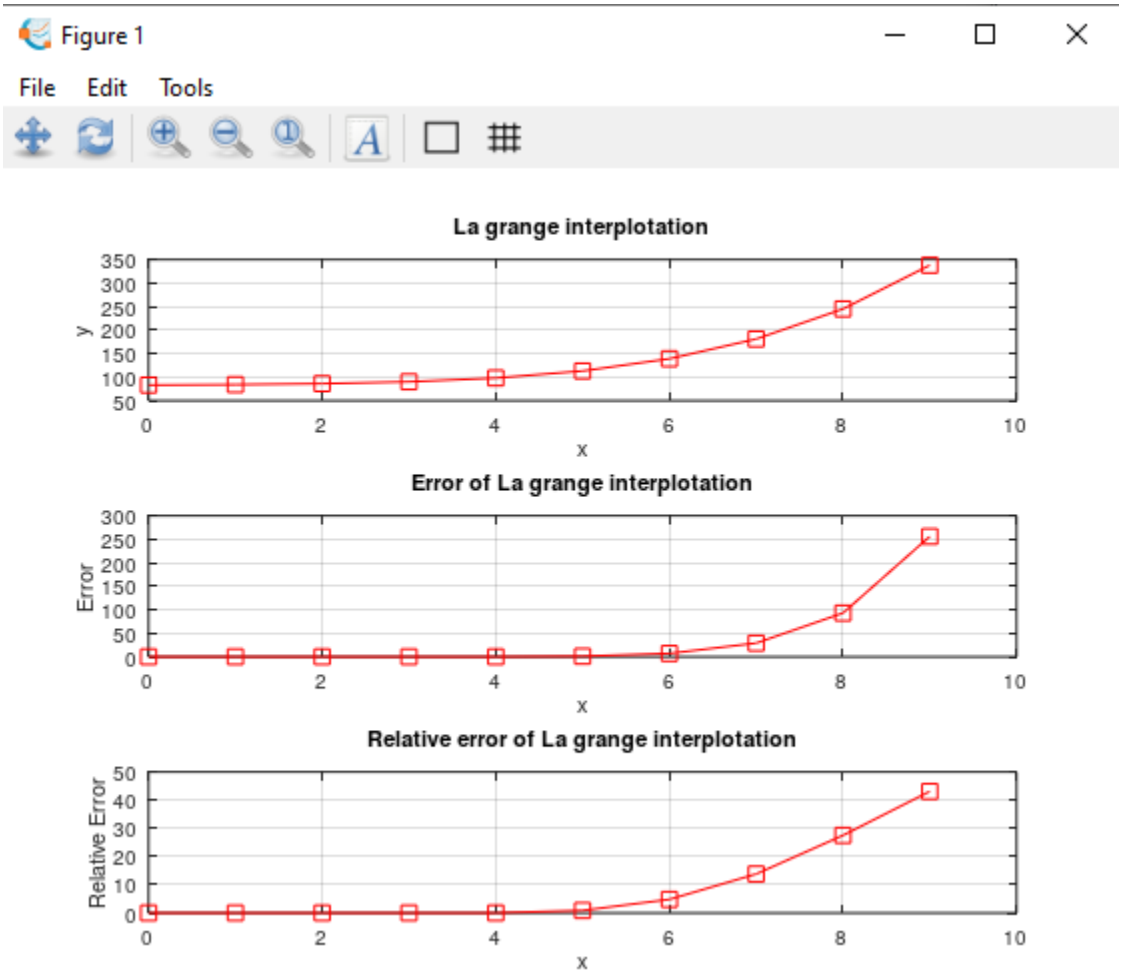

Figure 6: $h(x) = 2^x + C$

2.3 Splines

no

2.4 Ελαχιστα τετραγωνα

```
    function leastSquares(x,yi,n,xx)

sumGinomeno=0;sumPower2=0;sumX=0;sumY=0;

    %calculating the sums
    for i=1:n
        sumGinomeno= sumGinomeno + x(i)*yi(i);
        sumPower2= sumPower2 + x(i)*x(i);
        sumY=sumY+yi(i);
        sumX=sumX+x(i);
    endfor
    %a and b for the y=a*x+b
    a=  (n*sumGinomeno - sumX*sumY)/(n*sumPower2 - (sumX).^2);
    b= (sumPower2*sumY-sumX*sumGinomeno)/( n*sumPower2 - (sumX).^2);

    fun=@(x) a*x+b; %declaring our function via function handle

    plot(xx,fun(xx),"rs-");
    title('Least squares
       interplotation'),xlabel('x'),ylabel('y'),grid on
endfunction
```

- $f(x) = cos(x) + C$
- $g(x) = 5x^5 + 12x^4 - x^3 + 9x^2 + 4x + 12$
- $h(x) = 2^x + C$

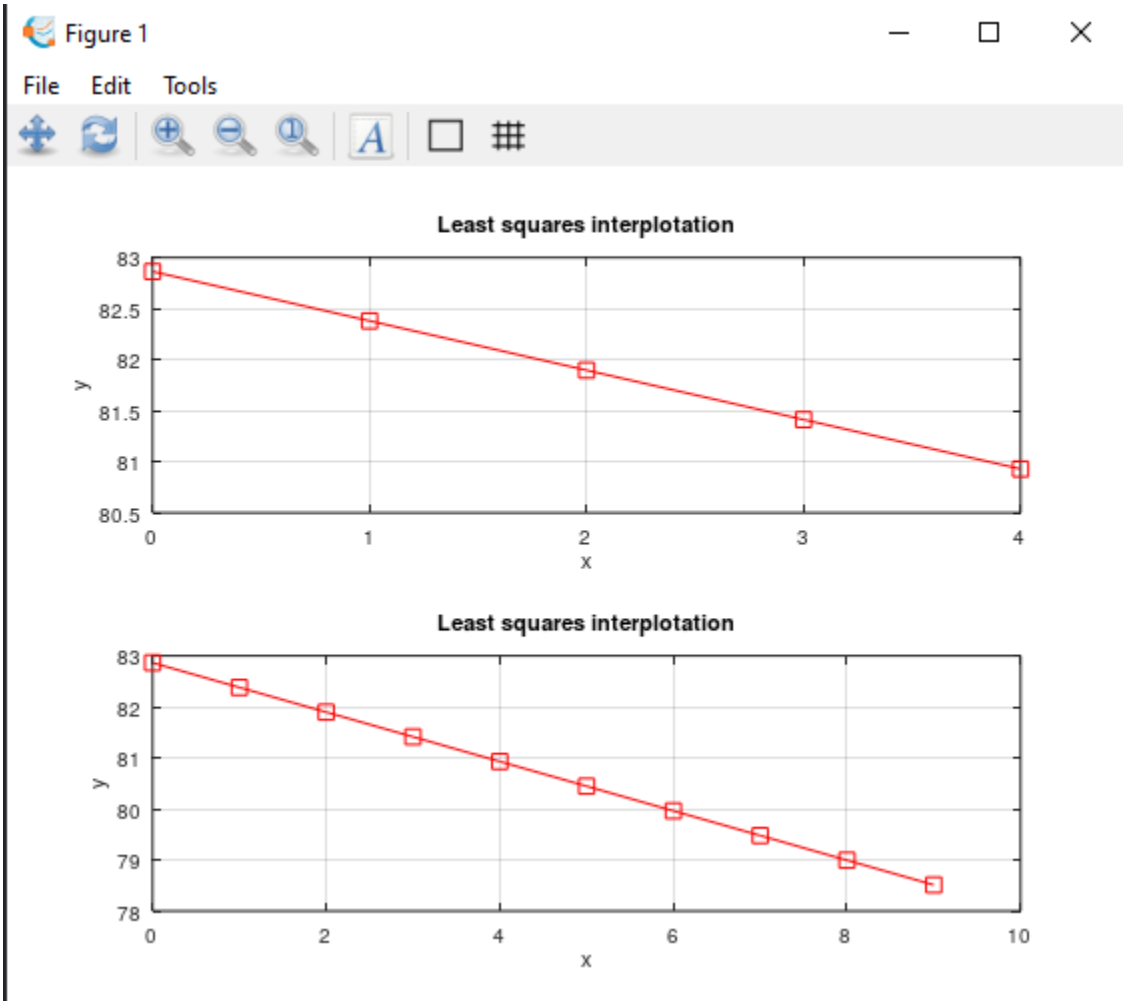

Figure 7: Ελασχιστα τετραγωνα για στο διαστημα 1)0-4 2)0-9

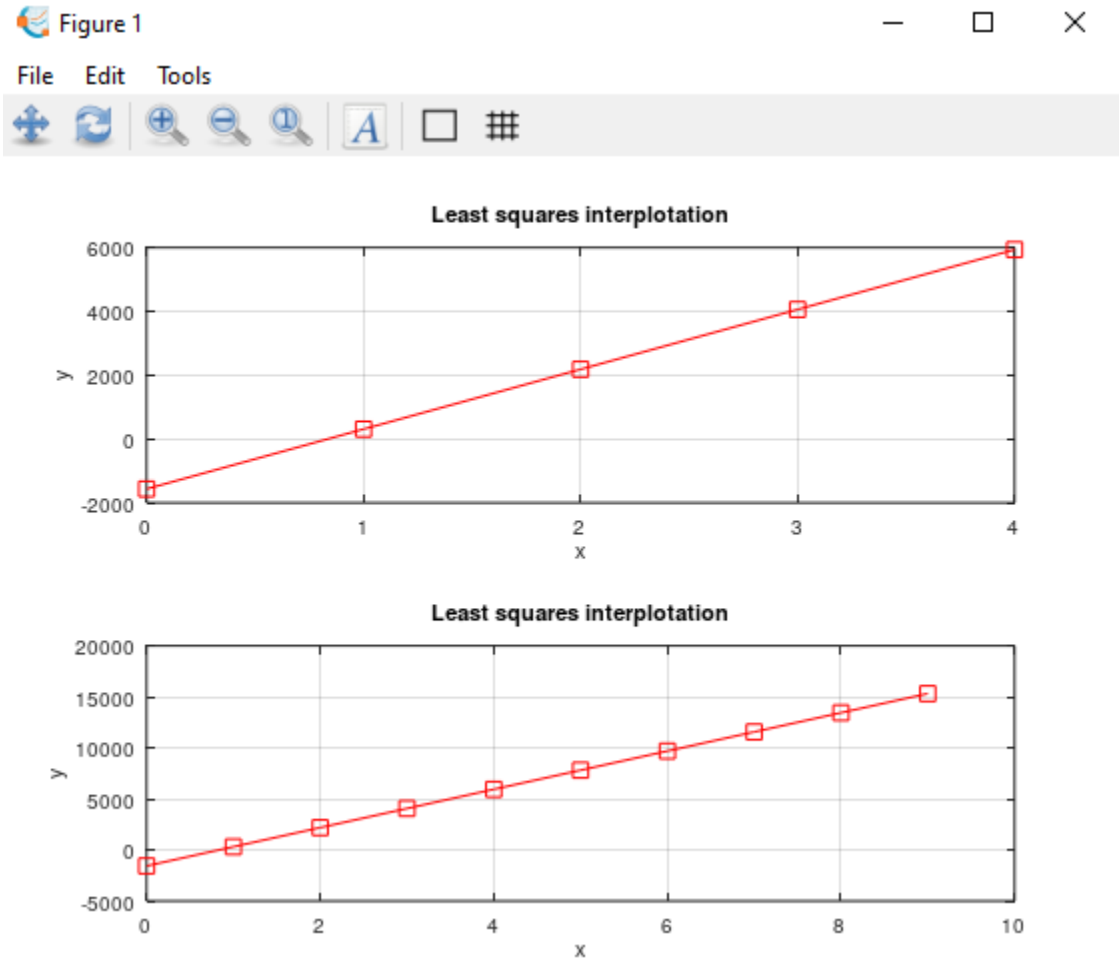

Figure 8: Ελασχιστα τετραγωνα για στο διαστημα 1)0-4 2)0-9

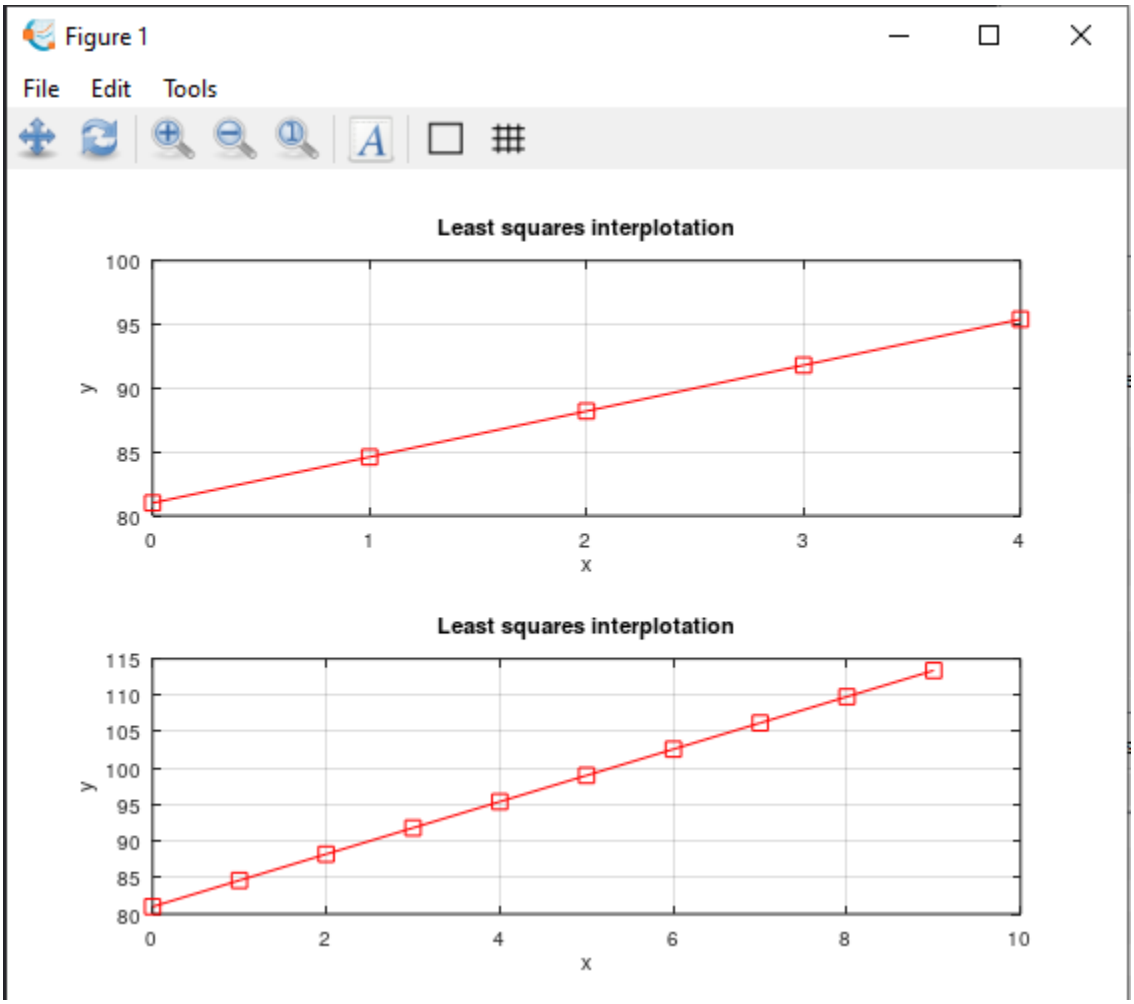

Figure 9: Ελασχιστα τετραγωνα για στο διαστημα 1)0-4 2)0-9