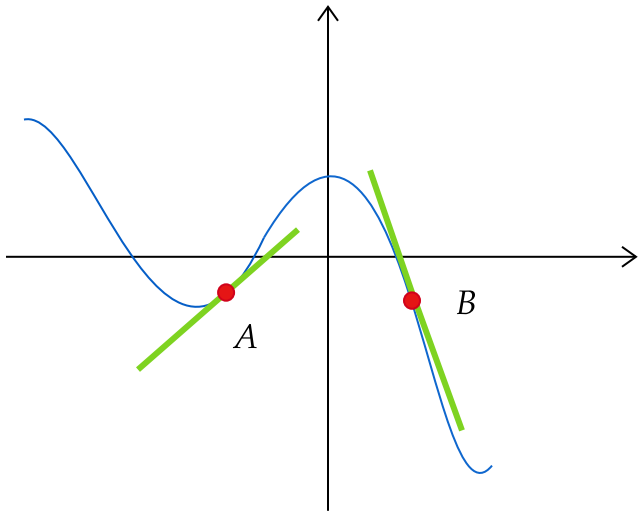
Link : [The Second Derivative]

- How does the derivative of a function tell us whether the function is increasing or decreasing on an interval?
- What can we learn by taking the derivative of the derivative (the second derivative) of a function f ?
- What does it mean to say that a function is concave up or concave down? How are these characteristics connected to certain properties of the derivative of the function ?
- What are the units of the second derivative ? How do they help us understand the range of change of the rate of change?

Given a differentiable function $u = f(x)$, we know that its derivative, $y = f'(x)$, is a related function whose output at $x = a$ tells us the slope of the tangent line to $y = f(x)$ at the point $(a, f(a))$. That is, heights on the derivative graph tell us the value of slopes on on the original function's graph.

At a point where $f'(x)$ is positive, the slope of the tangent line to f is positive. Therefore, on an interval where $f'(x)$ is positive, the function f is increasing (or rising). Similarly, if $f'(x)$ is negative on
an interval, the graph of f is decreasing (or falling).

The derivative of f tells us not only whether the function f is increasing or decreasing on an interval, but also how the function f is increasing or decreasing. Look at the two tangent lines as shown in the figure below. We see that near point A the value of $f'(x)$ is positive and relatively close to zero, and near that point the graph is rising slowly. By contrast, neat point B, the derivative is negative and relatively large in absolute value, and f is decreasing rapidly near B.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B