Bayesian analysis of VARs, State-space models and DSGEs Part V - DSGEs

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Lecture overview

- **A DSGE model**
- **The likelihood function**
- **Bayesian inference**
- **Words of wisdom**

Affärsvärlden om Ramses I, Maj 2005

INANS | RÄNTAN

Ekvationer på hal is

Riksbankens nylanserade ekonomiska modell som ska underlätta prognosarbetet och ligga till grund för räntepolitiken visar att dagens svenska styrränta på två procent är alldeles för låg. Nu är risken stor att riksbanksdirektionen faktiskt litat på modellresultaten och därför inte har sänkt räntan.

en stora faran för svenska bolånare ommer inte från att inflationen plötsligt a ta fart, lönerörelsen skena eller att onjunkturen blir så stark att Riksbanken ingas höja räntan.

Nej, den stora faran tycks vara att riksanksledningen börjar lita på resultaten än den ekonomiska modell som ban- der den här perioden försvunnit. Nu tycks det som om Riksbanken i högre grad har satsat på akademisk kompetens i stället för prognoskunnande när man nyrekryterat med superakademikern Anders Vredin i spetsen. Resultatet av den satsningen syns i den ekonomiska modell (DSGE) som den ekonomiska avdelningen på Riksban-

A DSGE MODEL (RAMSES I)

- **Small open economy**
- **15** observed variables
- **Many state variables**
- > 50 'deep' parameters.
- **Model parameters**:
  - Steady state parameters (calibrated in Ramses)
  - **Frictions**: ξ_w, ξ_d,b,...
  - **Shock processes**: ρ_z, σ_z^2,...
  - **Policy parameters** r_π, σ_R,...
  - VAR model for the exogenous variables (estimated separately)

Parameters in RAMSES I

Parameter	Prior distribution		
	Type	Mean	Std. dev./df
Calvo wages ξ_w	beta	0.750	0.050
Calvo domestic prices ξ_d	beta	0.750	0.050
Calvo import cons. prices $\xi_{m,c}$	beta	0.750	0.050
Calvo import inv. prices $\xi_{m,i}$	beta	0.750	0.050
Calvo export prices ξ_x	beta	0.750	0.050
Indexationwages κ_w	beta	0.500	0.150
Indexation prices κ_d	beta	0.500	0.150
Markup domestic λ_d	truncnormal	1.200	0.050
Markup imported cons. $\lambda_{m,c}$	truncnormal	1.200	0.050
Markup imported invest. $\lambda_{m,i}$	truncnormal	1.200	0.050
Investment adj. cost $\tilde{S}''$	normal	7.694	1.500
Habit formation b	beta	0.650	0.100
Subst. elasticity invest. η_i	invgamma	1.500	4
Subst. elasticity foreign η_f	invgamma	1.500	4
Technology growth μ_z	truncnormal	1.006	0.0005
Risk premium $\tilde{\phi}_a$	invgamma	0.010	2
UIP modification $\tilde{\phi}_s$	beta	0.500	0.15
Unit root tech. shock ρ_{μ_z}	beta	0.850	0.100
Stationary tech. shock ρ_{ε}	beta	0.850	0.100
Invest. spec. tech shock ρ_{Υ}	beta	0.850	0.100
Asymmetrictech. shock $\rho_{\tilde{z}*}$	beta	0.850	0.100
Consumption pref. shock ρ_{ζ_c}	beta	0.850	0.100
Labor supply shock ρ_{ζ_h}	beta	0.850	0.100
Risk premium shock $\rho_{\tilde{\phi}}$	beta	0.850	0.100
Unit root tech. shock σ_{μ_z}	invgamma	0.200	2
Stationary tech. shock σ_{ε}	invgamma	0.700	2
Invest. spec. tech. shock σ_{Υ}	invgamma	0.200	2
Asymmetric tech. shock $\sigma_{\tilde{z}*}$	invgamma	0.400	2
Consumption pref. shock σ_{ζ_c}	invgamma	0.200	2
Labor supply shock σ_{ζ_h}	invgamma	1.000	2
Risk premium shock $\sigma_{\tilde{\phi}}$	invgamma	0.050	2
Domestic markup shock σ_{λ_d}	invgamma	1.000	2
Imp. cons. markup shock $\sigma_{\lambda_{m,c}}$	invgamma	1.000	2
Imp.invest.markupshock $\sigma_{\lambda_{m,i}}$	invgamma	1.000	2
Export markup shock σ_{λ_x}	invgamma	1.000	2
Interest rate smoothing $\rho_{R,1}$	beta	0.800	0.050
Inflation response $r_{\pi,1}$	truncnormal	1.700	0.100
Diff. infl response $r_{\Delta\pi,1}$	normal	0.300	0.050
Real exch. rate response $r_{x,1}$	normal	0.000	0.050
Nominal exch. response r_S	normal	100	10
Output response $r_{y,1}$	normal	0.125	0.050
Diff. output response $r_{\Delta y,1}$	normal	0.063	0.050
Monetary policy shock $\sigma_{R,1}$	invgamma	0.150	2
Inflation target shock $\sigma_{\bar{\pi}^c,1}$	invgamma	0.050	2
Interest rate smoothing $\rho_{R,2}$	beta	0.800	0.050
Inflation response $r_{\pi,2}$	truncnormal	1.700	0.100
Diff. infl response $r_{\Delta\pi,2}$	normal	0.300	0.050
Real exch. rate response $r_{x,2}$	normal	0.000	0.050
Output response $r_{y,2}$	normal	0.125	0.050
Diff. output response $r_{\Delta y,2}$	normal	0.063	0.050
Monetarypolicy shock $\sigma_{R,2}$	invgamma	0.150	2
Inflation target shock $\sigma_{\bar{\pi}^c,2}$	invgamma	0.050	2

THE LIKELIHOOD FUNCTION

- Given a value for the parameter vector $\theta = \tilde{\theta}$, do the following:
  - **Compute the steady-state** of the model.
  - **Solve the log-linearized model** (e.g. AIM or Sims)
  - Set up **state-space model**

$$\xi_t = \mathbf{F}(\tilde{\theta})\xi_{t-1} + \mathbf{v}_t, \quad \mathbf{v}_t \overset{iid}{\sim} N\left(0, \mathbf{Q}(\tilde{\theta})\right) \quad \text{(state transition equation)}$$

$$\mathbf{y}_t = \mathbf{H}(\tilde{\theta})'\xi_t + \mathbf{w}_t, \quad \mathbf{w}_t \overset{iid}{\sim} N\left(0, \mathbf{R}\right) \quad \text{(measurement equation)}$$

    with

    - Transitions for latent states (e.g. technology shocks)
    - Matching states to observed variables through measurement equations
    - Decide on measurement errors ($\mathbf{R}$) or estimate them.

  - **Iterate the Kalman filter** forward to compute the (marginalized) likelihood:

$$p(\mathbf{y}_{1:T}|\mathbf{F}(\theta), \mathbf{H}(\theta), \mathbf{Q}(\theta), \mathbf{R}(\theta))$$

Bayesian analysis of DSGEs

- **Set up priors** for all model parameters. Use micro data, historical macro data before the current dataset, data from other countries, expert opinions, Larry Christiano's parameter values ...
- **Bayes' theorem** (posterior $\propto$ likelihood $\times$prior)

$$p(\theta|\mathbf{y}_{1:T}) \propto p(\mathbf{y}_{1:T}|\mathbf{F}(\theta), \mathbf{H}(\theta), \mathbf{Q}(\theta), \mathbf{R}(\theta)) \cdot p(\theta)$$

- **Optimize** numerically (fminunc with BFGS update of Hessian) to obtain $\hat{\theta}_{mode}$ and Hessian H at the mode.
- Initialize MCMC at $\hat{\theta}_{mode}$ and run **random walk Metropolis algorithm** with proposal

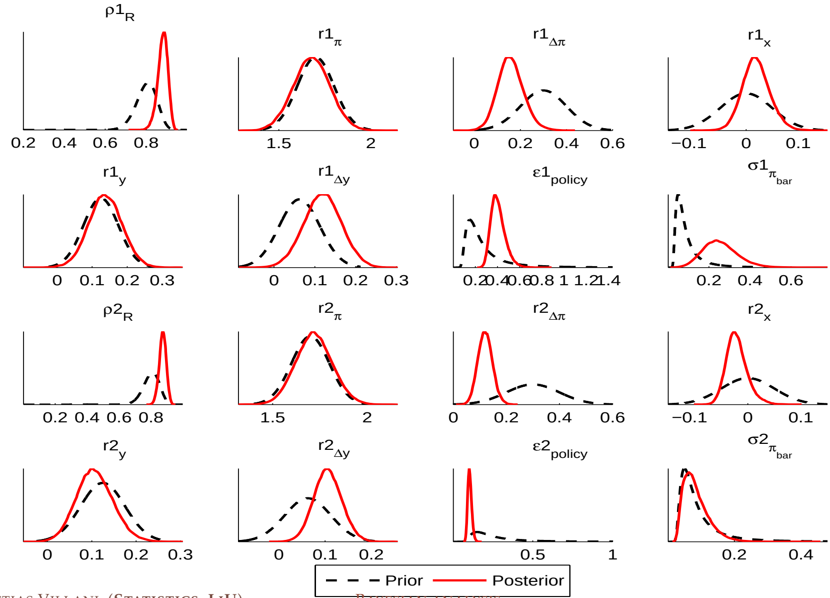
$$\theta_p|\theta^{(i-1)} \sim N\left(\theta^{(i-1)}, c \cdot \Sigma\right)$$

  with $\Sigma = -H^{-1}$ and c tuned so that accept. prob. is roughly 0.25.
- **Check for convergence**.

Bayesian analysis of DSGEs

- Compute functions of the parameter draws to approximate the posterior of other quantities (**IRs**).
- **Compute marginal likelihoods** with the modified Harmonic estimator (RWM) or Chib-Jeliazkov (IMH). Model comparison. Model averaging.
- **Predictions**.
- **Posterior predictive checks**.

Priors and posteriors - policy parameters



Prior sensitivity

Table A.6: Prior sensitivity

Parameter		Prior type	Benchmark prior						Vague prior					
			Prior distribution		Posterior distribution				Prior distribution		Posterior distribution			
			mean*	std /df	5%	mean	95%	std	mean*	std /df	5%	mean	95%	std
Calvo wages	ξ_w	beta	0.675	0.050	0.607	0.690	0.766	0.048	0.675	0.100	0.579	0.711	0.848	0.082
Calvo domestic prices	ξ_d	beta	0.675	0.050	0.862	0.891	0.921	0.018	0.675	0.100	0.934	0.961	0.981	0.015
Calvo import cons. prices	$\xi_{m,c}$	beta	0.500	0.100	0.345	0.444	0.540	0.059	0.500	0.200	0.260	0.366	0.477	0.066
Calvo import inv. prices	$\xi_{m,i}$	beta	0.500	0.100	0.641	0.721	0.792	0.046	0.500	0.200	0.965	0.985	0.996	0.011
Calvo export prices	ξ_x	beta	0.500	0.100	0.506	0.612	0.717	0.065	0.500	0.200	0.492	0.585	0.679	0.057
Calvo employment	ξ_e	beta	0.675	0.100	0.741	0.787	0.827	0.027	0.675	0.200	0.771	0.828	0.892	0.036
Indexation wages	κ_w	beta	0.500	0.150	0.258	0.497	0.739	0.145	0.500	0.200	0.118	0.378	0.689	0.173
Index. domestic prices	κ_d	beta	0.500	0.150	0.095	0.217	0.362	0.081	0.500	0.200	0.048	0.177	0.357	0.097
Index. import cons. prices	$\kappa_{m,c}$	beta	0.500	0.150	0.084	0.220	0.418	0.104	0.500	0.200	0.054	0.219	0.465	0.129
Index. import inv. prices	$\kappa_{m,i}$	beta	0.500	0.150	0.098	0.231	0.405	0.095	0.500	0.200	0.049	0.194	0.458	0.125
Indexation export prices	κ_x	beta	0.500	0.150	0.069	0.185	0.347	0.088	0.500	0.200	0.026	0.106	0.228	0.064
Markup domestic	λ_d	inv. gamma	1.200	2	1.122	1.222	1.383	0.084	1.200	2	1.126	1.248	1.463	0.109
Markup imported cons.	$\lambda_{m,c}$	inv. gamma	1.200	2	1.526	1.633	1.751	0.068	1.200	2	1.518	1.631	1.752	0.071
Markup.imported invest.	$\lambda_{m,i}$	inv. gamma	1.200	2	1.146	1.275	1.467	0.100	1.200	2	1.111	1.183	1.292	0.057
Investment adj. cost	$\tilde{S}''$	normal	7.694	1.500	6.368	8.670	10.958	1.396	7.694	3.000	2.793	7.047	11.488	2.644
Habit formation	b	beta	0.650	0.100	0.608	0.708	0.842	0.068	0.650	0.200	0.948	0.976	0.995	0.015
Subst. elasticity invest.	η_i	inv. gamma	1.500	4	1.393	1.696	2.142	0.235	1.500	4	1.315	1.477	1.699	0.121
Subst. elasticity foreign	η_f	inv. gamma	1.500	4	1.340	1.486	1.674	0.104	1.500	4	1.308	1.441	1.616	0.095
Technology growth	μ_z	trunc. normal	1.006	0.0005	1.004	1.005	1.006	0.000	1.006	0.001	1.004	1.005	1.005	0.001
Capital income tax	τ_k	beta	0.120	0.050	0.072	0.135	0.200	0.039	0.120	0.100	0.120	0.205	0.283	0.049
Labour pay-roll tax	τ_w	beta	0.200	0.050	0.118	0.197	0.286	0.051	0.200	0.100	0.060	0.194	0.379	0.098
Risk premium	$\tilde{\phi}$	inv. gamma	0.010	2	0.139	0.252	0.407	0.084	0.010	2	0.138	0.246	0.404	0.081

Impulse responses DSGE

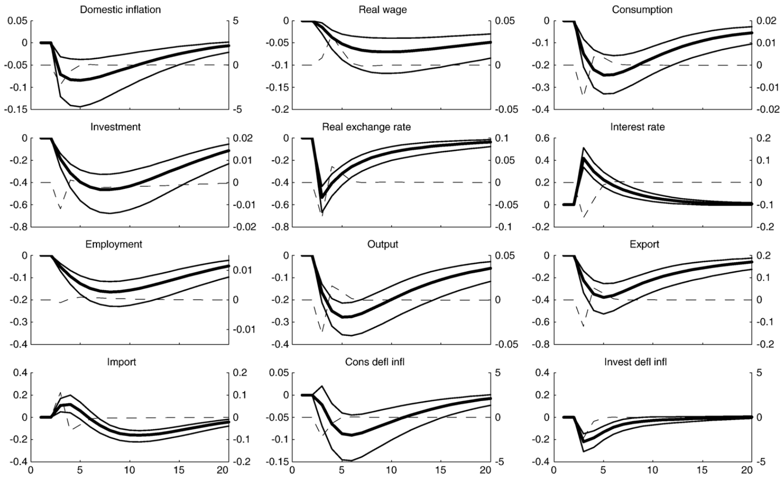
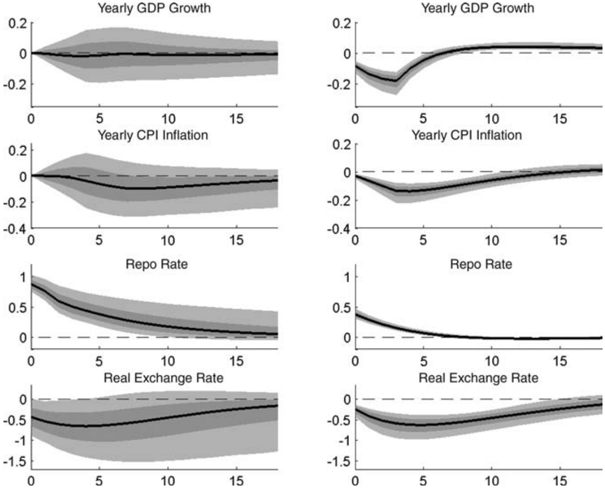


Fig. 3. Impulse responses (posterior median and 95% uncertainty intervals) to a one standard deviation monetary policy shock. Note: Benchmark (solid, left axis) and flexible prices and wages (dashed, right axis).

Impulse responses DSGE vs BVARs



Bayesian prediction with DSGEs

- **Predictive distribution** $p(\mathbf{y}_{T+1:T+h}|\mathbf{y}_{1:T})$ by simulation
  - Simulate a **parameter vector** $\tilde{\theta}$ from the posterior $p(\theta|\mathbf{y}_{1:T})$ by MCMC.
  - Draw the **current state** $\xi_T \sim N\left(\xi_{T|T}, P_{T|T}\right)$
  - Simulate **future states** for $t = T+1, ..., T+h$ from

$$\xi_t = \mathbf{F}(\tilde{\theta})\xi_{t-1} + \mathbf{v}_t, \quad \mathbf{v}_t \overset{iid}{\sim} N\left(0, \mathbf{Q}(\tilde{\theta})\right)$$

  - Simulate the **observed variables** for $t = T+1, ..., T+h$ conditional on the simulated states:

$$\mathbf{y}_t = \mathbf{H}(\tilde{\theta})'\xi_t + \mathbf{w}_t, \quad \mathbf{w}_t \overset{iid}{\sim} N\left(0, \mathbf{R}\right)$$

BAYESIAN PREDICTION WITH DSGES

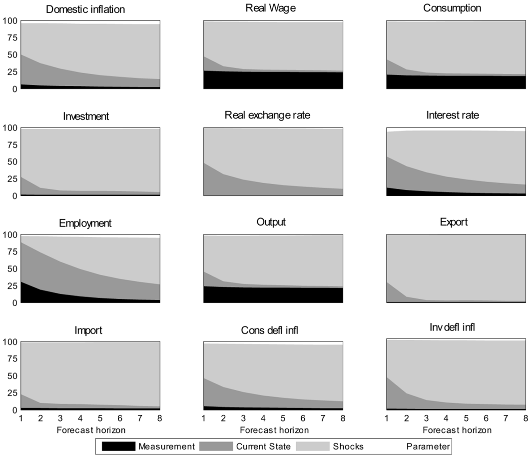


FIGURE 4 Decomposition of the forecast uncertainty. The subgraphs display the relative contribution to the predictive variances of the observed variables at different forecast horizons.

Forecasts DSGE

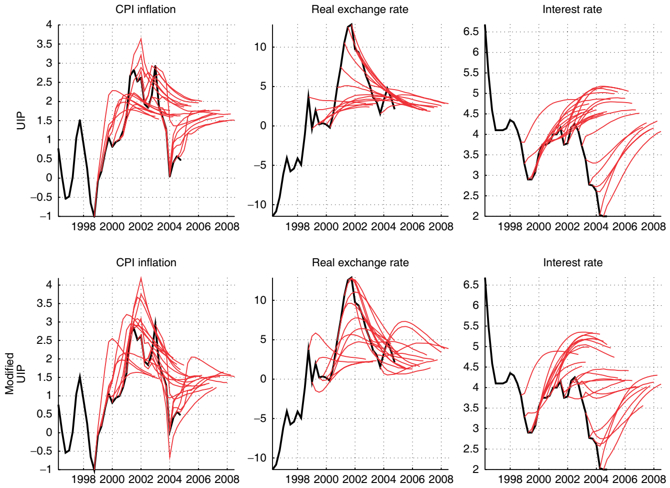
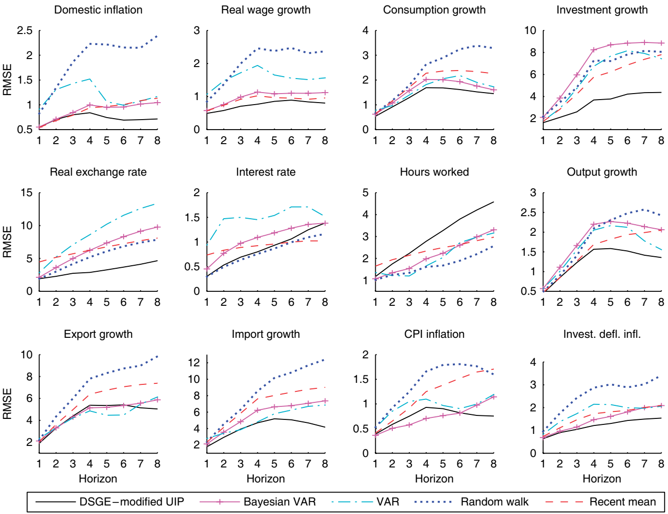


Fig. 3. Actual data (thick line) and forecasts (thin lines) 1999Q1–2004Q4 from the DSGE using different UIP specifications.

Forecast evaluation DSGE 1999Q1-2004Q4



Practical optimization

- Numerical optimization in DSGEs can be hard.
- RAMSES I started out with **fminsearch**. Derivative-free optimizer. Slow, but robust.
- Once fminsearch has obtained a decent point, switch to **fminunc** with **BFGS** update of Hessian. Less robust, but fast and reliable enough when you are not to far from mode.
- Repeat the fminsearch/fminunc procedure with **different starting values** to check for local modes.
- Once the mode has been reached in a benchmark model, **alternative specifications** can use that mode as good initial values and converges fast.

Practical optimization

- **Check the quality of the Hessian** by:
  1. Computing the exact log posterior by perturbing each parameter (one at a time). This produces a **slice** of the log posterior along each of the parameters.
  2. Compute the following approximation of the posterior from the optimization output:

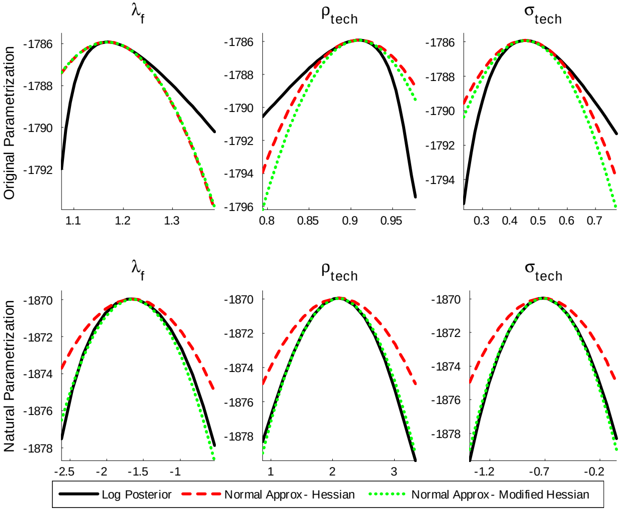
     $$\theta|\mathbf{y}_{1:T} \sim N\left(\hat{\theta}_{mode}, \Sigma\right)$$

     where $\Sigma = -H^{-1}$.
  3. **Slice the approximate posterior along the jth parameter**

     $$p(\theta_j|\mathbf{y}_{1:T}) \approx N\left(\hat{\theta}_{j,mode}, s_j^2\right)$$

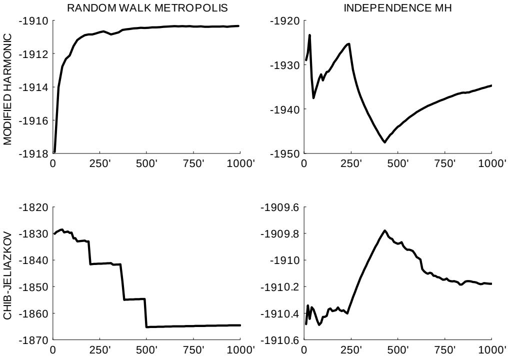
     where s_j^2 is the (approximate) posterior variance of θ_j **conditional on** the other parameters being at their mode (computed from Σ by a simple formula).
  4. Plot the slices from 1 and 3 in the same graph, one for each parameter.

Slicing the posterior



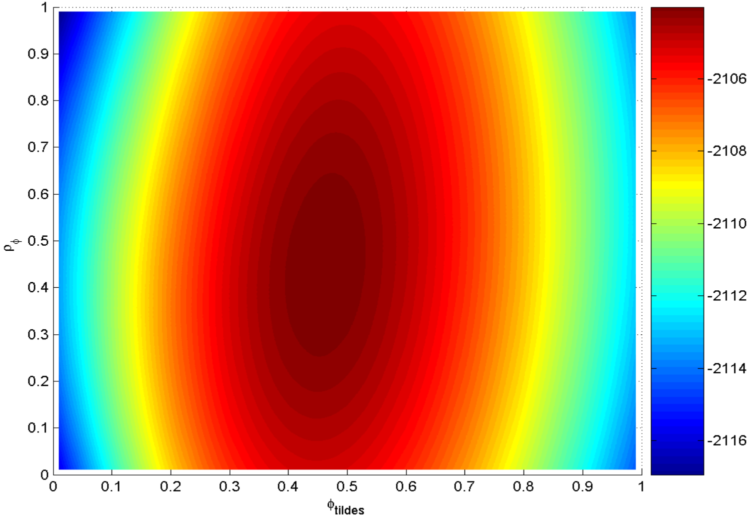
Marginal likelihood estimation

- The best marginal likelihood estimator depends on the MCMC algorithm [1]
  - **Modified harmonic estimator** works well with Random Walk Metropolis
  - Chib-Jeliazkov estimator works well with independence MH



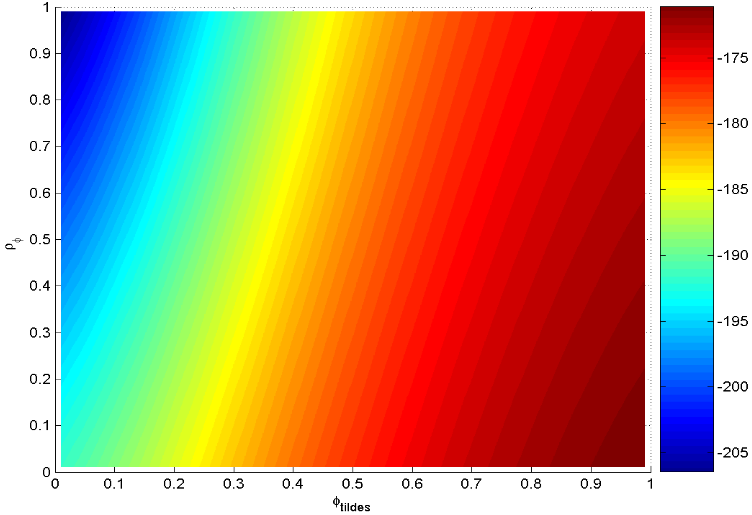
Choosing your measurements

Figure A.6a: Log likelihood contours in the $\{\widetilde{\phi}_s, \rho_\phi\}$-space, using all observable variables



Choosing your measurements

Figure A.6b: Log likelihood contours in the $\{\widetilde{\phi}_s,\ \rho_\phi\}$-space, only using the real exchange rate



Comments

- RAMSES I used **calibrated measurement error variances** (diagonal **R**). Ad hoc ... Calibrated measurement errors should depend on the properties of the measured variables. But, results were not very sensitive to the calibrated values.
- We also tried to estimate **R** (assuming it to be diagonal). Worked, but didn't change the posterior of the deep parameters very much.
- Measurement errors **do** have a crucial effect on the **marginal likelihood** comparison to reduced form models such as BVARs (and DSGE-VARs) [2].
- Posteriors can be bimodal (intrinsic and extrinsic frictions can produce similar fits). Not a problem per se, but the MCMC needs to visit both modes in correct proportions.

Comments

- DSGEs are relatively misspecified models. Marginal likelihoods are not (so) useful. Impulse responses, predictive checks and traditional forecasting evaluations more relevant.
- Multivariate measures of forecasting performance (log determinant MSFE matrix) can be **very** sensitive to poor predicitions in **very** specific directions of the data. [3]

PREDITIVE CHECK DSGE

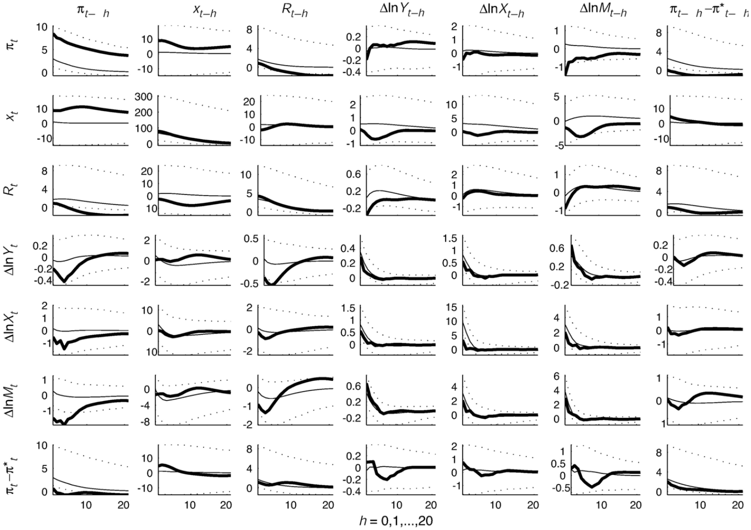


Fig. 2. Autocovariance functions in the data (thick) and the DSGE model (posterior predictive median; thin, and 95% posterior probability intervals; dotted).

M. Adolfson, J. Lindé, and M. Villani, "Bayesian analysis of dsge models - some comments," *Econometric Reviews*, vol. 26, no. 2-4, pp. 173–185, 2007.

M. Adolfson, S. Laséen, J. Lindé, and M. Villani, "Evaluating an estimated new keynesian small open economy model," *Journal of Economic Dynamics and Control*, vol. 32, no. 8, pp. 2690–2721, 2008.

M. Adolfson, J. Lindé, and M. Villani, "Forecasting performance of an open economy dsge model," *Econometric Reviews*, vol. 26, no. 2-4, pp. 289–328, 2007.