

CHE374

Cheat Sheet

Relevant: Weeks 1 - 8

Week 1 - Time Value of Money

Future Value	$F = P(1+i)$
Simple Interest	$F_N = P(1+iN)$
Compound Interest	$F = P(1+i)^N$
Subperiod Compounding	$F = P(1 + \frac{r}{m})^{mN}$
Eff. Annual Interest Rate	$i_e = (1 + \frac{r}{m})^m - 1 = (\frac{F}{P})^{\frac{1}{N}} - 1$ Eq. 2, only T-bills
Continuously Compounding	$i_e = \lim_{m \rightarrow \infty} (1 + \frac{r}{m})^m - 1 = e^r - 1$
Continuous Compounding	$r = \lim_{m \rightarrow \infty} ((i_e + 1) - 1)m = \ln(1 + i_e)$
Future Interest Rates	$(1 + r_{1,x})^{\frac{x}{12}} = (1 + r_{1,k})^{\frac{k}{12}} (1 + r_{k,x})^{\frac{x-k}{12}}$ $r_{a,b}$ is the rate from time a to b

Week 2 - Cash-flow Analysis

Arithmetic Gradient	$A_n = A' + (N-1)G$ A' : the original, G : amnt added, N : # of periods
Geometric Gradient	$A_n = A'(1+g)^{N-1}$
Compound Amount	$(F/P, i, N) = (1+i)^N, (P/F, i, N) = \frac{1}{(1+i)^N}$
Perpetuity	$(P/A, i) = \frac{1}{i}, (A/P, i) = i$
Annuity	$(P/A, i, N) = \frac{1}{i} (1 - \frac{1}{(1+i)^N})$
Arithmetic Growth	$(P/G, i, N) = \frac{1}{i^2} (1 - \frac{1+iN}{(1+i)^N})$
Geometric Growth	$(P/G, i, g, N) = \frac{1 - (\frac{1+g}{1+i})^N}{i-g}$

Week 3 - Mortgages and Bonds

Loan to Value Ratio	$LTV = \frac{\text{Principle}}{\text{House Value}}$
Amortization Period	# of years it takes to pay off the loan.
Mortgage Term	# of years the interest rate is fixed, after which the rate is renegotiated
Face/Par Value	Value received at bond maturity
Coupon Rate	Interest rate paid annually on the bond

Week 4 - Risk, Reward, and Arbitrage

Return vector $\vec{R}_i = \begin{bmatrix} r_{t1} \\ \vdots \\ r_{tn} \end{bmatrix}, r_{tj} = \frac{1}{\Delta t} \ln \left(\frac{P_{tj}}{P_{tj-1}} \right)$

A portfolio of n stocks, the returns of project i at t_i

Volatility $\sigma = \sqrt{\text{Var}(\vec{R}_i)} = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (r_i - \bar{r})^2}$

Average Return $\bar{r} = \frac{1}{n} \sum_{i=1}^n r_i$

Covariance $\text{Cov}(\vec{R}_i, \vec{R}_j) = \frac{1}{n-1} \sum_{i=1}^n (r_i - \bar{r}_i)(r_j - \bar{r}_j)$

Covariance Matrix $\Sigma = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \cdots & \sigma_{1n} \\ \sigma_{21} & \sigma_{22} & \cdots & \sigma_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{n1} & \sigma_{n2} & \cdots & \sigma_{nn} \end{bmatrix}$

Covariances $\sigma_{ij} = \sigma_{ji} = \text{Cov}(\vec{R}_i, \vec{R}_j)$
 $\sigma_{ii} = \text{Var}(\vec{R}_i)$

Portfolio $\vec{X} = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$ We invest x_i in each stock.

Portfolio Return $\mathbb{E}[\vec{R}_p] = \sum_{i=1}^n x_i \mathbb{E}[\vec{R}_i]$

Portfolio Volatility $\sigma_p = \sqrt{\vec{X}^T \Sigma \vec{X}}$

CAPM Assumptions

- Correlation & volatility of/between assets are fixed
- Investors aim to make as much money as possible
- Investors are rational and risk-averse
- Investors have the same info at the same time
- Investors have accurate conception of possible returns, i.e. the probability beliefs of investors match the true distribution of returns.
- No taxes or transaction costs
- Actions of investors do not influence prices
- Investors can lend and borrow unlimited amounts at the risk free rate
- Securities can be divided into any size

Efficiency Frontier & Capital Market Line

Minimize $\sigma_p^2 = \vec{X}^T \Sigma \vec{X}$

While $\sum_{i=1}^n x_i = 1, \sum_{i=1}^n x_i \mathbb{E}[R_i] = \mathbb{E}[R_p], \sum_{i=1}^n x_i \sigma_i = \sigma_p$

Week 4 - Contd.

Leveraging $\mathbb{E}[R_p] = w \cdot \mathbb{E}[R_m] + (1-w) \cdot r_f$ w is the weight of MP

Systematic Risk $\sigma_p = \sqrt{w^2 \sigma_m^2 + (1-w)^2 \cdot \sigma_f^2}, \sigma_f = 0$

Asset Expected Return $\beta_i = \frac{\text{Cov}(R_i, R_m)}{\text{Var}(R_m)} = \frac{\sigma_{i,m}}{\sigma_m^2} = \frac{\rho_{i,m} \sigma_i}{\sigma_m}$

Return $\mathbb{E}[R_i] = r_f + \beta_i (\mathbb{E}[R_m] - r_f)$

Security Market Line (Asset Expected Return Plotted)

Forwards/Futures Contract to buy/sell assets at a future date at a price agreed upon today

Expected Value $\mathbb{E}[P] = \sum_{i=1}^n P_i \cdot p_i$

Expected Return $\mathbb{E}[R] = \frac{\mathbb{E}[P]}{P_0} - 1 = \frac{\sum_{i=1}^n P_i \cdot p_i}{P_0} - 1$

p_i is the probability of P_i occurring