Math 2270: Linear Algebra
Section 1.1: Systems of Linear Equations

The Method of Elimination: We solve linear system of equations by performing elementary operations:

1. Multiply one equation by a nonzero constant.
2. Interchange two equations.
3. Add a constant multiple of (the terms of) one equation to (corresponding terms of) another equation.

Example 1: Solve the following system by the *method of elimination.*

$$\begin{array}{ll} \text{Eq1} & x + 5y + z = 2 \\ \text{Eq2} & 2x + y - 2z = 1 \\ \text{Eq3} & x + 7y + 2z = 3 \end{array}$$

$(\text{Eq2}) + (\text{Eq3}) \Rightarrow 3x + 8y = 4$ ①

$2(\text{Eq1}) + (\text{Eq2}) \Rightarrow 4x + 11y = 5$ ②

We eliminated "z"

Multiply ① by 4 : $12x + 32y = 16$

Multiply ② by -3 : $-12x - 33y = -15$

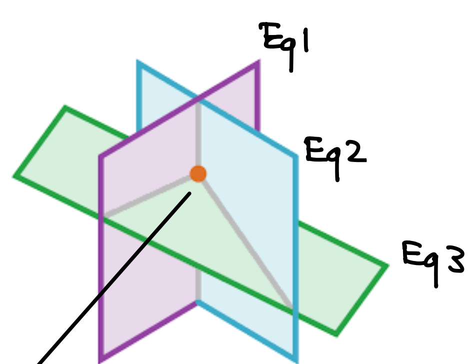
Add $0 - y = 1 \Rightarrow \boxed{y = -1}$

$3x + 8y = 4$

$3x - 8 = 4$ $\quad 3x = 12 \quad \boxed{x = 4}$

$x + 5y + z = 2$

$4 - 5 + z = 2$ $\quad \boxed{z = 3}$



$(x, y, z) = (4, -1, 3)$

Example 2: Solve the following system by the *method of elimination.*

$$\begin{array}{lrcl} \text{Eq1} & x+3y+2z &=& 5 \\ \text{Eq2} & x-\ \ y+3z &=& 3 \\ \text{Eq3} & 3x+\ \ y+8z &=& 10 \end{array}$$

$(\text{Eq2}) + (\text{Eq3}) \Rightarrow \quad 4x + 11z = 13$

$(\text{Eq1}) + 3(\text{Eq2}) \Rightarrow \quad 4x + 11z = 14$

We eliminated "y"

subtract $\quad 0 + 0 = -1$

$0 = -1$

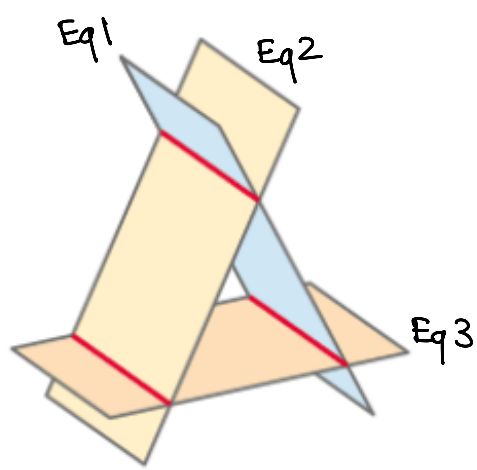
Contradiction ⁘

(Impossible equality)

There is no solution

The system is called "inconsistent"

Geometrically, the three planes do not intersect at a common point.



Example 3: Solve the following system by the *method of elimination.*

Eq1 $x + y - z = 5$
Eq2 $3x + y + 3z = 11$
Eq3 $4x + y + 5z = 14$

$(Eq2) - (Eq1) \Rightarrow 2x + 4z = 6$ ①
$(Eq3) - (Eq2) \Rightarrow x + 2z = 3$ ②

$2 \times$ ② $2x + 4z = 6$
① $2x + 4z = 6$

Subtract : $0 + 0 = 0$
trivial
(obvious equality)

① and ② are in fact the same equation ② $= \frac{1}{2} \times$ ①

We let z (or x) be a free parameter $\boxed{z = t}$

and find/represent all variables in terms of "t"

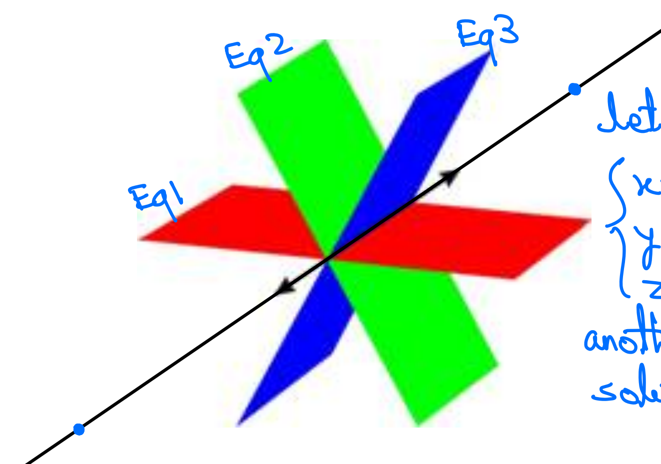
$x + 2z = 3$ $\quad x + 2t = 3$ $\quad \boxed{x = -2t + 3}$

Three planes intersect at a common line

$x + y - z = 5$
$(-2t + 3) + y - t = 5$
$\boxed{y = 3t + 2}$

$$\begin{cases} x = -2t + 3 \\ y = 3t + 2 \\ z = t \end{cases}$$

There are infinitly many solutions



let $t = 10$
$$\begin{cases} x = -17 \\ y = 32 \\ z = 10 \end{cases}$$
another solution

let $t = 0$
$$\begin{cases} x = 3 \\ y = 2 \\ z = 0 \end{cases}$$
one solution