settings

functions

initial parameters

```
In[∘]:= numberOfContainers = 8; (*количество контейнеров*)
        numberOfVehicles = 2;(*количество машин*)
        numberOfStacks = 4;(*количество стеков*)

In[∘]:= ϵ = 0.1;
        shareOfPairsBlocks = 0.05;
```

generate initial data

```
In[∘]:= vertices1 = Range@numberOfContainers;(*множество вершин V1*)
        vertices2 = vertices1 + numberOfContainers;(*множество вершин V2*)

In[∘]:= {pointsVertices1, pointsVertices2} = Table[RandomReal[{1, 5}, {numberOfContainers, 2}], 2];
        (*начальное и конечное положение контейнеров, случайные позиции на плоскости*)
        distanceMatrix = DistanceMatrix[pointsVertices1, pointsVertices2];
        (*временные затраты на перемещение между слоями*)

In[∘]:= δ1 = δ2 = Ceiling@Mean@Flatten@distanceMatrix;

In[∘]:= outArcs = MapThread[DirectedEdge, {vertices1, vertices2}] ;
        (*исходящие вершины первого слоя*)
        inArcs = DeleteCases[Flatten[Outer[DirectedEdge, vertices2, vertices1]],
           i_ •→ j_ /; i - numberOfContainers == j];(*входящие вершины первого слоя*)
        startArcs = Thread[DirectedEdge[0, vertices1]];(*выезд из 0 вершины*)
        endArcs = Thread[DirectedEdge[vertices2, 2 numberOfContainers + 1]] ;
        (*въезд в вершину 2n+1*)
        arcs = Join[startArcs, inArcs, outArcs, endArcs];(*множество всех дуг в графе*)

In[∘]:= stacks = TakeList[RandomSample@vertices1,
           getVarianteTwoArg[numberOfContainers, numberOfStacks]];(*разбитие вершин на стеки*)

In[∘]:= {pairsVertices1, pairsVertices2} =
          RandomSample[#, Floor[shareOfPairsBlocks Length@#]] &@Subsets[#, {2}] & /@
           {vertices1, vertices2}; (*множества PV1,PV2*)

In[∘]:= K = numberOfVehicles (Total[Max /@ distanceMatrix] + Total[Max /@ distanceMatrix^T]);
```

Переменные

```
In[◦]:= varsX2 = Flatten[x[#⟦1⟧, #⟦2⟧] & /@ Join[inArcs, outArcs]];
        varsX3 = Sort[Flatten[
            Table[Sort[x[#⟦1⟧, #⟦2⟧, k] & /@ Join[startArcs, endArcs]], {k, numberOfVehicles}]]];
        varsX = Join[varsX2, varsX3];
```

```
In[◦]:= varsU1 = u[#] & /@ Join[vertices1, vertices2];
        lstu2 = u[#] & /@ {0, 2 * numberOfContainers + 1};
        varsU2 = Sort[Table[Append[lstu2⟦i⟧, k], {i, Length@lstu2}, {k, 1, numberOfVehicles}]];
        varsU = Join[varsU1, Flatten@varsU2];
```

```
In[◦]:= varsy1 = y1[#⟦1⟧, #⟦2⟧] & /@ pairsVertices1;
        varsy2 = y2[#⟦1⟧, #⟦2⟧] & /@ pairsVertices2;
```

```
In[◦]:= vars = Join[varsX, varsU, varsy1, varsy2, {umax}];
```

Критериии оптимизации

```
In[◦]:= objFun1 = Total[Last@varsU2 - First@varsU2];
        objFun2 = umax;
```

```
In[◦]:= weights = {0.5, 0.5};
        objFun = Dot[weights, {objFun1, objFun2}];
        c = Last@CoefficientArrays[objFun, vars];
```

Ограничения

```
In[◦]:= con1 = Table[Total[Cases[varsX2, x[i, _]]], {i, vertices1}];
        rhs1 = ConstantArray[{1, 0}, Length@con1];
```

```
In[◦]:= con2 = Table[Total[Cases[varsX3, x[0, _, k]]], {k, numberOfVehicles}];
        rhs2 = ConstantArray[{1, -1}, Length@con2];
```

```
In[◦]:= con3 =
          Table[Total[Cases[varsX3, x[0, i, _]]] + Total[Cases[varsX2, x[_, i]]], {i, vertices1}];
        rhs3 = ConstantArray[{1, 0}, Length@con3];
```

```
In[◦]:= con4 = Table[Total[Cases[varsX3, x[i, 2 * numberOfContainers + 1, _]]] +
            Total[Cases[varsX2, x[i, _]]], {i, vertices2}];
        rhs4 = ConstantArray[{1, 0}, Length@con4];
```

```
In[◦]:= con5 = Table[Total[Cases[varsX3, x[0, _, k]]] -
            Total[Cases[varsX3, x[_, 2 * numberOfContainers + 1, k]]], {k, numberOfVehicles}];
        rhs5 = ConstantArray[{0, 0}, Length@con5];
```

```
In[◦]:= con6 = umax - Last@varsU2;
        rhs6 = ConstantArray[{0, 1}, Length@con6];
```

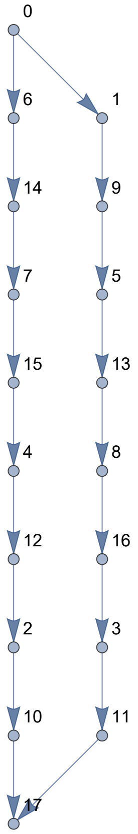
```
In[•]:= con7 = Flatten@Table[Cases[Flatten@varsU2, u[0, k]] -
             K * Total[Cases[varsX3, x[0, _, k]]], {k, numberOfVehicles}];
        rhs7 = ConstantArray[{0, -1}, Length@con7];

In[•]:= pairsOfPriorInStack = Flatten[Subsets[#, {2}] & /@ stacks, 1];
        con8 = Flatten[Cases[varsU1, u[#⟦1⟧]] - Cases[varsU1, u[#⟦2⟧]] & /@ pairsOfPriorInStack];
        rhs8 = ConstantArray[{δ1, 1}, Length@con8];

In[•]:= con9 = Flatten[
           -Cases[varsU1, u[#⟦1⟧]] + Cases[varsU1, u[#⟦2⟧]] - K * Cases[varsy1, y1[#⟦1⟧, #⟦2⟧]] & /@
            pairsVertices1];
        rhs9 = ConstantArray[{-δ2, -1}, Length@con9];

In[•]:= con10 = Flatten[
           Cases[varsU1, u[#⟦1⟧]] - Cases[varsU1, u[#⟦2⟧]] + Cases[varsy1, y1[#⟦1⟧, #⟦2⟧]] & /@
            pairsVertices1];
        rhs10 = ConstantArray[{K - δ2, -1}, Length@con10];

In[•]:= con11 = Flatten[
           -Cases[varsU1, u[#⟦1⟧]] + Cases[varsU1, u[#⟦2⟧]] - K * Cases[varsy2, y2[#⟦1⟧, #⟦2⟧]] & /@
            pairsVertices2];
        rhs11 = ConstantArray[{-δ2, -1}, Length@con11];

In[•]:= con12 = Flatten[
           Cases[varsU1, u[#⟦1⟧]] - Cases[varsU1, u[#⟦2⟧]] + Cases[varsy2, y2[#⟦1⟧, #⟦2⟧]] & /@
            pairsVertices2];
        rhs12 = ConstantArray[{K - δ2, -1}, Length@con12];

In[•]:= con13 = Flatten[
           -Cases[varsU1, u[#⟦1⟧]] + Cases[varsU1, u[#⟦2⟧]] - K * Cases[varsX2, x[#⟦1⟧, #⟦2⟧]] & /@
            Join[inArcs, outArcs]];
        rhs13 = Which[#⟦2⟧ > numberOfContainers,
             {distanceMatrix⟦#⟦1⟧⟧⟦#⟦2⟧ - numberOfContainers⟧ - K, 1}, #⟦1⟧ > numberOfContainers,
             {distanceMatrix⟦#⟦2⟧⟧⟦#⟦1⟧ - numberOfContainers⟧ - K, 1}] & /@ Join[inArcs, outArcs];

In[•]:= con14 = Flatten@
           Table[-Cases[varsU1, u[#⟦1⟧]] + Cases[Flatten@varsU2, u[2 * numberOfContainers + 1, k]] -
               K * Cases[varsX3, x[#⟦1⟧, 2 * numberOfContainers + 1, k]] & /@
             endArcs, {k, numberOfVehicles}];
        rhs14 = ConstantArray[{-K, 1}, Length@con14];

In[•]:= con15 = Flatten@Table[Cases[varsU1, u[#⟦2⟧]] - Cases[Flatten@varsU2, u[0, k]] -
               K * Cases[varsX3, x[0, #⟦2⟧, k]] & /@ startArcs, {k, numberOfVehicles}];
        rhs15 = ConstantArray[{-K, 1}, Length@con15];
```

LinearProgramming

```
In[∘]:= lu = Join[ConstantArray[{0, 1}, Length@varsX],
           ConstantArray[{0, K}, Length@Flatten[varsU]], ConstantArray[{0, 1}, Length@varsy1],
           ConstantArray[{0, 1}, Length@varsy2], {{0, K}}];
        domain = Join[ConstantArray[Integers, Length@varsX],
           ConstantArray[Reals, Length@Flatten[varsU]], ConstantArray[Integers, Length@varsy1],
           ConstantArray[Integers, Length@varsy2], {Reals}];
        matr = Last@CoefficientArrays[Join[con1, con2, con3, con4, con5, con6,
              con7, con8, con9, con10, con11, con12, con13, con14, con15], vars];
        bb = Join[rhs1, rhs2, rhs3, rhs4, rhs5, rhs6, rhs7, rhs8, rhs9,
           rhs10, rhs11, rhs12, rhs13, rhs14, rhs15];

In[∘]:= sol = LinearProgramming[c, matr, bb, lu, domain]

Out[∘]= {0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
         0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 1, 1, 1, 1, 1, 1, 1,
         0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0,
         0, 0., 11.1436, 11.5484, 7.17656, 3., 0., 4.83245, 7.16776, 0.92969, 12.6159, 13.5586,
         9.71758, 5.34421, 3.37338, 7.11979, 11.419, 0., 0., 12.6159, 13.5586, 1, 1, 13.5586}

In[∘]:= x2 = Sort[DeleteCases[sol⟦ ;; Length@varsX2⟧ * varsX2, 0]]

Out[∘]= {x[1, 9], x[2, 10], x[3, 11], x[4, 12], x[5, 13], x[6, 14], x[7, 15],
         x[8, 16], x[9, 5], x[12, 2], x[13, 8], x[14, 7], x[15, 4], x[16, 3]}

In[∘]:= x3 = DeleteCases[sol⟦Length@varsX2 + 1 ;; Length@varsX2 + Length@varsX3⟧ * varsX3, 0];
        GroupBy[x3, Last]

Out[∘]= <|2 → {x[0, 1, 2], x[11, 17, 2]}, 1 → {x[0, 6, 1], x[10, 17, 1]}|>

In[∘]:= vert = Join[{#⟦1⟧, #⟦2⟧} & /@ x2, {#⟦1⟧, #⟦2⟧} & /@ x3];
```

In[•]:=
```
Graph[#⟦1⟧ ⟶ #⟦2⟧ & /@ vert, VertexLabels → "Name"]
```

Out[•]=



GurobiOptimization

In[•]:=
```
Get[StringJoin[NotebookDirectory[], "\\Gurobi-main\\GurobiOptimization.wl"]];
directory = "C:\\gurobi912\\win64\\bin\\";
```

In[•]:=
```
solGurobi = GurobiOptimization[Normal@c, Normal@matr, bb, lu, domain, directory];
```

In[•]:=
```
solGurobi
```

Out[•]=
```
{0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0,
 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1,
 1, 1, 1, 1, 1, 1, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0,
 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 11.145, 11.55, 7.178, 3., 0, 4.833, 7.168, 0.93,
 12.618, 13.56, 9.719, 5.344, 3.374, 7.121, 11.42, 0, 0, 13.56, 12.618, 1, 1, 13.56}
```