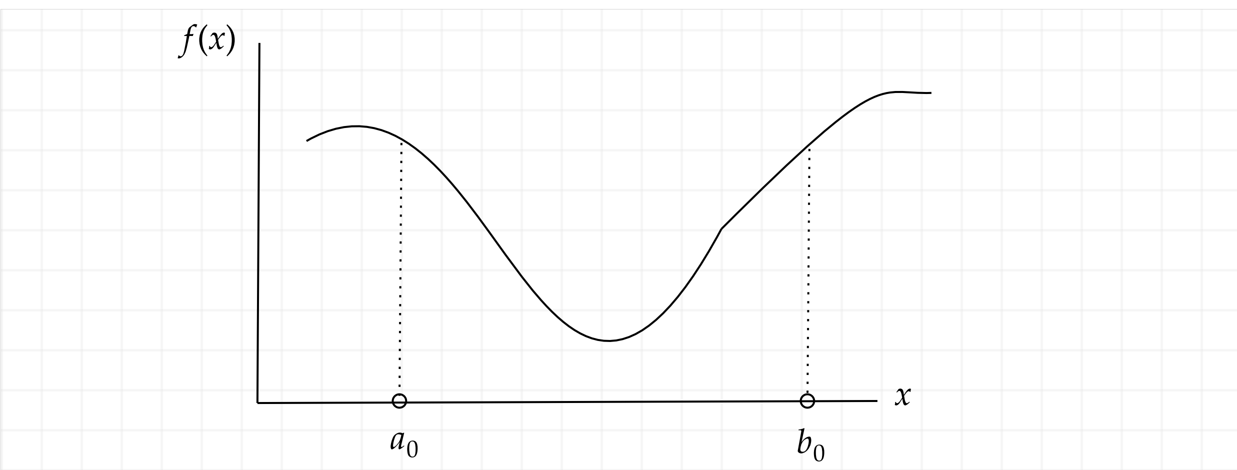
ONE_DIMENSIONAL SEARCH METHODS

❑ Introduction

In this chapter, we are interested in the problem of minimizing an objective function $f: R \rightarrow R(i.e.\ One - dimensional\ problem)$. The approach is to use an iterative search algorithm, also called a line-search method. One-dimensional search methods are of interest for the following reasons. First, they are special case of search methods used in multivariate algorithms.

In an iterative algorithm,we start with an initial candidate solution $x^{(0)}$ and generate a sequence of $iterates\ x^{(1)}, x^{(2)}, \ldots$ For each iteration $k = 0, 1, 2, \ldots,$ the next point $x^{(k+1)}\ depends\ on\ x^{(k)}$ and the objective function f. The algorithm may use the value of f at specific points, or perhaps its first derivative f', or even its second derivative f''.
The algorithms studies in this chapter :

1. Golden Section Method (uses only f)
2. Fibonacci Method(uses only f)
3. Bisection Method (uses only f')
4. Secant Method (uses only f')
5. Newton Method (uses f' and f'')



Unimodal Function

❑ Golden Section Search

The search methods we discuss in this and the next two section allow us to determine the minimizer of an objective function $f : R \to R$ over a closed interval, say $[a_0, b_0]$. The only property we assume of the objective function f is that it is $Unimodal$, which means f has only one local minimizer.

The methods we discuss are based on evaluating the objective function at different points in the interval $[a_0, b_0]$.

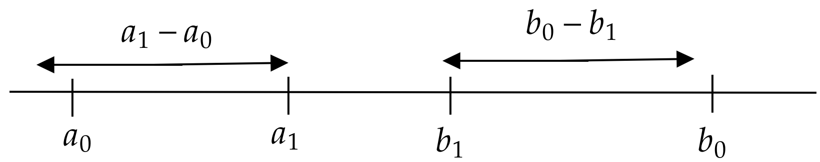
Consider a *unimodal* function f of one variable and the interval $[a_0, b_0]$. If we evaluate f at only one intermediate point of the interval, we cannot narrow the range within which we know the minimizer is located. *We have to evaluate f at two intermediate points. We choose the intermediate points in such a way that the reduction in the range is symmetric, in the sense that :*

$$a_1 - a_0 = b_0 - b_1 = \rho(b_0 - a_0),$$

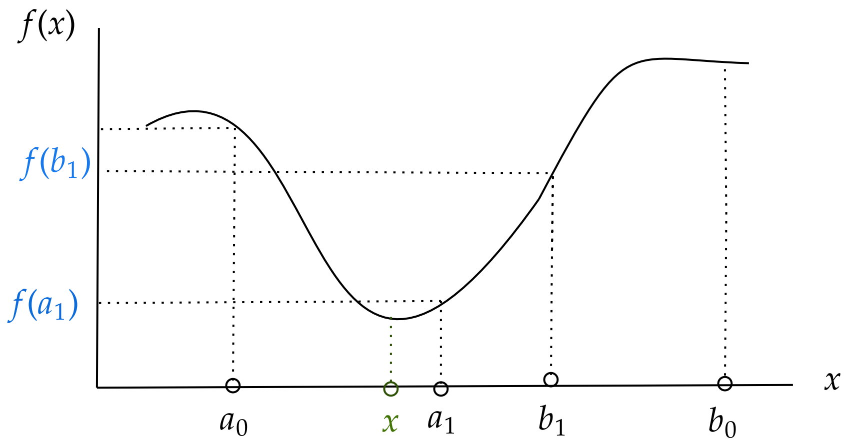
where

$$\rho < \frac{1}{2}$$

We then evaluate f at the intermediate points. If $f(a_1) < f(b_1)$, then the minimizer must lie in the range $[a_0, b_1]$. If, on the other hand, $f(a_1) > f(b_1)$, then the minimizer is located in the range $[a_1, b_0]$.



Evaluating the objective function at two intermediate points



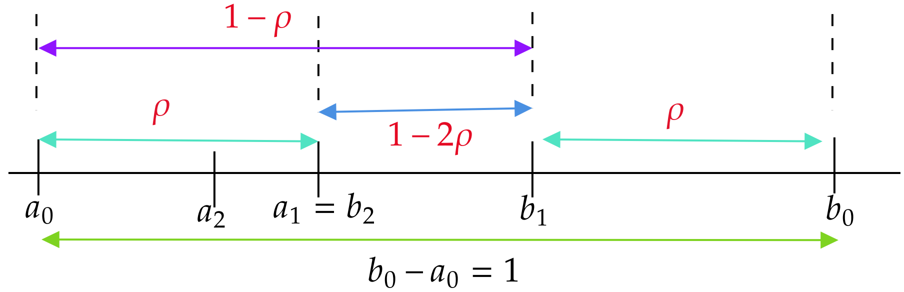
The case where $f(a_1) < f(b_1)$; *the minimizer* $x^* \in [a_0, b_1]$

Starting with the reduced range of uncertainty, we can repeat the process and similarly find two points, say a_2 *and* b_2, using the same value of $\rho < \frac{1}{2}$ as before. However, we would like to minimize the number of objective function evaluations while reducing the width of the uncertainty interval. Suppose, for example , that $f(a_1) < f(b_1)$. Then we know that $x^* \in [a_0, b_1]$. Because $a_1$is already in the uncertainty interval and $f(a_1)$ is already known, we can make a_1 coincide with b_2. Thus, only one new evaluation of f at a_2 would be necessary. To fine the value of ρ that results in only one new evaluation of f. Without loss of generality, imagine that the original range $[a_0, b_0]$ is of unit length. Then, to have only new evaluation of f it is enough to choose ρ so that:

$$\rho(b_1 - a_0) = b_1 - b_2$$

Because $b_1 - a_0 = 1 - \rho$ and $b_1 - b_2 = 1 - 2\rho, we\ have:$

$$\rho(1 - \rho) = 1 - 2\rho$$



Finding value of ρ resulting in only one new evaluation of f.

Fibonacci Method

Recall that the golden section method uses the same value of ρ *throughout*. Suppose now that we are allowed to vary the value ρ from state to state, so that at the kth stage in the reduction process we use a value ρ_k, at the next state we use $\rho_{k+1,}$ and so on.

As in the golden section search, our goal is to select successive values of $\rho_k, 0 \leq \rho_k \leq 1/2,$ such that only one new function evaluation is required at each state. To derive the strategy for selecting evaluation points, consider fig.

From this figure we see that it is suffieicent to choose the ρ_k such that :

$$\rho_{k+1}(1-\rho_k) = 1-2\rho_k$$

After some manipulations,we obtain:

$$\rho_{k+1} = 1 - \frac{\rho_k}{1 - \rho_k}$$

There are many sequences $\rho_1, \rho_2, \ldots$ that satisfy the law of formation above and the condition that 0