Northeastern University, Boston, MA

College of Engineering

Department of Civil and Environmental Engineering

CIVE 7381: Transportation Demand Forecasting and Model Estimation

Problem Set 5

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Problems

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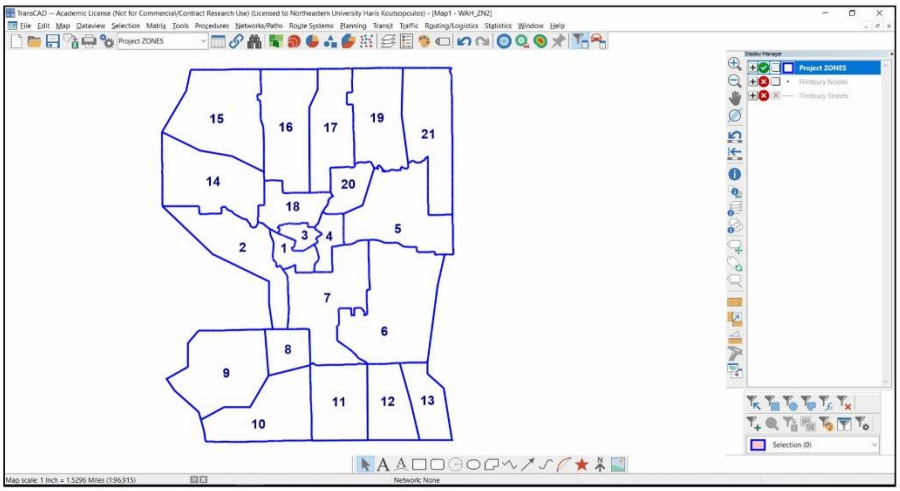
CIVE 7381 Transportation Demand
Mini Project Parts AB (Problem Set #5)
Due: Monday November 10, 2024

INSTRUCTIONS. For this assignment you can work in teams of at most 2 people. Each team will prepare one report, but each member of the team should submit a copy of the (same) report individually (for CANVAS bookkeeping purposes). Your report should be submitted in **pdf** format (supporting files can be in the native format, e.g. xlsx). **Make sure you include the names of the members of the team on the first page**. You can use Excel, or any software of your choice (e.g. python, R, matlab).

A town, illustrated in the figure below, has been divided into 21 zones (TAZ). The planning authority in the city is developing a travel demand forecasting model for the area. All the data that are needed for this mini project are in the files *project-data-part-AB.xlsx* and *project-data-part-C.xlsx* (the part C data will be used in a later assignment). These data files can be found on Canvas under the Assignments module, *Problem Set 5 (Mini project parts A and B)*, in the page *Mini Project Data*.



Part A: Trip Generation

The planning authority has developed a cross-classification model for home-based other (HBO) trip productions and attractions.

The **trip production** rates for HBO trips, in trips per household per day, are given below:

		Household Size		
		1	**2**	**3+**
auto ownership (# veh)	**0**	1.9	3.0	3.5
	1	1.8	3.5	4.8
	2+	1.6	3.4	5.3

The **trip attraction** rates for HBO trips, in trips per day, are given in the table below:

	Attractions per household	Attractions per retail employee	Attractions per non-retail employee
HBO	0.4	1.8	2.3

In addition to the trips generated according to the above attraction model, some of the zones have special attractors that generate additional trips. Hence, once the attractions have been calculated, based on the demographic/socioeconomic characteristics of the zones in the study area, the attractions of special generators must be calculated and added to the zone total. The special generators and the corresponding trip rates (per day) they attract are:

Zone	Generator	Size	Unit	Attracted trips per unit
6	Hospital	340	beds+employees	1.8
8	Port	395	employees	0.3
20	University	2,850	students	1.4
21	Shopping Center	46,500	ft^2	0.024

The demographic/socioeconomic data for the 21 zones are summarized in the table that follows (included in the file *project-data-part-AB.xlsx*).

	Households (HH) per group									total	retail	non-retail
Family Size	1			2			3+					
Auto ownership:	0	1	2+	0	1	2+	0	1	2+	HH	employees	employees
Zone 1	120	180	20	100	335	60	5	95	125	1,040	1,005	5,050
2	100	75	5	65	75	100	15	100	130	665	665	1,500
3	130	155	105	35	140	200	15	185	235	1,200	750	1,850
4	85	210	25	75	150	240	55	100	390	1,330	180	2,465
5	20	135	10	15	80	110	100	90	125	685	205	955
6	80	310	40	70	190	320	20	100	480	1,610	350	1,100
7	20	40	5	15	30	65	15	75	110	375	480	1,260
8	5	5	10	10	15	5	10	5	5	70	75	410
9	20	210	15	20	50	155	10	70	250	800	105	350
10	75	95	5	25	60	115	5	90	375	845	130	400
11	70	135	15	35	70	170	10	40	215	760	75	220
12	50	260	35	40	170	420	25	75	330	1,405	440	850
13	30	290	20	40	250	325	10	60	210	1,235	160	230
14	95	165	45	25	135	260	100	120	330	1,275	250	300
15	30	110	15	10	75	170	20	120	205	755	170	285
16	20	90	20	5	25	95	15	80	255	605	100	135
17	50	105	5	5	35	130	35	70	295	730	245	300
18	40	190	10	60	135	85	10	65	125	720	450	950
19	35	85	30	30	55	120	10	90	180	635	265	450
20	400	250	5	350	225	100	105	100	175	1,710	425	205
21	45	110	10	15	110	140	10	30	140	610	300	855

1. Calculate the total HBO trips produced by each zone. Using the demographic/socioeconomic data and the cross-classification model provided above calculate the productions for each zone.
2. Calculate the attractions for each zone. Using the attraction model and the special generators and other information above calculate the total attractions per zone.
3. Balance total productions and attractions. Once the total productions and attractions have been calculated, they must be balanced. Note that external trips are not included in the analysis. You may assume that they are 0.

This part of your report should include a table with columns of the calculated productions and attractions, as well as the corrected values for each zone. Mention all your assumptions and show the calculation of the adjustment factor.

Part B. Trip Distribution

In this part the productions and attractions that you calculated in the trip generation part are converted into zone-to-zone trips, using the gravity model.

$$T_{ij} = P_i \cdot \frac{A_j F_{ij}}{\sum_{j=1}^{J} A_j F_{ij}}$$

where, T_{ij} = the number of trips from zone "i" to zone "j"
P_i = the number of productions from zone "i"
A_j = the number of attractions to zone "j"
F_{ij} = the friction factor between zone "i" and zone "j"

The zone-to-zone travel times are given in the table below. Travel times include the terminal times within the production and attraction zone as well. The information is also available in the file: *project-data-part-AB.xlsx*.

The friction factor between zones i and j, F_{ij}, is a function of the travel time between the two zones i and j. The friction factors for different travel times and trip purposes are shown in the second table below. The table is also included in the file: *project-data-part-AB.xlsx*.

Zone-to-Zone Travel Times (minutes). Values include terminal times

To / From	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
1		10	12	15	24	11	10	13	19	22	21	17	25	16	20	21	27	23	29	25	31
2	14		10	13	17	22	19	23	26	29	27	28	32	8	12	11	16	12	17	15	20
3	16	13		8	14	15	15	18	24	26	27	25	28	12	15	11	15	11	15	14	19
4	19	16	11		10	10	14	17	25	27	28	23	25	15	18	14	15	13	12	10	14
5	28	20	17	12		10	12	16	21	22	19	14	13	15	19	16	15	14	15	11	12
6	15	25	18	13	12		12	15	19	20	18	13	14	17	19	18	18	16	17	15	17
7	14	22	18	16	13	13		8	13	12	10	11	17	16	18	16	17	14	18	16	20
8	17	26	21	19	17	17	10		10	10	11	14	18	18	20	19	19	16	20	19	22
9	23	29	27	27	22	21	14	12		12	13	16	18	22	25	26	27	25	28	27	33
10	26	32	29	29	23	22	13	11	12		10	13	15	21	23	24	25	24	27	26	32
11	25	30	30	30	22	20	11	12	13	10		10	14	20	22	23	24	23	26	24	30
12	21	31	28	25	15	15	12	15	16	13	10		12	18	20	21	22	21	24	22	28
13	29	35	31	27	14	16	18	19	18	15	14	12		22	25	26	27	26	28	26	33
14	20	12	15	17	16	19	17	19	22	21	20	18	22		13	16	18	17	21	20	24
15	24	15	18	20	20	21	19	21	25	23	22	20	25	13		12	15	18	21	19	23
16	25	14	14	16	17	20	17	20	26	24	23	21	26	16	12		10	12	14	15	17
17	31	19	18	17	16	20	18	20	27	25	24	22	27	18	15	10		10	12	13	15
18	27	15	14	15	15	18	15	17	25	24	23	21	26	17	18	12	10		14	12	15
19	33	20	18	14	16	19	19	21	28	27	26	24	28	21	21	14	12	14		12	14
20	29	18	17	10	12	17	17	20	27	26	24	22	26	20	19	15	13	12	12		14
21	35	23	22	16	12	19	21	23	33	32	30	28	33	24	23	17	15	15	14	14	

Friction Factors for Different Travel Times (minutes)

Travel Time	HBW Factor	HBO Factor	NHB Factor
1	30	70.00	33
2	20	40.00	20
3	15	22.00	15
4	10	15.00	10
5	8	9.60	7.5
6	5.6	8.00	6
7	4.5	5.20	4.5
8	3.5	3.80	3.4
9	2.8	2.70	2.8
10	2.2	2.00	2.2
11	1.8	1.80	1.8
12	1.5	1.40	1.5
13	1.3	1.00	1.2
14	1.1	0.85	1
15	0.9	0.80	0.8
16	0.74	0.60	0.7
17	0.62	0.52	0.5
18	0.55	0.45	0.5
19	0.5	0.40	0.42
20	0.44	0.34	0.36
21	0.38	0.30	0.31
22	0.35	0.26	0.24
23	0.32	0.24	0.25
24	0.29	0.21	0.22
25	0.28	0.18	0.19
26	0.25	0.16	0.16
27	0.24	0.15	0.16
28	0.23	0.13	0.14
29	0.22	0.13	0.13
30	0.2	0.12	0.13
31	0.19	0.10	0.1
32	0.18	0.09	0.1
33	0.17	0.08	0.09
34	0.17	0.07	0.09
35	0.16	0.07	0.08

Use (a) the zone-to-zone travel times, (b) the corresponding friction factors, and (c) the productions and attractions calculated in part A, to complete the trip distribution. Perform the following:

1. Complete the travel time matrix by adding intra-zonal travel times. To find these travel times, use the "nearest neighbor" technique *(half the travel time from the zone of interest to its nearest neighbor).*

2. Create the matrix of the inter-zonal friction factors, based on the travel times and friction factors provided.
3. Create a matrix of the zone-to-zone trip distribution using the gravity model. The inputs will be the balanced productions and attractions you calculated in part A and the matrices you developed in question 2) above.
4. Check if the trips calculated in question 3) satisfy the total attraction and production constraints. If not, use any of the methods discussed in class to balance productions and attractions for each zone.

Your report for part B should include the completed travel time matrix (with intra-zonal travel times), the matrix of friction factors, and the matrix of the zone-to-zone origins and destinations (including intermediate iterations).

Part A: Trip Generation

The total Home-Based-Other (HBO) trip productions from, and attractions to, each of the 21 traffic analysis zones (TAZs) are highlighted in yellow in ***Table 1***. These values were calculated using the demographic and socioeconomic data, the cross-classification model for trip production, the attraction model, and special attraction generators.

Table 1: HBO Trip Productions and Attractions

	Households (HH) per group									Total							
Family size	1			2			3+			Households	Retail employees	Non-retail employees	HBO Trip Productions	HBO Trip Attractions	Special Trip Attractions	Total Trip Attractions	Balanced attractions
Auto ownership	0	1	2+	0	1	2+	0	1	2+								
Zone 1	120	180	20	100	335	60	5	95	125	1,040	1,005	5,050	3,396.5	13,840.0		13,840.0	13,035.4
Zone 2	100	75	5	65	75	100	15	100	130	665	665	1,500	2,352.0	4,913.0		4,913.0	4,627.4
Zone 3	130	155	105	35	140	200	15	185	235	1,200	750	1,850	4,155.0	6,085.0		6,085.0	5,731.2
Zone 4	85	210	25	75	150	240	55	100	390	1,330	180	2,465	4,885.0	6,525.5		6,525.5	6,146.1
Zone 5	20	135	10	15	80	110	100	90	125	685	205	955	2,440.5	2,839.5		2,839.5	2,674.4
Zone 6	80	310	40	70	190	320	20	100	480	1,610	350	1,100	5,831.0	3,804.0	612.0	4,416.0	4,159.3
Zone 7	20	40	5	15	30	65	15	75	110	375	480	1,260	1,484.5	3,912.0		3,912.0	3,684.6
Zone 8	5	5	10	10	15	5	10	5	5	70	75	410	219.5	1,106.0	118.5	1,224.5	1,153.3
Zone 9	20	210	15	20	50	155	10	70	250	800	105	350	2,898.0	1,314.0		1,314.0	1,237.6
Zone 10	75	95	5	25	60	115	5	90	375	845	130	400	3,434.5	1,492.0		1,492.0	1,405.3
Zone 11	70	135	15	35	70	170	10	40	215	760	75	220	2,694.5	945.0		945.0	890.1
Zone 12	50	260	35	40	170	420	25	75	330	1,405	440	850	4,958.5	3,309.0		3,309.0	3,116.6
Zone 13	30	290	20	40	250	325	10	60	210	1,235	160	230	4,147.0	1,311.0		1,311.0	1,234.8
Zone 14	95	165	45	25	135	260	100	120	330	1,275	250	300	4,656.0	1,650.0		1,650.0	1,554.1
Zone 15	30	110	15	10	75	170	20	120	205	755	170	285	2,882.0	1,263.5		1,263.5	1,190.0
Zone 16	20	90	20	5	25	95	15	80	255	605	100	135	2,445.5	732.5		732.5	689.9
Zone 17	50	105	5	5	35	130	35	70	295	730	245	300	2,893.5	1,423.0		1,423.0	1,340.3
Zone 18	40	190	10	60	135	85	10	65	125	720	450	950	2,385.0	3,283.0		3,283.0	3,092.1
Zone 19	35	85	30	30	55	120	10	90	180	635	265	450	2,379.0	1,766.0		1,766.0	1,663.3
Zone 20	400	250	5	350	225	100	105	100	175	1,710	425	205	5,170.5	1,920.5	3,990.0	5,910.5	5,566.9
Zone 21	45	110	10	15	110	140	10	30	140	610	300	855	2,126.5	2,750.5	1,116.0	3,866.5	3,641.7
									Sum total:	19,060	6,825	20,120	67,835	66,185	5,837	72,022	67,835

The total trip productions did not initially match the total trip attractions. To reconcile this discrepancy, I kept the total productions constant and scaled down the attractions, based on the assumption that the attraction models are more prone to estimation errors than the production models, which are considered more reliable.

Given the constraints $\sum P_i = 67{,}835$ and $\sum A_j = 72{,}022$, I adjusted each attraction value, A_j, by multiplying it by an adjustment factor $\frac{\sum P_i}{\sum A_j} = \frac{67{,}835}{72{,}022} = 0.94186$. After applying this adjustment factor, the totals were balanced, yielding $\sum P_i = \sum A_j$. The corrected attractions are highlighted in green in ***Table 1***. External trips were excluded from these calculations.

Part B: Trip Distribution

The intrazonal travel times were calculated using **Equation 1.**

$$T_{ii} = \alpha \cdot T_{i\text{nearest}} = \alpha \cdot \min_{j \in N(i)} T_{ij}$$

Equation 1: Intrazonal Travel Time

Where:

T_{ii} = intrazonal travel time within Zone i

T_{ij} = travel time from Zone i to Zone j

$N(i)$ = the set of all neighboring zones j adjacent to Zone i

α = a proportionality constant which I set to 0.5

The full travel time matrix is shown in ***Table 2***.

Table 2: Travel Time Matrix

Interzonal Travel Times																					
	Zone 1	Zone 2	Zone 3	Zone 4	Zone 5	Zone 6	Zone 7	Zone 8	Zone 9	Zone 10	Zone 11	Zone 12	Zone 13	Zone 14	Zone 15	Zone 16	Zone 17	Zone 18	Zone 19	Zone 20	Zone 21
Zone 1	5	10	12	15	24	11	10	13	19	22	21	17	25	16	20	21	27	23	29	25	31
Zone 2	14	4	10	13	17	22	19	23	26	29	27	28	32	8	12	11	16	12	17	15	20
Zone 3	16	13	4	8	14	15	15	18	24	26	27	25	28	12	15	11	15	11	15	14	19
Zone 4	19	16	11	5	10	10	14	17	25	27	28	23	25	15	18	14	15	13	12	10	14
Zone 5	28	20	17	12	5	10	12	16	21	22	19	14	13	15	19	16	15	14	15	11	12
Zone 6	15	25	18	13	12	6	12	15	19	20	18	13	14	17	19	18	18	16	17	15	17
Zone 7	14	22	18	16	13	13	4	8	13	12	10	11	17	16	18	16	17	14	18	16	20
Zone 8	17	26	21	19	17	17	10	5	10	10	11	14	18	18	20	19	19	16	20	19	22
Zone 9	23	29	27	27	22	21	14	12	6	12	13	16	18	22	25	26	27	25	28	27	33
Zone 10	26	32	29	29	23	22	13	11	12	5	10	13	15	21	23	24	25	24	27	26	32
Zone 11	25	30	30	30	22	20	11	12	13	10	5	10	14	20	22	23	24	23	26	24	30
Zone 12	21	31	28	25	15	15	12	15	16	13	10	5	12	18	20	21	22	21	24	22	28
Zone 13	29	35	31	27	14	16	18	19	18	15	14	12	6	22	25	26	27	26	28	26	33
Zone 14	20	12	15	17	16	19	17	19	22	21	20	18	22	6	13	16	18	17	21	20	24
Zone 15	24	15	18	20	20	21	19	21	25	23	22	20	25	13	6	12	15	18	21	19	23
Zone 16	25	14	14	16	17	20	17	20	26	24	23	21	26	16	12	5	10	12	14	15	17
Zone 17	31	19	18	17	16	20	18	20	27	25	24	22	27	18	15	10	5	10	12	13	15
Zone 18	27	15	14	15	15	18	15	17	25	24	23	21	26	17	18	12	10	5	14	12	15
Zone 19	33	20	18	14	16	19	19	21	28	27	26	24	28	21	21	14	12	14	6	12	14
Zone 20	29	18	17	10	12	17	17	20	27	26	24	22	26	20	19	15	13	12	12	5	14
Zone 21	35	23	22	16	12	19	21	23	33	32	30	28	33	24	23	17	15	15	14	14	6

Using the full travel time matrix and the Home-Based-Other (HBO) friction factor values, which are calculated as a function of travel times, the friction factor matrix presented in ***Table 3*** was developed. This matrix was then used to perform trip distribution with the gravity model, following *Error! Reference source not found.*.

Table 3: Friction Factor Matrix

Interzonal Friction Factors																					
	Zone 1	Zone 2	Zone 3	Zone 4	Zone 5	Zone 6	Zone 7	Zone 8	Zone 9	Zone 10	Zone 11	Zone 12	Zone 13	Zone 14	Zone 15	Zone 16	Zone 17	Zone 18	Zone 19	Zone 20	Zone 21
Zone 1	9.60	2.00	1.40	0.80	0.21	1.80	2.00	1.00	0.40	0.26	0.30	0.52	0.18	0.60	0.34	0.30	0.15	0.24	0.13	0.18	0.10
Zone 2	0.85	15.00	2.00	1.00	0.52	0.26	0.40	0.24	0.16	0.13	0.15	0.13	0.09	3.80	1.40	1.80	0.60	1.40	0.52	0.80	0.34
Zone 3	0.60	1.00	15.00	3.80	0.85	0.80	0.80	0.45	0.21	0.16	0.15	0.18	0.13	1.40	0.80	1.80	0.80	1.80	0.80	0.85	0.40
Zone 4	0.40	0.60	1.80	9.60	2.00	2.00	0.85	0.52	0.18	0.15	0.13	0.24	0.18	0.80	0.45	0.85	0.80	1.00	1.40	2.00	0.85
Zone 5	0.13	0.34	0.52	1.40	9.60	2.00	1.40	0.60	0.30	0.26	0.40	0.85	1.00	0.80	0.40	0.60	0.80	0.85	0.80	1.80	1.40
Zone 6	0.80	0.18	0.45	1.00	1.40	8.00	1.40	0.80	0.40	0.34	0.45	1.00	0.85	0.52	0.40	0.45	0.45	0.60	0.52	0.80	0.52
Zone 7	0.85	0.26	0.45	0.60	1.00	1.00	15.00	3.80	1.00	1.40	2.00	1.80	0.52	0.60	0.45	0.60	0.52	0.85	0.45	0.60	0.34
Zone 8	0.52	0.16	0.30	0.40	0.52	0.52	2.00	9.60	2.00	2.00	1.80	0.85	0.45	0.45	0.34	0.40	0.40	0.60	0.34	0.40	0.26
Zone 9	0.24	0.13	0.15	0.15	0.26	0.30	0.85	1.40	8.00	1.40	1.00	0.60	0.45	0.26	0.18	0.16	0.15	0.18	0.13	0.15	0.08
Zone 10	0.16	0.09	0.13	0.13	0.24	0.26	1.00	1.80	1.40	9.60	2.00	1.00	0.80	0.30	0.24	0.21	0.18	0.21	0.15	0.16	0.09
Zone 11	0.18	0.12	0.12	0.12	0.26	0.34	1.80	1.40	1.00	2.00	9.60	2.00	0.85	0.34	0.26	0.24	0.21	0.24	0.16	0.21	0.12
Zone 12	0.30	0.10	0.13	0.18	0.80	0.80	1.40	0.80	0.60	1.00	2.00	9.60	1.40	0.45	0.34	0.30	0.26	0.30	0.21	0.26	0.13
Zone 13	0.13	0.07	0.10	0.15	0.85	0.60	0.45	0.40	0.45	0.80	0.85	1.40	8.00	0.26	0.18	0.16	0.15	0.16	0.13	0.16	0.08
Zone 14	0.34	1.40	0.80	0.52	0.60	0.40	0.52	0.40	0.26	0.30	0.34	0.45	0.26	8.00	1.00	0.60	0.45	0.52	0.30	0.34	0.21
Zone 15	0.21	0.80	0.45	0.34	0.34	0.30	0.40	0.30	0.18	0.24	0.26	0.34	0.18	1.00	8.00	1.40	0.80	0.45	0.30	0.40	0.24
Zone 16	0.18	0.85	0.85	0.60	0.52	0.34	0.52	0.34	0.16	0.21	0.24	0.30	0.16	0.60	1.40	9.60	2.00	1.40	0.85	0.80	0.52
Zone 17	0.10	0.40	0.45	0.52	0.60	0.34	0.45	0.34	0.15	0.18	0.21	0.26	0.15	0.45	0.80	2.00	9.60	2.00	1.40	1.00	0.80
Zone 18	0.15	0.80	0.85	0.80	0.80	0.45	0.80	0.52	0.18	0.21	0.24	0.30	0.16	0.52	0.45	1.40	2.00	9.60	0.85	1.40	0.80
Zone 19	0.08	0.34	0.45	0.85	0.60	0.40	0.40	0.30	0.13	0.15	0.16	0.21	0.13	0.30	0.30	0.85	1.40	0.85	8.00	1.40	0.85
Zone 20	0.13	0.45	0.52	2.00	1.40	0.52	0.52	0.34	0.15	0.16	0.21	0.26	0.16	0.34	0.40	0.80	1.00	1.40	1.40	9.60	0.85
Zone 21	0.07	0.24	0.26	0.60	1.40	0.40	0.30	0.24	0.08	0.09	0.12	0.13	0.08	0.21	0.24	0.52	0.80	0.80	0.85	0.85	8.00

$$T_{ij} = P_i \cdot \frac{A_j F_{ij}}{\sum_{j=1}^{J} A_j F_{ij}}$$

Equation 2: Gravity Model

Where:

P_i = total productions at Zone i

A_j = total attractions at Zone j

F_{ij} = friction factor from Zone i to Zone j

T_{ij} = number of trips from Zone i to Zone j

The resulting trip distribution matrix is shown in **Table** *4*. Both the total productions ($\sum P_i$) and total attractions ($\sum A_j$) matched the target value of 67,835, as expected. Each (P_i) value calculated from the gravity model also aligned with the target zonal productions. However, the (A_j) values did not fully match the target zonal attractions.

To balance the zonal attractions, the initial attraction values obtained after the first application of the gravity model were updated according to the equation displayed in **Figure** *1*. This iterative adjustment ensured that the distributed attractions in each zone aligned more closely with the target values. To obtain zonal attraction values that deviate from the target values by **no more than 5%**, the balancing process was performed twice.

Table 4: Initial Trip Distribution Matrix

Trip Distribution - Gravity Model																						Totals		
	Zone 1	Zone 2	Zone 3	Zone 4	Zone 5	Zone 6	Zone 7	Zone 8	Zone 9	Zone 10	Zone 11	Zone 12	Zone 13	Zone 14	Zone 15	Zone 16	Zone 17	Zone 18	Zone 19	Zone 20	Zone 21	ΣAjFij	ΣP	Target P
Zone 1	2,486.4	183.9	159.4	97.7	11.2	148.8	146.4	22.9	9.8	7.3	5.3	32.2	4.4	18.5	8.0	4.1	4.0	14.7	4.3	19.9	7.2	170,946.1	3,397	3,397
Zone 2	210.4	1,318.1	217.7	116.7	26.4	20.5	28.0	5.3	3.8	3.5	2.5	7.7	2.1	112.1	31.6	23.6	15.3	82.2	16.4	84.6	23.5	123,855.9	2,352	2,352
Zone 3	215.6	127.6	2,370.2	643.9	62.7	91.7	81.3	14.3	7.2	6.2	3.7	15.5	4.4	60.0	26.2	34.2	29.6	153.5	36.7	130.5	40.2	150,706.6	4,155	4,155
Zone 4	213.5	113.7	422.4	2,415.7	219.0	340.6	128.2	24.6	9.1	8.6	4.7	30.6	9.1	50.9	21.9	24.0	43.9	126.6	95.3	455.8	126.7	119,315.8	4,885	4,885
Zone 5	50.5	46.9	88.7	256.2	764.5	247.7	153.6	20.6	11.1	10.9	10.6	78.9	36.8	37.0	14.2	12.3	31.9	78.3	39.6	298.4	151.8	81,956.0	2,441	2,441
Zone 6	761.1	60.8	188.2	448.6	273.3	2,428.6	376.5	67.3	36.1	34.9	29.2	227.5	76.6	59.0	34.7	22.7	44.0	135.4	63.1	325.0	138.2	79,890.6	5,831	5,831
Zone 7	154.0	16.7	35.8	51.3	37.2	57.8	768.2	60.9	17.2	27.3	24.7	78.0	8.9	13.0	7.4	5.8	9.7	36.5	10.4	46.4	17.2	106,806.0	1,485	1,485
Zone 8	29.0	3.2	7.4	10.5	6.0	9.3	31.5	47.4	10.6	12.0	6.9	11.3	2.4	3.0	1.7	1.2	2.3	7.9	2.4	9.5	4.1	51,294.2	220	220
Zone 9	300.1	57.7	82.5	88.4	66.7	119.7	300.4	154.9	949.6	188.7	85.4	179.4	53.3	38.8	20.5	10.6	19.3	53.4	20.7	80.1	27.9	30,214.1	2,898	2,898
Zone 10	199.6	39.9	71.3	76.5	61.4	103.5	352.6	198.6	165.8	1,290.9	170.3	298.2	94.5	44.6	27.3	13.9	23.1	62.1	23.9	85.2	31.4	35,893.0	3,435	3,435
Zone 11	164.4	38.9	48.2	51.7	48.7	99.1	464.7	113.1	86.7	196.9	598.7	436.7	73.5	37.0	21.7	11.6	19.7	52.0	18.6	81.9	30.6	38,458.0	2,695	2,695
Zone 12	333.1	39.4	63.5	94.2	182.3	283.5	439.4	78.6	63.3	119.7	151.6	2,548.8	147.3	59.6	34.5	17.6	29.7	79.0	29.8	123.3	40.3	58,206.2	4,959	4,959
Zone 13	235.0	44.9	79.5	127.9	315.3	346.1	229.9	64.0	77.2	155.9	104.9	605.1	1,369.9	56.0	29.7	15.3	27.9	68.6	30.0	123.5	40.4	29,903.7	4,147	4,147
Zone 14	443.7	648.5	459.0	319.9	160.6	166.5	191.8	46.2	32.2	42.2	30.3	140.4	32.1	1,244.6	119.1	41.4	60.4	161.0	50.0	189.5	76.6	46,510.7	4,656	4,656
Zone 15	223.7	302.6	210.8	170.8	74.3	102.0	120.5	28.3	18.2	27.6	18.9	86.6	18.2	127.0	778.1	78.9	87.6	113.7	40.8	182.0	71.4	35,262.1	2,882	2,882
Zone 16	125.3	210.1	260.2	197.0	74.3	75.5	102.3	20.9	10.6	15.8	11.4	49.9	10.6	49.8	89.0	353.8	143.2	231.2	75.5	237.9	101.2	45,783.4	2,446	2,446
Zone 17	77.8	110.4	153.8	190.6	95.7	84.3	98.9	23.4	11.1	15.1	11.1	48.3	11.0	41.7	56.8	82.3	767.5	368.9	138.9	332.0	173.8	48,510.6	2,894	2,894
Zone 18	65.1	123.2	162.1	163.6	71.2	62.3	98.1	20.0	7.4	9.8	7.1	31.1	6.6	26.9	17.8	32.1	89.2	987.9	47.1	259.4	97.0	71,663.8	2,385	2,385
Zone 19	52.8	79.7	130.7	264.7	81.3	84.3	74.7	17.5	8.2	10.7	7.2	33.2	8.1	23.6	18.1	29.7	95.1	133.2	674.3	394.9	156.9	46,946.8	2,379	2,379
Zone 20	92.3	113.4	162.3	669.3	203.9	117.8	104.3	21.4	10.1	12.2	10.2	44.1	10.8	28.8	25.9	30.1	73.0	255.7	126.8	2,909.8	168.5	94,961.7	5,171	5,171
Zone 21	35.5	43.2	58.0	143.6	145.8	64.8	43.0	10.8	3.9	4.9	4.2	15.8	3.8	12.7	11.1	14.0	41.7	96.3	55.0	184.2	1,134.2	54,623.3	2,127	2,127
ΣA	6,469	3,723	5,432	6,599	2,982	5,054	4,334	1,061	1,549	2,201	1,299	4,999	1,984	2,145	1,396	859	1,658	3,278	1,690	6,554	2,659		67,835	67,835
Target A	13,035	4,627	5,731	6,146	2,674	4,159	3,685	1,153	1,238	1,405	890	3,117	1,235	1,554	1,190	690	1,340	3,092	1,663	5,567	3,642			

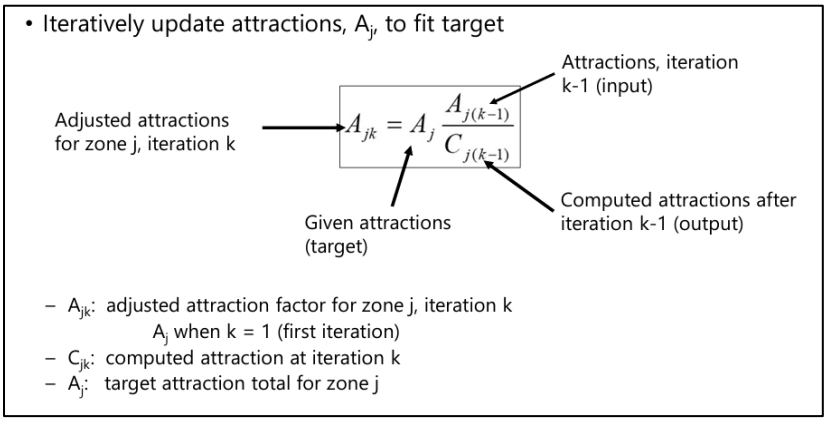


Figure 1: Formula to Update Attractions for Gravity Model

The final trip distribution matrix is shown in **Table 5**.

Table 5: Final Trip Distribution Matrix

Trip Distribution - after second balancing																						Totals		
	Zone 1	Zone 2	Zone 3	Zone 4	Zone 5	Zone 6	Zone 7	Zone 8	Zone 9	Zone 10	Zone 11	Zone 12	Zone 13	Zone 14	Zone 15	Zone 16	Zone 17	Zone 18	Zone 19	Zone 20	Zone 21	ΣAjFij	ΣP	Target P
Zone 1	2,948.3	124.8	87.9	45.7	4.8	60.8	59.4	11.7	3.7	2.0	1.6	8.7	1.2	6.8	3.4	1.6	1.6	6.9	2.2	8.2	5.2	336,405.4	3,397	3,397
Zone 2	384.3	1,377.7	184.9	84.0	17.5	12.9	17.5	4.1	2.2	1.5	1.2	3.2	0.9	63.4	20.9	14.6	9.3	59.2	13.1	53.5	26.3	158,242.4	2,352	2,352
Zone 3	462.3	156.5	2,363.6	544.1	48.7	67.8	59.6	13.2	4.9	3.1	2.0	7.5	2.1	39.8	20.3	24.8	21.1	129.6	34.3	96.8	52.7	164,034.1	4,155	4,155
Zone 4	504.6	153.8	464.4	2,250.6	187.6	277.6	103.7	24.9	6.8	4.8	2.8	16.5	4.8	37.3	18.7	19.2	34.6	117.9	98.3	372.8	183.2	117,791.0	4,885	4,885
Zone 5	129.1	68.6	105.6	258.4	708.7	218.5	134.4	22.6	9.0	6.5	6.9	45.9	21.1	29.3	13.1	10.7	27.2	78.9	44.2	264.1	237.5	74,758.2	2,441	2,441
Zone 6	1,719.1	78.6	197.8	399.4	223.7	1,891.7	290.9	65.3	25.9	18.5	16.8	116.9	38.8	41.3	28.3	17.3	33.1	120.5	62.2	254.1	190.9	82,538.8	5,831	5,831
Zone 7	369.9	23.0	40.0	48.5	32.4	47.9	631.2	62.8	13.1	15.4	15.1	42.6	4.8	9.6	6.5	4.7	7.8	34.6	10.9	38.6	25.3	103,772.8	1,485	1,485
Zone 8	64.6	4.0	7.6	9.2	4.8	7.1	24.0	45.3	7.5	6.3	3.9	5.7	1.2	2.1	1.4	0.9	1.7	7.0	2.4	7.3	5.5	53,751.5	220	220
Zone 9	746.5	82.1	95.4	86.7	60.1	102.7	255.6	165.4	748.5	110.1	54.0	101.5	29.7	29.9	18.5	8.9	16.0	52.3	22.5	68.9	42.5	28,341.7	2,898	2,898
Zone 10	592.5	67.7	98.4	89.5	66.1	105.9	358.1	253.1	155.9	899.1	128.6	201.5	62.9	41.0	29.3	13.9	22.8	72.7	30.9	87.6	56.9	28,213.3	3,435	3,435
Zone 11	473.2	64.1	64.5	58.6	50.8	98.4	457.6	139.8	79.1	133.0	438.3	286.1	47.4	33.0	22.5	11.3	18.9	59.0	23.4	81.6	53.9	31,177.1	2,695	2,695
Zone 12	1,014.5	68.7	89.9	113.1	201.1	297.7	457.8	102.7	61.0	85.5	117.5	1,766.4	100.5	56.2	37.9	18.2	30.1	94.8	39.5	129.9	75.1	44,599.7	4,959	4,959
Zone 13	711.2	77.8	111.9	152.5	345.7	361.2	238.1	83.1	74.1	110.7	80.8	416.8	928.7	52.5	32.5	15.7	28.1	81.8	39.6	129.4	74.8	23,056.4	4,147	4,147
Zone 14	991.8	829.7	477.3	281.9	130.1	128.4	146.7	44.3	22.8	22.1	17.2	71.4	16.1	861.8	96.2	31.3	45.0	141.8	48.7	146.6	104.7	48,549.6	4,656	4,656
Zone 15	510.1	394.7	223.5	153.5	61.4	80.2	93.9	27.7	13.2	14.7	11.0	44.9	9.3	89.7	640.8	60.9	66.6	102.2	40.6	143.6	99.6	36,091.9	2,882	2,882
Zone 16	292.4	280.5	282.4	181.2	62.8	60.8	81.7	21.0	7.8	8.6	6.8	26.5	5.5	36.0	75.0	279.2	111.4	212.6	76.9	192.1	144.3	45,788.9	2,446	2,446
Zone 17	193.5	157.2	178.0	187.0	86.3	72.4	84.2	25.0	8.7	8.8	7.1	27.4	6.2	32.1	51.0	69.3	636.6	361.7	150.8	285.9	264.4	45,495.1	2,894	2,894
Zone 18	156.1	169.1	180.9	154.7	61.9	51.5	80.5	20.6	5.6	5.5	4.3	17.0	3.5	20.0	15.4	26.1	71.3	933.9	49.2	215.3	142.3	69,708.9	2,385	2,385
Zone 19	124.9	107.9	143.7	246.7	69.7	68.7	60.4	17.8	6.1	5.9	4.3	17.8	4.3	17.3	15.4	23.8	74.9	124.1	695.4	323.1	226.8	46,342.5	2,379	2,379
Zone 20	240.0	168.8	196.3	686.2	192.2	105.6	92.8	23.8	8.3	7.5	6.7	26.1	6.3	23.2	24.4	26.4	63.3	241.6	143.9	2,619.0	268.1	85,194.4	5,171	5,171
Zone 21	69.2	48.2	52.5	110.2	102.8	43.5	28.7	9.0	2.4	2.2	2.1	7.0	1.7	7.7	7.8	9.2	27.1	73.9	46.8	124.1	1,350.6	65,465.9	2,127	2,127
ΣA	12,698	4,503	5,647	6,142	2,719	4,162	3,757	1,183	1,267	1,472	929	3,258	1,297	1,530	1,179	688	1,349	3,107	1,676	5,642	3,631		67,835	67,835
Target A	13,035	4,627	5,731	6,146	2,674	4,159	3,685	1,153	1,238	1,405	890	3,117	1,235	1,554	1,190	690	1,340	3,092	1,663	5,567	3,642			
Error	-2.6%	-2.7%	-1.5%	-0.1%	1.7%	0.1%	2.0%	2.6%	2.3%	4.8%	4.4%	4.5%	5.0%	-1.5%	-0.9%	-0.3%	0.6%	0.5%	0.7%	1.4%	-0.3%			