Problems selected (or modified) from the textbook by A. V. Oppenheim, A. S. Willsky, with S. H. Nawab, *Signals and Systems*, Second Edition, Prentice-Hall, 1996, are labelled with "OWN."

1. (OWN1.1) Express each of the following complex numbers in Cartesian form $x + y\,\mathtt{j}$: $\frac{1}{2}e^{\mathtt{j}\,\pi}$, $\frac{1}{2}e^{-\mathtt{j}\,\pi}$, $e^{\mathtt{j}\,\pi/2}$, $e^{-\mathtt{j}\,\pi/2}$, $e^{\mathtt{j}\,5\pi/2}$, $\sqrt{2}e^{\mathtt{j}\,\pi/4}$, $\sqrt{2}e^{\mathtt{j}\,9\pi/4}$.

2. (OWN1.2) Express each of the following complex numbers in polar form ($re^{\mathtt{j}\,\theta}$, with $r \geq 0$ and with $-\pi < \theta \leq \pi$: 5, -2, $-3\,\mathtt{j}$, $1 + \mathtt{j}$, $(1 - \mathtt{j})^2$, $\mathtt{j}(1 - \mathtt{j})$, $(1 + \mathtt{j})/(1 - \mathtt{j})$.

3. Classify each of the following functions as being injective, surjective, bijective (i.e., both injective and surjective) or neither injective nor surjective. In each case, determine the range of the function. If the function is bijective, determine its inverse.

   (a) $f : \mathbb{R} \to \mathbb{R}$ where $f(x) = x^2$.

   (b) $g : \mathbb{R} \to [0, \infty)$, where $g(x) = x^2$.

   (c) The complex exponential function $\exp : \mathbb{C} \to \mathbb{C}$.

   (d) The function $L : \mathbb{R}^2 \to \mathbb{R}^2$ where $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \overset{L}{\mapsto} \begin{bmatrix} x_1 + x_2 \\ x_1 - x_2 \end{bmatrix}$.

   (The notation $\mapsto$ means "maps to"; in particular, $\overset{L}{\mapsto}$ means "maps, according to the function L, to".)

4. Let A and B be nonempty finite sets containing $|A|$ and $|B|$ elements, respectively. How many elements does the set B^A contain? (In other words, how many distinct functions are there with domain A and codomain B?)

5. (OWN1.4) Let $x[n]$ be a signal with support $\{-2, -1, \ldots, 4\}$. Find the support of the following signals: (a) $x[n - 3]$, (b) $x[n + 4]$, (c) $x[-n]$, (d) $x[-n + 2]$, (e) $x[-n - 2]$.

6. (OWN1.5) Let $x(t)$ be a signal with support $[3, \infty)$. For each signal given below, determine the values of t for which it is guaranteed to be zero. (a) $x(1 - t)$, (b) $x(1 - t) + x(2 - t)$, (c) $x(1 - t)x(2 - t)$, (d) $x(3t)$, (e) $x(t/3)$.

7. (OWN1.54) The relations considered in this problem are used on many occasions in the study of signals and systems.

   (a) Prove the validity of the following expression:

   $$\sum_{n=0}^{N-1} \alpha^n = \begin{cases} N & \text{if } \alpha = 1, \\ \frac{1-\alpha^N}{1-\alpha}, & \text{for any complex number } \alpha \neq 1. \end{cases}$$

   (*Hint:* Let S be the desired sum. Consider subtracting αS from S.)

   (b) Show that if $|\alpha| < 1$, then $\sum_{n=0}^{\infty} \alpha^n = \frac{1}{1-\alpha}$.

   (c) Evaluate, for any integer k, $\sum_{n=k}^{\infty} \alpha^n$, assuming $|\alpha| < 1$.