**CIV102 Design Report - Team Work Log and Calculations (In Appendix)**

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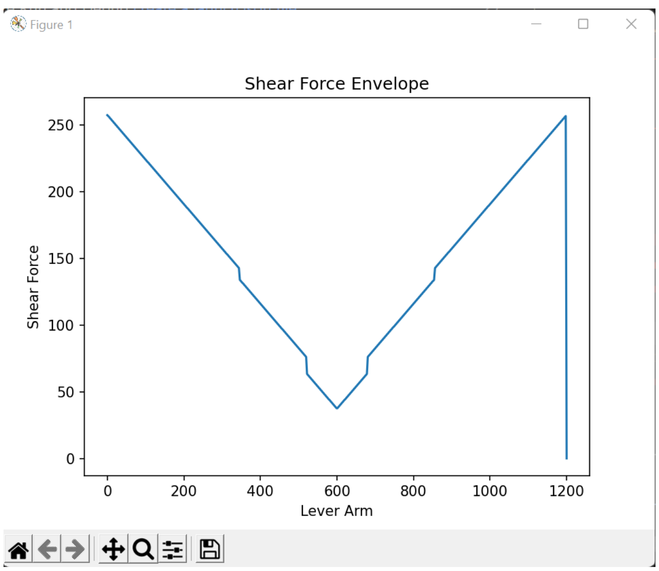
## 1. Design 0 - calculations and steps taken

Given a design 0 box girder design, calculations were made to verify whether the box girder would be feasible and whether it could withstand the bending moments and shear forces generated.

First, the maximum shear force and bending moments at each point along the bridge were generated by a python code (Figures 1 and 2). These shear force diagrams and bending moment diagrams assume that the train was 6 point loads being asserted on the bridge as it traversed through the bridge span.



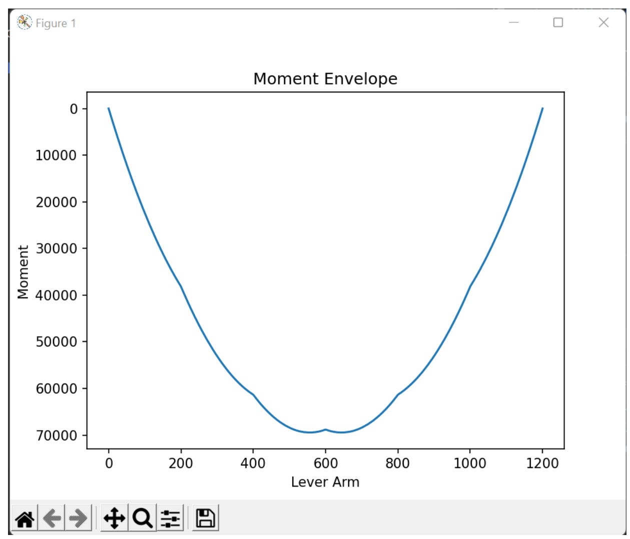




**Figure 1** (Left) Shear Force Envelope and **Figure 2 (Right)** Moment Envelope

The maximum shear force (257N) and bending moments (68800 Nmm) were applied for both hand and computed calculations as they are the maximum values - delineating the maximum forces subjected of which the bridge will fail.

### *Information on Hand Calculations and Computer Calculations*

Hand calculations for design 0 were conducted and are included in **Appendix A.** They coincide with the relevant computer calculations **(Appendix B).** Important details are taken into account to pave way for further investigation in **Figure 3**

***Direction taken from Design 0***

| | Design 0 | Equation | Eq for FOS | Variables to reduce | Variables to increase | FOS | Rank |
|---|---|---|---|---|---|---|---|
| Moment | Flexural Tension | $\frac{Myb_{ot}}{I}$ | $\frac{30I}{Mybot}$ | reduce $Mybot$ | Increase I | 4.4 | 7 |
| | Flexural compression | $\frac{Mytop}{I}$ | $\frac{6I}{Mytop}$ | reduce $Mybot$ | Increase I | 1.05 | 2 |
| Shear | Material shear | $\frac{VQ}{IB}$ | $\frac{4IB}{VQ}$ | Reduce VQ | Increase IB | 2.67 | 4 |
| | Glue Section Top | $\frac{VQ}{IB}$ | $\frac{2IB}{VQ}$ | Reduce VQ | Increase IB | 9.4 | 9 |
| | Glue Small Piece | $\frac{VQ}{IB}$ | $\frac{2IB}{VQ}$ | Reduce VQ | Increase IB | 19.8 | 10 |
| | Glue bottom | $\frac{VQ}{IB}$ | $\frac{2IB}{VQ}$ | Reduce Q | Increase IB | 2 | 3 |
| Buckling | Case 1 buckling | $\frac{4\pi^2 (4000)}{12(1-0.2)^2}(\frac{t}{b})^2$ | divide by flexural compression | Reduce b, flexcomp | Increase t | 0.623 | 1 |
| | Case 2 buckling | $\frac{0.425\pi^2 (4000)}{12(1-0.2)^2}(\frac{t}{b})^2$ | divide by flexural compression | Reduce b,flexcomp | Increase t | 3.63 | 6 |
| | Case 3 buckling | $\frac{6\pi^2 (4000)}{12(1-0.2)^2}(\frac{t}{b})^2$ | divide by flexural compression | Reduce b,flexcomp | Increase t | 4.9 | 8 |
| | Shear buckling | $\frac{5\pi^2 (4000)}{12(1-0.2)^2}((\frac{t}{a})^2 + (\frac{t}{l})^2)$ | divide by material shear | Reduce h,$l$ | Increase t | 3.39 | 5 |

**Figure 3 (Above)** **-** Chart of design 0 calculations. Full hand calculations are included in **Appendix A**

**Figure 3** shows design 0 will fail in case 1 buckling, followed by flexural compression and shear in glue at the top flange. Since most of the variables such as π, or k are constants, affecting variables were sketched out in the table to obtain an intuitive idea of what variables needed to increase/decrease.

In general, the prioritized calculations by rank require:

a. **An increase in the b:t ratio** - Because the maximum value of shear buckling increases as the $\frac{t}{b^2}$ value increases, the design would be best when the cross sections of shear buckling shared similar dimensions such that the $\frac{t}{b^2}$ ratio would be large - hence one approach could be to make the length and width of the cross sections to be square-like.
b. **A reduction of Q** - The first moment of area
c. **An increase in I - the second moment of area.** This could be achieved by either **increasing the height** such that there is a higher volume of material away from the centroid, or **increasing the area** on one specific side of the design to create a higher centroid and larger area. Since the ratio of compression to tension of the matboard was approximately 5:30, it is recommended to **allocate more mass to the top of the beam** (under compression) to balance out the flexural stresses in compression and tension, while increasing moment of inertia
d. **An increase in B - the width of which the glue is applied.** This would decrease the shear failure of the glue. Since the width of the glue is proportional to the centroid width, another approach would be to **increase the width of webs**
e. Case four (shear buckling) capacity will increase as the diaphragms are spaced closely together (*l* decreases). It is recommended to decrease the spaces between diaphragms

## 2. Design Process - Iterations and Justification

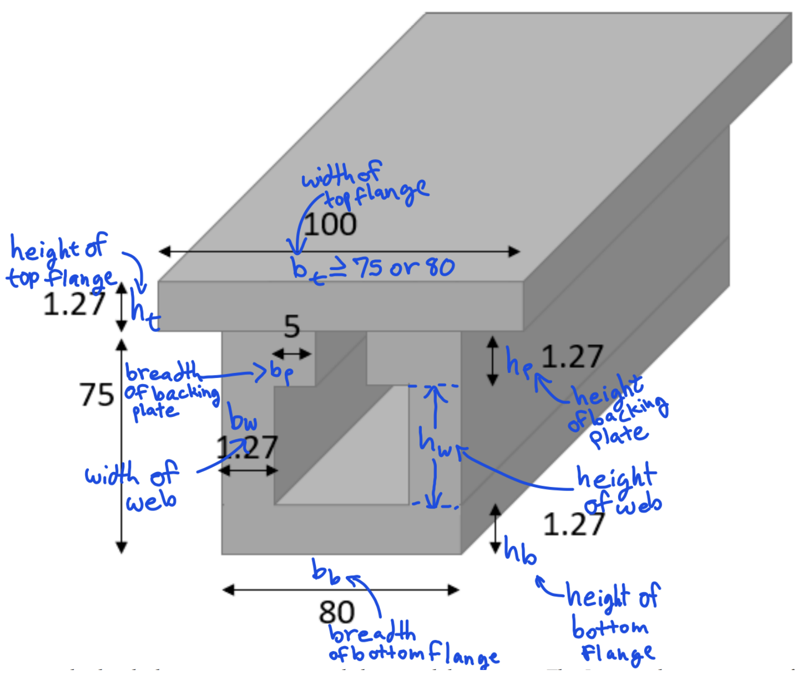



**Figure 4** variables iterated over for cross section optimization

Variables we chose included

1. **Thickness of box girder walls**:

Iterations were made changing only the thickness of the box girder.

2. **Height of box girder**

Iterations indicated that the higher the box girder, the higher the moment of inertia (I) and the lower the compressive failure until a point where case 3 or case 4 buckling would be the limiting factor.

3. **Thickness of the top flange**

Iterations showed that the higher the top flange, the higher the centroidal axis would be located.

**Iteration 1**

Our first iteration was a short double walled (doubled on the web) box girder of equal top and bottom flange widths with the following dimensions:

Width of top flange: 80
Width of bottom flange: 75
Width of web: 2.54
Width of backing plate: 5
Height of top flange: 1.27
Height of bottom flange: 1.27
Height of web: 120
Height of backing plate: 1.27
Diaphragm spacing: 200

The smallest FOS for this design was 1.52 due to flexural buckling case 1. The section concerned with case 1 buckling is the top flange. From this result, we realized that we would need to increase either the thickness of the top flange or decrease the width of the centered section due to the equation for case 1 buckling given in **figure 3**. We decided to restrict our width of the of the centered section of the top flange to fit the width of the train, so we could only increase the thickness. This gave way to our next design iteration, the dimensions of which are provided below.

**Iteration 2:**

Iteration 2 is a version of constant width with a double thickness of a top flange.
Width of top flange: 100
Width of bottom flange: 75
Width of web: 1.27

Width of backing plate: 15
Height of top flange: 2.54
Height of bottom flange: 1.27
Height of web: 100
Height of backing plate: 1.27
Diaphragm spacing: 200

This new design iteration now fails at a minimum FOS of 2.35 due to flexural compression. There are three variables associated with the flexural compression: max bending moment, distance between centroid and top of the beam, and the second moment of area. In order to decrease the applied compressive stress, we decided to make a taller cross section, because although the distance between the centroid and the top increases, the second moment of area increases by a greater amount, decreasing the overall stress. Other considerations were made in order to keep iteration 3 feasible given our amount of matboard, such as decreasing the backing plate width.

**Iteration 3**

Width of top flange: 90
Width of bottom flange: 75
Width of web: 1.27
Width of backing plate: 5
Height of top flange: 2.54
Height of bottom flange: 1.27
Height of web: 140
Height of backing plate: 1.27
Diaphragm spacing: 200

Iteration 3 has a minimum FOS of 2.51 and was going to fail due to shear buckling case 4. The parameters determining the shear buckling stress is the width of the web, height of the web and the distance between diaphragms. We could not make the cross section any shorter, because we would begin to fail in compression stress and we couldn't afford to increase the thickness of the webs due to material restrictions. Therefore, we decided to decrease the material in less significant areas to accommodate for more diaphragms, thereby decreasing the space between them and increasing the shear buckling stress.
This led us to our final design that gives a minimum FOS of 3.09:

**Final Iteration**
Width of top flange: 85
Width of bottom flange: 75
Width of web: 1.27
Width of backing plate: 5
Height of top flange: 2.54
Height of bottom flange: 1.27
Height of web: 140
Height of backing plate: 1.27
Diaphragm spacing: 150

## 3. Special Considerations:

***Consideration 1 - difficulty of cutting matboard***
Initially we had considered a trapezoidal side face as a significantly different top and bottom base would provide more effective optimisation for **I** and **b**. However, this would require tilting of the matboard pieces glued together, then sanding - reducing the accuracy of buckling capacity due to unevenness . Also, dimensions of matboard were kept at units of 1.27 as it would be unfavorable to halve matboards of 1.27mm.

***Consideration 2: Diaphragm locations***
There would be rigid support sections subjected to direct vertical compression. To prevent the bridge from buckling at such sections, multiple diaphragms were stuck together to form a solid "box-like-object" to prevent buckling at such areas

***Consideration 3: Splice connections***
Since the matboard will not be long enough, splice connections will be required to increase the length of the bridge. A stacking design was adopted such that the matboard would not all fail at the splices, rather at different splices. Remaining pieces of matboard were cut in strips and glue to the splices to resist vertical compression. The exact details of splicing are included in **Figure 8.**

# 4. Final Design

***Engineering Drawings***

***a. Elevation View***

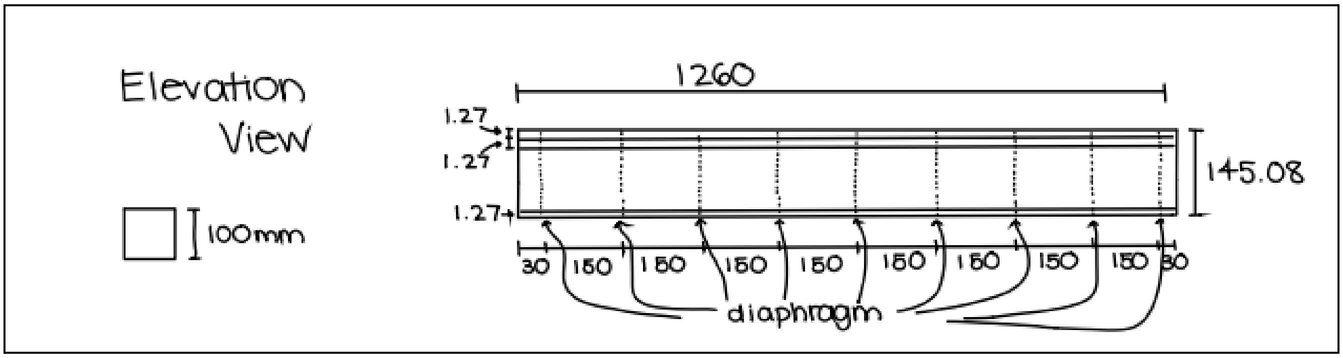
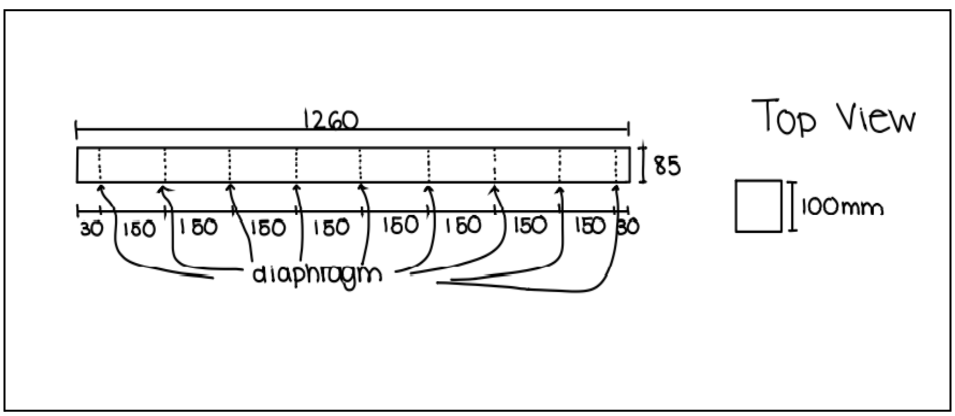



***Figure 5 -*** *elevation view of bridge*

***b. Top/Bottom View***



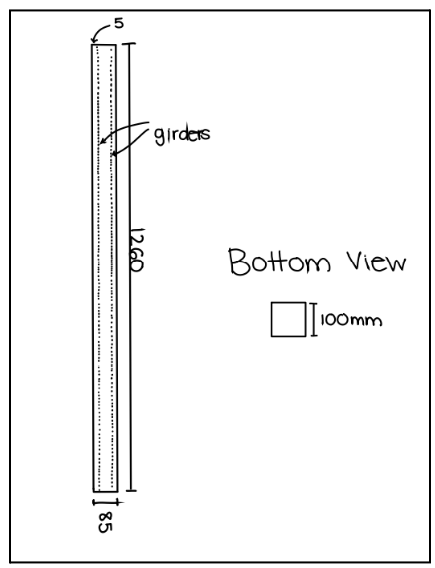




***Figure 6*** *(left) and* ***Figure*** *7 (right) - top and bottom view*

### c. Cross Section View at diaphragms and normal cross section view

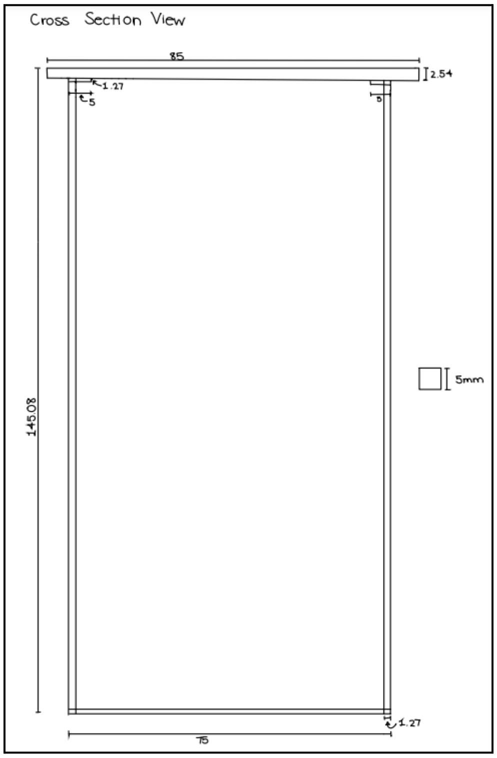



***Figure 8*** *- Section View*

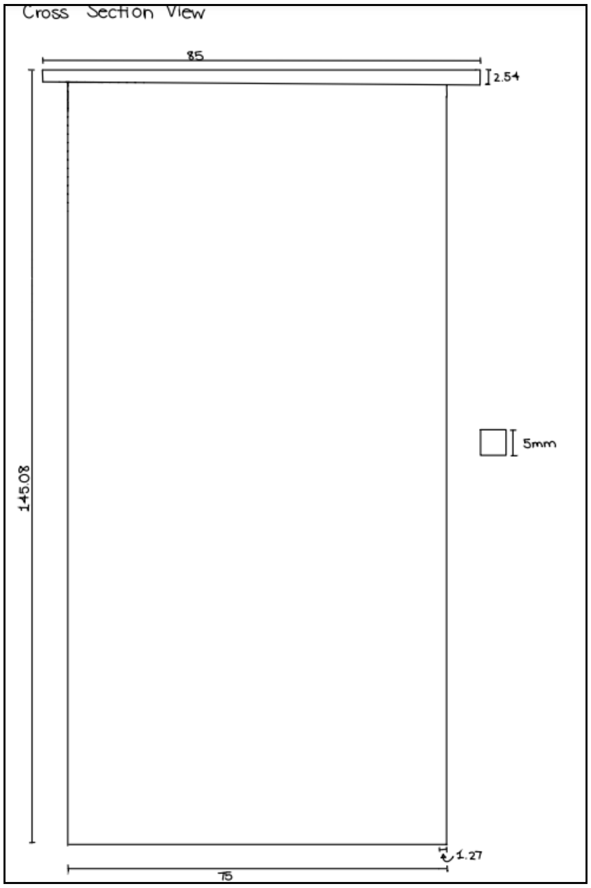



***Figure 8B -*** *Section View of Diaphragms*

### *d. Splice/connection details*

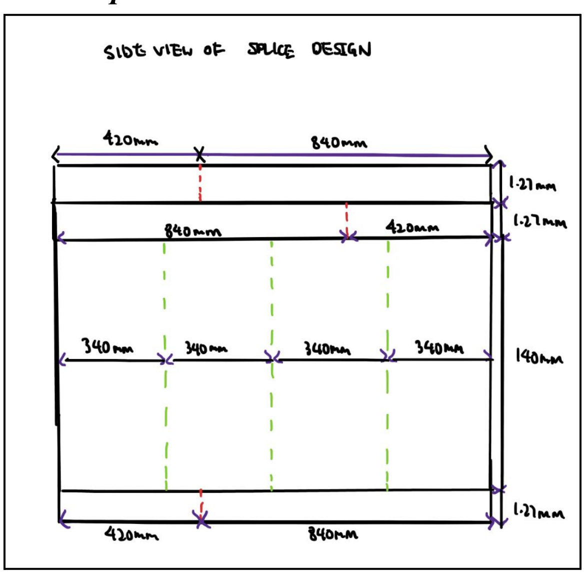



***Figure 9 -*** *Splice connection details of the final design*

### *e. Matboard Design*

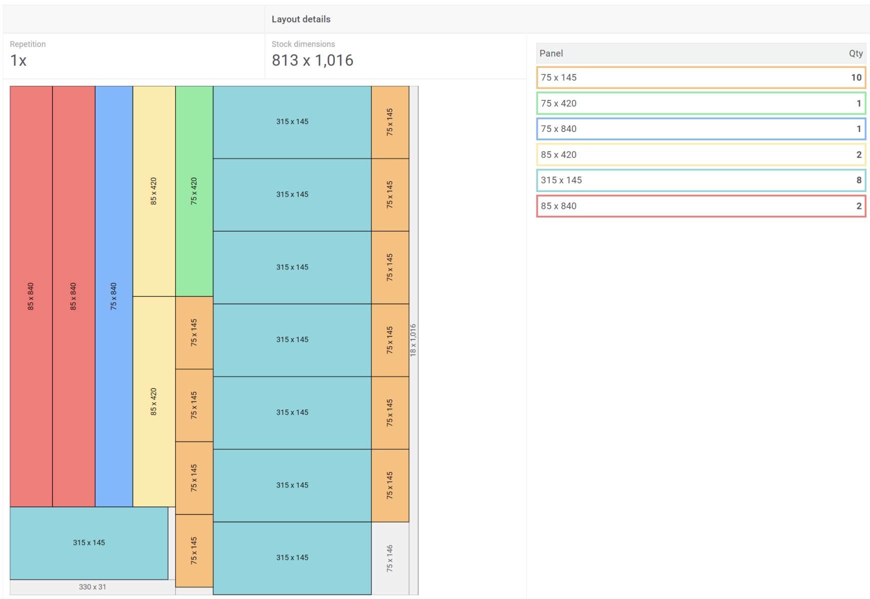



**Figure 10** - Matboard cutting

Application Opticutter was applied to optimise matboard area. An attachment of the cut matboard is included in **Figure 10.**

## 5. <u>Predictions for Final Design</u>

Failure stresses and factors of safety were evaluated by both computer and by hand. A summary of the results of the final design are included below in Figure

*Note: Hand calculations are attached in **Appendix C.** Computer calculations are included in **Appendix D.** There is a slight difference of computer and hand calculations due to rounding accuracy.*

**Figure 11** - chart showing failure and factor of safety of final design

| ***Prospect of failure*** | ***Factor of Safety*** |
|---|---|
| *σ tens* | 10.66 |
| σ compression | 3.08 |

| | |
|---|---|
| Shear material failure | 4.32 |
| Shear glue failure top | 13.0 |
| Shear glue failure between top | 219 |
| Shear glue failure bottom | 5.11 |
| Plate buckling case 1 | 8.40 |
| Plate buckling case 2 | 153 |
| Plate buckling case 3 | 5.07 |
| Plate buckling case 4 | 3.14 |
| Maximum FOS | 3.08 - bridge can sustain 791N |

The bridge is predicted to fail at 3.08 trains - or at 791N at compression failure.

## 6. Construction Process

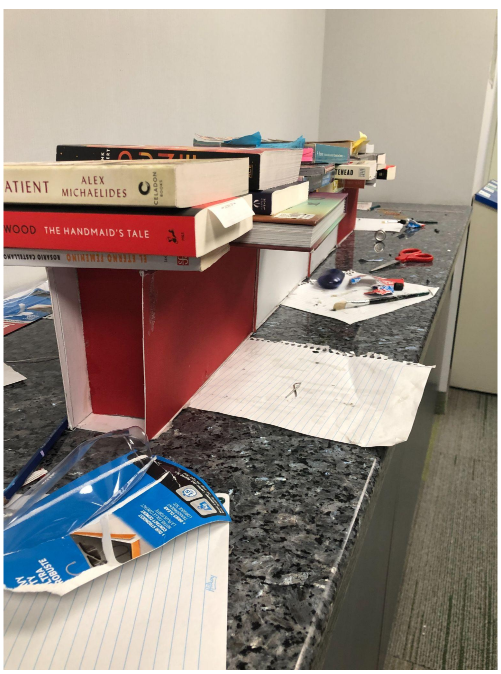
**Cutting of Matboard**

1. The matboard was traced using a meter rule, 30cm ruler and 15cm ruler to respective dimensions
2. Shapes of the matboard were labeled for simple recognition of pieces
3. Dimensions were traced using a retracting utility knife to provide good tracing, then cut out by scissors

(Left) **Figure 12** - Image of the team tracing dimensions

**Gluing Procedure**

1. Pieces of paper were torn apart and laid on the work bench to protect it from cement. Masks were worn to reduce the scent of cement and a well ventilated location was selected
2. The top flange was first glued together as they needed to dry. Parts where the webs and diaphragms located were marked on the top flange.
3. The webs were folded and glued to the top flange. Clips were added to the webs to secure the glue while it dried
4. The diaphragms were glued to the top flange
5. The webs were inserted into the top flange
6. Reinforcements such as matboard scraps were inserted along the sides of splices to reduce shear force
7. The bottom flange was glued onto the matboard. Books were added to pre-stress the matboard and secure empty spaces in glue



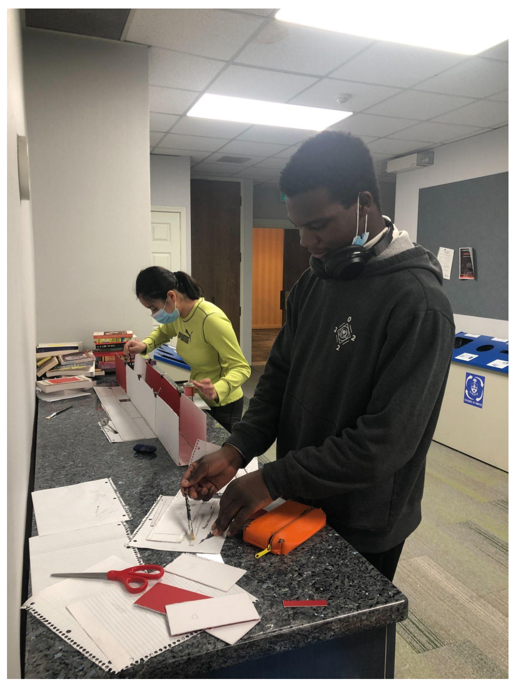


**Figure 13** (Left) stacking of books to pre-stress matboard and **Figure 14** (right) - gluing of matboard with clips

## 7. Time Log

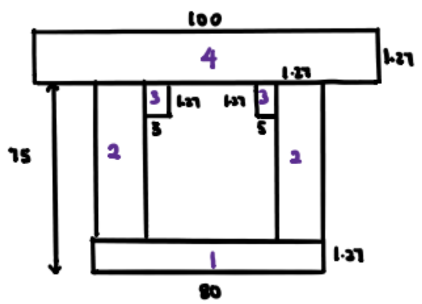
| Tasks | Date | Angela | Anthony | Nasrudeen | Total |
|---|---|---|---|---|---|
| **Coding** | 20-27 | Did a code for centroid and I (but not applied)<br><br>(1) | | Did code on optimisation and calculations (14) | 15 |
| **Drawing** | 23-27 Nov | Drew drawings for all of design 0 and final design calculations<br>Drew Engineering drawing splice section<br>(1) | Drew engineering drawings<br>(2) | | 3 |
| **Hand Calculations** | 25-27 Nov | Did design 0 hand calculations<br><br>Did hand calculations for iterative designs<br><br>Did final design hand calculations + checking<br><br>(10) | Checked hand calculations<br><br>(4) | | 12 |
| **Constructions** | 27 Nov | Drew matboard sketch<br><br>Cut and refined matboard<br><br>Glued matboard<br>(5) | Helped draw out matboard<br>(1)<br><br>Glued matboard (3) | Helped draw matboard sketch<br>(3)<br><br>Glued matboard<br>(5) | 14 |

| | | | | | |
|---|---|---|---|---|---|
| **Design Report** | | Typed and recorded design report (6) | Helped out typing design report (3) | Helped type design report (2) | 8 |
| **Total** | | 23 | 12 | 24 | 56 |

## Appendix A - Hand Calculations for Design 0

Design 0: $\bar{y} = 41.43\text{ mm}$  $M_y = 6.88\times10^4\text{ Nmm}$

$I = 418{,}358.38\text{ mm}^3$  $M_{shear} = 260\text{ N}$




All in mm

**CALC $\bar{y}$**

$$\bar{y} = \frac{\sum A_i d_i}{\sum A}$$

Piece 1 : $A = 80\times1.27$  $d_i = \frac{1.27}{2}$

2x Piece 2 : $A = (75-1.27)\times1.27$  $1.27+\frac{75}{2}$

2x Piece 3 : $A = 5\times1.27$  $= 75-\frac{1.27}{2}$

Piece 4 : $A = 100\times1.27$  $d_i = 75+\frac{1.27}{2}$

$$\bar{y} \quad \frac{\sum A_i d_i}{\sum A} = (80\times1.27)\left(\frac{1.27}{2}\right) + (2)\left((75-1.27)\times1.27\right)\left(1.27+\frac{75}{2}\right) + (2)(5\times1.27)\left(75-\left(\frac{1.27}{2}\right)\right) + \left(100\times1.27\times\left(75+\frac{1.27}{2}\right)\right)$$

$$\bar{y} = 41.43\text{ mm} = 41.4\text{ mm}$$

**CALC I**

$$\sum I = \sum \frac{bh^3}{12} + A_i d_i^2$$

$$= \frac{(80)(1.27)^3}{12} + (80)(1.27)\left(41.43-\frac{1.27}{2}\right)^2$$

$$+ (2)\left[\frac{(1.27)(75-1.27)^3}{12} + (1.27)(75)\left(41.43-\left(1.27+\frac{75}{2}\right)\right)^2\right]$$

$$+ (2)\left[\frac{(5)(1.27)^3}{12} + (5)(1.27)\left(75-\frac{1.27}{2}-41.43\right)^2\right]$$

$$+ \frac{(100)(1.27)^3}{12} + (100)(1.27)\left(75+\frac{1.27}{2}-41.43\right)^2$$

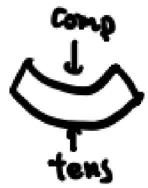
$$= 418{,}358.38\text{ mm}^4$$

$$= 0.418\times10^6\text{ mm}^4$$

## CALC For

### Flexural Tension / Compression

$\sigma = \frac{My}{I} \leftarrow M$ +ve




$$\sigma_{comp} = \frac{(6.83\times10^4)(y_{top})}{0.418\times10^6}$$

$$= \frac{(6.83\times10^4)(76.27-41.43)}{0.418\times10^6}$$

$= -5.734$ MPa // $\quad$ F.O.S $= \frac{6}{5.734} = 1.05$ //

$$\sigma_{tens} = \frac{(6.83\times10^4)(41.43)}{0.418\times10^6}$$

$= 6.82$ MPa // $\quad$ F.O.S $= \frac{30}{6.82} = 4.40$ //

### SHEAR (max at 257 N)

- At centroid:

Shear failure $= \frac{VQ}{Ib}$

$b = 1.27\times2 = 2.54$ mm

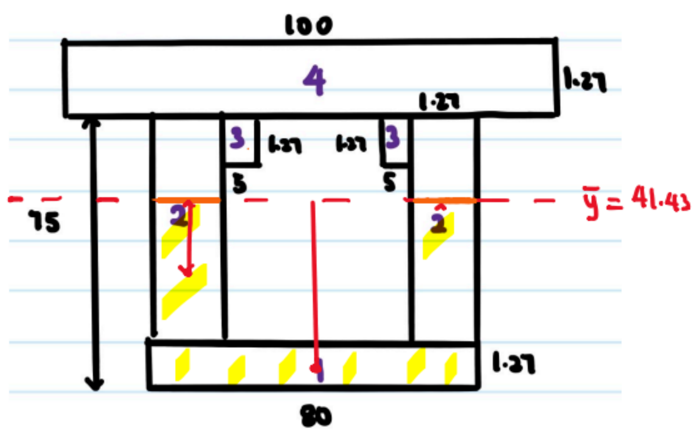
$Q = \Sigma A_i d_i$

$= \frac{(41.43-1.27)}{2}(41.43-1.27)(2)$

$+ (80)(1.27)(41.43-\frac{1.27}{2})$

$= 1,612.8 + 4,144$

$= 6193$ mm$^3$




$\frac{VQ}{IB} = \frac{(257)(6193)}{(0.418\times10^6)(2.54)} = 1.50$ MPa $\quad$ F.O.S $= \frac{4.00}{1.} = 2.67$ //

• Glue Sect. 1

$B = (1.27+5)(2) = 12.54$ mm

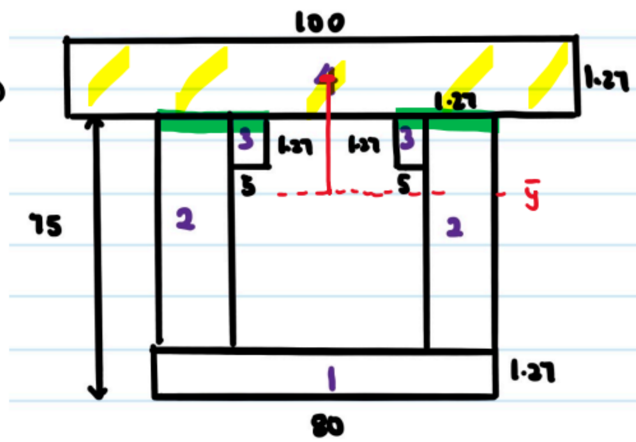
$Q = (100)(1.27)(75 + \frac{1.27}{2} - 41.43)$

$= 4344$

$\frac{VQ}{IB} = \frac{(257)(4344)}{(0.418\times10^6)(12.54)}$

$= 0.213$ MPa

$F.O.S = \frac{2}{0.213} = \underline{9.4}$ //





• GLUE SECT. 2

$b = 2\times1.27 = 2.54$ mm

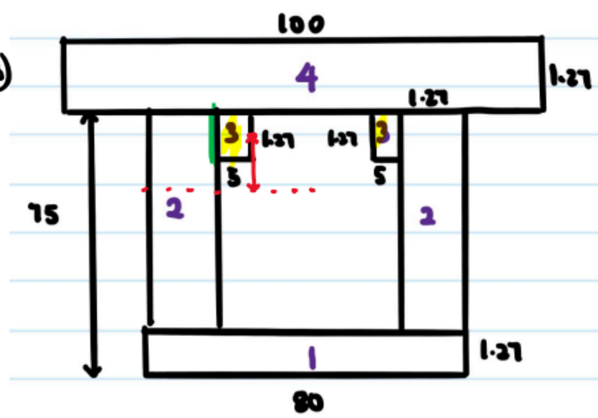
$Q = (2)(5)(1.27)(75 - \frac{1.27}{2} - 41.43)$

$= 418.2$ mm$^3$

$\frac{VQ}{Ib} = \frac{(257)(418.2)}{(0.418\times10^6)(2.54)}$

$= 0.101$ MPa

$F.O.S = \frac{2}{0.101} = \underline{19.8}$ //





• GLUE SECT. 3

$b = 2\times1.27 = 2.54$ mm

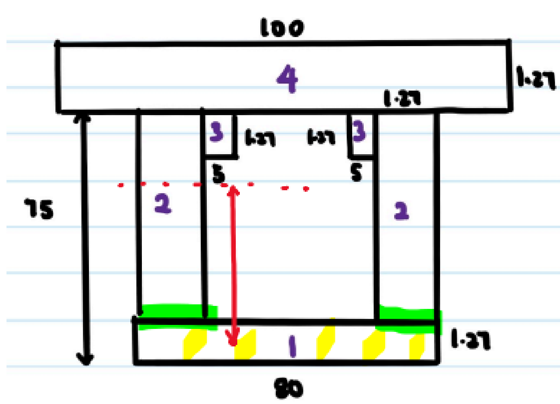
$Q = (80)(1.27)(41.43 - \frac{1.27}{2})$

$= 4144.8$ mm$^3$

$\frac{VQ}{Ib} = \frac{(257)(4144.8)}{(0.418\times10^6)(2.54)}$

$= 1.00$ MPa

$F.O.S = \frac{2}{1} = \underline{2}$ //



THIN PLATE BUCKLING CAPACITIES:

CASE 1 buckling

$k = 4$
$t = 1.27$ mm
$b = 80 - 1.27 = 78.73$ mm

$$\sigma_{buck} = \frac{k\pi^2 E}{12(1-\nu^2)}\left(\frac{t}{b}\right)^2$$

$$= \frac{(4)\pi^2(4000\text{ MPa})}{12(1-0.2^2)}\left(\frac{1.27}{78.73}\right)^2$$

$$= 13708 \times 2.60 \times 1$$
$$= 3.57 \text{ MPa} //$$

Also max moment at 600 mm

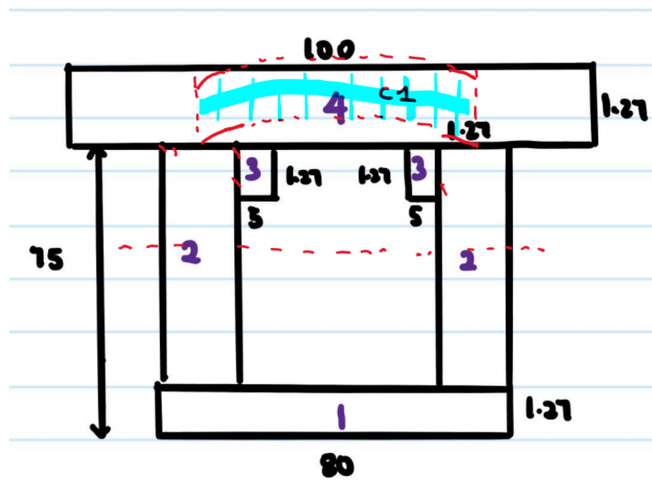
$$\frac{My}{I} = 3.57 \text{ MPa}$$

$$\frac{M\ (75+1.27-41.43)}{(0.418\times10^6)} = 3.57$$

$$M = 42{,}831$$

$$FOS = \frac{3.57}{5.73} = 0.623 //$$ FAIL

5.73




CASE 2 BUCKLING

$$\sigma_{buck} = \frac{(0.425)\pi^2(4000\text{ MPa})}{12\times(1-0.2^2)}\left(\frac{1.27}{10.635}\right)^2$$

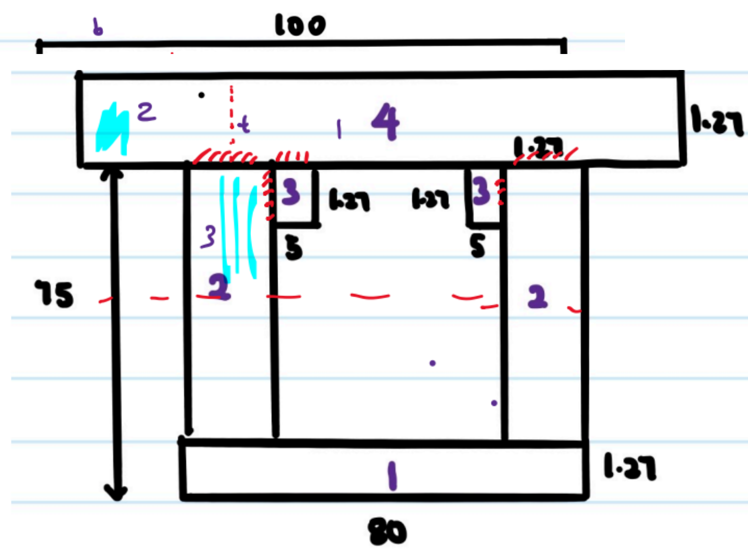
$$= 20.8 \text{ MPa}$$

(5.73)

$$\frac{(M)\ (75+1.27-41.43)}{0.418\times10^6} = 20.8$$

$$M = 0.250\times10^6$$

$$F.O.S = \frac{20.8}{5.73} = 3.63 //$$




CASE 3 BUCKLING

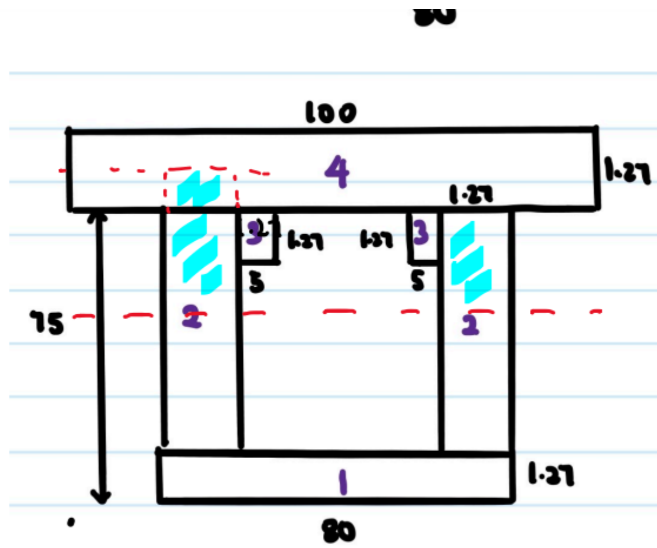
CASE 3 BUCKLING

Case 3:

$$\sigma_{buck} = \frac{(6)(\pi^2)(4000)}{12\times(1-0.2^2)}\left(\frac{1.27}{33.57+0.5(1.27)}\right)^2$$

$= 28.3$ MPa

$$FOS = \frac{28.3}{5.78} = 4.90$$




CASE 4 BUCKLING

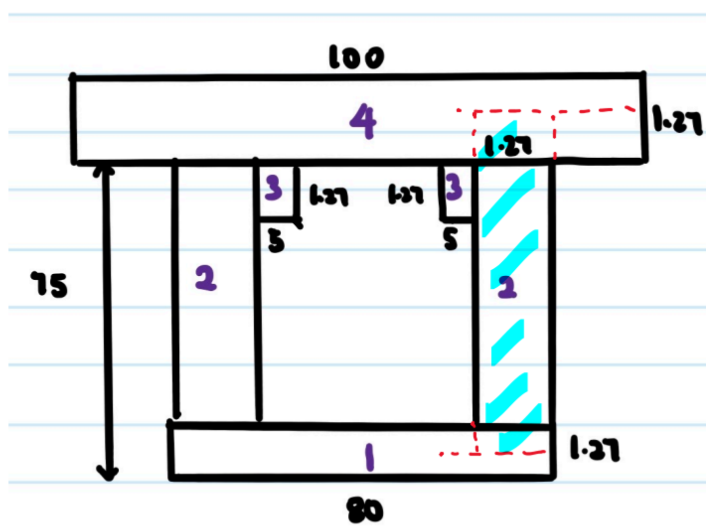
$a = 400$ mm
$b = 1.27$ mm
$h = 75$ mm

$$\tau_{cr} = \frac{5\pi^2 E}{12(1-\mu^2)}\left[\left(\frac{1.27}{75}\right)^2+\left(\frac{1.27}{400}\right)^2\right]$$

$$= \frac{5\pi^2 E}{12(1-\mu^2)}\left(2.968\right)\times10^{-4}$$

$= 5.09$ MPa

$$FOS = \frac{5.09}{1.50} = 3.39$$



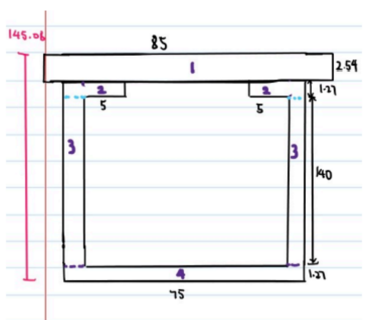

CONCLUSION DESIGN 0:

Smallest $FOS = 0.623 < 1 \rightarrow$ bridge fails by case 1 plate buckling.

## Appendix B: - Computer Calculations for Design 0

```
[[ 80.       1.27     1.     101.6      0.635 ]
 [  1.27    73.73     2.     187.2742  38.135 ]
 [  5.       1.27     2.      12.7     75.635 ]
 [100.       1.27     1.     127.      76.905 ]]
y_bar 41.84506934155159
Second Moment of Area 430599.961913091
Flexural Compression Failure : 1.0515051914784466 FOS , 5.706105921896427 MPa
Flexural Tension Failure : 4.48480615367884 FOS , 6.689252327080249 MPa
Material Shear Failure : 2.7105214479144575 FOS , 1.475730805626976 MPa
Glue Shear Failure Bottom : 2.0320415740351865 FOS , 0.9842318314523658 MPa
Glue Shear Failure Top : 9.433626736662974 FOS , 0.21200754024188523 MPa
Flexural Buckling Failure Case: 1 : 0.6251069951444227 FOS , 3.566926726812471 MPa
Flexural Buckling Failure Case: 2 : 3.6398946329734554 FOS , 20.769624320288855 MPa
Flexural Buckling Failure Case: 3 : 4.728289018905592 FOS , 26.980117971215044 MPa
Shear Buckling Failure Case 4 : 3.4463623924989917 FOS , 5.085903149965049 MPa
Minimum FOS: 0.6251069951444227
```

## Appendix C: Hand Calculations for Final Design




1) Calculation of $\bar{y}$

$$\bar{y} = \frac{\sum A_i d_i}{\sum A}$$

① $\sum A_i d_i = (85)(2.54) \times (1.27+140+1.27+1.27)$
$= 31048.6$

② $\sum A_i d_i = (2)(5)(1.27)(1.27+140+\frac{1.27}{2})$
$= 1802.2$

③ $\sum A_i d_i = (2)(140)(1.27)(1.27+70) = 25343.6$

④ $\sum A_i d_i = (75)(1.27)(\frac{1.27}{2}) = 60.48$

$$\sum A_i d_i = \frac{(31048.6 + 1802.2 + 25343.6 + 60.48)}{(75)(1.27)+(2)(140)(1.27)+(2)(1.27)(5)+(2.54)(85)}$$

$$= \frac{58,254}{679.45}$$

$$= 85.74 \text{ mm}$$

2) Calculation of $I$

$$I = \sum \frac{bh^3}{12} + A_i d_i^2$$

$$= \frac{(85)(2.54)^3}{12} + (85)(2.54)(140 + 1.27\times2 + 1.27 - 85.74)^2$$

$$+ (2)\left[\frac{(5)(1.27)^3}{12} + (5\times1.27)\left(1.27 + 140 + \frac{1.27}{2} - 85.74\right)^2\right]$$

$$+ (2)\left[\frac{(1.27)(140)^3}{12} + (1.27)(140)\left(85.74 - 1.27 - \frac{140}{2}\right)^2\right]$$

$$+ \left(\frac{(75)(1.27)^3}{12}\right) + (75)(1.27)\left(85.74 - \frac{1.27}{2}\right)^2$$

$$= 116.07 + (85)(2.54)(58.07)^2 \quad ]\ 728158$$

$$+ 2\left(0.853 + 6.35\times(56.165)^2\right) \quad ]\ 8696.2$$

$$+ (2)[(290406.7) + (37227.9)] \quad ]\ 655296.2$$

$$+ (12.80 + (689882.5)) \quad ]\ 689895.3$$

$$= 2097539$$
$$= 2.08\times10^6\ \text{mm}^4$$

Moment – Flexural stress

$$y_{top} = 145.06 - 85.74 = 59.32$$
$$y_{bot} = 85.74$$

$$\sigma_{comp} = \frac{(68800)(59.32)}{2.08\times10^6}$$

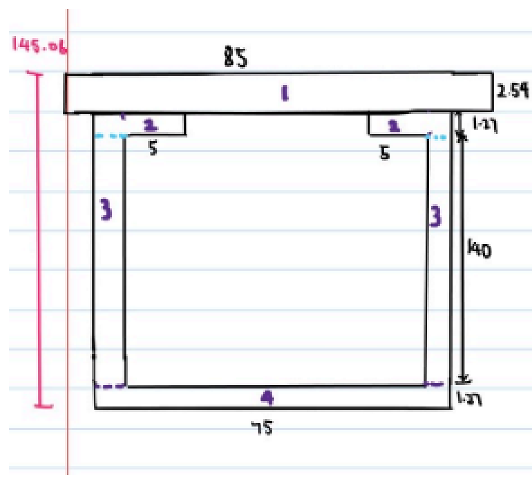

$$= 1.962$$

$$F.O.S = \frac{6}{1.962} = 3.057 = \boxed{3.06}$$

$$\sigma_{tens} = \frac{(68800)(85.74)}{2.08\times10^6}$$

$$= 2.84$$

$$FOS = \frac{30}{2.84} = 10.6$$



## SHEAR

$\frac{VQ}{Ib}$ (at centroid, material shear)

$Q = \Sigma A_i d_i$

$= (2)(140-85.74)\left(\frac{85.74-1.27}{2}\right)$

$+ (75)(1.27)\left(85.74-\frac{1.27}{2}\right)$

$= (2)(4583.34)$

$+ 8106.25$

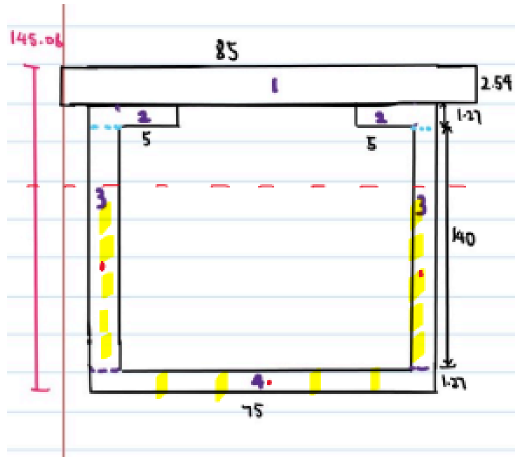
$= 17272.9 \text{ mm}^3$

$b = (2)(1.27)$

$\frac{VQ}{Ib} = \frac{(257)(17272.9)}{(2.08\times10^6)(2.54)}$

$= 0.8402$

$FOS = \frac{4}{0.8402} = 4.76$




## Glue shear 1

$Q = (85)(2.54)(140+1.27+1.27-85.74)$

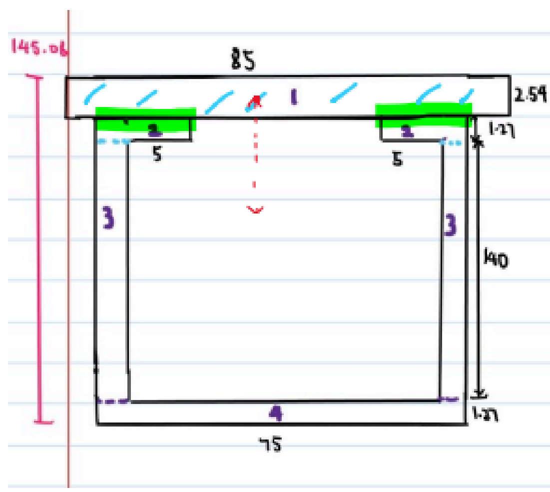
$= (85)(2.54)(56.8)$

$= 12263$

$\frac{VQ}{Ib} = \frac{(257)(12263)}{(2.08\times10^6)(10)}$

$= 0.152$

$FOS = \frac{4}{0.152} = 26.4$



## GLUE SHEAR 2

$$Q = (75)(1.27)\left(85.74 - \frac{1.27}{2}\right)$$

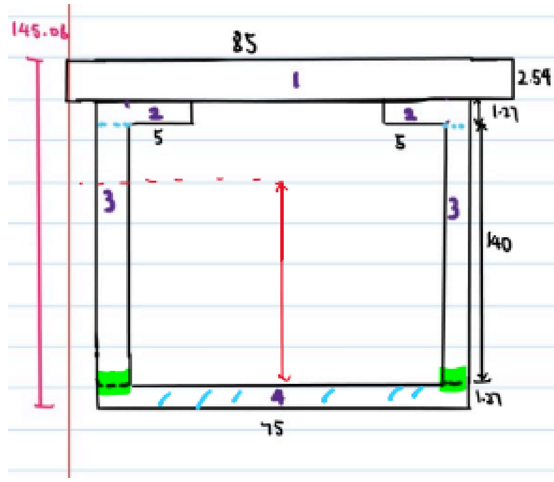
$$= 8106 \text{ mm}^3$$

$$b = 1.27 \times 2 = 2.54$$

$$\frac{VQ}{Ib} = \frac{(257)(8106)}{(2.08\times10^6)(2.54)}$$

$$= 0.394$$

$$FOS = \frac{4}{0.394} = 10.14$$
$$\approx \text{10.1}$$




## PLATE BUCKLING

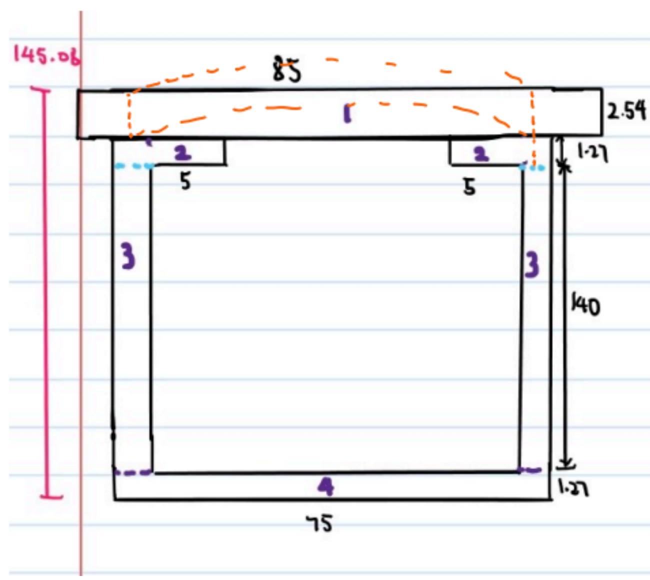
Case 1:

$k = 4$
$t = 2.54$
$b = 75 + \frac{1.27}{2} + \frac{1.27}{2}$
$= 76.27$

$$\tau_{buck} = \frac{4\pi^2(4000)}{12(1-0.2)^2}\left(\frac{2.54}{76.27}\right)^2$$

$$= 15.2$$

$$FOS = \frac{\tau_{buck}}{\tau_{calc}} = \frac{15.2}{1.962} = 7.74$$



## Case 2:

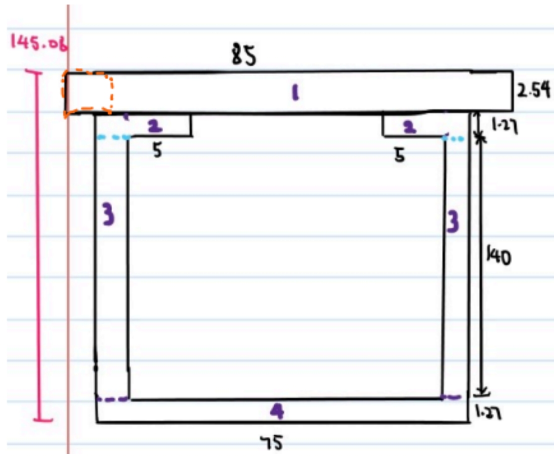
$k = 0.425$

$t = 2.54$
$b = 5 + 1.27$

$$\left(\frac{(0.425)\pi^2(4000)}{12 \times 0.96}\right)\left(\frac{2.54}{6.27}\right)^2$$

$= 239.0$

$\text{FOS} = \frac{239}{1.962} = 122 \quad \left(\frac{\text{capacity}}{\sigma_{comp}}\right)$



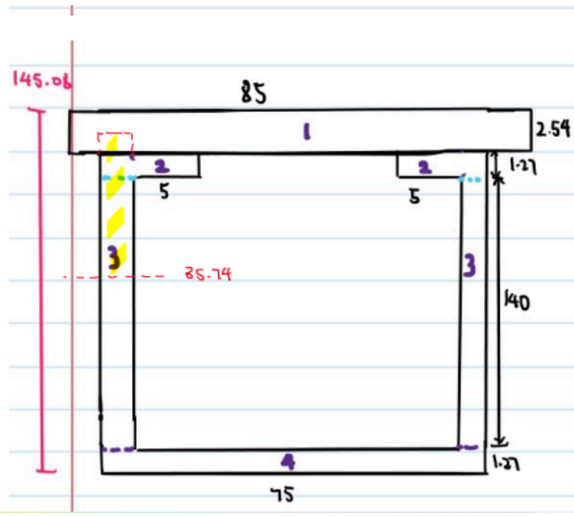

## CASE 3

$t = 1.27$
$b = 58.05$

$$\sigma_{buck} = \frac{(6\pi^2)(4000)}{12(1-0.2^2)}\left(\frac{1.27}{58.05}\right)^2$$

$= 9.84$

$\text{FOS} = \frac{9.84}{1.962} = 4.91 \;// \quad \left(\frac{\text{capacity}}{(\sigma_{comp})}\right)$




## SHEAR

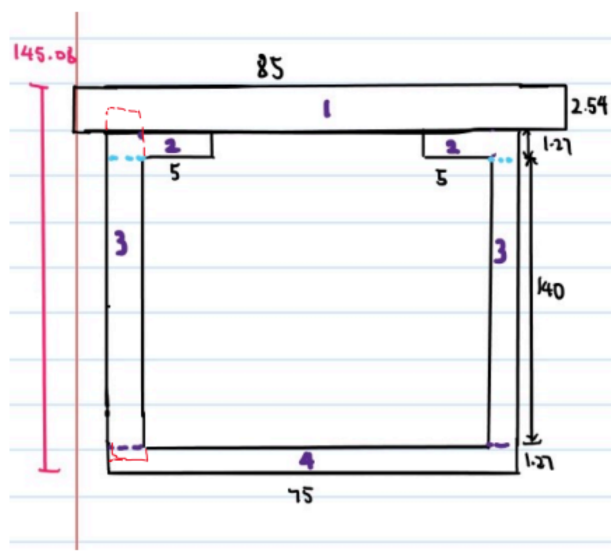
Diaphragm spacing =

height = 141.905
width = 1.27

$$\frac{5\pi^2(4000)}{12(1-0.2^2)}\left[\left(\left(\frac{1.27}{150}\right)^2 + \left(\frac{1.27}{141.905}\right)^2\right)\right]$$

$(1.518 \times 10^{-4})$ $\quad (0.8402)$

$= 2.60 \div 0.8402$
$= 3.09 \;//$




Minimum FOS = 3.09 //

## Appendix D: Computer Calculations for Final Design

```
[[ 75.      1.27    1.     95.25    0.635]
 [  1.27  140.      2.    355.6    71.27 ]
 [  5.      1.27    2.     12.7   141.905]
 [ 85.      2.54    1.    215.9   143.81 ]]
y_bar 85.7382710280374
Second Moment of Area 2113386.2840572246
Flexural Compression Failure : 3.1042927140344045 FOS , 1.9328074227260204 MPa
Flexural Tension Failure : 10.742816170483996 FOS , 2.792563842098064 MPa
Material Shear Failure : 4.864745517590416 FOS , 0.8222423938798883 MPa
Glue Shear Failure Bottom : 5.151376618453774 FOS , 0.3882457347100969 MPa
Glue Shear Failure Top : 16.44299636401484 FOS , 0.12163233243649918 MPa
Flexural Buckling Failure Case: 1 : 8.417005861010441 FOS , 16.268451405289397 MPa
Flexural Buckling Failure Case: 2 : 153.1042320852938 FOS , 295.92099607685236 MPa
Flexural Buckling Failure Case: 3 : 5.0880087807592735 FOS , 9.834141138346693 MPa
Shear Buckling Failure Case 4 : 3.1629611684239842 FOS , 2.600720762874065 MPa
Minimum FOS: 3.1042927140344045
```

## Appendix E: Iterations

```
[[ 75.      1.27    1.     95.25    0.635]
 [  2.54  120.      2.    609.6    61.27 ]
 [  5.      1.27    2.     12.7   121.905]
 [ 80.      1.27    1.    101.6   123.175]]
y_bar 62.83763565891473
Second Moment of Area 1515778.9063258623
Flexural Compression Failure : 2.1669395678710988 FOS , 2.7688820163520647 MPa
Flexural Tension Failure : 10.513080851300796 FOS , 2.853587870608649 MPa
Material Shear Failure : 7.7026699214281 FOS , 0.5193004556604948 MPa
Glue Shear Failure Bottom : 10.109920454540804 FOS , 0.19782549318691356 MPa
Glue Shear Failure Top : 29.005415820782886 FOS , 0.06895264016752917 MPa
Flexural Buckling Failure Case: 1 : 1.520805172164296 FOS , 4.2109300915809245 MPa
Flexural Buckling Failure Case: 2 : 59.692047867224076 FOS , 165.28023785878335 MPa
Flexural Buckling Failure Case: 3 : 13.159785784614071 FOS , 36.43789419806345 MPa
Shear Buckling Failure Case 4 : 15.805494957247138 FOS , 8.207800733238091 MPa
Minimum FOS: 1.520805172164296
```

## Appendix F: Code

```python
import numpy as np
import matplotlib.pyplot as plt

'''Checks to see if the entire train is on the bridge'''
def check_load(x, span, weight):
    P = weight/6
    pos = np.array([x+52,x+228,x+392,x+568,x+732,x+908])
    pos = pos[pos >= 0][pos <= span]
    return [P, pos]
```

```
def reaction(P, span, points):
    # rx1 is a pin, rx2 is a roller
    # gets the sum of the moments of the point loads then divides by the
lever-arm of rx2
    rx2 = (np.sum(points*P))/span
    # Uses vertical force equilibrium (F_y = 0) to find rx1
    rx1 = P*np.prod(points.shape) - rx2
    return [rx1, rx2]


'''Sets up the bridge, the values can be replaced with inputs'''
def setup():
    # length of bridge
    span = 1200

    # A train is not a point load, it's distributed over a distance
    distrib = 960

    # Was the position of the front of the train. But I added 960 to make
it the end of train
    x = int(input("what is your point of interest? "))

    if -960 >= x or x >= span:
        x = input('position must be within -960 and '+str(span))

    weight = 400


    # Applied load from train
    load = check_load(x, span, weight)
    rxs = reaction(load[0], span, load[1])
    return {'span':span,'load': load[0],'poi': x,'points': load[1],
'distrib': distrib, 'weight': weight, 'rx1': rxs[0], 'rx2': rxs[1]}

def sfd(points, rx1, rx2):
    V_val = np.zeros([points.shape[0]+3])
    V_val[2] = rx1
    V_val[2:-1] = np.cumsum(V_val[2:-1]-P)
    V_val[0:2] = [0,rx1]
    V_val[-1] = V_val[-2]+ rx2
```

```
    return V_val

def bmd(points, P, rx1, rx2, span):
    forces = np.append(rx1, -1*np.append(np.full(points.shape,P), rx2))
    lever_arm = np.append(0, np.append(points, span))
    lever_arm_matrix = np.array([lever_arm]*forces.size)

    for i in range(len(lever_arm_matrix)):
        lever_arm_matrix[i] = lever_arm_matrix[i] - lever_arm[i]
        lever_arm_matrix[i][lever_arm_matrix[i] < 0] = 0
    M_val = np.matmul(forces,lever_arm_matrix)
    return lever_arm,M_val

def plot(x,y,x_label,y_label,title,step,invert):
    if step == True:
        plt.step(x,y, where='post')
    else:
        plt.plot(x,y)
    if invert == True:
        plt.gca().invert_yaxis()
    plt.title(title)
    plt.xlabel(x_label)
    plt.ylabel(y_label)
    plt.show()

def envelope(distrib, span, weight, poi):
    train_pos = np.linspace(-52,span-960+52,500)
    bridge_pos= np.linspace(0,span,500)
    M = np.zeros(bridge_pos.shape)
    V = np.zeros(bridge_pos.shape)
    for x in train_pos:
        # Note that the load may not be the complete weight of the train
if x < 0 or x > span
        load, points = check_load(x, span, weight)
        rx1, rx2 = reaction(load, span, points)
        V_val = sfd(points, rx1, rx2)
        lever_arm, M_val = bmd(points, load, rx1, rx2, span)
        # plot(lever_arm, M_val, 'Lever Arm', 'Moment', 'Moment Diagram',
False, True)
        # Actually for each x value
```

```
        for i in range(len(bridge_pos)):

            new_V = V_val[np.append(0,lever_arm) <= bridge_pos[i]][-1]
            new_M = np.interp(bridge_pos[i], lever_arm, M_val)

            if abs(M[i]) < abs(new_M):
                M[i] = new_M
            if abs(V[i]) < abs(new_V):
                V[i] = abs(new_V)
        # plot(np.append(0,lever_arm),V_val,'Distance','Shear
Force','Shear Force Diagram',True, False)
    plot(bridge_pos, M, 'Lever Arm', 'Moment', 'Moment Envelope', False,
True)

    print("Max Bending Moment",np.interp(600,bridge_pos,M,))
    plot(bridge_pos, V, 'Lever Arm', 'Shear Force', 'Shear Force
Envelope', False, False)
    print("Max Shear Force",np.interp(0,bridge_pos,V,))

    return
def optimize_dim(span):
    girder_b_b = np.arange(75,150,5)
    girder_h_b = 1.27*np.arange(1,5,1)

    girder_b_w = 1.27*np.arange(1,3,1)
    girder_h_w = np.arange(50,200,5)

    girder_b_p = np.arange(5,36,5)
    girder_h_p = 1.27*np.arange(1,3,1)

    girder_b_t = np.arange(85,150,5)
    girder_h_t = 1.27*np.arange(1,3,1)
    # girder_dim = [[b_b, h_b,1],[b_w, h_w,2],[b_p, h_p,2],[b_t, h_t,1]]

    # pi_b_w = 1.27
    # pi_h_w = np.arange(0,200,5)

    # pi_b_p = np.arange(0,36,5)
    # pi_h_p = 1.27
```

```
    # pi_b_t = np.arange(0,200,5)
    # pi_h_t = 1.27
    # pi_dim = [[pi_b_w, pi_h_w,2],[pi_b_p, pi_h_p,2],[pi_b_t, pi_h_t,1]]

    max_FOS = 0
    max_area = 813*1016
    opt_dim = []
    opt_diaphragm_area = 0
    opt_diaphragm_spacing = 0
    opt_total_area = 0
    opt_num_diaphragms = 0
    for bottom_flange_width in girder_b_b:
        for bottom_flange_height in girder_h_b:
            for web_width in girder_b_w:
                for web_height in girder_h_w:
                    for plate_width in girder_b_p:
                        for plate_height in girder_h_p:
                            for top_flange_width in girder_b_t:
                                for top_flange_height in girder_h_t:
                                    dim =
dim_analysis([[bottom_flange_width, bottom_flange_height,1],[web_width,
web_height,2],[plate_width, plate_height,2],[top_flange_width,
top_flange_height,1]])
                                    y_bar = centroidal_axis(dim)
                                    E = 4000
                                    mu = 0.2
                                    M = 68834.67616595917
                                    I = second_moment_area(dim, y_bar)
                                    V = 257.10354041416167
                                    total_area = np.sum(dim[:,0][(dim[:,0]
!= 1.27*np.arange(1,5,1))]*(dim[:,1][(dim[:,0] !=
1.27*np.arange(1,5,1))]/1.27)*dim[:,2][(dim[:,0] !=
1.27*np.arange(1,5,1))]) + np.sum(dim[:,1][(dim[:,0] ==
1.27*np.arange(1,5,1))]*dim[:,2][(dim[:,0] == 1.27*np.arange(1,5,1))])
                                    total_area = total_area * 1260
                                    test_diaphragm_area =
(dim[0,0]-dim[1,0]*dim[1,2])*(dim[1,1]+5)
                                    num_diaphragms =
((max_area-total_area)//test_diaphragm_area)
```

```
                                    if  total_area + test_diaphragm_area*8
> max_area :

                                        break
                                    # if num_diaphragms < 4:
                                    #     break
                                    test_diaphragm_spacing = span/7
                                    test_FOS = FOS_check(M, V, dim, I, E,
mu, y_bar, test_diaphragm_spacing)[1]
                                    if test_FOS > max_FOS:
                                        max_FOS = test_FOS
                                        opt_dim = dim
                                        opt_diaphragm_area =
test_diaphragm_area

                                        opt_diaphragm_spacing =
test_diaphragm_spacing

                                        opt_total_area = total_area
                                        opt_num_diaphragms =
num_diaphragms
    return max_FOS, opt_dim, opt_diaphragm_area, opt_diaphragm_spacing,
opt_total_area, opt_num_diaphragms

def pi_beam_dim():
    # b_w = float(input("What is the width of the web? "))
    # h_w = float(input("What is the height of the web? "))

    # b_p = float(input("What is the width of the backing plate? "))
    # h_p = float(input("What is the height of the backing plate? "))

    # b_t = float(input("What is the width of the top flange? "))
    # h_t = float(input("What is the height of the top flange? "))

    # b_w = 1.27
    # h_w = 73.73

    # b_p = 5
    # h_p = 1.27

    # b_t = 100
    # h_t = 1.27
```

```python
    dim = [[b_w, h_w,2],[b_p, h_p,2],[b_t, h_t,1]]
    return dim

def box_girder_dim():
    # b_b = float(input("What is the width of the bottom flange? "))
    # h_b = float(input("What is the height of the bottom flange? "))

    # b_w = float(input("What is the width of the web? "))
    # h_w = float(input("What is the height of the web? "))

    # b_p = float(input("What is the width of the backing plate? "))
    # h_p = float(input("What is the height of the backing plate? "))

    # b_t = float(input("What is the width of the top flange? "))
    # h_t = float(input("What is the height of the top flange? "))

    b_b = 80
    h_b = 1.27

    b_w = 1.27
    h_w = 73.73

    b_p = 5
    h_p = 1.27

    b_t = 100
    h_t = 1.27

    dim = [[b_b, h_b,1],[b_w, h_w,2],[b_p, h_p,2],[b_t, h_t,1]]
    return dim

def dim_analysis(dim):
    dim_w_height = []
    # You could do this better using numpy
    for i in range(len(dim)):
        d_area = dim[i][0]*dim[i][1]*dim[i][2]
        d_height = 0
        # Wrong because some of the areas are at the same height
        for j in range(i):
            d_height += dim[j][1]
```

```
        d_height += dim[i][1]/2
        dim_w_height.append([dim[i][0],dim[i][1], dim[i][2], d_area,
d_height])
    return np.array(dim_w_height)

def centroidal_axis(dim):
    sum = np.sum(dim[:,3]*dim[:,4])
    y_bar = (sum)/(np.sum(dim[:,3]))
    return y_bar

def second_moment_area(dim,y_bar):
    I = np.sum(dim[:,2]*(dim[:,0]*dim[:,1]**3)/12 +
dim[:,3]*(dim[:,4]-y_bar)**2)
    return I

def flexural_failure(M, dim, I, y_bar, type):
    if type == "compression":
        max_comp_stress = 6
        y = dim[-1,4] + dim[-1,1]/2 - y_bar
        stress = M*y/I
        factor_of_safety = FOS(max_comp_stress, stress)
    if type == "tension":
        max_ten_stress = 30
        y = y_bar
        stress = M*y/I
        factor_of_safety = FOS(max_ten_stress, stress)
    return factor_of_safety, stress

def glue_shear_failure(V, I, dim, y_bar):
    # I don't think we will be gluing the bottom flage
    max_shear_stress = 2
    Q_1 = np.sum(dim[0,3]*dim[0,2]*(y_bar-dim[0,4]))
    # Check for optimized design!
    b_1 = dim[1,0]*dim[1,2]
    stress_1 = (V*Q_1)/(I*b_1)

    Q_2 = np.sum(dim[3,3]*dim[3,2]*(y_bar-dim[3,4]))
    b_2 = dim[1,0]*dim[1,2] + dim[2,0]*dim[2,2]
    stress_2 = (V*Q_2)/(I*b_2)
```

```
    return
[FOS(max_shear_stress,stress_1),FOS(max_shear_stress,stress_2)],
[stress_1,stress_2]
    # return FOS(max_shear_stress,stress_2), stress_2


def material_shear_failure(V, I, dim, y_bar):
    max_shear_stress = 4
    Q_1 = dim[1,0]*(y_bar - dim[0,1])*dim[1,2]*((y_bar -
dim[0,1])/2)+dim[0,3]*(y_bar - dim[0,4])
    b_1 = dim[1,0]*dim[1,2]
    shear_stress = (V*Q_1)/(I*b_1)
    return FOS(max_shear_stress,shear_stress),shear_stress

def flexural_buckling_failure(E,mu,applied_comp_stress,dim,y_bar):
    # Case 1
    k_1 = 4
    t_1 = dim[3,1]
    b_1 = dim[0,0] - dim[1,0]
    case_1_stress = ((k_1*np.pi**2*E)/(12*(1-mu**2)))*((t_1/b_1)**2)
    case_1_FOS = FOS(case_1_stress,applied_comp_stress)

    # Case 2
    k_2 = 0.425
    t_2 = dim[3,1]
    b_2 = (dim[3,0] - dim[0,0] + dim[1,0])/2
    case_2_stress = ((k_2*np.pi**2*E)/(12*(1-mu**2)))*((t_2/b_2)**2)
    case_2_FOS = FOS(case_2_stress,applied_comp_stress)

    # Case 3
    k_3 = 6
    t_3 = dim[1,0]
    b_3 = dim[1,1] + dim[3,1]/2 + dim[0,1] + dim[2,1] - y_bar
    case_3_stress = ((k_3*np.pi**2*E)/(12*(1-mu**2)))*((t_3/b_3)**2)
    case_3_FOS = FOS(case_3_stress,applied_comp_stress)
    return
[case_1_FOS,case_2_FOS,case_3_FOS],[case_1_stress,case_2_stress,case_3_str
ess]

def shear_buckling_failure(E,mu,applied_shear_stress,dim,a):
```

```
    # Case 4
    k = 5
    t = dim[1,0]
    b = dim[1,1] + dim[3,1]/2 + dim[0,1]/2
    case_4_stress = ((k*np.pi**2*E)/(12*(1-mu**2)))*((t/b)**2+(t/a)**2)
    case_4_FOS = FOS(case_4_stress,applied_shear_stress)
    return case_4_FOS,case_4_stress

def FOS(max_stress, applied):
    applied = max(abs(applied), 0.000001)
    return max_stress/applied

def FOS_check(M, V, dim, I, E, mu, y_bar, a,i):
    all_FOS = np.array([])
    comp = flexural_failure(M, dim, I, y_bar, "compression")
    all_FOS = np.append(all_FOS,comp[i])
    all_FOS = np.append(all_FOS,flexural_failure(M, dim, I, y_bar,
"tension")[i])
    shear = material_shear_failure(V, I, dim, y_bar)
    all_FOS = np.append(all_FOS,shear[i])
    all_FOS = np.append(all_FOS,glue_shear_failure(V, I, dim, y_bar)[i])
    all_FOS =
np.append(all_FOS,flexural_buckling_failure(E,mu,comp[int(not
i)],dim,y_bar)[i])
    all_FOS = np.append(all_FOS,shear_buckling_failure(E,mu,shear[int(not
i)],dim,a)[i])
    all_FOS = np.absolute(all_FOS)
    min_FOS = np.min(all_FOS)
    return all_FOS, min_FOS

'''
Create a calculation for y bar
Create a variable for each of the different lengths of the cross-sections
to calculate I
Create a bending moment envelop
Restrict the amount of volume of matboard

'''
if __name__ == "__main__":
    bridge = setup()
```

```
    span = bridge['span']
    P = bridge['load']
    points = bridge['points']
    distrib = bridge['distrib']
    weight = bridge['weight']
    rx1 = bridge['rx1']
    rx2 = bridge['rx2']
    poi = bridge['poi']

    # V_val = sfd(points, rx1, rx2)

    # lever_arm,M_val = bmd(points, P, rx1, rx2, span)
    # Max calcs should return the FOS and applied stress
    dim = dim_analysis([[ 75.    ,    1.27 ,    1.],
       [ 1.27  , 140   ,    2.],
       [  5.    ,    1.27 ,    2.],
       [ 85.    ,    2.54 ,    1.]])
    print(dim)
    y_bar = centroidal_axis(dim)
    E = 4000
    mu = 0.2
    M = 68834.67616595917
    I = second_moment_area(dim, y_bar)
    V = 257.10354041416167
    # CHANGE DEPENDING ON ITERATION
    diaphragm_spacing = 150
    checks = ["Flexural Compression Failure", "Flexural Tension Failure",
"Material Shear Failure", "Glue Shear Failure Bottom", "Glue Shear Failure
Top", "Flexural Buckling Failure Case: 1", "Flexural Buckling Failure
Case: 2", "Flexural Buckling Failure Case: 3", "Shear Buckling Failure
Case 4"]
    factors = FOS_check(M, V, dim, I, E, mu, y_bar, diaphragm_spacing,0)
    flex = FOS_check(M, V, dim, I, E, mu, y_bar, diaphragm_spacing,1)
    print("y_bar",y_bar)
    print("Second Moment of Area",I)
    for i in range(len(factors[0])):
        print(checks[i],":",factors[0][i],"FOS",",",flex[0][i], "MPa")
    print("Minimum FOS:",factors[1])
```

```
    # print(optimize_dim(span))
    # plot(np.append(0,lever_arm),V_val,'Distance','Shear Force','Shear Force Diagram',True, False)
    # plot(lever_arm,M_val,'Distance','Bending Moment','Bending Moment Diagram',False, True)
    # envelope(distrib, span, weight,poi)
```