Problem Definitions and Evaluation Criteria for the CEC 2005 Special Session on Real-Parameter Optimization

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In the past two decades, different kinds of optimization algorithms have been designed and applied to solve real-parameter function optimization problems. Some of the popular approaches are real-parameter EAs, evolution strategies (ES), differential evolution (DE), particle swarm optimization (PSO), evolutionary programming (EP), classical methods such as quasi-Newton method (QN), hybrid evolutionary-classical methods, other non-evolutionary methods such as simulated annealing (SA), tabu search (TS) and others. Under each category, there exist many different methods varying in their operators and working principles, such as correlated ES and CMA-ES. In most such studies, a subset of the standard test problems (Sphere, Schwefel's, Rosenbrock's, Rastrigin's, etc.) is considered. Although some comparisons are made in some research studies, often they are confusing and limited to the test problems used in the study. In some occasions, the test problem and chosen algorithm are complementary to each other and the same algorithm may not work in other problems that well. There is definitely a need of evaluating these methods in a more systematic manner by specifying a common termination criterion, size of problems, initialization scheme, linkages/rotation, etc. There is also a need to perform a scalability study demonstrating how the running time/evaluations increase with an increase in the problem size. We would also like to include some real world problems in our standard test suite with codes/executables.

In this report, 25 benchmark functions are given and experiments are conducted on some real-parameter optimization algorithms. The codes in Matlab, C and Java for them could be found at http://www.ntu.edu.sg/home/EPNSugan/. The mathematical formulas and properties of these functions are described in Section 2. In Section 3, the evaluation criteria are given. Some notes are given in Section 4.

1. Summary of the 25 CEC'05 Test Functions

- **Unimodal Functions (5):**
  - F_1: Shifted Sphere Function
  - F_2: Shifted Schwefel's Problem 1.2
  - F_3: Shifted Rotated High Conditioned Elliptic Function
  - F_4: Shifted Schwefel's Problem 1.2 with Noise in Fitness
  - F_5: Schwefel's Problem 2.6 with Global Optimum on Bounds

- **Multimodal Functions (20):**
  - **Basic Functions** (7):
    - F_6: Shifted Rosenbrock's Function
    - F_7: Shifted Rotated Griewank's Function without Bounds
    - F_8: Shifted Rotated Ackley's Function with Global Optimum on Bounds
    - F_9: Shifted Rastrigin's Function
    - F_{10}: Shifted Rotated Rastrigin's Function
    - F_{11}: Shifted Rotated Weierstrass Function
    - F_{12}: Schwefel's Problem 2.13
  - **Expanded Functions** (2):

- F_{13}: Expanded Extended Griewank's plus Rosenbrock's Function (F8F2)
- F_{14}: Shifted Rotated Expanded Scaffer's F6

- **Hybrid Composition Functions** (11):
  - F_{15}: Hybrid Composition Function
  - F_{16}: Rotated Hybrid Composition Function
  - F_{17}: Rotated Hybrid Composition Function with Noise in Fitness
  - F_{18}: Rotated Hybrid Composition Function
  - F_{19}: Rotated Hybrid Composition Function with a Narrow Basin for the Global Optimum
  - F_{20}: Rotated Hybrid Composition Function with the Global Optimum on the Bounds
  - F_{21}: Rotated Hybrid Composition Function
  - F_{22}: Rotated Hybrid Composition Function with High Condition Number Matrix
  - F_{23}: Non-Continuous Rotated Hybrid Composition Function
  - F_{24}: Rotated Hybrid Composition Function
  - F_{25}: Rotated Hybrid Composition Function without Bounds

- **Pseudo-Real Problems:** Available from `http://www.cs.colostate.edu/~genitor/functions.html`. If you have any queries on these problems, please contact Professor Darrell Whitley. Email: whitley@CS.ColoState.EDU

2. Definitions of the 25 CEC'05 Test Functions

2.1 Unimodal Functions:

2.1.1. F_1: *Shifted Sphere Function*

$$F_1(\mathbf{x}) = \sum_{i=1}^{D} z_i^{\,2} + f\_bias_1 \,,\, \mathbf{z} = \mathbf{x} - \mathbf{o} \,,\, \mathbf{x} = [x_1, x_2, ..., x_D]$$

D: dimensions. $\mathbf{o} = [o_1, o_2, ..., o_D]$: the shifted global optimum.

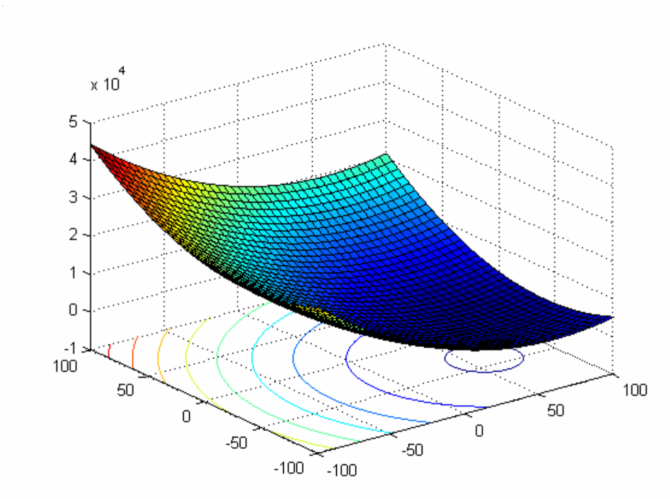

Figure 2-1 **3-*D*** map for 2-*D* function

Properties:

- Unimodal
- Shifted
- Separable
- Scalable
- $\mathbf{x} \in [-100,100]^D$, Global optimum: $\mathbf{x}^* = \mathbf{o}$, $F_1(\mathbf{x}^*) = f\_bias_1 = -450$

Associated Data files:

Name: sphere_func_data.mat
sphere_func_data.txt
Variable: **o** 1*100 vector the shifted global optimum
When using, cut **o**=**o**(1:*D*)

Name: fbias_data.mat
fbias_data.txt
Variable: **f_bias** 1*25 vector, record all the 25 function's f_bias_i

2.1.2. F_2: *Shifted Schwefel's Problem 1.2*

$$F_2(\mathbf{x}) = \sum_{i=1}^{D}(\sum_{j=1}^{i} z_j)^2 + f\_bias_2\,,\ \mathbf{z} = \mathbf{x} - \mathbf{o}\,, \mathbf{x} = [x_1, x_2, ..., x_D]$$

D: dimensions

$\mathbf{o} = [o_1, o_2, ..., o_D]$: the shifted global optimum

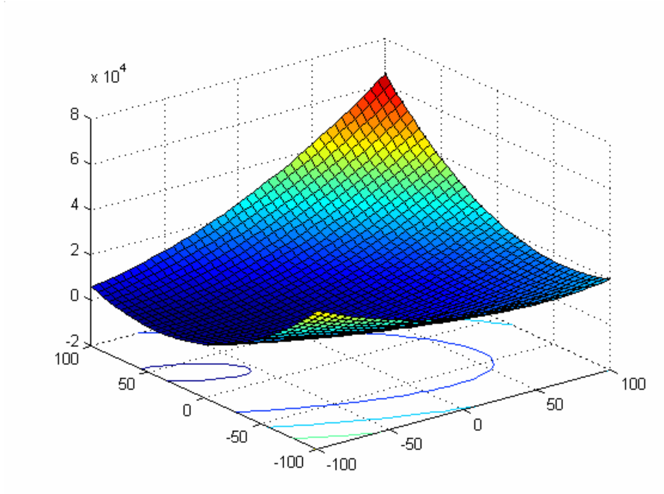

Figure 2-2 3-*D* map for 2-*D* function

Properties:

- Unimodal
- Shifted
- Non-separable
- Scalable
- $\mathbf{x} \in [-100,100]^D$, Global optimum $\mathbf{x}^* = \mathbf{o}$, $F_2(\mathbf{x}^*) = f\_bias_2 = -450$

Associated Data files:

Name: schwefel_102_data.mat
schwefel_102_data.txt

Variable: **o** 1*100 vector the shifted global optimum
When using, cut **o**=**o**(1:*D*)

2.1.3. F_3: *Shifted Rotated High Conditioned Elliptic Function*

$$F_3(\mathbf{x}) = \sum_{i=1}^{D} (10^6)^{\frac{i-1}{D-1}} z_i^2 + f\_bias_3, \ \mathbf{z} = (\mathbf{x} - \mathbf{o}) * \mathbf{M}, \mathbf{x} = [x_1, x_2, ..., x_D]$$

D: dimensions

$\mathbf{o} = [o_1, o_2, ..., o_D]$: the shifted global optimum

M: orthogonal matrix

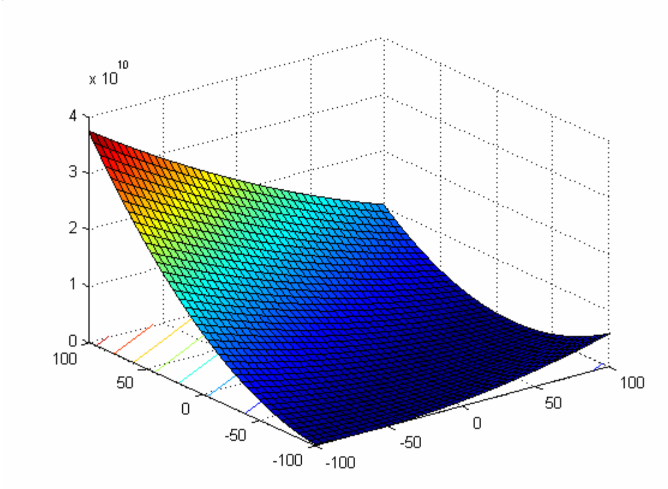

Figure 2-3 3-*D* map for 2-*D* function

Properties:

- Unimodal
- Shifted
- Rotated
- Non-separable
- Scalable
- $\mathbf{x} \in [-100,100]^D$, Global optimum $\mathbf{x}^* = \mathbf{o}$, $F_3(\mathbf{x}^*) = f\_bias_3$ =- 450

Associated Data files:

Name:	high_cond_elliptic_rot_data.mat		
	high_cond_elliptic_rot_data.txt		
Variable:	**o**	1*100 vector	the shifted global optimum
	When using, cut **o**=**o**(1:*D*)		
Name:	elliptic_M_D10 .mat		elliptic_M_D10 .txt
Variable:	**M**	10*10 matrix	
Name:	elliptic_M_D30 .mat		elliptic_M_D30 .txt
Variable:	**M**	30*30 matrix	
Name:	elliptic_M_D50 .mat		elliptic_M_D50 .txt
Variable:	**M**	50*50 matrix	

2.1.4. F_4: *Shifted Schwefel's Problem 1.2 with Noise in Fitness*

$$F_4(\mathbf{x}) = (\sum_{i=1}^{D}(\sum_{j=1}^{i} z_j)^2)*(1+0.4|N(0,1)|) + f\_bias_4,\ \mathbf{z} = \mathbf{x} - \mathbf{o},\ \mathbf{x} = [x_1, x_2, ..., x_D]$$

D: dimensions

$\mathbf{o} = [o_1, o_2, ..., o_D]$: the shifted global optimum

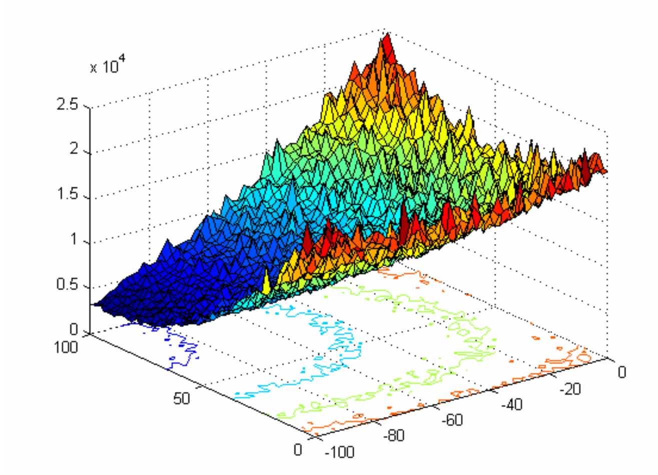

Figure 2-4 3-*D* map for 2-*D* function

Properties:

- Unimodal
- Shifted
- Non-separable
- Scalable
- Noise in fitness
- $\mathbf{x} \in [-100,100]^D$, Global optimum $\mathbf{x}^* = \mathbf{o}$, $F_4(\mathbf{x}^*) = f\_bias_4 = -450$

Associated Data file:

Name: schwefel_102_data.mat
schwefel_102_data.txt

Variable: **o** 1*100 vector the shifted global optimum
When using, cut **o**=**o**(1:*D*)

2.1.5. F_5: *Schwefel's Problem 2.6 with Global Optimum on Bounds*

$f(\mathbf{x}) = \max\{|x_1 + 2x_2 - 7|, |2x_1 + x_2 - 5|\}, i = 1,...,n$, $\mathbf{x}^* = [1,3]$, $f(\mathbf{x}^*) = 0$

Extend to D dimensions:

$F_5(\mathbf{x}) = \max\{|\mathbf{A}_i\mathbf{x} - \mathbf{B}_i|\} + f\_bias_5, i = 1,...,D$, $\mathbf{x} = [x_1, x_2,..., x_D]$

D: dimensions

A is a D*D matrix, a_{ij} are integer random numbers in the range [*-500, 500*], $\det(\mathbf{A}) \neq 0$, $\mathbf{A}_i$ is the i^{th} row of **A**.

$\mathbf{B}_i = \mathbf{A}_i * \mathbf{o}$, **o** is a D*1 vector, o_i are random number in the range [-100,100]

After load the data file, set $o_i = -100$, for $i = 1,2,...,\lceil D/4 \rceil$, $o_i = 100$, for $i = \lfloor 3D/4 \rfloor,...,D$

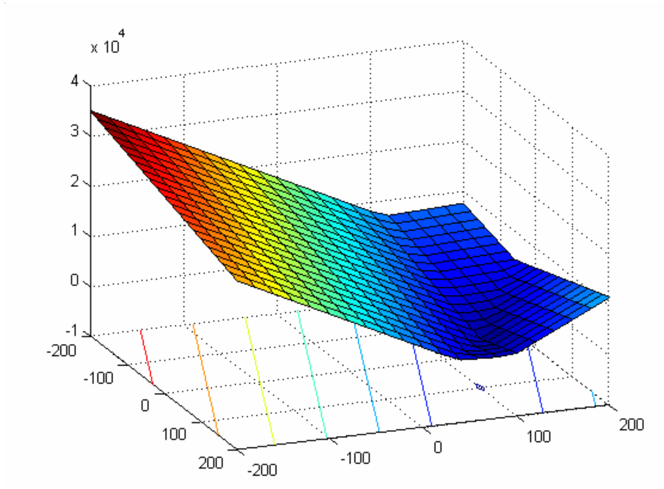

Figure 2-5 3-*D* map for 2-*D* function

Properties:

- Unimodal
- Non-separable
- Scalable
- If the initialization procedure initializes the population at the bounds, this problem will be solved easily.
- $\mathbf{x} \in [-100,100]^D$, Global optimum $\mathbf{x}^* = \mathbf{o}$, $F_5(\mathbf{x}^*) = f\_bias_5 = -310$

Associated Data file:

Name:	schwefel_206_data.mat		
	schwefel_206_data.txt		
Variable:	**o**	1*100 vector	the shifted global optimum
	A	100*100 matrix	

When using, cut **o**=**o**(1:*D*) **A**=**A**(1:*D*,1:*D*)

In schwefel_206_data.txt ,the first line is **o** (1*100 vector),and line2-line101 is **A**(100*100 matrix)

2.2 Basic Multimodal Functions

2.2.1. F_6: *Shifted Rosenbrock's Function*

$$F_6(\mathbf{x}) = \sum_{i=1}^{D-1}(100(z_i^2 - z_{i+1})^2 + (z_i - 1)^2) + f\_bias_6,\ \mathbf{z} = \mathbf{x} - \mathbf{o} + 1, \mathbf{x} = [x_1, x_2, ..., x_D]$$

D: dimensions

$\mathbf{o} = [o_1, o_2, ..., o_D]$: the shifted global optimum

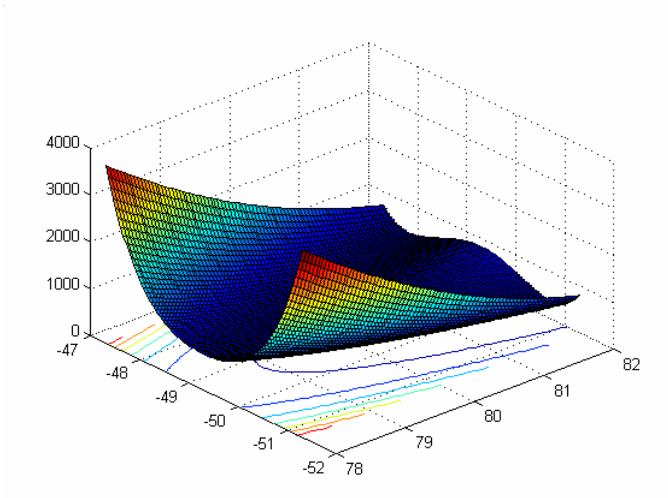

Figure 2-6 3-*D* map for 2-*D* function

Properties:

- Multi-modal
- Shifted
- Non-separable
- Scalable
- Having a very narrow valley from local optimum to global optimum
- $\mathbf{x} \in [-100,100]^D$, Global optimum $\mathbf{x}^* = \mathbf{o}$, $F_6(\mathbf{x}^*) = f\_bias_6 = 390$

Associated Data file:

Name:	rosenbrock_func_data.mat		
	rosenbrock_func_data.txt		
Variable:	**o**	1*100 vector	the shifted global optimum
	When using, cut **o**=**o**(1:*D*)		

2.2.2. *F_7: Shifted Rotated Griewank's Function without Bounds*

$$F_7(\mathbf{x}) = \sum_{i=1}^{D} \frac{z_i^{\ 2}}{4000} - \prod_{i=1}^{D} \cos(\frac{z_i}{\sqrt{i}}) + 1 + f\_bias_7 \ , \ \mathbf{z} = (\mathbf{x} - \mathbf{o}) * \mathbf{M} \, , \ \mathbf{x} = [x_1, x_2, ..., x_D]$$

D: dimensions

$\mathbf{o} = [o_1, o_2, ..., o_D]$: the shifted global optimum

M': linear transformation matrix, condition number=3

M =**M'**(1+0.3|N(0,1)|)

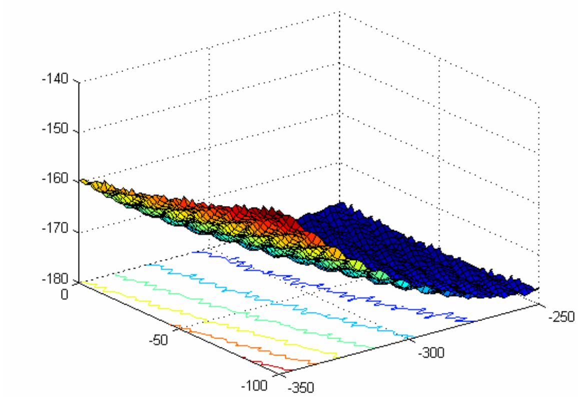

Figure 2-7 3-*D* map for 2-*D* function

Properties:

- Multi-modal
- Rotated
- Shifted
- Non-separable
- Scalable
- No bounds for variables *x*
- Initialize population in $[0, 600]^D$, Global optimum $\mathbf{x}^* = \mathbf{o}$ is outside of the initialization range, $F_7(\mathbf{x}^*) = f\_bias_7 = -180$

Associated Data file:

Name:	griewank_func_data.mat	griewank_func_data.txt
Variable:	**o** 1*100 vector When using, cut **o**=**o**(1:*D*)	the shifted global optimum
Name:	griewank_M_D10 .mat	griewank_M_D10 .txt
Variable:	**M** 10*10 matrix	
Name:	griewank_M_D30 .mat	griewank_M_D30 .txt
Variable:	**M** 30*30 matrix	
Name:	griewank_M_D50 .mat	griewank_M_D50 .txt
Variable:	**M** 50*50 matrix	

2.2.3. F_8: *Shifted Rotated Ackley's Function with Global Optimum on Bounds*

$$F_8(\mathbf{x}) = -20\exp(-0.2\sqrt{\frac{1}{D}\sum_{i=1}^{D} z_i^2}) - \exp(\frac{1}{D}\sum_{i=1}^{D}\cos(2\pi z_i)) + 20 + e + f\_bias_8, \quad \mathbf{z} = (\mathbf{x}-\mathbf{o})*\mathbf{M},$$

$\mathbf{x} = [x_1, x_2, ..., x_D]$, D: dimensions

$\mathbf{o} = [o_1, o_2, ..., o_D]$: the shifted global optimum;

After load the data file, set $o_{2j-1} = -32$ o_{2j} are randomly distributed in the search range, for $j = 1, 2, ..., \lfloor D/2 \rfloor$

M: linear transformation matrix, condition number=100

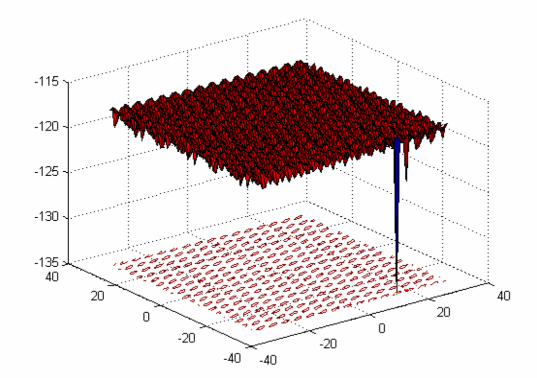

Figure 2-8 3-*D* map for 2-*D* function

Properties:

- Multi-modal
- Rotated
- Shifted
- Non-separable
- Scalable
- **A**'s condition number Cond(**A**) increases with the number of variables as $O(D^2)$
- Global optimum on the bound
- If the initialization procedure initializes the population at the bounds, this problem will be solved easily.
- $\mathbf{x} \in [-32, 32]^D$, Global optimum $\mathbf{x}^* = \mathbf{o}$, $F_8(\mathbf{x}^*) = f\_bias_8 = -140$

Associated Data file:

Name:	ackley_func_data.mat		ackley_func_data.txt
Variable:	**o**	1*100 vector	the shifted global optimum
	When using, cut **o**=**o**(1:*D*)		

Name:	ackley_M_D10 .mat		ackley_M_D10 .txt
Variable:	**M**	10*10 matrix	
Name:	ackley_M_D30 .mat		ackley_M_D30 .txt
Variable:	**M**	30*30 matrix	
Name:	ackley_M_D50 .mat		ackley_M_D50 .txt
Variable:	**M**	50*50 matrix	

2.2.4. F_9: *Shifted Rastrigin's Function*

$$F_9(\mathbf{x}) = \sum_{i=1}^{D}(z_i^{\,2} - 10\cos(2\pi z_i) + 10) + f\_bias_9\,,\ \mathbf{z} = \mathbf{x} - \mathbf{o}\,,\ \mathbf{x} = [x_1, x_2, ..., x_D]$$

D: dimensions

$\boldsymbol{o} = [o_1, o_2, ..., o_D]$: the shifted global optimum

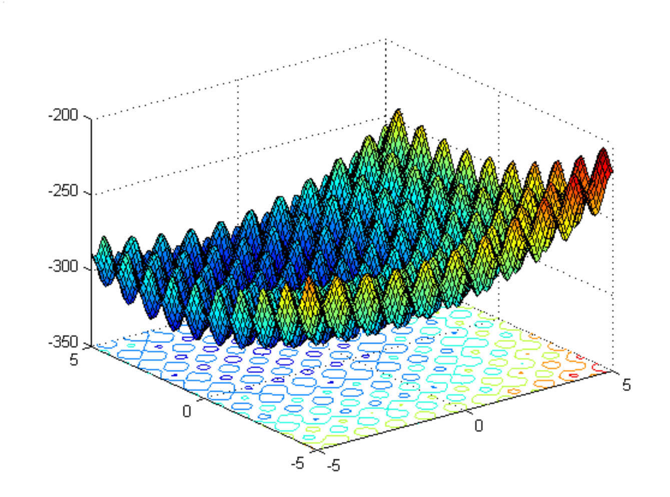

Figure 2-9 3-*D* map for 2-*D* function

Properties:

- Multi-modal
- Shifted
- Separable
- Scalable
- Local optima's number is huge
- $\mathbf{x} \in [-5, 5]^D$, Global optimum $\mathbf{x}^* = \mathbf{o}$, $F_9(\boldsymbol{x}^*) = f\_bias_9 = -330$

Associated Data file:

Name:	rastrigin_func_data.mat		
	rastrigin_func_data.txt		
Variable:	**o**	1*100 vector	the shifted global optimum
	When using, cut **o**=**o**(1:*D*)		

2.2.5. F_{10}: *Shifted Rotated Rastrigin's Function*

$$F_{10}(\mathbf{x}) = \sum_{i=1}^{D}(z_i^2 - 10\cos(2\pi z_i) + 10) + f\_bias_{10},\ \mathbf{z} = (\mathbf{x}-\mathbf{o})*\mathbf{M},\ \mathbf{x} = [x_1, x_2, ..., x_D]$$

D: dimensions

$\mathbf{o} = [o_1, o_2, ..., o_D]$: the shifted global optimum

M: linear transformation matrix, condition number=2

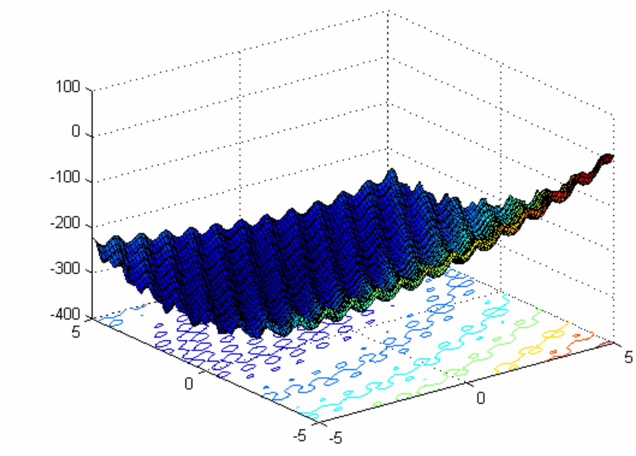

Figure 2-10 3-*D* map for 2-*D* function

Properties:

- Multi-modal
- Shifted
- Rotated
- Non-separable
- Scalable
- Local optima's number is huge
- $\mathbf{x} \in [-5, 5]^D$, Global optimum $\mathbf{x}^* = \mathbf{o}$, $F_{10}(\mathbf{x}^*) = f\_bias_{10} = -330$

Associated Data file:

Name:	rastrigin_func_data.mat rastrigin_func_data.txt	
Variable:	**o** 1*100 vector When using, cut **o**=**o**(1:*D*)	the shifted global optimum
Name:	rastrigin_M_D10 .mat	rastrigin_M_D10 .txt
Variable:	**M** 10*10 matrix	
Name:	rastrigin_M_D30 .mat	rastrigin_M_D30 .txt
Variable:	**M** 30*30 matrix	
Name:	rastrigin_M_D50 .mat	rastrigin_M_D50 .txt
Variable:	**M** 50*50 matrix	