# To Cross or Not to Cross the Spread: *That Is the Question*

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Traders have always empirically estimated the short-term dynamic of the market. Contrary to popular belief, building a quantitative estimate of the next trade is not some sort of "Holy Grail" only accessible to the darkest high-frequency traders. In fact, this knowledge is easily understood by anyone who observes the data attentively. Following the initial work on forecasting price changes from the order book (Avellaneda, Reed, and Stoikov [2011]) and research on estimating the cost of latency in trading (Stoikov [2014]), we focused our research on predicting the next trade side.

Thanks to our tick database (see the Appendix for details), we find that order book imbalances allow us to make reliable forecasts regarding the side (bid/ask) of the next trade; in short, on average, aggressive trades will mainly target the smallest limit (in size). We also find that aggressive trades will consume a larger share when the smaller limit is hit rather than the larger limit.

Using this insight regarding the next trade enables us to estimate the risk induced by a passive posting and thereby helps to decide whether to cross the spread using objective criteria based on the Sharpe ratio. Similar rules can be applied to trading algorithms to help eliminate many unnecessary aggressive trades and thus significantly increase trading performances.

## ORDER BOOK CHARACTERISTICS OBSERVABLE BEFORE TRADING

The order book gathers bid and ask orders at different price levels, which are referred to as *limits*. The best bid and best ask prices are called *first limits* and are illustrated in Exhibit 1. This article will focus on the state of the first limits and on order book imbalance in particular, which we will define later.

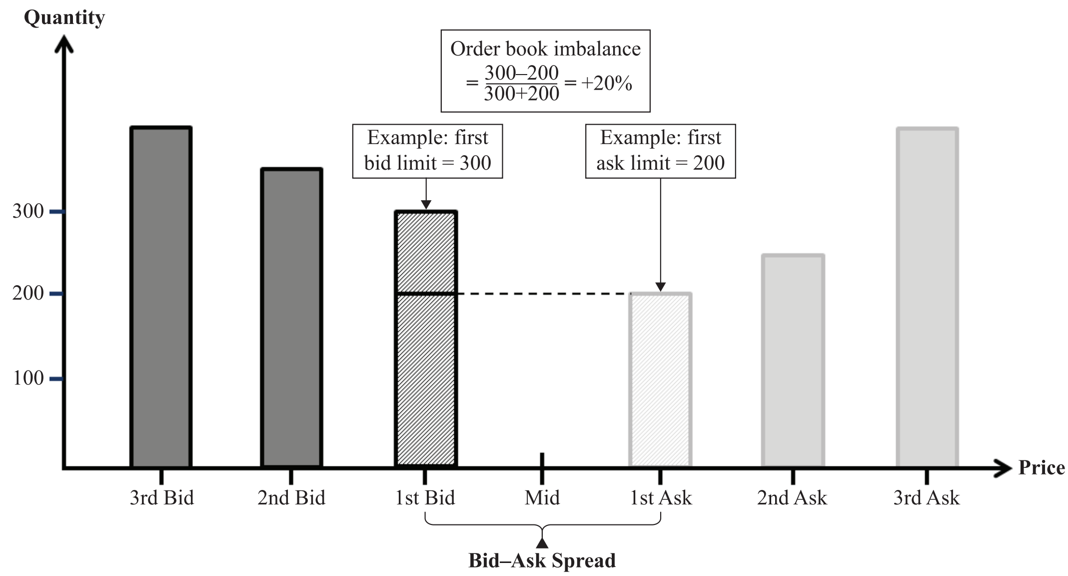
Order books change in real time. In this report, we study the last order book before a trade. Thus, we highlight the specific link between the last order book available and the subsequent aggressive trade. We will not discuss practical topics concerning the latency of operating systems and the delay with which one observes the order book in reality, which depends on the practical settings of each specific setup.

We characterize the state of the order book based on the *order book imbalance*, defined as the difference between quantities present on the best bid limit and quantities present on the best ask limit:

$$\textit{Order book imbalance (shares)} = \text{Bid size} - \text{Ask size} \tag{1}$$

$$\textit{Order book imbalance (\%)} = \frac{\text{Bid size} - \text{Ask size}}{\text{Bid size} + \text{Ask size}} \tag{2}$$

## EXHIBIT 1
**Elementary View of a Limit Order Book**




*Note: A schematic of the first three bid and ask limits.*

The order book imbalance expressed as a percentage varies from −100% to +100%. This measure, as opposed to the simple difference in shares, enables us to compare different stocks. The more positive the order book imbalance, the greater the number of passive buyers (bid side). The more negative the order book imbalance, the greater the number of passive sellers (ask side).

## ESTIMATING THE SIDE OF THE NEXT TRADE

The state of the last order book strongly influences the side of the next trade: bid or ask. The major contributing factors are the sizes available on the bid and the ask side.

### Order Book Shape Drives the Side of the Next Trade

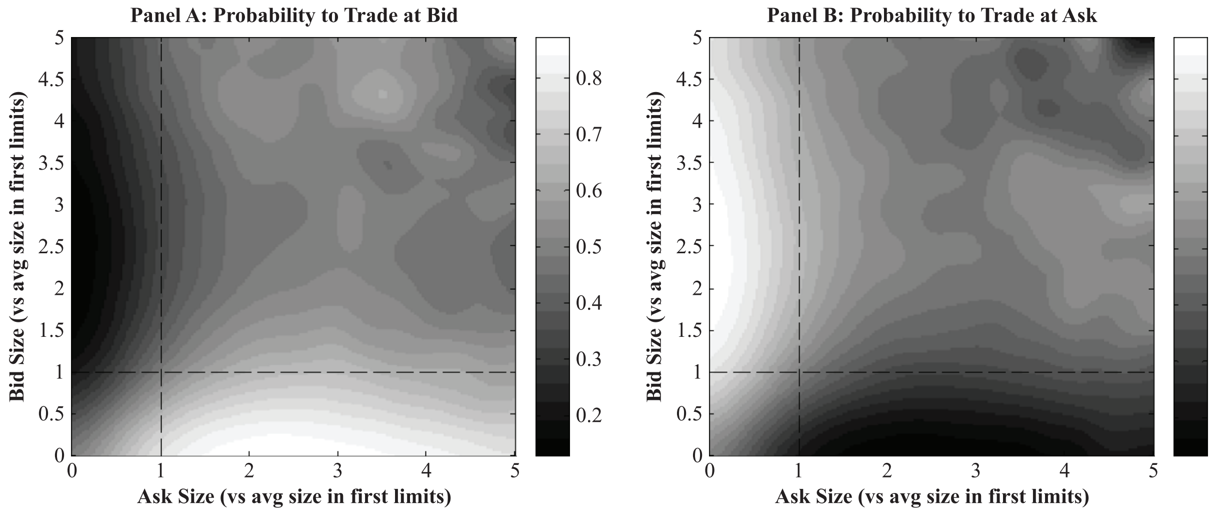
In Exhibit 2, we represent the empirical probability of seeing a trade at bid (Panel A) or at ask (Panel B) according to the size available in the first limits. For each stock, we compute the ask limit in proportion to the average first limit size over the period ($x$-axis). Similarly, relative bid sizes are shown on the $y$-axis. The greatest probabilities are represented in light gray and the smallest in dark gray. We make the following observations:

- When the ask size is large compared to the bid size (bottom-right areas in each graph), a very high probability exists of observing a trade on the bid: between 80% and 90% (see light areas in Exhibit 2, Panel A). This indicates that significant, passive selling pressure on the ask is likely to induce an aggressive selling order.
- In contrast, when the bid size is large compared to the ask size (upper left area), the probability of seeing a trade on the bid is low (dark areas in Exhibit 2, Panel A).
- When sizes are similar, the probability of seeing a bid trade is roughly 50% (in medium gray).
- Symmetrically, when the bid size is large compared to the ask size, there is an 80%–90% probability of seeing a trade on the ask side (light areas in Exhibit 2, Panel B).

On average, the next aggressive trade will target the smallest limit (in size). We can also see that the constant probability level lines (same color zones) are distributed according to constant relative order-book imbalance levels.

## EXHIBIT 2

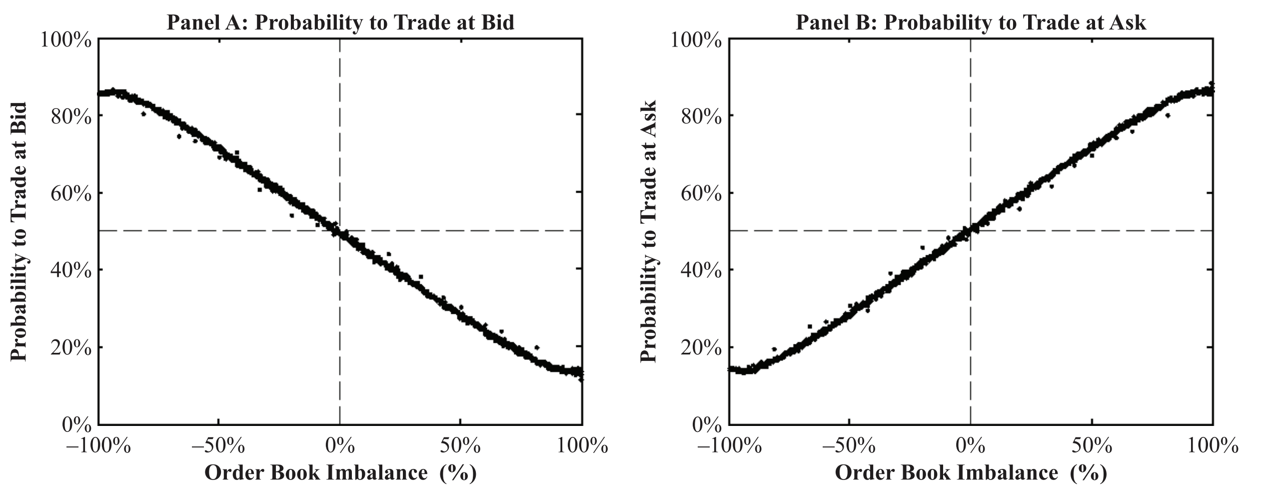
**Empirical Probability to Trade on One Side according to the Size of the First Limits**




*Source: Data on DJ Stoxx Large 200 components from February 2016.*

## EXHIBIT 3

**Empirical Probability to Trade on One Side according to the Order Book Imbalance**




*Note: We consider 1,000 intervals of order book imbalance (1,000 dots) over which we calculate the empirical probability.*

*Source: Data on DJ Stoxx Large 200 components from February 2016.*

### Order Book Imbalance Predicts the Side of the Next Trade

The conclusion highlighted by the two-variable exhibit in the previous section can be summarized by the order-book imbalance measure (computed as the difference between the bid and the ask size, in percentages). Exhibit 3 shows the probability of seeing a trade on one side ($y$-axis), depending on the order book imbalance ($x$-axis).

When the order book imbalance is strongly negative, the average probability of the next trade being at the bid reaches 86% (Exhibit 3, Panel A). Alternatively, when the order book imbalance is strongly positive, the probability of the next trade being at the ask reaches 87% (Exhibit 3, Panel B).

In addition, the effect of the order book imbalance on the probability of the trade occurring on a given side is very linear: the more negative (positive) the order book imbalance, the greater the probability of observing a bid (ask) trade. As soon as the order book imbalance exceeds 60% in absolute terms, the predictability of the trade side is greater than 76%. This occurs in 40% of cases. Note that when both first limits are equal, bid and ask trades are equally probable.

### Findings Remain True from Large Caps to Small Caps

The previous charts concentrate on the 200 largest European capitalizations. Small and mid-sized capitalizations share the same behavior, albeit to a lesser extent. In fact, Exhibit 4 shows that one can forecast the next trade side with good accuracy on large, mid, and small caps.

### Predictability Remains High When the Trading Is Slow

For discretionary traders, the time at which the next trade occurs after the last order book update (called dt in Exhibits 5 and 6) is a key element. If the dt is too small, traders may not have time to place their order manually, and they will not be able to seize the opportunity of a predictable trade. This possibility will only remain true for trading algorithms. In fact, several changes in the order book can occur between two aggressive trades (see vertical lines in Exhibit 5). Thus, traders and algorithms have, on average, a time dt to react and send their order before the next aggressive trade.

Exhibit 6 shows the empirical probability of a trade on the bid or the ask side depending on the dt. We see that even when the next aggressive trade occurs, more than half a second after the last order book update (light gray dots), the probability of being on the bid side remains at around 73% for a −60% order book imbalance (Exhibit 6, Panel A). This should be compared with an 80% probability when the dt is smaller than 0.1 seconds (dark gray dots).

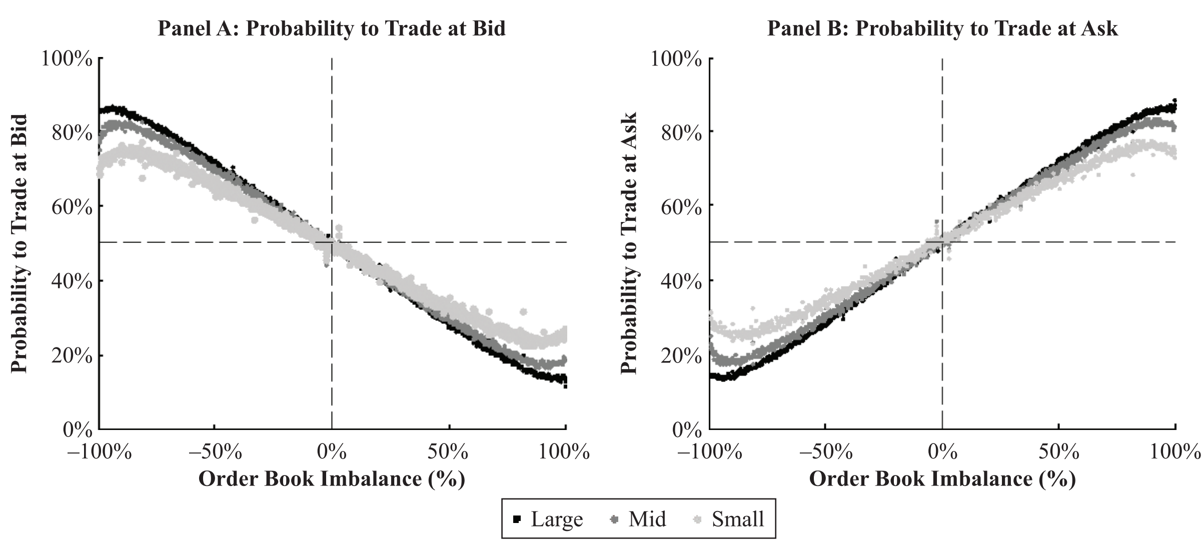
This shows that both traders and algorithms have time to react and to take advantage of the predictable side of the next trade.

### Practical Example on a Large Cap

Let us consider Unilever (ULVR LN), which belongs to the DJ STOXX 50 Index. It has, on average,

## EXHIBIT 4

**Empirical Probability to Trade on One Side, depending on Market Capitalization**




*Note: We consider 1,000 intervals of order book imbalance (1,000 dots) over which we calculate the empirical probability.*

*Source: Data on DJ Stoxx 600 components from February 2016.*

**EXHIBIT 5**

**Aggressive Trades and Order Book Updates: Detailed Timeline**

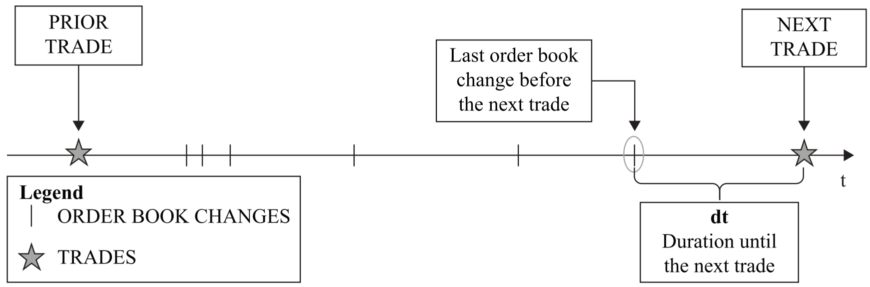
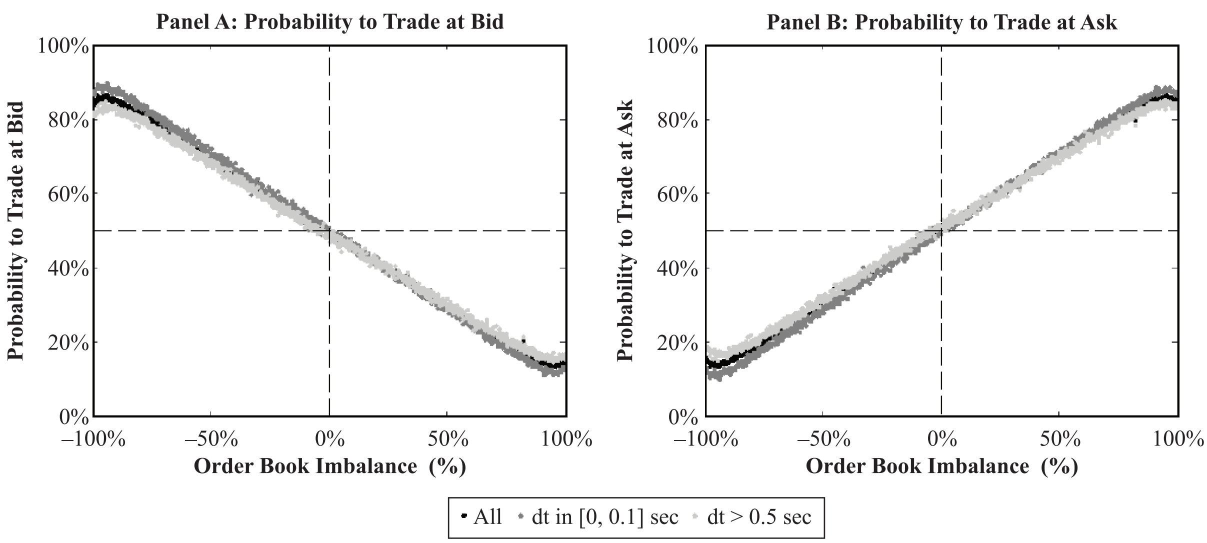



**EXHIBIT 6**

**Probability of Trading on One Side Upon Different Time dt (timing of the next aggressive trade)**




*Note: We consider 1,000 intervals of order book imbalance (1,000 dots) over which we calculate the empirical probability.*
*Source: Data on DJ Stoxx Large 200 components from June 2016.*

1,200 shares available in its first limits. If the ask is large (4,800 shares) but the bid remains at 1,200 shares, then the order book imbalance equals −60%. We can then estimate that the probability of the future trade happening at the bid is 76% (see line 2 in Exhibit 7).

## ESTIMATING THE SIZE OF THE NEXT TRADE

In addition to anticipating the side of the next aggressive trade, one can also anticipate the next trade's size. This aspect is key in estimating how much of the posted volume will be executed by the next aggressive trade. We show that the state of the order book enables us to predict what percentage of the first limit will be consumed, which helps traders estimate the quantity they should post on the first limit.

### Order Book Imbalance Predicts the Size of the Next Trade

To reach a more precise estimation of the next traded quantity, the last observable order book imbalance is invaluable. Aggressive trades will consume a larger share of the smaller limit (in size) when hit than when the larger limit is hit. For example, for an order book imbalance of 60% in absolute terms, if the smaller

## Exhibit 7

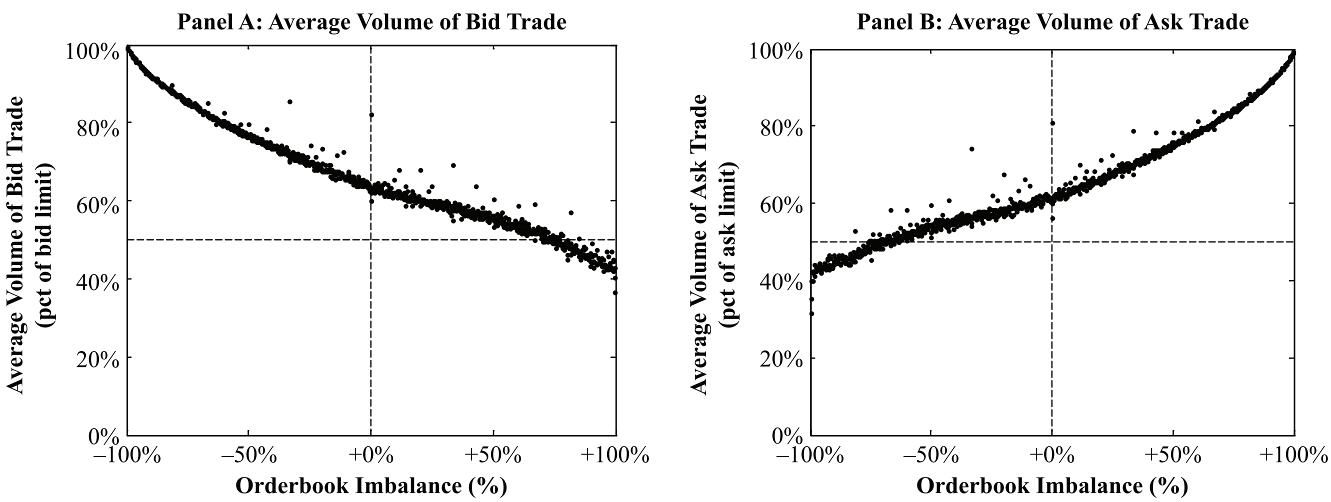
**Probability of Trading on the Bid or Ask Side: A Large Cap Example**

| Ticker | Name | Bid Size (shares) | Ask Size (shares) | Order Book Imbalance | Probability to Trade at Bid | Probability to Trade at Ask |
|---|---|---|---|---|---|---|
| ULVR LN | UNILEVER | 10 | 20,000 | –100% | 86% | 14% |
| ULVR LN | UNILEVER | 1,200 | 4,800 | –60% | 76% | 24% |
| ULVR LN | UNILEVER | 1,200 | 1,200 | 0% | 50% | 50% |
| ULVR LN | UNILEVER | 4,800 | 1,200 | 60% | 24% | 76% |
| ULVR LN | UNILEVER | 20,000 | 10 | 100% | 13% | 87% |

*Source: Data on DJ Stoxx Large 200 components from February 2016.*

## Exhibit 8

**Average Volume of Bid and Ask Trades (percentage of bid or ask limit)**




*Note: We consider 1,000 intervals of order book imbalance (1,000 dots) over which we calculate the average percentage of the limit that is consumed.*
*Source: Data on DJ Stoxx Large 200 components from February 2016.*

limit (in size) is hit, on average, the next trade will hit 78% of the limit. In contrast, if the larger limit is hit, then only 53% of its size will be consumed, as shown in Exhibit 8.

This can easily be understood by the fact that, for example, a large positive order-book imbalance will correspond to a strong passive buying pressure. The stronger the pressure, the more probable it is that the opposite limit will be fully consumed.

### Our Finding Remains True from Large Caps to Small Caps

The same phenomenon is observed for large, mid-sized, and small caps. Exhibit 9 shows the average percentage of the limit that will be consumed for three examples: Unilever, ThyssenKrupp, and Euronext.

In the case of Unilever, if the ask is large (4,800 shares), but the bid remains at 1,200 shares (i.e., a −60% order book imbalance), we see that the most probable trade will be at the bid (76% probability). In this case, 78% of the bid size will be consumed on average—that is, 936 shares (see line 2 in Exhibit 9).

## EMPIRICAL RULE FOR PRACTICAL TRADING

Long before the emergence of algorithmic trading, traders wondered how to best execute a client order, deciding when to cross the spread and become aggressive and when it would be more advantageous to wait

## EXHIBIT 9
**Percentage of the Limit That Will Be Consumed: Examples**

| Ticker | Name | Bid Size (shares) | Ask Size (shares) | Order Book Imbalance | Most Probable Trade (proba.) | Pct of Bid Limit Consumed if Bid is Hit | Pct of Ask Limit Consumed if Ask is Hit | Expected Volume on Most Probable Side |
|---|---|---|---|---|---|---|---|---|
| ULVR LN | UNILEVER | 10 | 20,000 | –100% | BID (86%) | 100% | 43% | 10 |
| ULVR LN | UNILEVER | 1,200 | 4,800 | –60% | BID (76%) | 78% | 53% | 936 |
| ULVR LN | UNILEVER | 1,200 | 1,200 | 0% | BID/ASK (50%) | 63% | 63% | 756 |
| ULVR LN | UNILEVER | 4,800 | 1,200 | 60% | ASK (76%) | 53% | 78% | 936 |
| ULVR LN | UNILEVER | 20,000 | 10 | 100% | ASK (87%) | 43% | 100% | 10 |
| TKA GY | THYSSENKRUPP | 5 | 10,000 | –100% | BID (77%) | 100% | 48% | 5 |
| TKA GY | THYSSENKRUPP | 1,600 | 6,400 | –60% | BID (74%) | 81% | 57% | 1,296 |
| TKA GY | THYSSENKRUPP | 1,600 | 1,600 | 0% | BID/ASK (50%) | 69% | 69% | 1,104 |
| TKA GY | THYSSENKRUPP | 6,400 | 1,600 | 60% | ASK (74%) | 57% | 81% | 1,296 |
| TKA GY | THYSSENKRUPP | 10,000 | 5 | 100% | ASK (81%) | 48% | 100% | 5 |
| ENX FP | EURONEXT | 2 | 4,000 | –100% | BID (70%) | 100% | 48% | 2 |
| ENX FP | EURONEXT | 200 | 800 | –60% | BID (69%) | 81% | 57% | 162 |
| ENX FP | EURONEXT | 200 | 200 | 0% | BID/ASK (50%) | 69% | 69% | 138 |
| ENX FP | EURONEXT | 800 | 200 | 60% | ASK (69%) | 57% | 81% | 162 |
| ENX FP | EURONEXT | 4,000 | 2 | 100% | ASK (74%) | 48% | 100% | 2 |

*Source: Data on DJ Stoxx 600 components from February 2016.*

for the bid to be executed with a passive order. These problems can be summed up as whether to pay the spread or try to save the spread and hold the risk of a deferred execution.

### Empirical Estimates of Waiting Time in Number of Trades and Risk Incurred by Passive Posting

To estimate the risk incurred by posting a passive order at the first limit in the order book, we evaluate the number of trades before the passive order is executed. To gain a realistic estimate, we make two alternative assumptions: first, an optimistic one, which provides us with a lower-bound risk estimate; second, a pessimistic one, which provides us with an upper-bound risk estimate.

### Lower Risk Estimate: Passive Order Always at the Top of the First Limit

Assuming that we posted a limit order in the order book, we suppose that our passive order is entirely executed as soon as an aggressive order hits our side of the order book. This assumption is optimistic—it is rare to be posted directly in the first position on the first limit—and would only be the case for the fastest traders in the market. This scenario provides us with a lower bound of the risk incurred while posting a passive order.

### Higher Risk Estimate: Passive Order Always at the Bottom of the First Limit

Our second assumption is to consider that the passive order is posted at the bottom of the first limit, which would be the case if we were the last trader in the market on this limit. Our order will be executed only after all posted orders in the specific limit are executed (*full limit consumption*). This pessimistic assumption provides us with a higher estimate of the waiting time before execution and thus gives us an upper bound of the risk.

### Estimating the Passive Waiting Time in Number of Trades

Based on these two assumptions, we study the variations of waiting time before the next execution depending on the available quantities on the first limits. We represent the average waiting time in number of trades on a heat map according to the ask size ($x$-axis) and the bid size ($y$-axis). Because different stocks have different sizes on the first limits, on average, bid and ask sizes will be normalized by the daily averages of the first limit sizes.

We observe in Exhibit 10, Panel A, that the lower estimate (orders posted on the top of the limit) of the expected number of trades before next execution at the bid side varies between 1.4 and 3.4 depending on the available quantity at the bid and at the ask sides. Furthermore, the observable pattern reveals that the waiting time in the number of trades before the next execution relies more on the difference between the bid and ask sizes, the so-called order book imbalance, than on the actual bid and ask sizes. Thus, the greater the order book imbalance, the greater the waiting time in number of trades before the next execution at the bid. If, all things equal, the posted quantity at the bid increases, the expected number of trades before the next trade at the bid before execution increases. Therefore, for a 0.5 ask size and a 2 bid size (+60% order book imbalance), the expected waiting time at the bid is 3.2 trades (see Exhibit 10, dark area in Panel A). This can be easily understood by the fact that the larger the size of the passive buyers, the longer the passive buyers will have to wait for a passive execution. Conversely, in the case of a negative order book imbalance (more passive sellers than buyers)—for example, an ask size of 2 and a bid size of 0.5 (−60% order book imbalance)—the expected passive waiting time at the bid is only 1.6 trades (see Exhibit 10, light area in Panel B).

Exhibit 10, Panel B shows the upper estimate (orders posted at the bottom of the limit) of the expected number of trades before the next trade depletes the bid limit according to the bid and ask sizes. Our estimate varies between 2.4 and 6, which is slightly higher than our optimistic estimate. The pattern we observed in Panel A of Exhibit 10 does not appear so clear-cut in Panel B, but it is still present. We see a rather strong dependence on the bid size and a rather low dependence on the ask size. This seems natural, as the greater the quantity at the bid, the longer it will take for the aggressive order flow to consume the entire limit, regardless of the quantity posted at the ask.

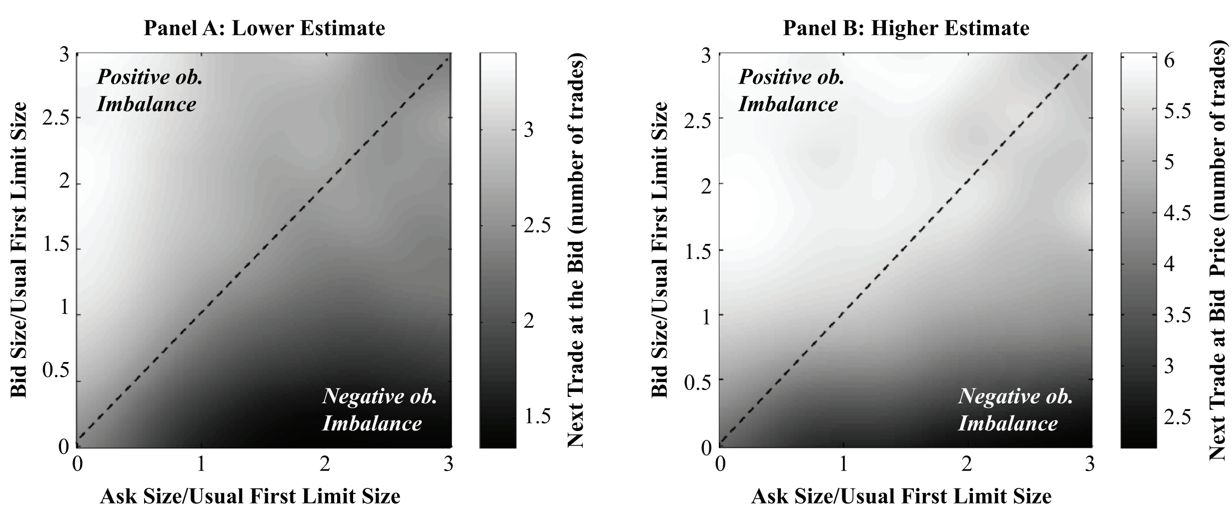
One observes that for a 0.5 ask size and a 2 bid size (+60% order book imbalance), the expected number of trades before the entire bid size depletion is 5.9, while for a 2 ask size and a 0.5 bid size (−60% order book imbalance), it is 3.6 (see Exhibit 10, Panel B).

### Order Book Imbalance Drives the Passive Waiting Time on the Limit

We have previously seen that the expected number of trades until the next execution could be accurately

## Exhibit 10

**Lower and Higher Estimates of the Waiting in the Number of Trades for a Bid Order**




*Notes: Lower estimate: average number of trades before next trade at the bid. Higher estimate: the average number of trades before the next trade consumes all of the bid quantities. "ob. Imbalance" refers to the order book imbalance.*

*Source: Stoxx Europe 50, February 2016.*

described using only order book imbalances without considering the actual bid and ask sizes. We study the variation of the expected number of trades relative to the order book imbalance in Exhibit 11. The charts show the variations of these values when considering trades only at the bid (Panel A) or trades at the ask (Panel B).

In both charts, the black curve gives the expected number of trades before the next aggressive order consumes all of the quantity available at the first limit on our side (pessimistic assumption). The gray curve gives the expected number of trades before the next trade hits the first limit on our side (optimistic assumption). The dashed gray curve gives the average between the two solid curves. We make the following observations:

- For a +60% order book imbalance and a passive order posted at the best bid, the trader has to wait for 3.2 (optimistic assumption) to 5.7 trades (pessimistic assumption) before execution (see Exhibit 11, Panel A).
- For the same order book imbalance with a passive order at the best ask, the trader has to wait for 1.6 (optimistic assumption) to 3.4 trades (pessimistic assumption) before execution (see Exhibit 11, Panel B).

We can thus easily estimate the risk incurred by a passive posting as a function of the current order book imbalance.

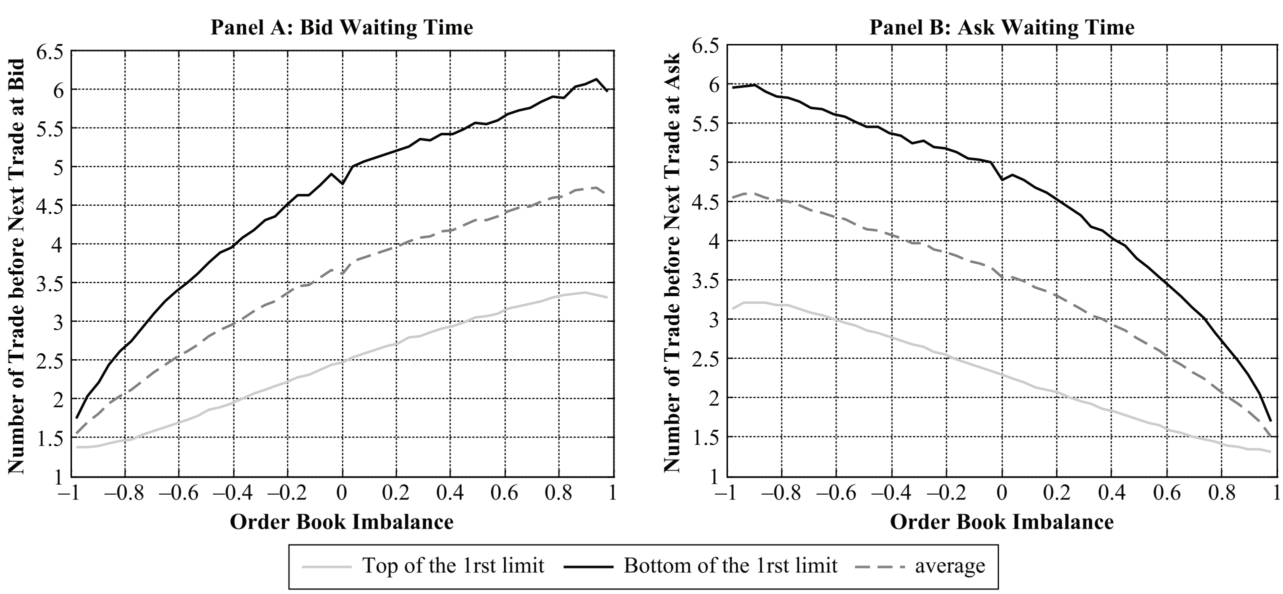
### From the Estimated Waiting Times in Number of Trades to the Risk Estimate

In the previous paragraph, we described how to estimate the number of trades before a passive order is executed according to the order book imbalance. In the following paragraph, we explain how to obtain a risk estimate from the expected number of trades.

If the only information we have about trade distribution in time is the expected number of trades on a daily basis, our best estimate of the waiting time between two trades will be the duration of the trading day divided by the daily number of trades. Thus, a good estimate of the average waiting time until the next execution is the ratio of the expected number of trades before execution to the expected daily number of trades. Average waiting time until next execution (AVGWT) is therefore:

$$\text{AVGWT} = \frac{3{,}600 \times 8.5 \times \text{Waiting time in nb of trades}}{\text{Daily nb of trades}} \quad (3)$$

## EXHIBIT 11
**Bid and Ask Waiting Time in Number of Trades**




*Source: Stoxx Europe 50, February 2016.*

# EXHIBIT 12

**Passive Posting: Favorable or Unfavorable Order Book Imbalances**

| | Negative Order Book Imbalance (bid size < ask size) | Positive Order Book Imbalance (bid size > ask size) |
|---|---|---|
| Passive buy | FAVORABLE (lower waiting time → lower execution risk) | UNFAVORABLE (higher waiting time → higher execution risk) |
| Passive sell | UNFAVORABLE (higher waiting time → higher execution risk) | FAVORABLE (lower waiting time → lower execution risk) |

where $3{,}600 \times 8.5$ is the number of seconds for a 8.5-hour trading day. This normalization gives us AVGWT in seconds.

Once we have estimated the expected waiting time until the next execution, we just need an estimate of the stock price volatility to obtain an estimate of the risk incurred when waiting for the execution of a passive order rather than going aggressive immediately. To that end, we take the average daily volatilities in basis points over a second; thus, we reach a good estimate of the risk by multiplying the average daily volatility by the square root of the average waiting time until the next execution. We denote $\sigma$ as the estimated stock volatility (in bps per second). The expression of our risk estimate is therefore:

$$\text{Risk} = \sqrt{\text{AVGWT}} \times \sigma \tag{4}$$

Exhibit 12 summarizes the cases in which the order book imbalance is favorable to a passive posting in terms of risk.

A passive order will be all the more interesting when it sits on the smaller first limit (in size). This means that this order goes against the larger passive interests. For traders, it is a well-known fact that it is better to trade against the market than to chase the market.

### Example of a Bid Order on ThyssenKrupp (TKA GY)

Let us take a look at an example using TKA GY. In April 2016, TKA GY's average daily volatility was 257 bps/day. Its average daily number of trades was 4,635. The stock's volatility per second is then:

$$\sigma = \frac{257}{\sqrt{8.5\,\text{h} \times 60\,\text{min} \times 60\,\text{sec}}} = 1.47\ \text{bps/sec} \tag{5}$$

In Exhibit 13, we have plotted two points with x-coordinates of 2.5 and 4.4 expected trades and both y-coordinates as 257 bps daily volatility. These two values correspond to the average waiting number of trades at the bid for −60% and +60% order book imbalances, respectively. The associated risk values are as follows:

- For a −60% order book imbalance:

$$\text{Waiting time in nb of trades} = 2.5 \tag{6}$$

$$\text{Waiting time in seconds (AVGWT)} = \frac{3{,}600 \times 8.5 \times 2.5}{4{,}635} = 16.5\ \text{sec} \tag{7}$$

$$\text{Risk} = 1.47 \times \sqrt{16.5} = 6\ \text{bps} \tag{8}$$

- For a +60% order book imbalance:

$$\text{Waiting time in nb of trades} = 4.4 \tag{9}$$

$$\text{Waiting time in seconds (AVGWT)} = \frac{3{,}600 \times 8.5 \times 4.4}{4{,}635} = 29.0\ \text{sec} \tag{10}$$

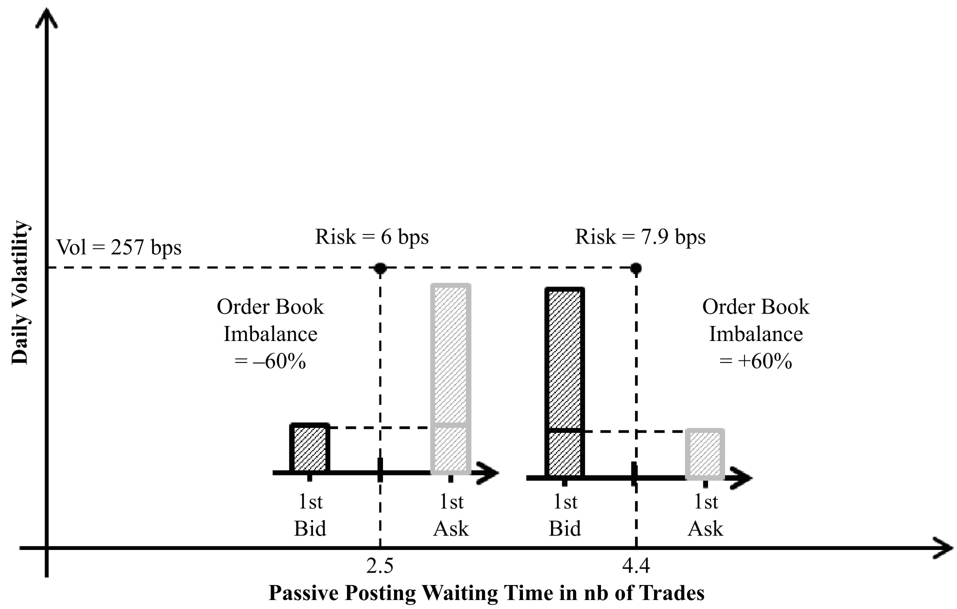
$$\text{Risk} = 1.47 \times \sqrt{29.0} = 7.9\ \text{bps} \tag{11}$$

### Sharpe Trading Rule to Help Discretionary Traders to Efficiently Decide whether to Cross the Spread

**The passive posting price improvement versus risk trade-off.** At every moment, traders have to make a decision whether to post a passive order in the order book and wait for the next opposite side aggressive order to hit the passive order. Alternatively, traders can send an aggressive order, which is executed immediately but incurs the cost of crossing the spread. The decision is not straightforward, and, in the light of the above, taking under consideration the information embedded in the order book may help traders make consistent choices.

We propose a methodology based on a risk–reward analysis. Here, the ratio is a *price improvement versus risk ratio*, which is quite similar to the Sharpe ratio approach commonly used by asset managers. The trade-off between being aggressive with no execution risk and

# EXHIBIT 13
**Estimated Passive Posting Risk at the Bid**




*Note: Example of TKA GY—average daily number of trades = 4,635.*

being passive with an execution risk and a local price improvement can be stated as follows:

- If traders decide to be aggressive, they will see no price improvement and will obtain an execution at the best opposite price.
- If the traders decide to post a passive order at the best price, they obtain a local price improvement that corresponds to the size of the spread when the order is executed; however, the price might have deviated from its initial position, thus incurring some market risk because of the deferred execution.

The Sharpe ratio of a passive execution can thus be defined as the ratio of the bid–ask spread (price improvement) to the passive posting market risk:

$$\text{Passive posting Sharpe ratio} = \frac{\text{Bid–ask spread}}{\text{Passive posting market risk}} = S \qquad (12)$$

- If $S > 1$, the order book imbalance is favorable to a passive posting, given the level of the bid–ask spread.
- If $S < 1$, the order book imbalance is unfavorable to a passive posting, given the level of the bid–ask spread; aggressive trading is preferable.

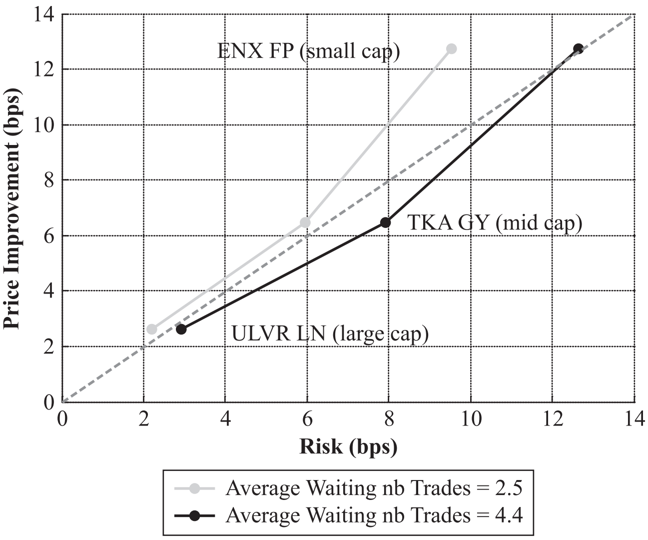
**Three examples on large, mid-sized, and small caps.** Exhibit 14 shows three examples for different market caps: Unilever (large cap), ThyssenKrupp (mid-sized cap) and Euronext (small cap). For each of these market caps, we have computed the average daily number of trades, the average daily volatility, and the average bid–ask spread. Furthermore, we have considered two order book imbalance scenarios:

- Favorable (light gray curve): the expected number of trades until the next execution is 2.5 (for a passive order at the bid with a −60% order book imbalance);
- Unfavorable (black curve): the expected number of trades until the next execution equals 4.4 (for a passive order at the bid with a +60% order book imbalance).

We then computed the associated market risk and marked the price improvement for each stock and scenario with a dot in Exhibit 14. The black dots correspond to an

## EXHIBIT 14

**Passive Posting Price Improvement versus Risk Trade-off of the Next Passive Execution**




expected number of trades equal to 4.4 and the gray dots to an expected number of trades equal to 2.5. Interestingly, we see that for a given scenario, the dots associated with different market caps are almost aligned, indicating that it is not more favorable to be passive on some market caps than on others. We also see that, depending on the order book imbalance, a discretionary trader will favor either a passive posting or an aggressive posting, depending on the corresponding Sharpe ratio associated with a passive posting.

For example, for a buy order on ThyssenKrupp (TKA GY):

- If the order book imbalance is favorable (−60%), the average waiting time in number of trades is 2.5 (middle gray dot in Exhibit 14); thus, the trader should remain passive.
- If the order book imbalance is unfavorable (+60%), the average waiting time in the number of trades is 4.4 and aggressive trading should be favored (middle black dot in Exhibit 14).

### Improving Trading Algorithm Performance Using Our Findings on the Order Book Imbalance

**The case of the percentage of volume algorithms.** A common trading strategy used in algorithmic trading consists of trading a fixed percentage of the total traded volume in the market. To that end, traders often resort to the percentage of volume (POV) trading algorithm specifically designed to target a fixed participation rate. If the trading algorithm execution lags behind its execution target, aggressive orders are sent to catch up with the target participation. Most standard algorithms will behave in such a way, not taking into account the current order book imbalance before sending aggressive orders.

### Taking into Account the Order Book Imbalance Helps Improve Trading Slippage

The most commonly used indicator to evaluate trading algorithm performances is the *slippage*, which corresponds to the relative difference between the algorithm's average traded price versus the market's volume weighted average price (VWAP) during the trading period.

In this section, we show how our risk estimate of the next passive execution can be leveraged to improve trading performances, while preventing the participation rate from deviating too far from its target. We simulate executions using different passive posting methodologies and show that our order book imbalance–driven passive posting methodology outperforms the standard approach.

The three methodologies tested in this section are the following:

- The first is a *fully aggressive* strategy, which compensates for the lag between the algorithm's real participation and its target participation as soon as it differs from zero. In this case, an aggressive order is sent to the market.
- The second methodology, *standard passive posting*, consists of sending passive orders with sizes equal to the lag of the algorithm and sending aggressive orders to catch up as soon as the lag exceeds 5 ATS ("mediAn Trade Size"). When the lag is null, the posted quantity will match the target participation.
- The last methodology, *advanced passive posting*, is similar to the standard one, with the difference being that orders are not sent aggressively to the market if their passive execution probability is greater than 30% (as estimated by our model). This advanced model enables us not to waste aggressive

orders when the order book imbalance tells us that a passive posting is likely to catch up to our execution lag. In the following section, we estimate the potential gains of this new feature.

**An estimated performance of simulated POV algorithms (advanced, standard, and aggressive versions).** We have simulated the performance of POV algorithm occurrences on a sample consisting of components of the Stoxx Europe 50 index over a period of one month. This corresponds to 1,000 client orders and 1,000,248 individual trades. We use tick-by-tick data, composed of the sizes and prices of each trade and the prices and quantities of the first limits in the order book at the time of trades. For each trade in the database, we decide either to send an aggressive order or to post a passive order:

- In the case of a passive posting, we assume that the executed quantity equals the minimum between the posted quantity and the size of the incoming aggressive trade under a given execution probability. If the trade does not hit our side of the order book, we do not execute.
- In the case of an aggressive order, we assume that the executed quantity equals the minimum between the best opposite size and our aggressive order size.

The decision to be aggressive or passive is based on the type of POV algorithm we test (fully aggressive, standard passive, or advanced passive) and on the current lag between the algorithm's effective participation and its target participation. In our simulations, we choose a target participation of 20% and 5% of the average daily volume quantity to be executed. This would result in two-hour trades on average.

Performances are measured with the slippage versus the market VWAP over the trading period. For a buying occurrence of the POV the slippage can be written as:

$$\text{Slippage} = \frac{\text{Market VWAP} - \text{POV average price}}{\text{Market VWAP}}, \quad (13)$$

often expressed in percentage of the stock's bid–ask spread.

Another execution performance measure is the *passive fill rate*, which corresponds to the percentage of the occurrence amount executed through passive limit orders. The passive fill rate is written as:

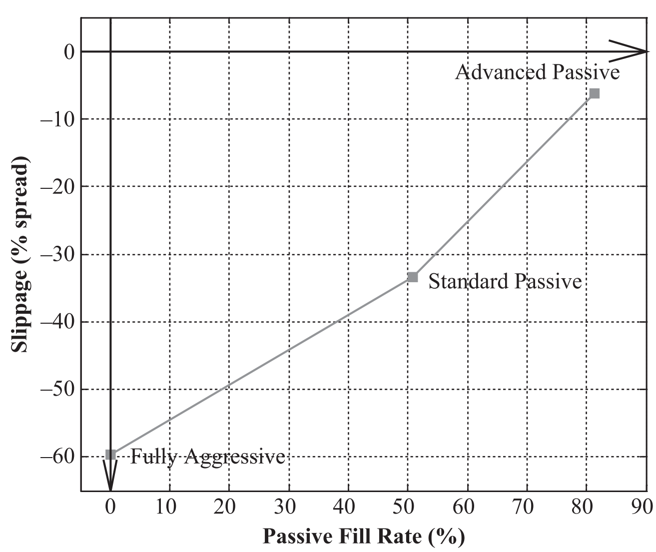
$$\text{Passive fill rate} = \frac{\text{Passive executed quantity}}{\text{Passive executed quantity} + \text{Aggressive executed quantity}} \quad (14)$$

There is a positive correlation between the passive fill rate and the slippage. Indeed, the passive fill rate measures the algorithm's ability to capture the passive flow and avoid crossing the spread. The higher the passive fill rate, the less it crossed the spread and the smaller the slippage.

Exhibit 15 shows the average slippages and passive fill rates across our simulated executions sample for our three methodologies. The slippage of each methodology as a percentage of the spread is on the $y$-axis and the average passive fill rate in percentage is on the $x$-axis. As anticipated, the relation between the slippage and the passive fill rate is positive; therefore, the more passive the algorithm, the less negative the

**EXHIBIT 15**

**Strategies' Slippages Related to Their Passive Fill Rates**




*Source: Stoxx Europe 50, February 2016.*

slippage. If we compare the respective performances of our three strategies, we see that

- the fully aggressive strategy has −60% of the spread slippage with a 0% passive fill rate;
- the standard passive strategy has −33% of the spread slippage and a 50% passive fill rate;
- the advanced passive posting strategy has −6% of the spread slippage and an 81% passive fill rate.

As expected, our advanced strategy enables us to decrease unnecessary aggressive trading and thus increase the passive fill rate from 50% to 81%, which improves the execution slippage from a −0.33 spread to a −0.06 spread.

**The impact of our advanced passive posting strategy on effective participation.** As previously mentioned, traders use POV algorithms to target a fixed participation rate (ratio of the executed quantity to the market volume) over the trading period:

$$\text{Effective participation rate} = \frac{\text{Executed quantity}}{\text{Market volume over the trading period}} \tag{15}$$

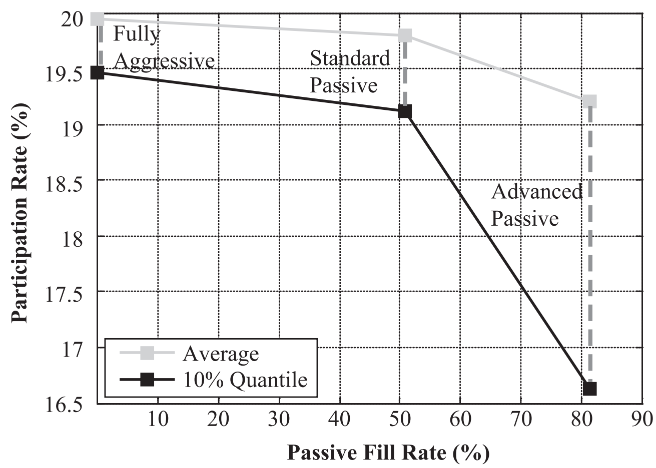
While our advanced passive posting strategy benefits from capturing the spread more often, it comes at the expense of closely tracking the participation rate. In effect, this strategy may not immediately catch up with the executed volume target, as a passive posting always induces an execution risk.

Exhibit 16 shows the relationship between the passive fill rate and the effective participation rate for our three strategies. The participation rates as a percentage are shown on the $y$-axis and the passive fill rate, also as a percentage, on the $x$-axis. For all three strategies, we have plotted both the average participation versus the average passive fill rate (in gray) and the 10% quantile of the participation rate versus the average passive fill rate (in black).

We see in Exhibit 16 that the fully aggressive strategy has, by definition, a 0% passive fill rate, a 19.9% average participation rate, and a 19.5%, 10% quantile, which is very close to the 20% initial target. The standard passive strategy has a 50% average passive fill rate, a 19.8% average participation rate, and a 10% quantile of 19.1%. The advanced passive strategy has an 81% average passive fill rate, a 19.2% average participation rate, and

## E X H I B I T 16

**Strategies' Participation Rates Related to their Passive Fill Rates**




*Source: Stoxx Europe 50, February 2016.*

16.6% in the 10% worst cases. Thus, as expected, the more passive the strategy, the larger the fluctuations in the participation rate. A slight widening, however, in the final participation range can induce a much-improved performance, from a −0.33 spread to a −0.06 spread using our advanced passive posting strategy.

These results will be implemented in new generations of forward-looking POV and VWAP algorithms to enable investors to benefit in terms of execution performances from the predictive power of the order book imbalance that we exhibited in this article. We are currently completing this work for Kepler Cheuvreux's algorithms.

## A P P E N D I X

### FINDING THE "NEXT TRADE": HISTORICAL DATASET AND CONSISTENCY

This article mainly focuses on the 600 largest European capitalizations (members of the DJ STOXX 600) over the month of February 2016. It represents 28.2 million trades (primary market data only). In some charts, we highlight the behavior of mid-sized and small caps by considering members of the DJ STOXX MID 200 and the DJ STOXX SMALL 200 indexes compared with members of the DJ STOXX LARGE 200. These results are estimated on primary markets, but they hold true for multilateral trading facilities as well.

We examine the question of forecasting the next aggressive trade, knowing the previous state of the order book. It is thus particularly important to determine if two consecutive trades result from the same aggressive order or if they result from two independent aggressive trades, each corresponding to a different state of the order book. In effect, a single "large" aggressive trade can be matched to several smaller orders, which sometimes generate multiple simultaneous trades in our dataset. However, the same large, aggressive trade induced these multiple trades and therefore the multiple trades reacted to the same state of the order book. In these cases, the splitting of the initial trade into smaller trades is somewhat artificial. To address this issue, we aggregated all trades that occurred within 100 microseconds on the same side. In so doing, the "next trade" refers to a distinct state of the order book and is originated by a different aggressive order.

In concrete terms, with a 100-microsecond threshold, we aggregated 68% of trades on average on DJ STOXX LARGE 200 members (70% of their traded volume).

## CAVEAT

To be statistically rigorous, it should be noted that in this article, we based our estimations on the state of the last available order book before a trade. As such, it represents a specific sample of all order book updates that can be observed during trading hours. Therefore, the estimations of the probabilities concerning the next aggressive trade are not theoretically fully transposable to those one could draw from any order book state.

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