Northeastern University, Boston, MA

College of Engineering

Department of Civil and Environmental Engineering

CIVE 7380: Performance Models and Simulation of Transportation Networks

Problem Set III

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CIVE 7380
Problem Set #3
Due: Friday, February 21, 2025

1. An airport services three types of airplanes: Heavy (H), Large (L) and Small (S). The approach to the runway is n =5 n. miles. The planes have the following characteristics:

Type	Approach Speed (knots)	Mix (%)	Occupancy (sec)
H	150	25	40
L	120	50	40
S	90	25	30

The minimum separation between the various airplane types is given in the table below:

		Trailing aircraft		
		H	L	S
Leading aircraft	H	5	5	6
	L	4	4	4
	S	4	4	4

The above separation distances include a safety buffer, so there is no need to add extra time.

a. Find the landing capacity of the runway, assuming a random sequencing of landings.
b. Find the corresponding average delay per airplane (assuming an arrival rate of 20 arrivals per hour).
c. Assuming that the approach speed for small airplanes can increase to 120 knots find the corresponding capacity.
d. Assume that the minimum separation criteria for S-L, S-H, L-H cases (i.e cases with no wake vortex considerations) is decreased to 3 n. miles. Find the corresponding capacity.
e. Assume now that the ATC can deliberately control the sequence of landings. Air traffic controllers can land small aircraft first at intervals equal to 160 sec, then transition to large planes with one 120-sec interval, and land the sequence of large planes at 120-sec intervals; then transition to heavy aircrafts with a 96-sec interval followed by 120-sec intervals between all the heavy aircrafts. For this sequence and intervals find the corresponding capacity (assuming that 100 aircrafts landed, i.e., 99 intervals).
f. Based on the above results identify different approaches to improve runway capacity. Discuss relative advantages and disadvantages of the various approaches.

2. An airline operates 2 check-in counters. The service time is exponentially distributed with an average of 2 min per customer. Customers arrive according to a Poisson process at a rate of 40 customers per hour. The airline is considering two alternative modes of operation.

   a) customers form a single line and use the first available check-in counter.

   b) customers form two separate lines, one for each check-in counter. Each arriving customer is assigned randomly to one of the check-in counters (regardless of its status).

   Compare the two alternatives in terms of average waiting time. Make a recommendation in terms of the preferred option.

3. Show that:

$$k = \frac{\%o}{100} * \frac{1}{L_v + L_D}$$

   where,
   % o: percent occupancy
   k: corresponding density
   L_v: Average vehicle length
   L_D: detection zone length

4. Answer the following:
   a) A traffic stream displays average vehicle headways of 2.2 sec. at a speed of 50mi/h. Compute the density and flow for this traffic stream.
   b) At a given section, the space mean speed is measured as 40 mi/h and the flow as 1,600pc/h/ln. What is the density for the analysis period?

5. A single loop detector has a length of 9 feet. It recorded 6 vehicles over a period of 142 sec. The occupancy times of the vehicles are (in seconds): 0.46, 0.48, 0.53, 0.42, 0.52, 0.45. The lengths of the vehicles are known and given by (in feet): 18, 21, 20, 23, 17, 19.

   a) Find the occupancy of the detector
   b) Find the speeds of the vehicles and the average speed
   c) Find the flow and the density
   d) Assume now that the lengths of the vehicles are not known but the average length is known (given by the average of the lengths given above). Calculate vehicle speeds, flow and density. Comment on the difference with the values found in b) and c).

Solution to Q1

Minimum Time Separations

For each leading-trailing aircraft combination, I wrote a simple Python code to check the difference in speeds to determine whether to use the opening case formula or the closing case formula (**Equation 1**). The result is displayed in **Table 1**.

Equation 1: Minimum Time Separation

$$T_{ij} = \begin{cases} max\left(\frac{n+s_{ij}}{v_j} - \frac{n}{v_i},\ \ o_i\right),\ \ if\ v_i > v_j\ \ (opening\ case) \\ max\left(\frac{s_{ij}}{v_j},\ \ o_i\right),\ \ if\ v_i \le v_j\ \ (closing\ case) \end{cases}$$

Where:

$n = 5\ nautical\ miles$ (approach path length)

$s_{ij} = minimum\ separation$ (nautical miles)

$v_i, v_j = approach\ speeds$ (knots)

$o_i = runway\ occupancy\ time$ (seconds)

Table 1: Minimum Time Separation Matrix in Seconds

T_{ij}	H	L	S
H	120	180	320
L	96	120	210
S	96	120	160

Expected Time Separation ($E(T_{ij})$)

The expected time separation is given by **Equation 2**, where p_i is the proportion of each aircraft type. I wrote a Python code to calculate the expected time separation using the given aircraft mix percentages and the results in **Table 1**. The result I had was **149.25 seconds**.

Equation 2: Expected Time Separation

$$E(T_{ij}) = \sum_i \sum_j p_i p_j T_{ij}$$

Runway Landing Capacity

Using **Equation 3**, the runway landing capacity is found to be **24.12 landings/hour**.

Equation 3: Runway Landing Capacity

$$C = \frac{3,600}{E(T_{ij})}$$

Average Delay Per Aircraft

The average delay per aircraft is given by **Equation 4**. It is found to be **6.94 minutes**.

Equation 4: Average Delay Per Aircraft

$$W_q = \frac{\rho^2 + \lambda^2\sigma^2}{2\lambda(1-\rho)}$$

Where:

$\rho = \frac{\lambda}{C}$ (utilization factor)

$\lambda = 20\ aircraft/hr$ (arrival rate)

$C = 24.12\ landings/hr$ (runway landing capacity)

$\sigma^2 = Var(T_{ij})$ (variance of separation times)

$Var(T_{ij})$ is calculated using **Equation 5** and is found to be $3,314.94\ s^2$.

Equation 5: Variance of Separation Times

$Var(T_{ij}) = E\left[{T_{ij}}^2\right] - (E\left[T_{ij}\right])^2$, where $E\left[{T_{ij}}^2\right] = \sum_i \sum_j p_i p_j T_{ij}$

Runway Capacity Assuming $v_{small} = 120\ knots$

I just modified the value of the v_{small} variable in the code. The corresponding capacity becomes **27.99 landings/hour**.

Runway Capacity After Decreasing s_{SL}, s_{SH}, and s_{LH}

I just changed the values of s_{SL}, s_{SH}, and s_{LH} in my code to 3 nautical miles. The corresponding capacity becomes **25.53 landings/hour**.

Runway Capacity Under ATC Sequencing

Small and heavy aircraft each are 25% of the total and the remaining are large aircraft. Hence, interval counts in **Table 2**.

Table 2: Interval Counts

Interval Type	Count
S-S intervals	24
S-L transition	1
L-L intervals	49
L-H intervals	1
H-H intervals	24
Total	99

From **Table 2**, I calculated the total interval time to be 12,816 s. This implies an average interval time of **129.45 s**. From **Equation 3**, the corresponding capacity is **27.81 landings/hr**.

Other Ways of Improving Runway Capacity

Aside from increasing approach speeds, reducing separation distances, and optimizing landing sequences, I'll propose the following two additional approaches:

1. Investing in advanced ATC systems, runway infrastructure, and aircraft technology (e.g., precision landing systems, wake turbulence detection systems). This approach can potentially result in faster approach speeds and is particularly effective for small aircraft, which have lower separation requirements. The drawback is that it will require additional pilot training.

2. The second approach will be to get high-speed exit taxiways for situations where runway occupancy time limits how quickly the next aircraft can land. This strategy allows for closer sequencing of landings without compromising safety. The two disadvantages I can think of are that the existing taxiway configuration may not permit such infrastructure modifications or there may be certain aircraft types that cannot perform high-speed exits.

Solution to Q2

Solution to Q2a

Arrival time distribution: Poisson process

Service time distribution: exponential

Number of servers, $\boldsymbol{s}$: 2

This single-queue operational mode follows the $\boldsymbol{M/M/2}$ queueing model with:

$\lambda = 40\ customers/hour$

$$\mu = \frac{60\ min/hr}{2\ min/customer/server} = 30\ customers/hour/server$$

Utilization ratio, ρ

Using **Equation 6**, $\rho = 0.\dot{6}$.

Equation 6: Utilization Ratio for M/M/s

$$\rho = \frac{\lambda}{s\mu}$$

Probability that all servers are idle, P_0

Using **Equation 7**, $P_0 = 0.2$.

Equation 7: Probability that all servers are idle for M/M/s

$$P_0 = \left[\sum_{n=0}^{s-1} \frac{(\lambda/\mu)^n}{n!} + \frac{(\lambda/\mu)^s}{s!\,(1-\rho)}\right]^{-1}$$

Average queue length, $\boldsymbol{L_q}$

From **Equation 8**, $L_q = 1.0\dot{6}\ customers$.

Equation 8: Average Queue Length M/M/s

$$L_q = \frac{(\lambda/\mu)^s \rho}{s!\,(1-\rho)^2} P_0$$

Average waiting time in queue, $\boldsymbol{W_q}$

Using Little's formula (**Equation 9)**, $W_q = 1.6\ minutes$.

Equation 9: Little's Formula

$$W_q = \frac{L_q}{\lambda}$$

Solution to Q2b

Arrival time distribution: Poisson process

Service time distribution: exponential

Number of servers, $\boldsymbol{s}$: 2

This two-separate-queues operational mode can be modeled as **two $\boldsymbol{M/M/1}$** models with:

$$\lambda' = \frac{40\ customers/hour}{2\ servers} = 20\ customers/hour/server$$

$$\mu = \frac{60\ min/hr}{2\ min/customer/server} = 30\ customers/hour/server$$

Average waiting time in queue, $\boldsymbol{W_q}$

Using **Equation 10**, $W_q = 4.0\ minutes$.

Equation 10: Little's Formula

$$W_q = \frac{\lambda'}{\mu(\mu - \lambda')}$$

- Based on the results above, the single queue system ($M/M/2$) is the better option since it results in a shorter waiting time for customers by utilizing both counters efficiently.

- I think Case 2 resulted in longer average delay due to the chance of a counter sitting idle while there are customers waiting at the other counter. This leads to inefficient resource utilization because customers at the busy counter cannot be served by the idle counter.

Solution to Q3

Percent occupancy, $\%o$, is the percentage of time (out of the total observation time, T_{obs}) that the detection zone is occupied by vehicles. The time a single vehicle occupies the detection zone, T_{occ}, is equal to the time it takes for the vehicle to travel a distance of $L_v + L_D$ as illustrated in **Figure 1**.

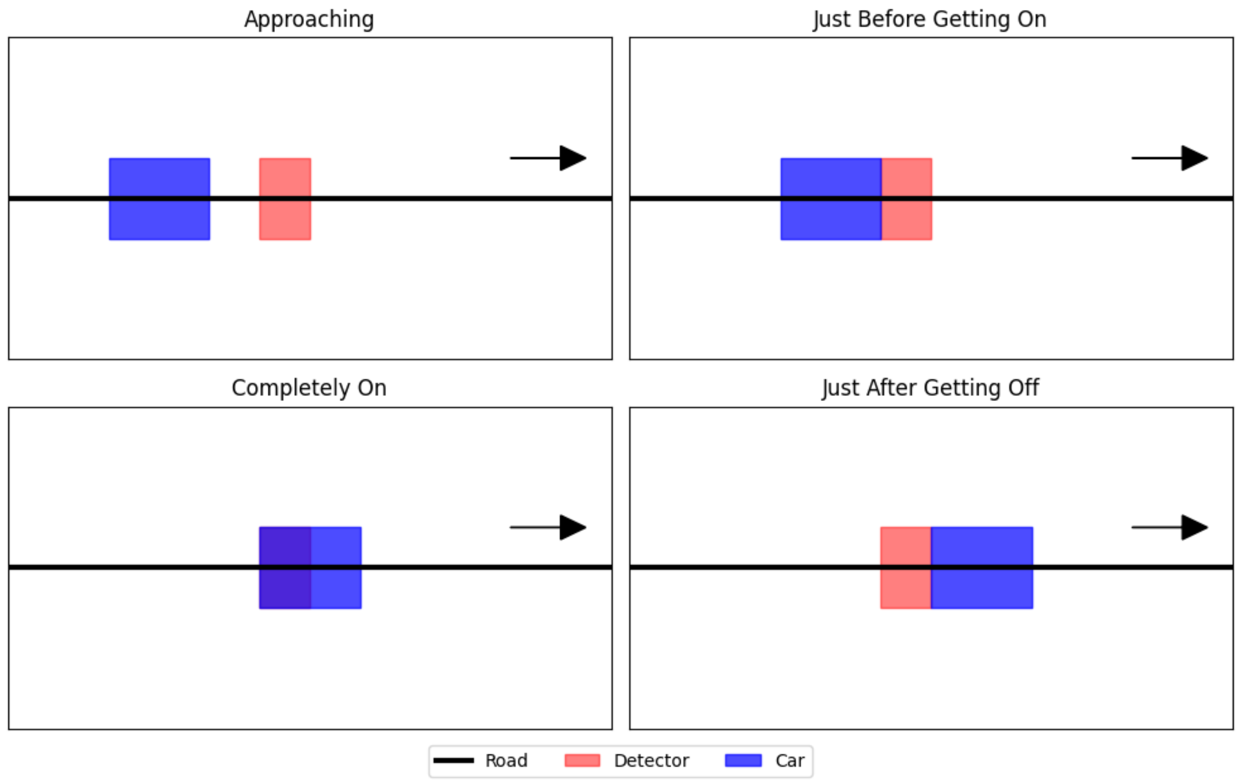

Figure 1: Illustration of a Car Passing Over a Detector

Assuming this vehicle is traveling at a speed of u, then, we have:

$$T_{occ} = \frac{L_v + L_D}{u}$$

If N vehicles pass over the detection zone during the observation period, the total occupancy time, T_{total}, assuming all of them are traveling at a speed of u, will be:

$$T_{occ\ total} = N \times T_{occ} = N \times \frac{L_v + L_D}{u}$$

Now, we have:

$$\%o = \frac{T_{occ\ total}}{T_{obs}} \times 100$$

$$\%o = \frac{N \times \frac{L_v + L_D}{v}}{T_{obs}} \times 100$$

Now, the vehicle flow rate, q, which is essentially the number of vehicles passing the detector per unit time can be expressed as:

$$q = \frac{N}{T_{obs}}$$

Substituting q into the $\%o$ equation, we'll now have:

$$\%o = \frac{q}{u} \times L_v + L_D \times 100$$

From the traffic flow equation (**Equation 11**), we can substitute k for $\frac{q}{u}$ in the above equation.

Equation 11: Traffic Flow Equation

$$q = uk$$

That'll result in:

$$\%o = k(L_v + L_D) \times 100$$

Rearranging the above equation, we have:

$$k = \frac{\%o}{100} \times \frac{1}{L_v + L_D}$$

Solution to Q4

Solution to Q4a

Given headway, h, in seconds, flow rate, q, can be calculated using **Equation 12**. The flow rate, q, is found to be **1,636 veh/hr**.

Equation 12: Flow Rate

$$q = \frac{3{,}600}{h}$$

From the traffic flow equation (**Equation 11**), the corresponding density, k, is calculated to be **32.73 veh/mi**.

Solution to Q4b

From the traffic flow equation (**Equation 11**), the corresponding density for the given analysis period, k, is calculated to be **40 veh/mi/ln**.

Solution to Q5

Percentage Occupancy

$$T_{occ\ total} = 0.46 + 0.48 + 0.53 + 0.42 + 0.52 + 0.45 = 2.86\ s$$

Given $T_{obs} = 142\ s$, then:

$$\%o = \frac{T_{occ\ total}}{T_{obs}} \times 100 = \frac{2.86}{142} \times 100 = 2.014\%$$

Individual Vehicle Speeds and Average Speed

From **Equation 12**, the vehicle speeds are 58.7, 62.5, 54.7, 76.2, 50.0, and 62.2 ft/s. The average vehicle speed is 60.7 ft/s.

Equation 13: Vehicle Speeds

$$u = \frac{L_v + L_D}{T_{occ}}$$

Flow and Density

$$q = \frac{N}{T_{obs}} = \frac{3{,}600 \times 6}{152} = 152.11\ veh/hr$$

From the traffic flow equation (**Equation 11**), the corresponding density, k, is calculated to be **3.67 veh/mi**.

Average Vehicle Length

$$L_{oavg} = \frac{18 + 21 + 20 + 23 + 17 + 19}{6} = 19.\dot{6}\ ft$$

Vehicle Speeds Using Average Vehicle Length

From **Equation 12**, the vehicle speeds assuming each vehicle is $19.\dot{6}\ ft$ long are:

62.3, 59.7, 54.0, 68.3, 55.1, and 63.7 ft/s. The average vehicle speed is 60.5 ft/s.

Flow and Density Using Average Vehicle Length

Flow remains the same: $152.11\ veh/hr$.

Density increases slightly to $3.69\ veh/mi$.

Comment on the Difference

- Flow remains the same in either case because it depends only on the number of vehicles and the observation time.

- Average speed is slightly lower in the second case due to the minor errors that are introduced as a result of using a single average vehicle length for the calculations, which does not account for the variations in individual vehicle sizes.

- Since density is inversely proportional to speed according to the traffic flow equation, if speed is underestimated, then density will be overestimated. Intuitively, lower speed implies that the same number of vehicles will take up more space per mile.

In practice, we may not know all the vehicle lengths. I think it'll be okay to use average vehicle length (or weighted average or something of that sort). That's sufficient to provide a close approximation.