Utilice la serie de Taylor para encontrar una serie de potencias centrada en cero para la función

$$f(x) = \ln\left(x^2 + 1\right)$$

$$In(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$$

$$In(1 + x^2) = x^2 - \frac{x^{2^2}}{2} + \frac{x^{2^3}}{3} - \frac{x^{2^4}}{4} + \cdots$$

$$In(1 + x^2) = x^2 - \frac{x^4}{2} + \frac{x^6}{3} - \frac{x^8}{4} + \cdots$$

$$\boldsymbol{In(1 + x^2) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}\, x^{2n} = x^2 - \frac{x^4}{2} + \frac{x^6}{3} - \frac{x^8}{4} + \cdots\ R/}$$

Utilice el resultado anterior para obtener la serie de la función

$$f(x) = \frac{\ln\left(x^2 + 1\right)}{x^2}$$

$$\frac{ln(1 + x\ 2)}{x^2} = \frac{x^2}{x^2} - \frac{\frac{x^4}{2}}{x^2} + \frac{\frac{x^6}{3}}{x^2} - \frac{\frac{x^8}{4}}{x^2} + \cdots$$

$$\frac{ln(1 + x\ 2)}{x^2} = \frac{x^2}{x^2} - \frac{x^4}{2}\left(\frac{1}{x^2}\right) + \frac{x^6}{3}\left(\frac{1}{x^2}\right) - \frac{x^8}{4}\left(\frac{1}{x^2}\right) + \cdots$$

$$\frac{ln(1 + x\ 2)}{x^2} = \frac{x^2}{x^2} - \frac{x^4}{2(x^2)} + \frac{x^6}{3(x^2)} - \frac{x^8}{4(x^2)} + \cdots$$

$$\frac{ln(1 + x\ 2)}{x^2} = \frac{x^2}{x^2} - \frac{x^4}{2x^2} + \frac{x^6}{3x^2} - \frac{x^8}{4x^2} + \cdots$$

$$\frac{ln(1 + x\ 2)}{x^2} = \frac{\cancel{x^2}}{\cancel{x^2}} - \frac{\cancel{x^4}}{\cancel{2x^2}} + \frac{\cancel{x^6}}{\cancel{3x^2}} - \frac{\cancel{x^8}}{\cancel{4x^2}} + \cdots$$

$$\frac{ln(1 + x\ 2)}{x^2} = 1 - \frac{x^{4-2}}{2} + \frac{x^{6-2}}{3} - \frac{x^{8-2}}{4} + \cdots$$

$$\boldsymbol{\frac{In(1+x^2)}{x^2} = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} x^{2n} = 1 - \frac{x^2}{2} + \frac{x^4}{3} - \frac{x^6}{4} + \cdots}$$

Utilice el resultado anterior para estimar el límite

$$\lim_{x \to 0} \frac{\ln (x^2+1)}{x^2}$$

$$\lim_{x=0} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} x^{2n}$$

$$\lim_{n \to \infty} \left(\left| \frac{a_{n+1}}{a_n} \right| \right)$$

$$\lim_{n \to \infty} \left(\left| \frac{\frac{(-1)^{n+1+1}}{n+1} x^{2(n+1)}}{\frac{(-1)^{n+1}}{n} x^{2n}} \right| \right)$$

$$\lim_{n \to \infty} \left(\left| \frac{\frac{(-1)^{n+2}}{n+1} x^{2n+2}}{\frac{(-1)^{n+1}}{n} x^{2n}} \right| \right)$$

$$\lim_{n \to \infty} \left(\left| \frac{\frac{(-1)^{n+2}}{n+1} x^{2n+2-2n}}{\frac{(-1)^{n+1}}{n}} \right| \right)$$

$$\lim_{n \to \infty} \left(\left| \frac{\frac{(-1)^{n+2}}{n+1} x^{2}}{\frac{(-1)^{n+1}}{n}} \right| \right)$$

$$\lim_{n \to \infty} \left(\left| \frac{\frac{(-1)^{n+2} (x)^2}{n+1}}{\frac{(-1)^{n+1}}{n}} \right| \right)$$

$$\lim_{n \to \infty} \left(\left| \frac{\cancel{(-1)^{n+2}} (x)^2 (n)}{n+1 \cancel{(-1)^{n+1}}} \right| \right)$$

$$\lim_{n\to\infty}\left(\left|\frac{-1(x)^2(n)}{n+1}\right|\right)$$

$$\lim_{n\to\infty}\left(\left|-\frac{1x^2n}{n+1}\right|\right)$$

$$\lim_{n\to\infty}\left(\left|-\frac{x^2n}{n+1}\right|\right)$$

$$\lim_{n\to\infty}\left(\left|-x^2\frac{n}{n+1}\right|\right)$$

$$\lim_{n\to\infty}\left(|-x^2|\left(\left|\frac{n}{n+1}\right|\right)\right)$$

$$|-x^2|\lim_{n\to\infty}\left(\left|\frac{n}{n+1}\right|\right)$$

$$x^2\lim_{n\to\infty}\left(\frac{n}{n+1}\right)$$

$$x^2\left(\frac{\lim_{n\to\infty} n}{\lim_{n\to\infty}(n+1)}\right)$$

$$x^2\left(\frac{\infty}{\infty}\right)$$

$$x^2\lim_{n\to\infty}\left(\frac{n}{n\left(1+\frac{1}{n}\right)}\right)$$

$$x^2\lim_{n\to\infty}\left(\frac{\cancel{n}}{\cancel{n}\left(1+\frac{1}{n}\right)}\right)$$

$$x^2\lim_{n\to\infty}\left(\frac{1}{1+\frac{1}{n}}\right)$$

$$x^2\left(\frac{\lim_{n\to\infty}1}{\lim_{n\to\infty}1+\lim_{n\to\infty}\frac{1}{n}}\right)$$

$$x^2\left(\frac{1}{1+0}\right)$$

$$x^2\left(\frac{1}{1}\right)$$

$$x^2(1)$$

$$x^2$$

$$\lim_{x\to\infty} \frac{In(1+x^2)}{x^2} = x^2$$

$$\lim_{x\to\infty} x^2$$

$$\lim_{x\to\infty} (0)^2$$

$$0$$

$$\boldsymbol{\lim_{x\to\infty} \frac{In(1+x^2)}{x^2}}$$

$$\underline{\boldsymbol{\approx\ 0}}$$