Northeastern University, Boston, MA

College of Engineering

Department of Civil and Environmental Engineering

CIVE 7380: Performance Models and Simulation of Transportation Networks

Problem Set IV

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Questions

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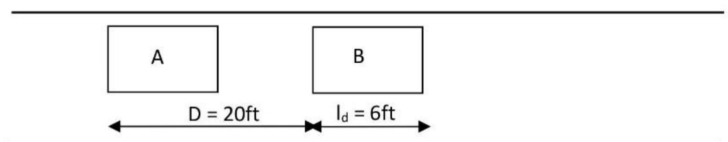
CIVE 7380
Problem Set #4
Due: March 12, 2025

1. Two vehicles pass over a double detector. Each detector has length of 6 ft and the distance between the upstream edges of the two detectors is 20 feet.



The time on, t_{on}, and time off, t_{off}, for each vehicle and each detector, in units of 1/30-second, are shown in the table below.

Vehicle	Detector	Time on, t_{on}	Time off, t_{off}
1	A	16	27
	B	25	36
2	A	141	154
	B	149	163

a) Find the time headway between the two vehicles as measured by each detector
b) Find the occupancy time (in seconds) of each detector (for each vehicle)
c) Find the length of each vehicle
d) Find the average speed of each vehicle (over the distance D)
e) Find the speed of each vehicle as it is measured by each detector
f) Find the acceleration of each vehicle between the two detectors

2. The speed density relationship for a highway segment is given by:
$u = 44.4 – 0.234k$
where,
u: space-mean speed in miles/hour (mph)
k: density in veh/lane-mile

a. Find the flow-density (q = f(k)) and speed-flow (u=f(q)) relationships
b. Find the optimum density, jam density, and capacity
c. Find the average spacing (space headway) corresponding to the jam density
d. Determine the speeds for
   1. flow near zero
   2. flow at capacity
e. Comment on the reasonableness of the values of the parameters found in b), c) and d).

3. A section of the freeway has the following flow-density relationship:
$q = 50k – 0.156k^2$

What is the capacity of the highway section, the speed at capacity, and the density when the highway flow is at one-quarter of its capacity?

4. The following data represents observations on speed and corresponding density:

Speed (mph)	Density (veh/mile-lane)
46.0	22.2
56.0	29.3
48.0	30.1
42.0	40.7
22.9	90.0
56.6	29.2
9.2	113.3
17.9	88.2
14.5	103.1
29.5	60.2
25.0	67.4
32.0	70.6
35.5	41.1
36.5	49.6

   a. Plot the data (speed-density)
   b. Suggest an appropriate single regime model (speed-density) from the ones discussed in class that fits the data. Manually or using Excel, estimate the parameters of your model.
   c. Discuss the meaning of the various parameters and comment on how well they capture the data.

Solution to Q1

Time Headway Between Two Vehicles

I'll measure time headway with respect to the upstream edges of the detectors and the front bumpers of the vehicles. This implies that:

Time Headway at Detector A, $\boldsymbol{h_A}$

$$h_A = \frac{141 - 16}{30} = 4.1\dot{6}\ s$$

Time Headway at Detector B, $\boldsymbol{h_B}$

$$h_B = \frac{149 - 25}{30} = 4.1\dot{3}\ s$$

Occupancy Time of Each Detector for Each Vehicle

Occupancy time is the duration for which each vehicle is on a detector.

Let $o_{n,X} = occupancy\ time\ for\ vehicle\ n\ at\ detector\ X$.

This implies that:

$$o_{1,A} = \frac{27-16}{30} = 0.3\dot{7}\ s \qquad o_{1,B} = \frac{36-25}{30} = 0.3\dot{7}\ s$$

$$o_{2,A} = \frac{154-141}{30} = 0.4\dot{3}\ s \qquad o_{2,B} = \frac{163-149}{30} = 0.4\dot{6}\ s$$

Average Vehicle Speed Over a Distance, D

Vehicle speed over $D = 20ft$ will be D (the distance between the upstream edges of the detectors) divided by the time taken for each vehicle to travel this distance.

Hence:

$$\overline{u_1} = \frac{20}{\frac{1}{30}(25-16)} = 66.\dot{6}\, ft/s \; or \; 45.45\, mph$$

$$\overline{u_2} = \frac{20}{\frac{1}{30}(149-141)} = 75 ft/s \; or \; 51.14\, mph$$

Vehicle Lengths

Length of Vehicle 1

$L_{v,1} = u_1 \times o_{1,A} - L_d$

$L_{v,1} = 66.\dot{6} \times 0.3\dot{7} - 6$

$L_{v,1} = 19.2\, ft$

Length of Vehicle 2

$L_{v,2} = u_2 \times o_{2,A} - L_d$

$L_{v,2} = 75 \times 0.4\dot{3} - 6$

$L_{v,2} = 26.5\, ft$

Speed of Each Vehicle as Measured by Each Detector

Speed of Vehicle 1 at Detector A

$$u_{1,A} = \frac{L_{v,1} + L_d}{o_{1,A}}$$

$$u_{1,A} = \frac{19.2 + 6}{0.3\dot{7}}$$

$$u_{1,A} = 66.7\, ft/s \; or \; 45.45\, mph$$

Speed of Vehicle 1 at Detector B

$$u_{1,B} = \frac{L_{v,1} + L_d}{o_{1,B}}$$

$$u_{1,B} = \frac{19.2 + 6}{0.3\dot{7}}$$

$$u_{1,B} = 66.7\, ft/s \; or \; 45.45\, mph$$

Speed of Vehicle 2 at Detector A

$$u_{2,A} = \frac{L_{v,2} + L_d}{o_{1,A}}$$

$$u_{2,A} = \frac{26.5 + 6}{0.4\dot{3}}$$

$$u_{2,A} = 75\, ft/s \; or \; 51.14\, mph$$

Speed of Vehicle 2 at Detector B

$$u_{2,B} = \frac{L_{v,2} + L_d}{o_{2,B}}$$

$$u_{2,B} = \frac{26.5 + 6}{0.4\dot{6}}$$

$$u_{2,B} = 69.64\, ft/s \; or \; 47.48\, mph$$

Acceleration of Each Vehicle Between the Two Detector

Acceleration of Vehicle 1

$$a_1 = \frac{{u_{1,B}}^2 - {u_{1,A}}^2}{2D} = \frac{66.7^2 - 66.7^2}{2 \times 20} = 0$$

Acceleration of Vehicle 2

$$a_2 = \frac{{u_{2,B}}^2 - {u_{2,A}}^2}{2D} = \frac{69.64^2 - 75.00^2}{2 \times 20} = -19.38\ ft/s^2 = -13.21\ mph/s$$

Solution to Q2

Flow-Density Relationship

Given: $u = 44.4 - 0.234k$. Also, we know that $q = uk$. Substituting the first into the second equation, we have:

$$q = (44.4 - 0.234k)k$$

$$q = 44.4k - 0.234k^2$$

Speed-Flow Relationship

Given: $u = 44.4 - 0.234k$, $k = \frac{44.4-u}{0.234}$. Substitute k into $q = uk$. We'll now have:

$$q = u\left(\frac{44.4 - u}{0.234}\right)$$

$$u^2 - 44.4u + 0.234q = 0$$

Using the quadratic formula, we'll now have:

$$u = 22.2 \pm 0.5\sqrt{1{,}971.36 - 0.936q}$$

Optimum Density, k_o

The optimum density occurs when the throughput (or flow) is maximized ($dq/dk = 0$).

$$\frac{dq}{dk} = 44.4 - 0.468k = 0$$

Hence, $k_0 = 94.87\ veh/lane - mile$.

Jam Density, k_j

Jam density occurs when speed is zero.

$$44.4 - 0.234k = 0$$

Hence, $k_j = 189.74\ veh/mile - lane$.

Capacity, q_{max}

This is the maximum flow, and it occurs at the optimal density.

$$q_{max} = 44.4(94.87) - 0.234(94.87)^2 = 2{,}106\ veh/hr - lane$$

Space Headway, s

$$s = \frac{1}{k_j} = \frac{1}{189.74} = 0.00527\ mi/veh = 27.83\ ft/veh$$

Speeds for Flow Near Zero

$$u = 22.2 \pm 0.5\sqrt{1{,}971.36 - 0.936(0)}$$

$$u = 22.2 \pm 0.5\sqrt{1{,}971.36}$$

$$u = 0\ and\ u = 44.4\ mph$$

Speeds for Flow at Capacity

$$u = 22.2 \pm 0.5\sqrt{1{,}971.36 - 0.936(2{,}106)}$$

$$u = 22.2 \pm 0.5\sqrt{0}$$

$$u = 22.2\ mph$$

Reasonableness of Results

Optimum density: Roughly 95 veh/mi or about 12 veh per 100-m section is reasonable. Lower density than this could result in lower throughput. Higher density could lead to congestion where each vehicle's speed is affected by the vehicle ahead of it.

Jam density: Roughly 190 veh/mi implies about 12 vehicles per 100-m section. That's a reasonable value for jam density. I imagine this as a situation where the vehicles are very closely packed with very little space in between them. In such a situation, speed is about zero.

Capacity: Roughly 2,100 veh/hr-lane implies that one car passes about every 1.7 s. That's a reasonable value for capacity. Shorter headway than that means that cars will be crashing into each other (depending on reaction time). It could also feel uncomfortable. Longer headway implies the lane is being underutilized (not at capacity).

Space headway at jam density: Spacing is the sum of clearance (distance separation between rear bumper of leading vehicle and front bumper of trailing vehicle) and average vehicle length. 28 ft spacing implies 13 ft clearance assuming an average vehicle length of 15 ft. 13 ft clearance (cannot even fit one vehicle) is reasonable at jam density.

Speeds for flow near zero: Flow near zero means almost nobody is on the highway (which means speed is equivalent to free flow speed) or the highway is jammed (which means speed is almost zero. 44.4 mph as free flow speed is reasonable.

Speed for flow at capacity: 22.2 mph is reasonable. Traffic isn't moving quite as fast (as compared to free flow speed) but the throughput is maximized. Speed at capacity is half of the free flow speed.

Solution to Q3

Given $q = 50k - 0.156k^2$.

Capacity

Capacity is maximum flow, q_{max}. For maximum flow, $\frac{dq}{dk} = 0$.

$$\frac{dq}{dk} = 50 - 0.312k = 0$$

Hence:

$$k_0 = 160.26\ veh/mi$$

Now, we have:

$$q_{max} = 50(160.26) - 0.156(160.26)^2$$

$$q_{max} = 4{,}006\ veh/hr$$

Speed at Capacity

$$q_{max} = u_c k_0$$

$$u_c = \frac{4{,}006}{160} = 25\ mph$$

Density When Flow is One-Quarter of Capacity

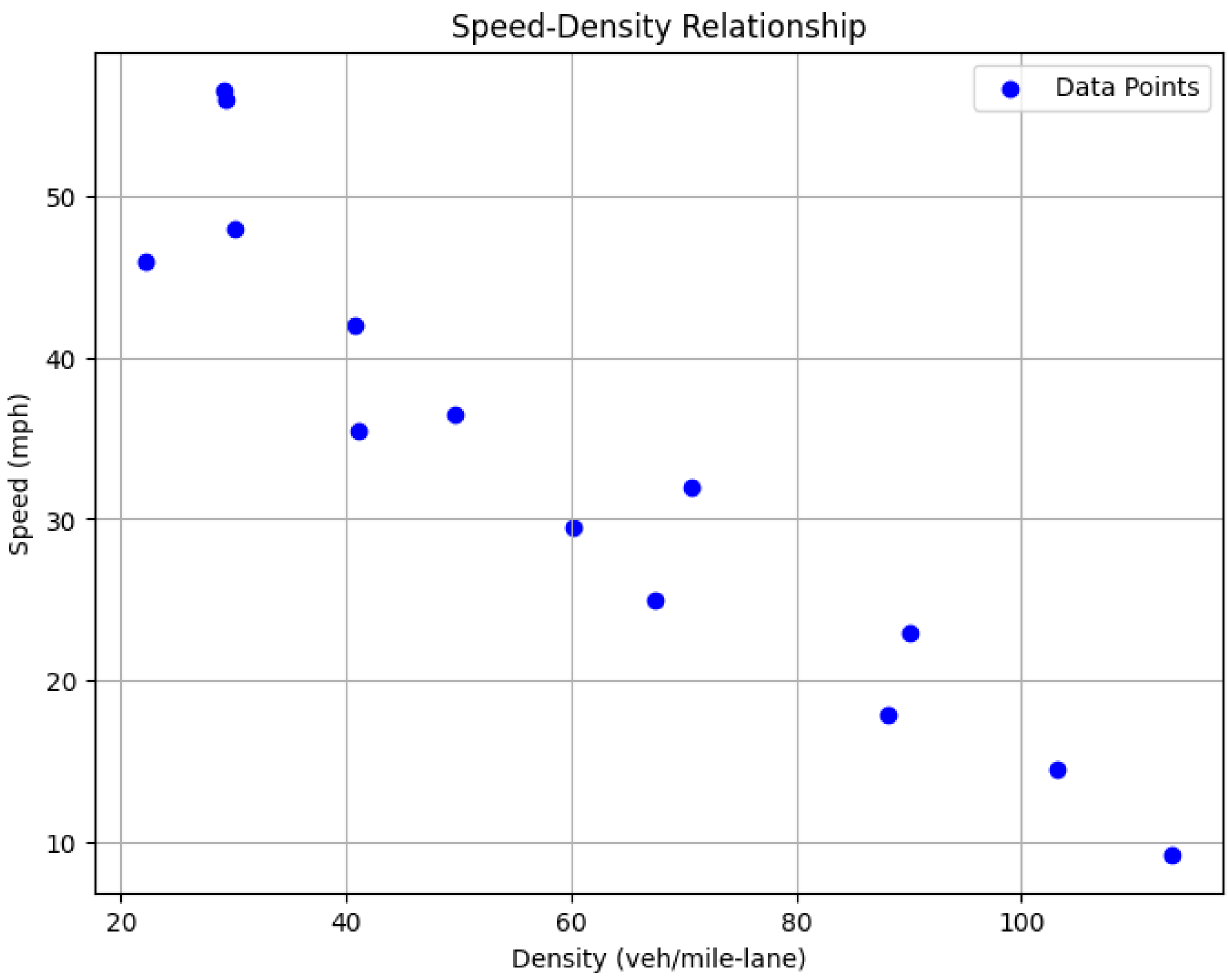
$$\frac{1}{4}(4{,}006) = 50k - 0.156k^2$$

$$0.624k^2 - 200k + 1{,}001.5 = 0$$

Density will be **5 veh/mi** for uncongested flow and **315 veh/mi** for congested flow.

Solution to Q4

Plot the Data (Speed-Density)

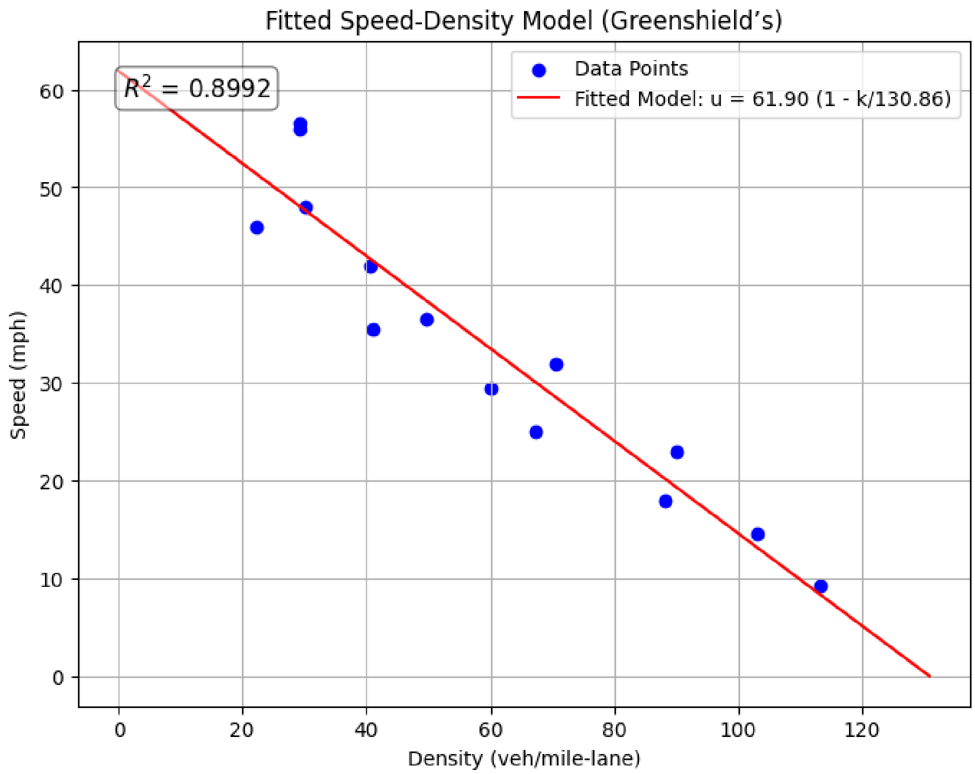


Single-Regime Model

Looks like a single line (rather than a curve) can fit the data points pretty well. Hence, I'll use the Greenshield model (below).

$$\overline{u_s} = u_f - \frac{u_f}{k_j}k = u_f\left(1 - \frac{k}{k_j}\right)$$

From Excel, the equation of the line of best fit is $\overline{u_s} = -0.473k + 61.90$.



Discussion of the Model Parameters

Rearranging the line of best fit, we'll have:

$$\overline{u_s} = 61.90\left(1 - \frac{k}{130.86}\right)$$

This implies that:

1. the free flow speed $u_f = 61.90\ mph$
2. the jam density $k_j = 130.86\ veh/mile - lane$

The above parameters are reasonable

From MS Excel, R^2 is approximately 0.9. This means that the model is a strong fit because it's able to explain about 90% of the variation in speed. It's not 100% due to external factors like driver behavior, road conditions, etc. The model may not predict the speed well for densities beyond those in the dataset.