(a) Multi-Expert Reasoning

Query

Given $x + y = 10$ and $x^3 + y^3 = 370$, find the value of $x^2 + y^2$.

Expert 1 — **Solution** Factor out the common factor $(x + y)$... **142**

Expert 2 — **Solution** Calculate the sum of cubes divided by the sum ... **142**

Expert 3 — **Solution** Use standard algebraic transformation formulas ... **58**

Majority Voting

reasoning synthesis

Expert 1: Factor out the common factor $(x+y)$ ... ans: **142**

Expert 2: Calculate the sum of cubes divided by the sum ... ans: **142**

Final Solution

Step 1: Establish relationships.
$x^3 + y^3 = (x + y)(x^2 + y^2 + 5xy)$

Step i: Solve xy.
$xy = (370 - 1000)/30$

Step n: Final results.
$x^2 + y^2 = (x + y)^2 - xy = 142$

(b) Logic-Preserving Query Mutation

Query Mutator — As a math generator, create variations with identical logic and structure, updating numbers to ensure reasonable values and geometric validity.

Query — Given $x + y = -1$ and $x^3 + y^3 = 370$, find the value of $x^2 + y^2$. Note: Contradictory conditions; no solution.

Mutation Validator — As a "variant validity auditor", your task is to discover internal logic errors by verifying domain violations, geometric constraints, algebraic consistency, and logical flow.

Query — Given $x + y = 6$ and $x^3 + y^3 = 72$, find the value of $x^2 + y^2$. Note: Consistent conditions; solvable.

(c) Step-wise Reasoning Abstraction
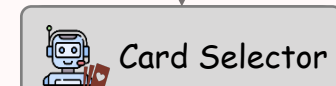
Logic Card Synthesizer

Card Generator — Act as a Mathematical Logic Analyzer. For every individual solution step, you must output '4 Distinct Logical Perspectives (Cards A-D)' and relevant 'math Equations' strictly in the specified '||' delimited format.

step 1

	Abstraction Logical Expression	Computational Grounding
Card A	Cubic Value Decomposition	$x^3 + y^3 = 10 * (x^2 + y^2 - xy)$
Card B	Sum of Cubes Identity	$x^3 + y^3 = (x + y) * (10^2 - 3xy)$ ✓
Card C	Sum Calculation	$x^3 + y^3 = 10* (x + y)^2 - 630$
Card D	Trinomial factorization	$(x^3 + y^3) / (x + y)^2 = 3.7$

step i

	Abstraction Logical Expression	Computational Grounding
Card A	Difference factorization	$x^3 + y^3 - (x + y)^3 = 3xy$
Card B	Conjugate Pair Law	$(x^3 + y^3)/(x + y) = 100 - 2xy$ ✗
Card C	Net Result Computation	$xy = (x^3 + y^3)/(x + y) - 16$
Card D	Sum Term Extraction	$xy = 2(x + y) + 1$

step n

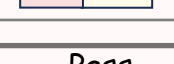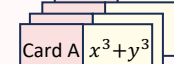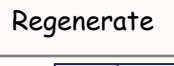|	Abstraction Logical Expression	Computational Grounding
Card A	Square Value Analysis	$x^2 + y^2 = (x + y)^2 - 2xy = 58$ ✓
Card B	Binomial Square Rule	$x^2 + y^2 = (x^3 + y^3)/10 + xy = 58$
Card C	Target Value Verification	$t^2 - 10t + 21 = 0$
Card D	Root Term Calculation	$x \approx y \approx 10/2 = 5$ $x^2 + y^2 = 58$

Best-Card Router

+ 📖 Cards selected in Steps 1 ~ i-1
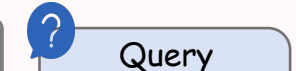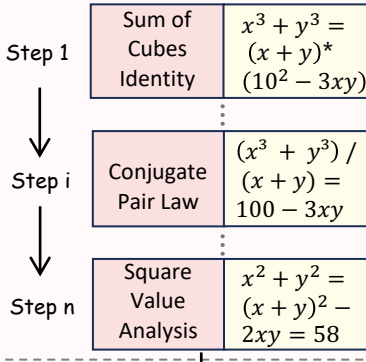
Card Selector — Your task is selecting the BEST logic card (A, B, C, or D) that correctly applies to the MUTATED problem, distinguishing between cards that rely on old numbers (Invalid) and those that are general (Valid).

Card B from Step i — Conjugate Pair Law $(x^3 + y^3)/(x + y) = 100 - 2xy$ + Query
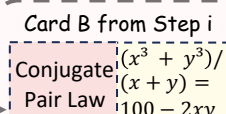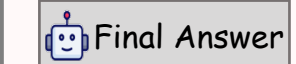
Logic Validator — You are a Logic Auditor responsible for verifying a solution trace for consistency and validity by identifying errors such as premise conflicts, calculation mistakes ...

Query + step i → **Validator Feedback** → Regenerate → step i Card A $x^3 + y^3$ ... → **Pass**

Reasoning Chain

Step 1	Sum of Cubes Identity	$x^3 + y^3 = (x + y)* (10^2 - 3xy)$
Step i	Conjugate Pair Law	$(x^3 + y^3)/(x + y) = 100 - 3xy$
Step n	Square Value Analysis	$x^2 + y^2 = (x + y)^2 - 2xy = 58$

→ **Logic Validator**

Query — Given $x + y = 10$ and $x^3 + y^3 = 370$, find the value of $x^2 + y^2$.

Final Answer — **58** (verfied) 🏅