Northeastern University, Boston, MA

College of Engineering

Department of Civil and Environmental Engineering

CIVE 7380: Performance Models and Simulation of Transportation Networks

Basic Cyclical Operations

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CIVE 7380
Problem Set #1
Due: January 22, 2025

1. An approach to an intersection is controlled by a traffic light with cycle 60 sec. and effective green time of 40 sec. Vehicles approach at a uniform (deterministic) rate of 600 veh/hr (vehicles per hour). Vehicles are being served during the green period (when a queue is present) at a (deterministic) rate of 1800 veh/hr. The system is deterministic.
   a. Plot a diagram showing cumulative arrivals and departures
   b. Find the following quantities:
      i. Duration of queue
      ii. Number of vehicles in queue
      iii. Maximum queue length
      iv. Maximum delay
      v. Total delay
      vi. Average delay per vehicle in queue

2. A three-lane directional freeway has a capacity of 6,000 veh/hr. The flow of vehicles in the freeway is 4,800 veh/hr. An accident occurs that lasts for 0.75 hrs. The operating authority is interested in finding out the impact of accidents on the operations of the freeway. Perform a sensitivity analysis for different reduction capacity scenarios because of the incident, assuming that the capacity is reduced from 6,000 veh/hr to 2,000 veh/hr in steps of 1,000 veh/hr. For each case find the following quantities of interest assuming deterministic queuing:
   a. Duration of queue
   b. Number of vehicles in queue
   c. Maximum queue length
   d. Maximum delay
   e. Total delay
   f. Average delay per vehicle in queue

3. Passengers arrive to board a plane at steady (deterministic) rates given by:

$$\lambda = \begin{cases} 2\ pax/\min & 0 \le t < 30 \\ 3\ pax/\min & 30 \le t < 40 \\ 2\ pax/\min & 40 \le t < 55 \end{cases}$$

Passengers start boarding during the interval between 32 and 55 minutes at a maximum rate of 6 pax/min. Find:

a. The total delay before boarding
b. The maximum queue length
c. The longest delay of any customer using a FIFO policy
d. Assume that the boarding area has a capacity of 50 pax. What time should the boarding start to ensure that the capacity is not exceeded?

Solution to Q1

Let:

C = cycle time (s)

t_g = effective green time (s)

t_r = effective red time (s)

λ = arrival rate (veh/s)

μ = service rate (veh/s)

t_q = duration of queue (s)

Q_{max} = maximum queue length (s)

d_{max} = maximum delay (s)

d_{total} = total delay (s)

N_q = number of vehicles caught in the queue (veh)

$\bar{d}$ = average delay per vehicle in queue (s)

Given:

$$C = 60\ s$$

$$t_g = 40\ s$$

$$\lambda = 600 \frac{veh}{hr} \times \frac{1\ hr}{3{,}600\ s} = \frac{1}{6}\ veh/s$$

$$\mu = 1{,}800 \frac{veh}{hr} \times \frac{1\ hr}{3{,}600\ s} = \frac{1}{2}\ veh/s$$

Evaluate:

$$t_r = C - t_g$$

$$t_r = 60 - 40$$

$$t_r = 20\ s$$

Solution to Q1a

Figure 1 shows the cumulative arrivals and departures curve. Time $t = t_0$ corresponds to the start of the effective red period.

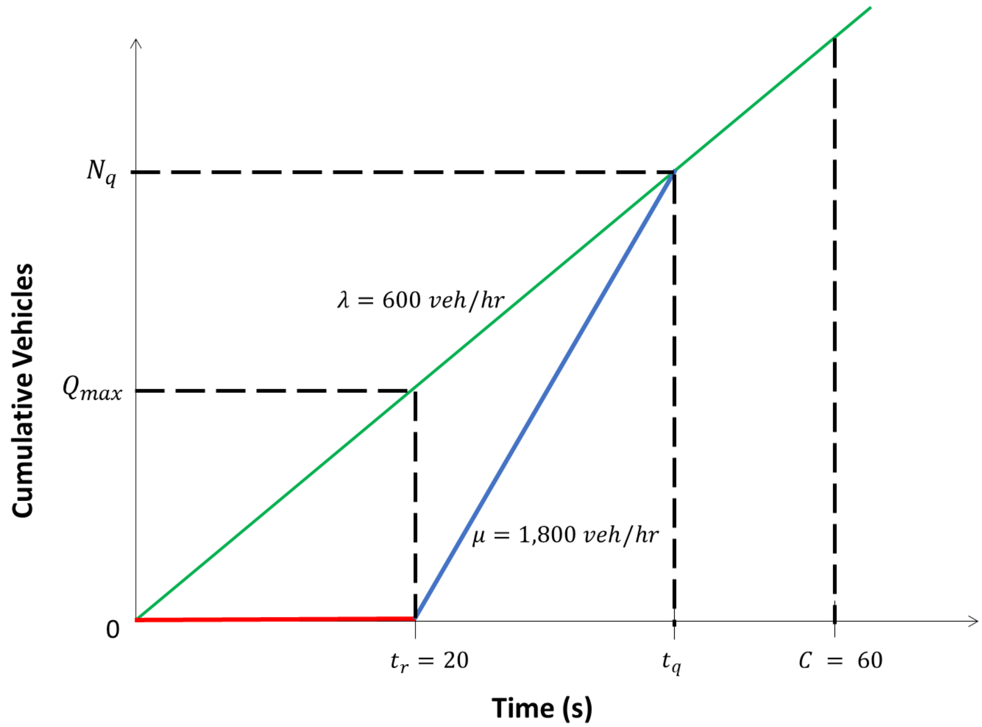

Figure 1: Cumulative Arrivals and Departures Curve at a Traffic Intersection

Solution to Q1b

Duration of the Queue, t_q

$$t_q = t_r \frac{\mu}{\mu - \lambda}$$

$$t_q = 20 \times \frac{1{,}800}{1{,}800 - 600}$$

$$t_q = 30\ s$$

Number of vehicles caught in queue, N_q

$$N_q = t_q \lambda$$

$$N_q = 30/6$$

$$N_q = 5\ veh$$

Maximum Queue Length, $\boldsymbol{Q_{max}}$

$$Q_{max} = t_r \lambda$$

$$Q_{max} = 20 \times \frac{1}{6}$$

$$Q_{max} = 3.\dot{3}\ veh$$

Maximum delay, $\boldsymbol{d_{max}}$

$$d_{max} = t_r$$

$$d_{max} = 20$$

$$d_{max} = 20\ s$$

Total delay, $\boldsymbol{d_{total}}$

$$d_{total} = \frac{1}{2} t_r t_q \lambda$$

$$d_{total} = \frac{20 \times 30 \times \frac{1}{6}}{2}$$

$$d_{total} = 50\ veh \cdot s$$

Average delay per vehicle in queue, $\boldsymbol{\bar{d}}$

$$\bar{d} = \frac{1}{2} t_r$$

$$\bar{d} = \frac{1}{2} \times 20$$

$$\bar{d} = 10\ s$$

Solution to Q2

The results of the sensitivity analysis have been presented in **Table 1**. The results are zero unless vehicle flow exceeds capacity. The full analysis can be found in the excel workbook attached to this submission. **Figure 2** illustrates the cumulative arrivals & departures curve.

Table 1: Sensitivity Analysis Results

μ_r	t_q	N_q	Q_{max}	d_{max}	d_{total}	$\bar{d}$
6,000	0	0	0	0	0	0
5,000	0	0	0	0	0	0
4,000	1.250	6,000	600	0.12500	3750.0	0.625
3,000	1.875	9,000	1,350	0.28125	8437.5	0.938
2,000	2.500	12,000	2,100	0.43750	15000.0	1.250

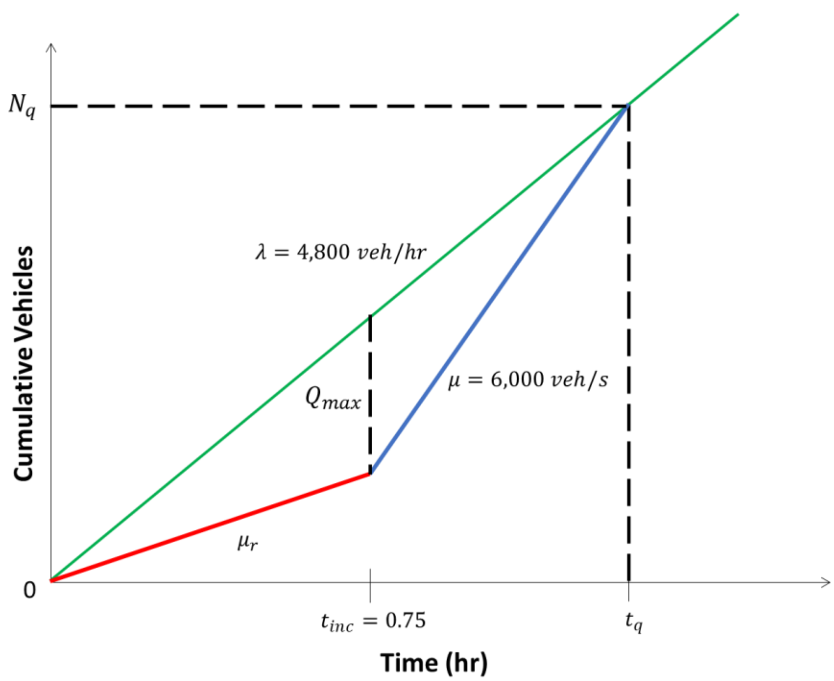

Figure 2: Cumulative Arrivals and Departures Curve at a Freeway Segment

Solution to Q3

$$\lambda = \begin{cases} 2 \ pax/\min & 0 \le t < 30 \\ 3 \ pax/\min & 30 \le t < 40 \\ 2 \ pax/\min & 40 \le t < 55 \end{cases}$$

Figure 3 shows the cumulative arrivals and departures curve at the airport gate.

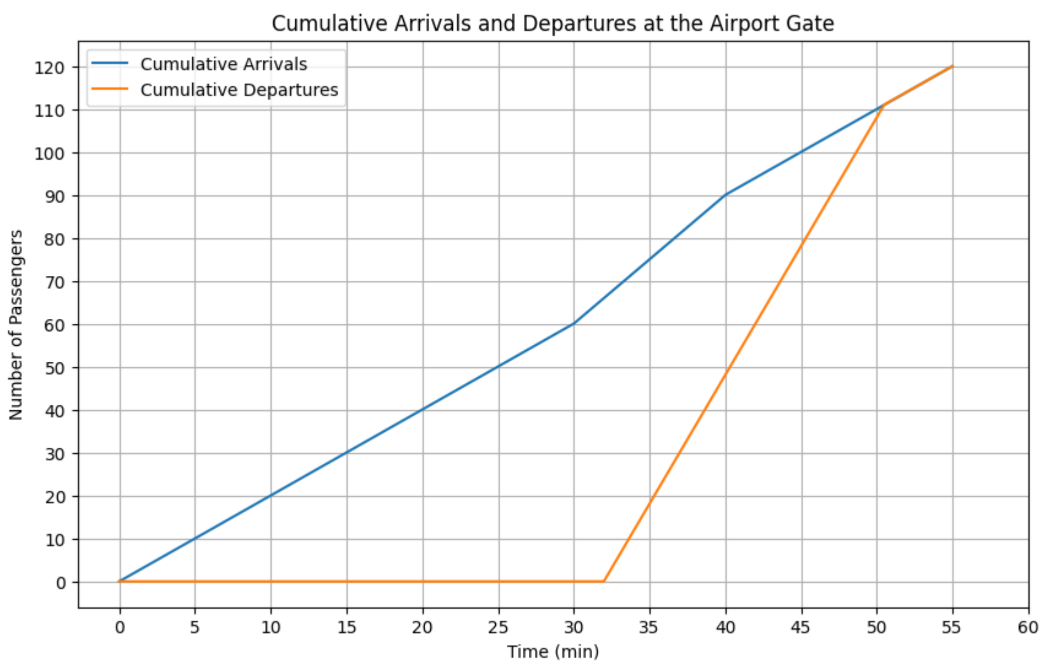

Figure 3: Cumulative Arrivals and Departures Curve at the Airport Gate

Total Delay Before Boarding

This is the area under the curve from time t = 0 to time t = 32 minutes

$$d_{pre-boarding} = d_{0-30} + d_{30-32}$$

$$d_{0-30} = \int_{0}^{30} 2t \, dt = 900 \, pax \cdot min$$

$$d_{30-32} = \int_{30}^{32} (3t - 30) \, dt = 126 \, pax \cdot min$$

$$d_{pre-boarding} = 900 + 126 = 1{,}026 \, pax \cdot min$$

Maximum Queue Length

This is the total number arrivals before boarding (time t = 32 minutes)

$$\boldsymbol{A_{0-32}} = \int_{0}^{30} 2\,dt + \int_{30}^{32} 3\,dt = 66\,pax$$

Longest Delay of any Customer

This is the time until boarding commenced. The first passenger arrived at time $t = 0$ and wasn't served until boarding began at time $t = 32\,minutes$. That passenger was delayed the longest and it was for **32 minutes**.

Assume that the boarding area has a capacity of 50 pax. What time should the boarding start to ensure that the capacity is not exceeded?

I will use common sense to solve this one:

- To ensure the boarding area's capacity of 50 passengers is not exceeded, boarding must begin as soon as the 50th passenger arrives.

- Once boarding starts, the boarding rate will always exceed or match the arrival rate, ensuring the queue length does not grow beyond 50 passengers.

- With an arrival rate of 2 pax/min for the first 30 minutes, the 50th passenger will arrive at $t = 25\,minutes$. Hence, to prevent the queue length from exceeding 50 passengers, boarding should begin no later than $t = 25\,minutes$.