COMPUTATIONAL PHYSICS

HOMEWORK #1

Garcia / Numerical Methods for Physics, chapters 1,2

1. Use the computer you have, and find the following:
   1. The largest and smallest positive numbers that you can write.
   2. The smallest ϵ which does not give $1 + \epsilon = 1$.
   3. Calculate $\sqrt{1.1}$ using the relevant function on your computer language (e.g. SQRT(x) with *Fortran*). Use the Taylor expansion to express $\sqrt{1+\epsilon}$ as a sum of polynomials in ϵ. How many terms in the Taylor expansion you need to include to reach the accuracy of your computer for $\epsilon = 0.1$?

2. Take an interesting analytic $f(x)$ of your choice. Derive the analytic expression for the first derivative, df/dx. Calculate numerically the first derivative, using

$$\frac{\Delta f}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{\Delta x},$$

   for some specific value of x and Δx of your choice. Derive the relative error R in the derivative,

$$R = \frac{df/dx - \Delta f/\Delta x}{df/dx}.$$

   That is the difference between the accurate numerical value of df/dx from the analytic expression, and the one derived numerically from $f(x)$. Make a plot of $\log R$ versus $\log \Delta x$ for a large range of values of Δx. The range of Δx values should be large enough to probe both the error at large Δx, and the roundoff errors at very small Δx. Find Δx which gives the most accurate numerical calculation.

3. Ballistic motion. Make a plot of the analytic solution for a ballistic trajectory, $y(x)$, i.e. height versus distance (no friction), for initial conditions of your choice, for some time range $t = 0 - T$. Add a plot of the numerical solution for $y(x)$ using the Euler method with 10 time steps of size $T/10$, and 100 time steps of size $T/100$. Plot all the orbits in one figure, and compare the numerical solutions to the analytic solution.

   Derive the numerical solution above using the Midpoint method, with a time step of your choice. Repeat the calculations with a time step which is much smaller, and with a time step which is much larger. Repeat the calculation with just a single time step of size T. Make all the plots above in a single figure. Compare the different solutions to the analytic solution. Explain the results.

Please submit on Moodle by Sunday, November 7th, 2022.
Behatzlacha!
Ari