

PhD Thesis codes

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1 Pre-impact phase of droplet impact onto a viscoelastic surface.

Sec. 2.1.2.

Code: 'pre_impact_viscoelastic_code.m'

$$\begin{aligned} F_{TT} &= \frac{1}{\pi}(PV) \int_{-\infty}^{\infty} \frac{\tilde{e}_3 G_{\zeta}(\zeta, T) + \tilde{e}_5 G_{\zeta T}(\zeta, T)}{X - \zeta} d\zeta, \\ ((F - G)^3(\tilde{e}_3 G_X + \tilde{e}_5 G_{XT}))_X &= 12(F - G)_T, \\ F \rightarrow \frac{X^2}{2} - T, G \rightarrow 0, \quad &\text{as } T \rightarrow -\infty, |X| \rightarrow \infty. \end{aligned}$$

2 Pre-impact phase of droplet impact onto a flexible surface

Sec. 2.1.3.

Code: 'pre_impact_flexible_code.m'

$$\begin{aligned} F_{TT} &= \frac{1}{\pi}(PV) \int_{-\infty}^{\infty} \frac{P_{\zeta}(\zeta, T)}{X - \zeta} d\zeta, \\ ((F - G)^3 P_X)_X &= 12(F - G)_T, \\ \tilde{e}_1 G_{XXXX} + \tilde{e}_2 G_{XX} + \tilde{e}_3 G + \tilde{e}_4 G_{TT} + \tilde{e}_5 G_T &= P, \\ F \rightarrow \frac{X^2}{2} - T, G \rightarrow 0, P \rightarrow 0 \quad &\text{as } T \rightarrow -\infty, |X| \rightarrow \infty. \end{aligned}$$

3 Pre-impact phase of droplet impact onto a lubricant-infused surface

Sec. 2.2.

Code: 'pre_impact_LIS_code.m'

$$\begin{aligned}
F_{TT} &= \frac{1}{\pi}(PV) \int_{-\infty}^{\infty} \frac{P_{\zeta}(\zeta, T)}{X - \zeta} d\zeta, \\
\left(\frac{F^3(F + 4\Lambda)}{F + \Lambda} P_X \right)_X &= 12F_T, \\
F &\rightarrow \frac{X^2}{2} - T, P \rightarrow 0 \quad \text{as } T \rightarrow -\infty, |X| \rightarrow \infty.
\end{aligned}$$

4 Boundary layer jet on a flat lubricant-infused surface

Sec. 4.2.2.

Code: 'BL_flat_LIS_code.m'

$$\begin{aligned}
h^2 u u_X + 2X h \tilde{v} u_Y &= \frac{2X}{\tilde{Re}} u_{YY}, \\
(hu)_X + \tilde{v}_Y &= 0,
\end{aligned} \tag{1}$$

$$u = \Lambda u_Y, \quad \tilde{v} = 0, \quad \text{at } Y = 0, \tag{2}$$

$$u_Y = 0, \quad \tilde{v} = 0, \quad Y = 1, \tag{3}$$

$$h = 1, \quad u = \frac{3}{2 + 6\Lambda} (2\Lambda + 2Y - Y^2), \quad \tilde{v} = 0, \quad \text{at } X = 0 \tag{4}$$

5 Boundary layer jet on a deformable lubricant meniscus.

Sec. 4.3.2.

Code: 'BL_meniscus_LIS_code.m'

$$\begin{aligned}
h^2 X_x u u_X + h \tilde{v} u_Y &= \frac{1}{\varepsilon^2 \text{Re}} u_{YY}, \\
X_x (hu)_X + \tilde{v}_Y &= 0, \\
H'''(x) &= -\frac{2\mu \text{Ca}}{\varepsilon^3} \frac{u_s(3A + B)}{(H + 1)^2} + \frac{\rho \text{ReCa}}{\varepsilon} u_s(u_s(A + B + C))',
\end{aligned} \tag{5}$$

$$u = \frac{H + 1}{\mu h(3A + 2B + C)} u_Y, \quad \tilde{v} = 0, \quad \text{at } Y = 0,$$

$$u_Y = 0, \quad \tilde{v} = 0, \quad \text{at } Y = 1,$$

$$u = \frac{3Y}{2} (2 - Y), \quad \text{at } X = 0,$$

$$h \int_0^1 u \, dY = h_0, \quad H(0) = H(1) = \int_0^1 H \, dx = 0.$$

6 Droplet deformation: flow in air

Sec. 5.3.1.

Code: 'droplet_air_code.m'

$$\begin{aligned}
 \psi_{\xi\xi} + \psi_{\theta\theta} &= -e^{2\xi}\zeta, \\
 \zeta_{\xi\xi} + \zeta_{\theta\theta} &= \text{Re}(\psi_{\theta}\zeta_{\xi} - \psi_{\xi}\zeta_{\theta} + e^{2\xi}\zeta_t) \\
 \psi &= \psi_{\xi} = 0, \quad \text{at } \xi = 0, \\
 \psi &\rightarrow 2\sinh\xi\sin\theta, \quad \zeta \rightarrow \infty, \quad \text{as } \xi \rightarrow \infty, \\
 \psi &= \zeta = 0, \quad \theta = 0, \pi.
 \end{aligned}$$

7 Droplet deformation: flow in droplet

Sec. 5.3.2.

Code: 'droplet_inside_code.m'

$$\begin{aligned}
 r^2 P_{rr} - rP_r + P + P_{\theta\theta} &= 0 \\
 r^2 U_t &= -r^2 P_r + rP + \frac{1}{\mu_1 \text{Re}} (r^2 U_{rr} - rU_r + U_{\theta\theta} - 2V_{\theta}) \\
 r^2 V_t &= -rP_{\theta} + \frac{1}{\mu_1 \text{Re}} (r^2 V_{rr} - rV_r + V_{\theta\theta} + 2U_{\theta}) \\
 U &= V = P = 0, \quad \text{at } r = 0, \\
 P &= p_g|_{r=1} + \frac{1}{\text{We}}, \quad V_r - V = \mu_1 \zeta_g|_{r=1} - U_{\theta}, \quad U_r = -V_{\theta}, \quad \text{at } r = 1, \\
 U_{\theta} &= V = P_{\theta} = 0, \quad \text{at } \theta = 0, \pi.
 \end{aligned}$$