PhD Thesis codes

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1 Pre-impact phase of droplet impact onto a viscoelastic surface.

Sec. 2.1.2.
Code: 'pre_impact_viscoelastic_code.m'

$$F_{TT} = \frac{1}{\pi}(PV)\int_{-\infty}^{\infty} \frac{\tilde{e}_3 G_\zeta(\zeta,T) + \tilde{e}_5 G_{\zeta T}(\zeta,T)}{X-\zeta}\mathrm{d}\zeta,$$
$$\left((F-G)^3(\tilde{e}_3 G_X + \tilde{e}_5 G_{XT})\right)_X = 12(F-G)_T,$$
$$F \to \frac{X^2}{2} - T, G \to 0, \qquad \text{as } T \to -\infty, |X| \to \infty.$$

2 Pre-impact phase of droplet impact onto a flexible surface

Sec. 2.1.3.
Code: 'pre_impact_flexible_code.m'

$$F_{TT} = \frac{1}{\pi}(PV)\int_{-\infty}^{\infty} \frac{P_\zeta(\zeta,T)}{X-\zeta}\mathrm{d}\zeta,$$
$$\left((F-G)^3 P_X\right)_X = 12(F-G)_T,$$
$$\tilde{e}_1 G_{XXXX} + \tilde{e}_2 G_{XX} + \tilde{e}_3 G + \tilde{e}_4 G_{TT} + \tilde{e}_5 G_T = P,$$
$$F \to \frac{X^2}{2} - T, G \to 0, P \to 0 \qquad \text{as } T \to -\infty, |X| \to \infty.$$

3 Pre-impact phase of droplet impact onto a lubricant-infused surface

Sec. 2.2.
Code: 'pre_impact_LIS_code.m'

$$F_{TT} = \frac{1}{\pi}(PV)\int_{-\infty}^{\infty} \frac{P_\zeta(\zeta,T)}{X-\zeta}\mathrm{d}\zeta,$$

$$\left(\frac{F^3(F+4\Lambda)}{F+\Lambda}P_X\right)_X = 12F_T,$$

$$F \to \frac{X^2}{2} - T, P \to 0 \qquad \text{as } T \to -\infty, |X| \to \infty.$$

4 Boundary layer jet on a flat lubricant-infused surface

Sec. 4.2.2.
Code: 'BL_flat_LIS_code.m'

$$h^2 u u_X + 2Xh\tilde{v}u_Y = \frac{2X}{\tilde{R}e}u_{YY},$$

$$(hu)_X + \tilde{v}_Y = 0, \tag{1}$$

$$u = \Lambda u_Y, \qquad \tilde{v} = 0, \qquad \text{at } Y = 0, \tag{2}$$

$$u_Y = 0, \qquad \tilde{v} = 0, \qquad Y = 1, \tag{3}$$

$$h = 1, \qquad u = \frac{3}{2+6\Lambda}(2\Lambda + 2Y - Y^2), \qquad \tilde{v} = 0, \qquad \text{at } X = 0 \tag{4}$$

5 Boundary layer jet on a deformable lubricant meniscus.

Sec. 4.3.2.
Code: 'BL_meniscus_LIS_code.m'

$$h^2 X_x u u_X + h\tilde{v}u_Y = \frac{1}{\varepsilon^2 \mathrm{Re}}u_{YY},$$

$$X_x(hu)_X + \tilde{v}_Y = 0,$$

$$H'''(x) = -\frac{2\mu\mathrm{Ca}}{\varepsilon^3}\frac{u_s(3A+B)}{(H+1)^2} + \frac{\rho\mathrm{ReCa}}{\varepsilon}u_s(u_s(A+B+C))', \tag{5}$$

$$u = \frac{H+1}{\mu h(3A+2B+C)}u_Y, \qquad \tilde{v} = 0, \qquad \text{at } Y = 0,$$

$$u_Y = 0, \qquad \tilde{v} = 0, \qquad \text{at } Y = 1,$$

$$u = \frac{3Y}{2}(2-Y), \qquad \text{at } X = 0,$$

$$h\int_0^1 u \ \mathrm{d}Y = h_0, \qquad H(0) = H(1) = \int_0^1 H \ \mathrm{d}x = 0.$$

6 Droplet deformation: flow in air

Sec. 5.3.1.
Code: 'droplet_air_code.m'

$$\psi_{\xi\xi} + \psi_{\theta\theta} = -e^{2\xi}\zeta,$$
$$\zeta_{\xi\xi} + \zeta_{\theta\theta} = \frac{1}{\mathrm{Re}}(\psi_\theta\zeta_\xi - \psi_\xi\zeta_\theta - e^{2\xi}\zeta_t)$$
$$\psi = \psi_\xi = 0, \qquad \text{at } \xi = 0,$$
$$\psi \to 2\sinh\xi\sin\theta, \qquad \zeta \to \infty, \qquad \text{as } \xi \to \infty,$$
$$\psi = \zeta = 0, \qquad \theta = 0, \pi.$$

7 Droplet deformation: flow in droplet

Sec. 5.3.2.
Code: 'droplet_inside_code.m'

$$r^2P_{rr} - rP_r + P + P_{\theta\theta} = 0$$
$$r^2U_t = -r^2P_r + rP + \frac{1}{\mu_1\mathrm{Re}}(r^2U_{rr} - rU_r + U_{\theta\theta} - 2V_\theta)$$
$$r^2V_t = -rP_\theta + \frac{1}{\mu_1\mathrm{Re}}(r^2V_{rr} - rV_r + V_{\theta\theta} + 2U_\theta)$$
$$U = V = P = 0, \qquad \text{at } r = 0,$$
$$P = p_g|_{r=1} + \frac{1}{\mathrm{We}}, \qquad V_r - V = \mu_1\zeta_g|_{r=1} - U_\theta, \qquad U_r = -V_\theta, \qquad \text{at } r = 1,$$
$$U_\theta = V = P_\theta = 0, \qquad \text{at } \theta = 0, \pi.$$